ANOMALOUS MAGNETIC MOMENT OF THE MUON QED/QCD free

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ABSTRACT

The theoretical determination of the muon magnetic anomaly is usually carried out through QED techniques up to five loops, adding electroweak contributions and hadronic contributions from QCD via lattice numerical simulations. Recent experimental results (BNL E821, Fermilab E989) support the theoretical predictions.

The present work, in analogy with the determination of the electron magnetic anomaly, proposes a theoretical determination based exclusively on the structure of the muon, within a deterministic and non-local modified Bohmian (dBBZ) framework.

This structure is carefully defined through a matrix Mz containing all the information related to the orbitals constituting the muon itself. Subsequent refinement stages of the matrix are achieved through the repeated imposition of entanglement among the constituent masses.

The method follows that reported in viXra: 2505.0128, replacing the quantities of the muon hybrid orbitals (weak components and electric components) with the purely electric components of the electron case.

The structure of the muon contains that of the electron, with its mass, total charge, and charge source.

The compensation of the weak components at the three-loop calculation level, for each orbital, is performed and justified by two parameters, dw and iw, whose determination leads to relative errors on the total mass of the order of 10^-8 and on the magnetic anomaly of the order of 10^-7.

The aforementioned errors, as functions of the parameters dw and iw, lead to the definition of a structural error space, within which it is possible to identify a very rigid but non-zero local minimum, a circumstance that foreshadows the instability of the muon and lays the foundations for the structural calculation of its decay time.

INTRODUCTION[a]

The work proceeds in parallel with that carried out for the determination of the electron magnetic anomaly, to which explicit reference is made. A necessary framework can be obtained by consulting previous works.[1][2][3][4]
This document is structured in the following phases:

A) Identification of an initial structure of the matrix $M_{z_\mu}^{*}(W)$, structurally containing the electron, in which its mass and charge are reproduced, and which is used for a first estimate α_W, with the aim of determining, through $M_{z_\mu}^{*}(W)$, the radii of the weak sub-orbitals with which, through three-loop running calculations, the weak coupling constants are evaluated for each sub-orbital considered.

In this way the final matrix $M_{z_\mu}(W_i)$ is obtained, which has the same structure as the initial matrix but contains the calculated values α_{W_i} for each orbital.

This phase makes use, where necessary, of a fit of the α_{wi} with the total muon mass, thereby realizing a functional entanglement among the α_{wi} themselves and therefore among the corresponding orbital masses.

B) The knowledge of the α_{wi} allows us to determine all the principal magnitudes for each orbital, both the electric ones, already known, and the weak ones, such as masses, radii, velocities, and the derived quantities such as kinetic energies and potential energies.

Unlike the electronic case, primary and derived quantities of a total nature are considered, containing the electric contribution together with the weak one.

C) In order to calculate, for each orbital, the effect of the simultaneous presence of kinetic energies and potential energies on the velocity of the mass elements present, we set up energy balances having such velocity as the variable to be determined.
The determination of the anomalous orbital moments, the calculation of their entanglement coefficients, their sum, and the obtaining of the theoretical magnetic anomaly to be compared with the experimental value follow the same methodological scheme used in the electron case, to which reference is made.

D) The calculation of the α_{wi}, carried out with a three-loop running, is affected by a theoretical error estimable at 1–3%, due to the missing contribution of subsequent loops; moreover, the logarithmic structure of the loops performed, through the orbitals, produces an error that can be compensated only by an exponential form distributed over the orbitals themselves.
For these reasons we introduce two correction factors for the quantities: a multiplicative one dw and an exponential one, of the form exp[-(i−1)/iw], distributed over all the orbitals.
dw and iw are determined with a tight fit on mass and magnetic anomaly.
The relative errors on the mass err_{m_μ} and on the magnetic anomaly err_{a_μ} constitute a parameter space that presents a local minimum which can be calculated for specific dw and iw.

IDENTIFICATION OF THE MATRIX Mz_μ AND OF THE α_{wi}:

We construct a matrix, hypothesizing a possible structure, which contains the already known structure of the electron and therefore respects its mass and charge values.

	$\alpha_w^2/3.5$	$\alpha_w^{2.5}/4$	$\alpha_w^3/4.5$	$\alpha_w^{3.5}/5$	$\alpha_w^4/5.5$	.....	$\alpha_w^n/n+1.5$
$\alpha_w^{1.5}/2.5$	$\alpha_w^2/3$	$\alpha_w^{2.5}/3.5$	$\alpha_w^3/4$	$\alpha_w^{3.5}/4.5$	$\alpha_w^4/5$	.....	$\alpha_w^n/n+1$
$\alpha_w^2/2$	$\alpha_w^2/2.5$	$\alpha_w^{2.5}/3$	$\alpha_w^3/3.5$	$\alpha_w^{3.5}/4$	$\alpha_w^4/4.5$	.....	$\alpha_w^n/n+0.5$
↑	$\alpha_e^2/2$	$\alpha_e^{2.5}/2.5$	$\alpha_e^3/3$	$\alpha_e^{3.5}/3.5$	$\alpha_e^4/4$	.....	α_e^n/n
K_{w0}	↑	$\alpha_w^{2.5}/2$	$\alpha_w^3/2.5$	$\alpha_w^{3.5}/3$	$\alpha_w^4/3.5$	.....	$\alpha_w^n/n-0.5$
	K_{T1}	↑	$\alpha_e^3/2$	$\alpha_e^{3.5}/2.5$	$\alpha_e^4/3$	.....	$\alpha_e^n/n-1$
		K_{T2}	↑	$\alpha_e^{3.5}/2$	$\alpha_e^4/2.5$	.....	$\alpha_e^n/n-1.5$
			K_{T3}	↑	$\alpha_e^4/2$	.....	$\alpha_e^n/n-2$
				K[illegible]	↑		

Such matrix Mz^{*}_{μ}, although coarse (it does not distinguish the α_{w_i} for each orbital but uses only α_w), identifies an orbital structure that can also be expressed in analytic form, distinguishing sub-orbitals endowed with weak charge from those endowed with electric charge.(b)

Weak-charge sub-orbitals:	Electric-charge sub-orbitals
$Kw0=(1/2+1/2.5)\cdot\alpha_w^{1.5}$	
$Kw1=(1/2.5+1/3+1/3.5)\cdot\alpha_w^{2}$	$Ke1=(1/2)\cdot\alpha_e^{2}$
$Kw2=(1/2+1/3+1/3.5+1/4)\cdot\alpha_w^{2.5}$	$Ke2=(1/2.5)\cdot\alpha_e^{2.5}$
$Kw3=(1/2.5+1/3.5+1/4+1/4.5)\cdot\alpha_w^{3}$	$Ke3=(1/2+1/3)\cdot\alpha_e^{3}$
$Kw4=(1/3+1/4+1/4.5+1/5)\cdot\alpha_w^{3.5}$	$Ke4=(1/2+1/2.5+1/3.5)\cdot\alpha_e^{3.5}$
$Kw5=(1/3.5+1/4.5+1/5+1/5.5)\cdot\alpha_w^{4}$	$Ke5=(1/2+1/2.5+1/3+1/4)\cdot\alpha_e^{4}$
$Kw6=(1/4+1/5+1/5.5+1/6)\cdot\alpha_w^{4.5}$	$Ke6=(1/2+1/2.5+1/3+1/3.5+1/4.5)\cdot\alpha_e^{4.5}$
$Kw7=(1/4.5+1/5.5+1/6+1/6.5)\cdot\alpha_w^{5}$	$Ke7=(1/2+1/2.5+1/3+1/3.5+1/4+1/5)\cdot\alpha_e^{5}$
$Kw8=(1/5+1/6+1/6.5+1/7)\cdot\alpha_w^{5.5}$	$Ke8=(1/2+1/2.5+1/3+1/3.5+1/4+1/4.5+1/5.5)\cdot\alpha_e^{5.5}$
$Kw9=(1/5.5+1/6.5+1/7+1/7.5)\cdot\alpha_w^{6}$	$Ke9=(1/2+1/2.5+1/3+1/3.5+1/4+1/4.5+1/5+1/6)\cdot\alpha_e^{6}$
$Kw10=(1/6+1/7+1/7.5+1/8)\cdot\alpha_w^{6.5}$	$Ke10=(1/2+1/2.5+1/3+1/3.5+1/4+1/4.5+1/5+1/5.5+1/6.5)\cdot\alpha_e^{6.5}$

We calculate the coupling constant α_w, in a first approximation, through a three-loop running equation at the length scale corresponding to the mass of the first weak orbital.

For this purpose, we estimate the mass of the first weak orbital as approximately 80% of the total weak mass of the muon.

It is not important to have an exact estimate, because in the subsequent refinement stages a non-excessive error is suppressed by the procedure itself.

(1) $K_{w0} = \hbar / [0.8\,(m_\mu^{sp} - m_e)\cdot c] \simeq 2.34584\cdot 10^{-15}$ [m]

At that distance, we determine the coupling constant α_w by means of a two-loop running and an approximate three-loop extension.

We temporarily adopt the Natural System of Units.

The necessary initial parameters(6) are determined at the mass of the Z boson : mz $\simeq$ 91.1876 [GeV]

- $\alpha_e(mz) \simeq 0.00781543$ electromagnetic interaction constant calculated at mz,
- $\sin^2\theta_z \simeq 0.23121$ with θ_z, Weinberg angle calculated at the mass mz.

Other parameters appearing in the running formulas are: $b_0 = 19.6$; $b_1 = 35/6$; $b_2 = 147/6$

We determine the reference energy at which to calculate the running:

(2) $Q_0 = \hbar c / K_{w0} \simeq 0.0841179$ GeV ; with: $\hbar\cdot c = 0.197326904$ [GeV·Fm]

We determine the weak coupling constant at the reference mass mz:

(3) $\alpha_w(mz) = \alpha_e(mz) / \sin^2\theta_z \simeq 0.033803$

We calculate the α_W by means of a two-loop running at the energy Q_0 :

(4) $$\frac{1}{\alpha_W(Q_0)_{2L}} = \frac{1}{\alpha_W(m_Z)} + \left(\frac{b_0}{2\pi}\right)\cdot \ln\left(\frac{m_Z}{Q_0}\right) + \left(\frac{b_1}{2\pi b_0}\right)\cdot \ln\left[1+\left(b_0\cdot\left(\frac{\alpha_W(m_Z)}{2\pi}\right)\cdot \ln\left(\frac{m_Z}{Q_0}\right)\right)\right]$$

(4') $$\alpha_W(Q_0)_{2L} = 1/\left(1/\alpha_W(Q_0)_{2L}\right) \simeq 0.030176035$$

We calculate the additional term through an approximate three-loop running:

(5) $$\frac{1}{\alpha_W(Q_0)_{agg.}} = \left(\frac{\alpha_W(m_Z)}{(4\pi)^2}\right)\cdot\left(\frac{b_2}{b_0}\right)\cdot\left(\frac{b_0\cdot\alpha_W(m_Z)}{2\pi}\right)\cdot \ln\left(\frac{m_Z}{Q_0}\right) \simeq 0{,}00019717$$

From (3) and (5) we obtain:

(6) $$\frac{1}{\alpha_W(Q_0)_{3L}} = \frac{1}{\alpha_W(Q_0)_{2L}} + \frac{1}{\alpha_W(Q_0)_{agg}}$$ and therefore:

(7) $$\alpha_W(Q_0)_{3L} = 1/\left(1/\alpha_W(Q_0)_{3L}\right) \simeq 0.030175856$$

Having determined $\alpha_W(Q_0)_{3L} \equiv \alpha_{W0} \cong \alpha_W$ we can proceed, through the corresponding expressions, to a first calculation of the muon mass, considering both weak and electric charges: (c)

(8) $$m_\mu^{th} = \hbar c\left(\sum_{i=1}^{10} K_{ei} + \sum_{i=1}^{10} K_{ei}^2/2\pi + \sum_{i=0}^{10} K_{Wi}\right)$$

We can carry out a first refinement of the α_W by reiterating the calculation of the total muon mass,

through $M^{*}_{Z\mu}$ and imposing a relative error analogous to that of the electron:

(9) $$err_{m_\mu} = \frac{(m_\mu^{SP} - m_\mu^{th})}{m_\mu^{SP}} \simeq 3.636\cdot 10^{-8}$$

This procedure is equivalent to imposing a functional entanglement on the individual masses through α_W:

(10) $$\alpha_W \simeq 0.030267524$$

Using the relation : $K_\alpha = 1/(K_W\cdot c^2)$ we can determine the radii of the individual weak sub-orbitals of

the matrix $M^{*}_{Z\mu}$.

By applying the methodology for the calculation of the approximate three-loop running used previously

(formulas from (2) to (7)), we can determine a first set of weak coupling constants

specific to each sub-orbital:

(11) $\alpha_{W0} \simeq 0{,}0301755$; $\alpha_{W1} \simeq 0.0294417$; $\alpha_{W2} \simeq 0{,}0288148$; $\alpha_{W3} \simeq 0{,}0280282$; $\alpha_{W4} \simeq 0.0272936$;
$\alpha_{W5} \simeq 0.0266034$; $\alpha_{W6} \simeq 0.0259526$; $\alpha_{W7} \simeq 0.025336$; $\alpha_{W8} \simeq 0{,}0247509$; $\alpha_{W9} \simeq 0.0241947$; $\alpha_{W10} \simeq 0.0236648$.

The same matrix $M^{*}_{Z\mu}(W)$, replacing the calculated α_{Wi} in place of the initial ones, assumes its

definitive character $M_{Z\mu}(W_i)$.

DERIVED QUANTITIES

The coherent superposition of the sub-orbitals, and therefore of the Kei with the Kwi, constitutes the basis of the functional entanglement between the weak and electric masses that constitute the muon mass.

All the quantities used are defined as functions of the K_{e_i} and K_{w_i}, and therefore as functions of the corresponding parameters α_e and α_{w_i}.

$$m_{e_i} = \hbar_e K_{e_i} \; ; \; m_{w_i} = \hbar_e K_{w_i} \; ; \; t_{e_i} = 1/(K_{e_i} \cdot c^2) \; ; \; t_{w_i} = 1/(K_{w_i} \cdot c^2) \; ;$$

$$v_{e_i} = c \cdot \sqrt{2 K_{e_i}} \; ; \; v_{w_i} = c \cdot \sqrt{2 K_{w_i}}$$

We also define total quantities, starting from: $K_{T_i} = K_{e_i} + K_{w_i}$

$$m_{T_i} = \hbar_e K_{T_i} \; ; \; t_{T_i} = 1/(K_{T_i} \cdot c^2) \; ; \; v_{T_i} = c \sqrt{2 K_{T_i}}$$

Furthermore, we also have total kinetic energies and total potential energies :

$$E_{c_{T_i}} = E_{c_{e_i}} + E_{c_{w_i}} = \frac{c^2}{2} (m_{e_i} + m_{w_i})$$

$$E_{p_{T_i}} = E_{\epsilon_i} + E_{p_{w_i}}$$

Where the quantities $E_{p_{w_i}}$ represent the internal self-interaction components of the weak part of the i-th orbital with respect to the charge center.
These components can be seen as a modulation of the internal electromagnetic field on the various orbitals.
The resonance with the electromagnetic field implies a form of proportionality with the latter, weighted by the ratio of weak energy densities with respect to the electric ones.

ENERGY BALANCES

The energy balances of the i-th orbital are a means of equating, from the point of view of total energy, one model to the subsequent, more sophisticated and therefore more performant one.

We use what has already been calculated in the case of the electron magnetic anomaly in order to have a model that takes into account the presence of electric charge in the muon.
For these reasons we briefly recall the essential points and the results obtained in previous works.

The source charge q_s is given by the choice of the $\bar{\alpha}_{ew} \simeq 0.0326292827 8$ at partial electroweak unification in an allowed field configuration.

This value makes it possible to write the source potential at a given distance r :

$$(12) \quad U_{q_s} = \frac{q_s}{4\pi \varepsilon_0 r}$$

We take into account, in the charge seen by the i-th orbital, the stationarity constraints of each individual orbital (the “Poincaré belt” problem).

By applying Gauss’s theorem of electrostatics, bringing each individual charge considered to the Compton distance, we can write the electrostatic energy defined on the orbital:

(13) $E_{E_i} = U_{q_s} \cdot q_{i\tau} = \frac{q_s}{4\pi\varepsilon_0} \cdot \left(e - 2\sum_{l=1} d_l \cdot q_l\right) / \left(\lambda_{e_i}/nq\right)$ with: $d_l = \frac{\lambda_e}{\lambda_{e_l}} = \frac{q_l}{q_T}$

From (13) we can derive the magnetic energy relative to the i-th orbital:

(14) $E_{B_i} = E_{E_i}\, v_i^2 / 4\pi c^2$

The weak potential U_{w_i} represents the internal cohesion component of the entangled EM−weak field. E_{Pw_i} is generated by a form of internal energy and not by a charge.
Its form is parallel to that of the E_{E_i}, but with an energy-type source and with coupling to the weak interaction rather than to the electromagnetic one.
We therefore hypothesize the following correlation between the weak and electric potentials energy :

(15) $E_{Pw_i} = \frac{\alpha_{w_i}}{\alpha_e} \cdot E_{E_i} \cdot \frac{\rho_{w_i}}{\rho_{e_i}}$ with: $\rho_{w_i} = \frac{m_{w_i} c^2}{\frac{4}{3}\pi \lambda_{w_i}^3}$; $\rho_{e_i} = \frac{m_{e_i} c^2}{\frac{4}{3}\pi \lambda_{e_i}^3}$

Considering the relation : $\lambda = \hbar / m \cdot c$ one obtains:

(16) $E_{Pw_i} = \frac{\alpha_e}{\alpha_{w_i}} \cdot \left(\frac{m_{w_i}}{m_{e_i}}\right)^4 \cdot E_{E_i}$

The electric potential represents an electric polarization energy, whereas the weak potential represents an internal confinement energy.
The weak potential energy is treated as a formal extension of the electric potential energy, with analogous structure, the same units, but different scale and intensity.

We can therefore write the energy balance, for an i-th orbital, between a purely dynamical model and a model characterized by kinetic and potential energies.
In writing this balance, we use total quantities instead of the purely electric ones of the electron case, so that the subsequent treatment will also be formally analogous to that used for the electron.

(17) $0.5\, m_{T_i} \cdot {}_A v_{T_i}^2 = 0.5\, m_{T_i} \cdot {}_B v_{T_i}^2 - E_{P_{T_i}} \cdot {}_B v_{T_i}^2 / 4\pi c^2$ (d)

From which one can extract:

(18) ${}_B v_{T_i} = {}_A v_{T_i} \cdot \sqrt{0.5\, m_{T_i} / \left(0.5\, m_{T_i} - E_{P_{T_i}} / 4\pi c^2\right)}$

We can set : $Y_i = {}_B v_{T_i} / {}_A v_{T_i}$ obtaining the index of velocity variations due to potential energies :

$Y_1 \simeq 0.170886$; $Y_2 \simeq 0.0297306$; $Y_3 \simeq 0.0460452$; $Y_4 \simeq 0.040046$; $Y_5 \simeq 0.0287995$; $Y_6 \simeq 0.0190603$; $Y_7 \simeq 0.0121086$; $Y_8 \simeq 0.00754706$; $Y_9 \simeq 0.00467208$; $Y_{10} \simeq 0.00289475$.

From the values of the Y_i, comparing them with the analogous values in the electron case, one can note the considerable additional slowing down due to the presence of the weak component.

Analogously to the case of the electron magnetic anomaly, we write the anomalous moment of the i-th orbital, in relation to the total components of the hybrid orbitals:

(19) $L_{T_i} = m_{T_i} \cdot {}_B v_{T_i} \cdot \lambda_{T_i}$

We impose entanglement on the terms L_{T_i} through the determination and subsequent normalization of the corresponding coefficients.

The sum of the entangled terms constitutes the total orbital anomalous moment L_T

The parameters that define L_{T_i}: m_{T_i}; λ_{T_i}; ${}_B\vec{V}_{T_i}$ are in turn dependent on K_{T_i}

(20) $m_{T_i} = \hbar c\, K_{T_i}$; $\lambda_{T_i} = 1/(K_{T_i} \cdot c^2)$; ${}_B\vec{V}_{T_i} = c \cdot \sqrt{2 K_{T_i} \cdot Y_i^2}$.

The appropriate way to calculate the normalization coefficients, in this case, is to use the squared probabilities relative to the individual parameters defined on the orbitals themselves, after a change of variables to make their probability distributions compatible.

We calculate the derivatives with respect to K_{T_i} of equations (20):

(21) $dm_{T_i}/dK_{T_i} = \hbar c$; $d({}_A V_{T_i} \cdot Y_i^2)/dK_{T_i} = c \cdot Y_i \cdot \sqrt{2 K_{T_i}}$; $d\lambda_{T_i}/dK_{T_i} = 1/(K_{T_i}^2 \cdot c^2)$

We obtain the probabilities relative to the individual parameters:

(22) $P_{m_{T_i}} = P(K_{T_i})/(dm_{T_i}/dK_{T_i}) = P(K_{T_i})/\hbar c$

(23) $P_{{}_A V_{T_i}} = P^*(K_{T_i})/(d({}_A V_{T_i} \cdot Y_i^2)/dK_{T_i}) = P^*(K_{T_i}) \cdot \sqrt{2 K_{T_i}}/c \cdot Y_i$

(24) $P_{\lambda_{T_i}} = P(K_{T_i})/(d\lambda_{T_i}/dK_{T_i}) = P(K_{T_i}) \cdot c^2 \cdot K_{T_i}^2$

We determine the square of the total probability for each i-th orbital:

(25) $P_{T_i}^2 = P_{m_{T_i}}^2 \cdot P_{{}_A V_{T_i}}^2 \cdot P_{\lambda_{T_i}}^2 = P^2(K_{T_i}) \cdot P^{*2}(K_{T_i}) \cdot \dfrac{2}{\hbar^2 c^2} \cdot \dfrac{K_{T_i}}{c^2 \cdot Y_i^2} \cdot c^4 K_{T_i}^4 = 2 P^4(K_{T_i}) \cdot P^{*2}(K_{T_i}) \cdot K_{T_i}^5/(\hbar^2 \cdot Y_i^2)$

We can calculate the sum of the squares extended to all the orbitals and obtain the normalization constants:

(26) $C_i = \sqrt{P_{T_i}^2 / \sum_{i=1}^{10} P_{T_i}^2} = \dfrac{\sqrt{2} \cdot P^2(K_{T_i}) \cdot P^*(K_{T_i})/\hbar}{\sqrt{2}\; P^2(K_{T_i}) \cdot P^*(K_{T_i})/\hbar} \cdot \sqrt{\dfrac{K_{T_i}^5}{Y_i^2} \Big/ \sum_{i=1}^{10} \left(\dfrac{K_{T_i}^5}{Y_i^2}\right)} = \left(\dfrac{K_{T_i}^{2.5}}{Y_i}\right) \Big/ \sqrt{\sum_{i=1}^{10} (K_{T_i}^5/Y_i^2)}$

It is immediate to verify that: $\sum_{i=1}^{10} C_i^2 = 1$

Once the modified velocities of the total masses have been obtained, we write the expression of the total orbital anomalous moment:

(27) $L_T = \sum_{i=1}^{10} C_i \cdot m_{T_i} \cdot {}_B V_{T_i} \cdot \lambda_{T_i} \simeq 7.725467623446 \cdot 10^{-37}$ $[J \cdot s]$

We recall the definition of the anomalous magnetic moment of the muon:

(28) $\mu_\mu^{ST} = \dfrac{e}{m} \cdot \dfrac{(g_\mu - 2)}{2} \cdot \hbar$ where:

(29) $a m_\mu^{ST} = \dfrac{(g_\mu - 2)}{2}$ is defined as the standard muon magnetic anomaly.

We define rhe magnetic anomaly of proposed model as the ratio : L_T/h

(30) $a m_\mu^{th} = L_T/h$ therefore:

(31) $\dfrac{(g_\mu - 2)}{2} = L_T/h$ finally:

(32) $\mu_\mu^{th} = \dfrac{e}{m_\mu} \cdot \dfrac{L_T}{2\pi}$

We can calculate the relative error:

$(a m_\mu^{SP} - a m_\mu^{th})$

$$(33)\quad err_{a_\mu} = \frac{\ldots}{a_{\mu}^{SP}}$$

REFINEMENT PARAMETERS

As recalled in point (D), the truncation at three loops of the running of the coupling constants α_{W_i} produces a percentage error that must be taken into account.

Moreover, since the running of the coupling constants α_{W_i} is logarithmic in the corresponding scales λ_{W_i} the variation induced by a systematic error is not linearly distributed among the levels.

The inner orbitals, corresponding to smaller distances, show a more strongly amplified sensitivity compared to the outer ones.

Therefore, the most effective percentage correction to simultaneously stabilize mass and anomaly is an exponentially decreasing function of the orbitals, consistent with the effective compensation of the original logarithmic behavior.

For these reasons we introduce two correction factors on the coupling constants, one multiplicative : dw and the other exponential : iw, distributed over all the orbitals.

The parameters dw and iw are determined by a tight fit on the imposed relative mass error and on the relative error of the magnetic anomaly.

We therefore modify the coupling constants α_{W_i} as follows:

$$(34)\quad \alpha'_{W_i} = \alpha_{W_i} \cdot \left(1 + dw \cdot e^{-[(i-1)/iw]}\right)$$

Where the orbital corresponding to the first level is affected only by the truncation error and acts as the initial base of logarithmic errors.

By imposing a mass error equal to the analogous error on the electron masses, a first fit was sought, also on the anomaly, which produced:
dw = 0.0132 and iw = 1.196.

To characterize the local stability of the fitting with respect to the attenuation parameter iw, a one-dimensional scan of iw was performed, keeping the parameter dw fixed at the best-fit value
dw = 0.013251478

The local slopes of the relative mass error. $M_{er} \equiv err_{m_\mu}$ and of the relative magnetic anomaly error $A_{er} \equiv err_{a_\mu}$ with respect to iw were estimated by central finite differences, using typical increments $\Delta iw = \pm 0.02$ e ± 0.05 around the value iw ≈ 1.196, corresponding to relative variations of 1.7–4.2%. These increments allow probing the local response of the model without inducing macroscopic variations of the fit.

From these slopes, a scalar sensitivity indicator (or local rigidity) of the best-fit with respect to perturbations of iw was introduced, representing a quadratic norm of the gradient.

$$(35)\quad S_\mu(iw) = M_{er}^2(iw) + A_{er}^2(iw)$$

In the vicinity of the best-fit, the indicator assumes a characteristic value $S_\mu \simeq 2.7746 \cdot 10^{-4}$ indicating that the fitting presents a well-defined local basin along the iw direction and allowing a quantification of the relative error under controlled variations of the parameter.

The presence of a local minimum of S identifies a neighborhood in which the fitting is well conditioned.

The numerical value of S depends on the adopted step scale and must therefore be interpreted as a local diagnostic.

A complete two-dimensional scan in the (dw, iw) space is conceptually possible but much more demanding and not necessary for the purposes of local best-fit characterization, which is dominated by the iw direction.
Such an analysis can be carried out through the study of the Hessian of a cost function associated with the relative errors of mass and magnetic anomaly.

The constraints of these relative errors with analogous parameters of other leptons (electron and tau), together with constraints on the source value and on decay times (muon and tau), impose harmonized values of the structural parameters, without the need for extreme best-fits.

For this reason we report the already harmonized best-fit determination of the two parameters dw and iw and of the relative error of the magnetic anomaly, which turns out to be higher than in the case in which the muon were considered as a single particle not belonging to, and therefore not constrained by, its leptonic family.

(36) $dw \doteq 0.013251478$; $iw \simeq 1.196127$

(37) $A_{er} \simeq 6.15 \cdot 10^{-7}$; $M_{er} \simeq 3.673 \cdot 10^{-8}$

The relative error of the magnetic anomaly determined with QED/QCD is: $A_{er}^{ST} \simeq 3.22 \cdot 10^{-7}$

The quantities numerically expressed in the text d_{ew} ; Y_i ; L_T were anticipated assuming the constraining values of (36).

NOTES

a) The International System of Units (SI) is adopted and the constants are consistent with CODATA 2022.
The sign of the electric charges is not considered unless explicitly indicated.
The value of the spin is always considered as a positive scalar.

b) Analogously to the electron, the matrix $M_{z\mu}$ can be written as a mass formula; only the first terms are reported:

(38) $$m_\mu^{th} = D \cdot \hbar c \left(\sum_{i=1}^{n} K_{ei} + \sum_{i=0}^{n} K_{wi} \right)$$

We neglect the magnetic mass part of the electron and consider D = 1 as customary:

(39) $$m_\mu^{th} \doteq \hbar \cdot c \Big[\left(\tfrac{1}{2} + \tfrac{1}{2.5}\right) \alpha_{w0}^{1.5} + \tfrac{1}{2} \alpha_e^{2} + \left(\tfrac{1}{2.5} + \tfrac{1}{3} + \tfrac{1}{3.5}\right) \alpha_{w1}^{2} + \tfrac{1}{2.5} \alpha_e^{2.5} + \left(\tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{3.5} + \tfrac{1}{4}\right) \alpha_{w2}^{2.5} + \\ + \left(\tfrac{1}{2} + \tfrac{1}{3}\right) \alpha_e^{3} + \left(\tfrac{1}{2.5} + \tfrac{1}{3.5} + \tfrac{1}{4} + \tfrac{1}{4.5}\right) \cdot \alpha_{w3}^{3} + \dots \Big]$$

c) The innermost weak orbital assumes very high velocities and, due to relativistic effects, would undergo a mass increase of about : $m_{w0} \cdot 10^{-3}$
This modest increase is not taken into account as it is subsequently suppressed by the refinement method.

d) We report a more explicit version :

40) $0.5\, m_{T_i} \cdot {}_{A}v_{T_i}^{2} = 0.5\, m_i \cdot {}_{B}v_{T_i}^{2} - \left[\left(1 + \frac{\alpha_e}{\alpha_{w_i}} \cdot \left(\frac{m_{W_i}}{m_{e_i}} \right)^4 \right) \cdot \frac{q_s}{4\pi\varepsilon_0} \cdot \left(e - 2 \sum_{l=1}^{l} d_l \cdot q_l \right) / \left(r_{e_i} / n_q \right) \right] \cdot \frac{{}_{B}v_{T_i}}{4\pi c^2}$

CONCLUSIONS

The determination of the muon magnetic anomaly fits perfectly within the modeling developed for the calculation of the electron magnetic anomaly.

The two models share the same procedural method, the same value of the source parameter, and an analogous structure, charge, and mass of the electron, whether isolated or as part of the muon.

The hybrid nature of the latter required additional effort in describing the structure and in establishing the energy balances for each orbital, due to the entangled nature of the variables involved.

This integrated structure, which accounts for the dynamical contribution, the electromagnetic contribution together with its weak potential, and the entanglement of anomalous orbital moments, can be denoted as:
H.E.E.D.M. (Hybrid Entangled Electro-Dynamic Model).

The introduction of two convergence parameters for the weak coupling constants allows exploration of a space of relative errors of mass and anomaly, identifying a convergence basin where such errors assume best-fit values, compatible with parameters proper to the electron and tau, to which they are connected.

The methodology used has no connection with QED and QCD techniques, overcoming their computational difficulties.

Moreover, it has the merit of assigning a precise physical meaning to the quantities and parameters used, within a deterministic and non-local dBBZ (modified de Broglie–Bohm) theory that describes the muon and electron as distributed entangled structures.

This work opens the way both to an analogous determination of the tau magnetic anomaly and to the calculation of the decay time of the muon and tau as a derivation of their internal structure and dynamics.

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