

General equations of the galaxy dynamic gravitational field that correspond to reality

P Danylchenko

SPE "GeoSystem", Vinnytsia, Ukraine

E-mail: pavlodanylchenko@gmail.com

Abstract

The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter n is found. The additional variable parameter n determines in General Relativity (GR) the distribution of the average mass density mainly in the galactic friable nucleus. The velocity of the orbital motion of stars is close to Keplerian one only for $n > 2^{25}$. At $n < 2^{15}$, it is slightly less than the highest possible velocity even at the edge of the galaxy. According to the GR equations and the Relativistic Gravithermodynamics (RGTD) equations, the configuration of the dynamic gravitational field of a galaxy in a quasi-equilibrium state is standard (canonical in RGTD). That is because it is not determined at all by the spatial distribution of the average mass density of its non-continuous matter. After all, this spatial distribution of the average mass density of the galaxy's matter is itself determined by the standard configuration of its dynamic gravitational field. The standard value of the average mass density of matter at the edge of a galaxy is determined by the cosmological constant Λ and the difference between unity and the maximum value of the parameter b_c . And it is a non-zero standard value, despite the gravitational radius at the edge of a galaxy takes the zero value. Therefore, in the RGTD, in contrast to the GR, there can be no shortage of baryonic mass.

Keywords: General Relativity, Kepler's law, galaxy, gravitational potential, non-baryonic dark matter, orbital velocity, Relativistic Gravithermodynamics.

1. Logarithmic gravitational potential

Physical laws are based only on increments of metrical distances and not on increments of coordinates. Therefore, gravitational field strength k is determined via its gravitational potential φ in the following way:

$$k = -\text{grad}(\varphi) = -\frac{1}{\sqrt{a}} \frac{\partial \varphi}{\partial r} = -\sqrt{1 - \frac{r_g}{r}} \frac{\Lambda r^2}{3} \frac{\partial \varphi}{\partial r},$$

where: $\Lambda = 3H^2 c^{-2}$ is cosmological constant, H is Hubble constant, a is square of the ratio between increment of metrical segment and increment of radial coordinate r , and r_g is gravitational radius of astronomical body, from where observation takes place.

Nowadays, the following gravitational potential is used in General Relativity (GR) and in practical calculations:

$$\varphi = cv_{cvj} = c^2 \sqrt{1 - r_g / r},$$

where: v_{cv} is the coordinate vacuum velocity of light.

When $\Lambda = 0$ that potential forms the same spatial distribution of gravitational field strength as in classical physics:

$$k = -c^2 r_g / 2r^2 = -GM r^{-2} \quad (r_g = 2GM c^{-2}).$$

However, it does not correspond to Einstein's opinion that free fall of bodies in gravitational field is inertial motion. According to this potential the kinetic energy of falling body is less than the difference between rest energies of the body in the starting point of the falling and in the point of its instantaneous disposition. Wrong opinion that gravitational field has own energy corresponds to that gravitational potential [1].

In contrast to this potential, the potential that is in a form of logarithm of the rest inert free energy of matter corresponds to inertial motion of freely falling body [2] with the conservation of Lagrangian L of its ordinary internal energy $W_{0j} = W_{00} c / v_{cvj} = m_{gr0} c^2 = m_{00} c^3 / v_{cvj}$, and of Hamiltonian H of its inert free energy $E_{0j} = E_{00} v_{cvj} / c = m_{in0} c^2 = m_{00} c v_{cvj}$ [3]:

$$\varphi_j = -c^2 \ln(W_{0j} / W_{00}) = c^2 \ln(E_{0j} / E_{00}) = c^2 \ln(v_{cvj} / c) = c^2 \ln b_j / 2 \quad (1)$$

Such representation of potential is based on the possibility of proportional synchronization of all quantum clocks and on proportionality of pseudo-forces of inertia and gravitation to the Hamiltonian of matter. This is in good correspondence with the principle of mass and energy equivalence. Such representation also makes the proof of equivalence of inert and gravitational masses redundant. Logarithmic gravitational potential forms the following spatial distribution of gravitational field strength:

$$k = \frac{F_{gr}}{m_{gr0r}} = \frac{v_{cvr} F_{gr}}{m_{00} c} = \mathbf{grad}(c^2 \ln W_0) = -\mathbf{grad}(c^2 \ln E_0) = -\mathbf{grad}(c^2 \ln v_{cv}) = -\frac{G_{00} M_{gr0} - H_E^2 r^3}{r^2 b_j \sqrt{a}} = -\frac{G_j M_{gr0} - H_E^2 r^3 / b_j}{r^2 \sqrt{a}}.$$

The equivalent value of strength of gravitational field adjusted to the inert mass of rest of the body that is moving in gravitational field will be as follows:

$$k_{eq} = \frac{F_{gr}}{m_{in0r}} = \frac{L}{H} k = \frac{m_{gr0r}}{m_{in0r}} k = \frac{c^2}{v_{cvr}^2} k = -\frac{c^4}{v_{cvr}^2} \mathbf{grad}(\ln v_{cv}) = -\frac{G_j(r) M_{gr0} - H_E^2 r^3 / b_r}{r^2 b_r \sqrt{a}}.$$

According to this the effective value of gravitational parameter ("constant"), where z is redshift:

$$G_{eff} = (c^2 / v_{cvr}^2) G_j = b_j^{-2} G_{00} = k(z, \mu_{os}) G_{00} \approx (1+z)^4 (1+2z)^{-2} G_{00} \quad (2)$$

tends to infinity while approaching the event pseudo-horizon as well as the Schwarzschild sphere and is continuously decreasing while distancing from the gravity center. And, of course, this should successfully prevent the false conclusions about the deficit of baryonic matter in the centers of the galaxies.

Usage of logarithmic gravitational potential does not require the adjustment of the values of mass of the Sun and the planets. If gravitational radius of Sun is 2.95 km then its mass should be decreased on just two millionth parts of it. It is 35 times less than the determination error of Sun mass. On the Mercury orbit the strength of Sun gravitational field should be decreased on just 20 billionth parts of it. The Earth itself has very small gravitational radius 0.887 cm. Due to this fact Earth mass should be decreased on just one billionth part of it. At the same time, Earth mass determination error is 100000 bigger. Unlike for the Solar System, the usage of logarithmic gravitational potential can be very essential for the far galaxies.

2. The inconsistency of the motion of galaxies with Kepler's laws

Laws of motion of single astronomical objects, found by Kepler, are based on gravitational influence of mainly central massive body. According to those laws, the velocity of rotation of galactic objects should decrease in inverse ratio to the square root of the distance to galaxy center. However, observations reveal the different picture: this velocity remains quasi constant on quite far distance from galaxy center for many galaxies, including ours [4].

When single objects and their aggregates form big collection (cluster) their total mass can essentially exceed the mass of central astronomical body (supermassive neutron star or quasar). The attraction of astronomical objects of the internal spherical layers of the galaxy can be much stronger than the attraction to the central body of the galaxy. Then, their collective gravitational influence can essentially distort the correspondence of the motion of peripheral astronomical objects to Kepler's laws. And, therefore, according to astronomical observations the velocities of rotation of galaxy's peripheral astronomical objects required for prevention of joint collapse of all matter of the galaxy are much higher than the velocities of rotation of the separate peripheral astronomical objects required for prevention of the independent fall of those objects onto the central astronomical body.

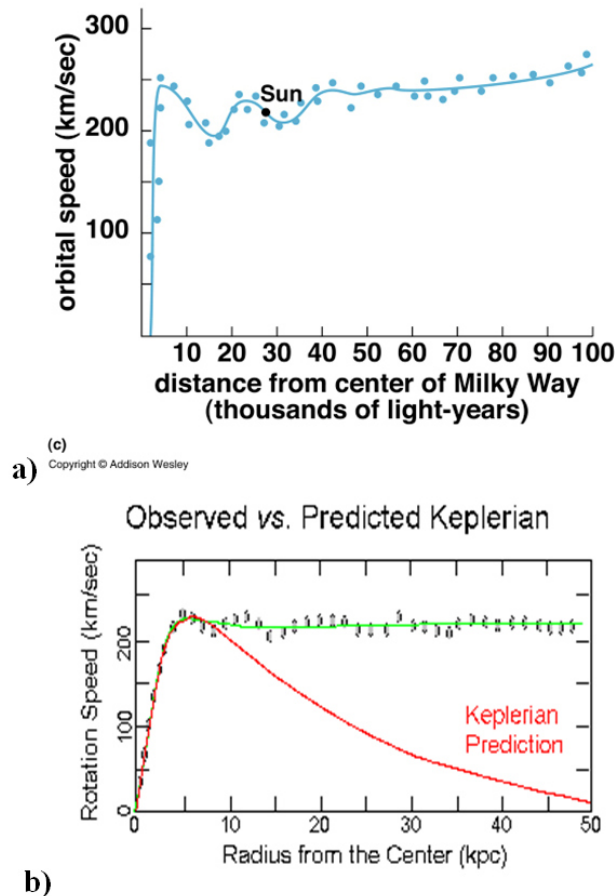


Figure: Dependencies of velocity of rotation of astronomical objects on the distance to gravity center: **a)** our Milky Way galaxy [4, 5], **b)** comparing to prognosed Keplerian velocities [6].

The quite close dependency to the observed one is the following dependence of really metrical value $\hat{v} = v/\sqrt{b} = v c/v_{cv}$ of galactic velocity of rotation v of astronomical objects on the distance to the galaxy center. It is determined by the common galactic clock when the radial distribution of the average relativistic density of corrected relativistic mass of matter in the galaxy is the following:

$$\mu_{inc} = \frac{\mu_{in0} + p\hat{v}^2/c^2}{1 - \hat{v}^2/bc^2} = \frac{\eta + \chi_0 r}{\kappa c^2 r^2} = \frac{\mu_{00}}{r^2} \left\{ r_e^2 \left[1 - \left(1 - \frac{r}{r_e} \right) \exp\left(-\frac{r}{r_e}\right) \right] + \sigma_m^2 \left[\sin\left(\frac{2\pi r}{r_m}\right) + \frac{2\pi r}{r_m} \cos\left(\frac{2\pi r}{r_m}\right) \right] \right\}, \quad (3)$$

where:

$$\eta = \frac{\kappa c^2}{r} \int_0^r \mu_r r^2 dr = \kappa c^2 \mu_0 \left\{ r_e^2 \left[1 - \exp\left(-\frac{r}{r_e}\right) \right] + \sigma_m^2 \sin\left(\frac{2\pi r}{r_m}\right) \right\}$$

$$\chi_0 = \kappa \mu_0 c^2 [r_e \exp(-r/r_e) + 2\pi \sigma_m \cos(2\pi r/r_m)],$$

μ_0, r_e, r_m, σ is constants.

In this case on the large distances to the central astronomical body with the radius r_e ($r \gg r_e$) the parameter η is only weakly sinusoidally modulated. And, also, the square of velocity of orbital rotation of astronomical objects of the galaxy, that can be found from the condition of equality of centrifugal pseudo force of inertia $\mathbf{F}_{in} = H \hat{v}^2 c^{-2} a^{-1/2} / r$ and pseudo force of gravity $\mathbf{F}_{gr} = L c^{-2} a^{-1/2} d[\ln(v_{cv}/c)]/dr$:

$$\frac{[\hat{v}^2]_{GR}}{c^2} = \frac{L r}{H} \frac{d \ln(v_c / c)}{dr} = \frac{r b'}{2 b b_s} = \frac{a}{2 b_s} [1 - 1/a + (\kappa p - \Lambda) r^2] = \frac{[\eta + (\kappa p - 2\Lambda/3) r^2]}{2 b_s (1 - \eta - \Lambda r^2/3)} \quad (4)$$

very slightly depends on $r \gg r_e$ due to the smallness of $\exp(-r/r_e)$, pressure p in the outer space of the galaxy and cosmological constant Λ . And its value can only slightly increase together with increasing of r due to the gradual increasing of the parameter η .

Here “galactic” value of coordinate velocity of light $^u v_{cvg} \equiv v_l = c b^{1/2}$, Lagrangian and Hamiltonian

$$L = m_{gr} c^2 = m_{gr0} c^2 (1 - \hat{v}^2 c^{-2})^{1/2} = H (1 - \hat{v}^2 c^{-2}) / b = H / b_s,$$

$$H = m_{in} c^2 = m_{in0} c^2 (1 - \hat{v}^2 c^{-2})^{-1/2} = m_{00} c^2 b^{1/2} (1 - \hat{v}^2 c^{-2})^{-1/2}$$

and increment of the metric radial distance $d\check{r} = a^{1/2} dr$ are determined by the parameters b , $a = 1/(1 - \eta - \Lambda r^2/3)$ and $b_s = (v_{ls}/c)^2 = b/(1 - \hat{v}^2 c^{-2})$ of the equations of GR gravitational field:

$$b' / a b r - r^{-2} (1 - 1/a) + \Lambda = \kappa p = \kappa \gamma \mu_{in} c^2 / b = \kappa \gamma \mu_{00} c^2 / \sqrt{b},$$

$$a' / a^2 r + r^{-2} (1 - 1/a) - \Lambda = \kappa (\mu_{in0} c^2 + p \hat{v}^2 c^{-2}) / (1 - \hat{v}^2 c^{-2}) = \kappa \mu_{in} c^2 [1 + \gamma \hat{v}^2 / b (c^2 - \hat{v}^2)],$$

$$[\ln(ba)]' / ar = \kappa \mu_{gr} (b + \gamma) c^4 / (c^2 - \hat{v}^2) = \kappa \mu_{in} (1 + \gamma/b) c^4 / (c^2 - \hat{v}^2) = \kappa \mu_{00} (\sqrt{b} + \gamma/\sqrt{b}) c^4 / (c^2 - \hat{v}^2).$$

However, instead of eigenvalues of density of the mass μ_{00} and pressure p_{00} their coordinate values in FR are used in tensor of energy-momentum $\mu_{in0} = \mu_{00} \sqrt{b}$ and $p = p_{00} / \sqrt{b}$ ($p / \mu_{gr0} c^2 = p_{00} / \mu_{00} c^2 = \gamma = \text{const}(r)$). This is related to temporal invariance of really metrical mechanical and thermodynamic parameters and characteristics of matter. An insufficient amount of the mass in the Universe denotes the fact that not only in RGTD but also in GR the tensor of energy-momentum should be based on the ordinary internal energy of matter that includes not only inert free energy but also bound energy of matter.

The defined by the same spatial distribution (3) average relativistic density of corrected relativistic mass of galaxy matter in GR has the following form:

$$\mu_{inc} = \mu_{00} \sqrt{b} [1 + \gamma \hat{v}^2 / b (c^2 - \hat{v}^2)],$$

where:

$$p_{00} = \gamma \mu_{00} c^2, \quad \sqrt{b} = \frac{v_l}{c} = \frac{1}{\sqrt{a}} \left(1 + \frac{\kappa c^2}{2} \int_{r_e}^r \frac{m_{00} a^{3/2} r dr}{V [1 - \hat{v}^2 c^{-2}]} \right), \quad \mu_{00} = m_{00} / V;$$

¹ Here and further, we consider the minimum radial distance r from the center of the galaxy to the point on the trajectory of rotation of the astronomical object at which equilibrium is achieved, and therefore, its radial displacement is absent ($dr/dt=0$).

V is volume of matter; $m_{00}=m_{in0}b^{-1/2}=m_{gr0}b^{1/2}$ is intrinsic value of the mass of matter that corresponds to “critical” equilibrium value of the ordinary internal energy of matter ($b=1$), and $v_l=v_{cv}$ is maximum possible (extreme) value of velocity of matter in the outer space of the galaxy.

As we can see, exactly the logarithmic potential of gravitational field and the spatial distribution of gravitational strength defined by it in the extremely filled by stellar substance space of the galaxy correspond to these astronomical observations. The quite significant decreasing of the average density of matter when distancing from the center of the galaxy towards the periphery also corresponds to these astronomical observations. Together with the deepening into cosmological past ($\tau_p < \tau_e$) the average density of matter in the gravithermodynamic frame of reference of spatial coordinates and time (GT-FR) of the galaxy is decreasing on its periphery proportionally to the square of radial coordinate r_p . In the picture plane of astronomical observation this radial decreasing of the density of matter is even more significant:

$$\mu_{gr0cpobs} = \mu_{gr0p}(r_p/r_{pobs})^3 = \mu_{gr0cp} \exp[-3H(\tau_e - \tau_p)] = \mu_{gr0} r_e^2 r_p^{-2} \exp[-\sqrt{3\Lambda}(r_p - r_e)],$$

since, in contrast to GT-FR of the central astronomical object of the observed galaxy, in GT-FR of terrestrial observer all other astronomical objects of this galaxy belong to the same moment of cosmological time $\tau_p = \tau_e$.

And, therefore, the quantity of baryonic matter currently present in galaxies can be quite enough for examined here justification for observed velocities of astronomical objects of galaxies. The one more contributing fact is that having the same quantity of matter ($m_{00p}=m_{00e}$) its inertial mass of rest $m_{in0}=m_{00}b^{1/2}$ on the galaxy periphery is bigger than in its center since $b_p > b_e$.

The GR gravitational field equations de facto correspond to spatially inhomogeneous thermodynamic states of only utterly cooled down matter. The similar to them equations of relativistic gravithermodynamics (RGTD) [3, 7, 8] correspond to spatially inhomogeneous thermodynamic states of gradually cooling down matter. Therefore, in RGTD for matter that cools quasi-equilibrally, the four-momentum must obviously be formed in the extended system not by enthalpy, but by the intranuclear Gibbs free energy (which in RGTD is an alternative to the inert free energy). The Lagrangian of the ordinary internal energy of the matter (the multiplicative component of its total energy) forms the four-momentum not with the Hamiltonian momentum, but with the Lagrangian momentum. The GR gravitational field equations de facto correspond to spatially inhomogeneous thermodynamic states of only utterly cooled down matter. In addition, in RGTD, unlike GR, bodies that move by inertia in a gravitational field, influence (by their movement) the configuration of the dynamic gravitational field surrounding them. At the same time, in equilibrium processes, along with the usage of ordinary Hamiltonians and Lagrangians, in RGTD it is also possible to use GT-Hamiltonians and GT-Lagrangians. Therefore, in RGTD for matter that cools quasi-equilibrally, the Hamiltonian (GT-Hamiltonian) four-momentum must obviously be formed in the extended system not by enthalpy, but by the inert free energy, and Lagrangian (GT-Lagrangian) four-momentum must obviously be formed by the multiplicative component of total energy and also by the Gibbs free energy (which in RGTD is an alternative to the inert free energy). The GT-Lagrangian of the ordinary internal energy of the matter (the multiplicative component of its total energy):

$$L_c = m_{gr} c^2 = m_{gr0} c^2 (1 + v^2 v_l^{-2})^{-1/2} = m_{00} c^3 / v_{lc} = H_c / b(1 + \hat{v}^2 c^{-2}) = H_c / b(1 + v^2 v_l^{-2}) = H_c / b_c$$

forms the four-momentum not with the GT-Hamiltonian momentum, but with the GT-Lagrangian momentum:

$$P_{Lc} = m_{gr0} v (1 + v^2 v_l^{-2})^{-1/2} = m_{00} c v / v_{lc} = m_{00} v c (v_l^2 + v^2)^{-1/2} = m_{00} v c / v_{lc} = m_{00} \hat{v},$$

where: $W_0^2 = L_c^2 + c^4 v_l^{-2} P_{Lc}^2 = m_{00}^2 c^6 v_l^{-2} / (1 + v^2 v_l^{-2}) + m_{00}^2 c^6 v_l^{-4} v^2 / (1 + v^2 v_l^{-2}) = m_{00}^2 c^6 v_l^{-2} = m_{gr0}^2 c^4$,

$$(ds_c)^2 = v_{lc}^2(dt)^2 - (d\bar{x})^2 - (d\bar{y})^2 - (d\bar{z})^2 = b_c c^2 (dt)^2 - (d\bar{l})^2 = (v_l^2 + v^2)(dt)^2 - (d\bar{l})^2 = bc^2(dt)^2 = \mathbf{inv},$$

$$\hat{v} = v b_c^{-1/2} = v c / v_{lc} = v c / v_l \hat{\Gamma}_c, \quad \hat{\Gamma}_c = (1 + v^2 v_l^{-2})^{1/2}, \quad v_{lc}^2 = b_c c^2 = bc^2 + v^2 = v_l^2 + v^2 = \mathbf{const}(t),$$

$$b_c = b \hat{\Gamma}_c^2 = (v_l^2 + v^2) c^{-2} = b + v^2 c^{-2} = v_{lc}^2 c^{-2} = \mathbf{const}(t).$$

And therefore, the condition of equilibrium precisely in the dynamic gravitational field of the galaxy of all its objects moving by inertia leads to both the absence of relativistic deceleration of the flow of their own time and the invariance of their own time with respect to relativistic transformations. The spatial homogeneity of the rate of flow of proper time in entire gravithermodynamically bound matter is consistent with the single frequency of change of its collective spatially inhomogeneous Gibbs microstates, which is not affected by either a decrease (during approaching gravity center) in the frequency of intranuclear interaction or a increase (during approaching gravity center) in the frequency of extranuclear intermolecular interactions. Moreover, this is ensured even without conformal transformations of the space-time interval s . Therefore, like the parameters v_b , v_{lc} , b and Γ_m in thermodynamics [3], the parameter b_c (or its analogous parameter b_s) in the RGTD is a hidden internal parameter of the moving matter. And the usage of this parameter in the equations of the dynamic gravitational field of the RGTD allows us not to additionally use the velocity of matter in those equations, as in the equations of thermodynamics.

A similar dependence of the parameter v_{lc} on the velocity also occurs for distant galaxies that are in the state of free fall onto the pseudo-event horizon of the expanding Universe: $v_{l_{cg}}^2 \equiv c^2 = v_{lg}^2 + v^2$. After all, according to Hubble's law and the Schwarzschild solution of the gravitational field equations with a non-zero value of the cosmological constant $\Lambda = 3H_E^2 c^{-2}$ and a zero value of the gravitational radius:

$$v_{lg}^2 = c^2 (1 - \Lambda r^2 / 3) = c^2 - H_E^2 r^2 = c^2 - v_g^2.$$

The use of the parameter $b_s = b \Gamma_s^2 = b / (1 - v^2 c^{-2} / b) = v_{ls}^2 c^{-2} = \mathbf{const}(t)^2$, built on the basis of relativistic size shrinkage $\Gamma_s = (1 - v^2 v_l^{-2})^{-1/2}$, in the equations of the dynamic gravitational field of the RGTD is also possible. However, in order to ensure the absence of deceleration of the flow of the proper time of matter moving in a gravitational field by inertia, it will be necessary to use conformal Lorentz transformations (instead of the usual Lorentz transformations) of the increments of spatial coordinates and time. The solutions of the equations of dynamic gravitational field of the RGTD do not depend on the usage of the parameter b_c or the parameter b_s in them. The only parameters that will differ are the parameters of hypothetical static gravitational fields (which are reproduced on the basis of those parameters b_c and b_s).

According to this, in the tensor of energy-momentum of the RGTD not only intranuclear pressure p_N but also intranuclear temperature T_N is taken into account (where S_N is intranuclear entropy [3]):

$$b'_c / a_c b_c r - r^{-2} (1 - 1/a_c) + \Lambda = \kappa (T_N S_N - p_N V_N) / V = \kappa (m_{gr} - m_{in}) c^2 / V = \kappa m_{00} c^2 (1/\sqrt{b_c} - \sqrt{b_c}) / V, \quad (5)$$

$$a'_c / a_c^2 r + r^{-2} (1 - 1/a_c) - \Lambda = \kappa E / V = \kappa m_{in} c^2 / V = \kappa m_{00} c^2 \sqrt{b_c} / V,$$

$$[\ln(b_c a_c)]' / a_c r = \kappa W / V = \kappa m_{gr} c^2 / V = \kappa m_{00} c^2 / \sqrt{b_c} V,$$

where: b_c and a_c are the parameters of the dynamic gravitational field equations of the non-continuous matter of the galaxy; $p_V V_N = \tilde{\beta}_{pVN} E = b_c \tilde{\beta}_{pVN} m_{gr} c^2 = \tilde{\beta}_{pVN} m_{in} c^2$, $\tilde{\beta}_{pVN} \neq \mathbf{const}(r)$, $S_N = m_{gr} c^2 / T_N = m_{00} c^2 / T_{00} = \mathbf{const}(r)$, $T_{00N} = T_N \sqrt{b_c} = \mathbf{const}(r)$, $m_{00} = m_{gr} \sqrt{b_c} = m_{in} / \sqrt{b_c} = \mathbf{const}(r)$,

² Apparently, this parameter is inherent only to the equilibrial (pseudo-inertial uniform) motion of matter of bodies that are evolutionarily self-contracting in the frame of references of spatial coordinates and time which is comoving with the expanding Universe.

$\mu_{00} = m_{00}/V \neq \mathbf{const}(r)$, $\mu_{gr} = m_{00}/\sqrt{b_c}V = \mu_{in}/b_c \neq \mathbf{const}(r)$, $\mu_{in} = m_{00}\sqrt{b_c}/V \neq \mathbf{const}(r)$, $V \neq \mathbf{const}(r)$ and $V_N \neq \mathbf{const}(r)$ are molar and intranuclear volume of matter, respectively.

In addition, according to the RGTD equations, the configuration of the dynamic gravitational field of a galaxy in a quasi-equilibrium state is standard (canonical in RGTD). That is because it is not determined at all by the spatial distribution of the average mass density of its non-continuous matter. After all, this spatial distribution of the average mass density of the galaxy's matter is itself determined by the standard configuration of its dynamic gravitational field:

$$S' = \frac{d[r/a_c(1-b_c)]}{dr} = \frac{1-r'_g - \Lambda r^2}{(1-b_c)} + \frac{(r-r_g - \Lambda r^3/3)}{(1-b_c)^2} b'_c = -\frac{b_c S}{r(1-b_c)} + \frac{(1-\Lambda r^2)}{(1-b_c)^2}, \quad (6)$$

$$S = \frac{r}{a_c(1-b_c)} = \frac{r-r_g - \Lambda r^3/3}{1-b_c} = \exp \int \frac{-b_c dr}{(1-b_c)r} \times \int \left[\frac{(1-\Lambda r^2)}{(1-b_c)^2} \exp \int \frac{b_c dr}{(1-b_c)r} \right] dr,$$

where the parameter S can be conditionally considered as the distance from the event pseudo-horizon.

The trivial solution of this equation, which takes place at:

$$b_c = b_{ce} \left(\frac{3 - \Lambda r_e^2}{3 - \Lambda r_e^2} \right), \quad S_0 = \frac{r - \Lambda r^3/3}{1-b_c} = \frac{(r - \Lambda r^3/3)(3 - \Lambda r_e^2)}{3 - \Lambda r_e^2 - b_{ce}(3 - \Lambda r^2)}, \quad r_g = \frac{(1-b_c)r_{ge}}{(1-b_{ce})} \exp \int_{r_{ge}}^{r_g} \frac{b_c dr}{r(1-b_c)} =$$

$$= \frac{(1-b_c)r_{ge}}{(1-b_{ce})} \exp \frac{2b_{ce} \ln(r/r_e) - (1 - \Lambda r_e^2/3) \{ \ln[r^2 + (3/\Lambda - r_e^2)/b_{ce} - 3/\Lambda] - \ln[(1/b_{ce} - 1)(3/\Lambda - r_e^2)] \}}{2(1 - \Lambda r_e^2/3 - b_{ce})},$$

does not correspond to physical reality. After all, because of $b'_c = -2b_{ce}\Lambda r/(3 - \Lambda r_e^2) \neq 0$ at $r \neq 0$, the solution does not imply the presence of event pseudo-horizon in the FR of matter. And the parameter b_c , unlike the parameter a_c , does not depend on the gravitational radius r_g . And therefore, gravity is absent in the FR corresponding to this trivial solution.

According to the non-identity of the gravitational and inert masses of matter we find the square of the rotation velocity of astronomical object relatively to the galaxy center according to the equations (5, 6) of dynamic gravitational field of RGTD:

$$[\hat{v}^2]_{RGTD} = \frac{c^2 r}{b_c} \frac{d \ln(v_{lc}/c)}{dr} = \frac{c^2 r b'_c}{2b_c^2} = \frac{c^2 a_c}{2b_c} \left\{ (1 - 1/a_c) + [\kappa m_{00} c^2 (1/\sqrt{b_c} - \sqrt{b_c})/V - \Lambda] r^2 \right\} \gg [\hat{v}^2]_{GR} \quad (7)$$

As we can see, at the same radial distribution of the average density of the mass $\mu_{00} = m_{00}/V$ of baryonic matter the circular velocities of rotation of astronomical objects relatively to the galaxy center are much bigger in RGTD than in GR. And this is, of course, related to the fact that:

$$(T_N S_N - p_N V_N)/V \equiv (m_{gr} - m_{in})c^2/V = \mu_{00}c^2(1/\sqrt{b_c} - \sqrt{b_c}) \gg p.$$

Therefore, we can get rid of the imaginary necessity of dark non-baryonic matter in galaxies that follows from the equations of GR gravitational field if we analyze the motion of their astronomical objects using the equations of gravitational field of RGTD.

If we do not take into account local peculiarities of distribution of average density of the mass in galaxies and examine only the general tendency of typical dependence of the orbital velocity of their objects on radial distance to the galaxy center, then the following dependencies of this velocity on parameter b_c and, thus on radial distance r , can be matched with the graphs on Fig.:

$$\tilde{v} = \frac{v}{\sqrt{b_c}} = \sqrt{\frac{2LH_e(b_c/b_{ce})^n}{HL_e[1+(b_c/b_{ce})^{2n}]}} \hat{v}_e = \sqrt{\frac{2b_{ce}(b_c/b_{ce})^n}{b_c[1+(b_c/b_{ce})^{2n}]}} \hat{v}_e = \sqrt{\frac{2}{b_c[(b_{ce}/b_c)^n + (b_c/b_{ce})^n]}} v_e = \frac{v_{\max}}{\sqrt{b_c}} \left\{ 1 + \left[\frac{2m v_e^2}{c^2} \ln\left(\frac{r}{r_e}\right) \right]^2 \right\}^{-1/4},$$

$$\hat{v} = \frac{v}{\sqrt{b_c}} = \sqrt{\frac{2LH_e(b_c/b_{ce})^n}{HL_e[1+(b_c/b_{ce})^{2n}]}}\hat{v}_e = \sqrt{\frac{2(b_c/b_{ce})^n}{b_c[1+(b_c/b_{ce})^{2n}]}}v_e = \frac{v_{\max}}{\sqrt{b_c}} \left\{ 1 + \frac{4n^2v_e^4}{c^4} \left[\ln\left(\frac{r-\Lambda r^3/3}{r_e-\Lambda r_e^3/3}\right) - u \ln\left(\frac{1-b_c}{1-b_{ce}}\right) \right]^2 \right\}^{-1/4}, \quad (8)$$

where: $(dv/db_c)_e = (dv/dr)_e = 0$.

In the first approximate dependence [3, 8], the evolutionary self-contraction of matter in infinite fundamental space of CFREU is conditionally not taken into account. And therefore, there is no limitation of the galaxy's intrinsic space by the pseudo-event horizon in it. After all, according to it, the coordinate velocity of light continuously increases along with the increase in the radial coordinate r at the gravitational radius of the galaxy:

$$\begin{aligned} r_g &= r - \frac{\Lambda r^3}{3} - (1-b_c)^{1+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4} \exp\left\{ \frac{c^2}{4v_e^2} \left[b_{ce}^{-n} \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} + b_{ce}^n \left(\sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\} \times \\ &\times \int_{r_e}^r (1-\Lambda r^2)(1-b_c)^{-2+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4} \exp\left\{ -\frac{c^2}{4v_e^2} \left[b_{ce}^{-n} \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} + b_{ce}^n \left(\sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\} dr = \\ &= r - \frac{\Lambda r^3}{3} - \frac{c^2 r_e B}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{1}{b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] M_r db_c + \frac{\Lambda c^2 r_e^3 B}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{1}{b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] M_\Lambda db_c = \\ &= r - \frac{\Lambda r^3}{3} - (1-b_c)^{1+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4} \exp\left\{ \frac{c^2}{4v_e^2} \left[b_{ce}^{-n} \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} + b_{ce}^n \left(\sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\} \times \\ &\times \left\{ \frac{c^2 r_e}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{\left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] \left\{ \exp\left[\frac{c^2 b_{ce}^{-n}}{4v_e^2} \left(\frac{b_c^n}{n} - \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} \right) - \frac{c^2 b_{ce}^n}{4v_e^2} \left(\frac{1}{nb_c^n} + \sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\}}{b_c (1-b_c)^{2+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4}} db_c - \right. \\ &\left. - \frac{\Lambda c^2 r_e^3}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{\left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] \left\{ \exp\left[\frac{c^2 b_{ce}^{-n}}{4v_e^2} \left(\frac{3b_c^n}{n} - \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} \right) - \frac{c^2 b_{ce}^n}{4v_e^2} \left(\frac{3}{nb_c^n} + \sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\}}{b_c (1-b_c)^{2+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4}} db_c \right\} = \\ &= \frac{2\Lambda r^3}{3} \left[1 + \frac{3c^2}{v_e^2} (1-b_c) \right] + (1-b_c)^{1+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4} \exp\left\{ \frac{c^2}{4v_e^2} \left[b_{ce}^{-n} \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} + b_{ce}^n \left(\sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\} \times \\ &\times \left\{ \frac{c^2 r_e}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{\left\{ \exp\left[\frac{c^2 b_{ce}^{-n}}{4v_e^2} \left(\frac{b_c^n}{n} - \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} \right) - \frac{c^2 b_{ce}^n}{4v_e^2} \left(\frac{1}{nb_c^n} + \sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\}}{(1-b_c)^{2+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4}} db_c - \right. \\ &\left. - \frac{\Lambda c^2 r_e^3}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{\left\{ 1 + \frac{c^2(1-b_c)^2}{b_c v_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\} \left\{ \exp\left[\frac{c^2 b_{ce}^{-n}}{4v_e^2} \left(\frac{3b_c^n}{n} - \sum_{k=0}^{n-1} \frac{b_c^{n-k}}{n-k} \right) - \frac{c^2 b_{ce}^n}{4v_e^2} \left(\frac{3}{nb_c^n} + \sum_{k=1}^{n-1} \frac{b_c^{k-n}}{n-k} - \ln b_c \right) \right] \right\}}{(1-b_c)^{2+c^2v_e^{-2}(b_{ce}^n+b_{ce}^{-n})/4}} db_c \right\}, \end{aligned}$$

³ Here and below, definite integrals are equal to unity when the upper limit of integration is equal to the lower limit. ($b_c=b_{ce}$).

where actually:

$$\frac{dM_r}{db_c} = \frac{c^2}{4v_e^2 b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] M_r, \quad \frac{dM_\Lambda}{db_c} = \frac{c^2 [1 + 2(1 - b_e)]}{4v_e^2 b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] M_\Lambda.$$

When $n=1$:

$$\begin{aligned} r_g &= r - \frac{\Lambda r^3}{3} - (1 - b_c)^{1+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4} \exp \left\{ \frac{c^2}{4v_e^2} \left[\frac{b_c}{b_{ce}} - b_{ce} \ln b_c \right] \right\} \int_{r_e}^r \frac{(1 - \Lambda r^2) \exp[-c^2 v_e^{-2}(b_c/b_{ce} - b_{ce} \ln b_c)/4]}{(1 - b_c)^{2+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4}} dr = \\ &= r - \frac{\Lambda r^3}{3} - (1 - b_c)^{1+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4} \exp \left\{ \frac{c^2}{4v_e^2} \left[\frac{b_c}{b_{ce}} - b_{ce} \ln b_c \right] \right\} \frac{c^2 r_e}{4v_e^2} \int_{b_{ce}}^{b_c} \frac{[(b_c/b_{ce}) + (b_{ce}/b_c)]}{(1 - b_c)^{2+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4}} \exp \left\{ \frac{c^2 b_{ce}}{4v_e^2} \left(\ln b_c - \frac{1}{b_c} \right) \right\} - \\ &- \Lambda r_e^2 \exp \left\{ \frac{c^2}{4v_e^2} \left(\frac{2b_c}{b_{ce}} + b_{ce} \ln b_c - \frac{3b_{ce}}{b_c} \right) \right\} db_c \left\{ = (1 - b_c)^{1+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4} \exp \left\{ \frac{c^2}{4v_e^2} \left[\frac{b_c}{b_{ce}} - b_{ce} \ln b_c \right] \right\} \frac{c^2 r_e}{4v_e^2} \times \right. \\ &\times \int_{b_{ce}}^{b_c} \frac{\exp \left\{ \frac{c^2 b_{ce}}{4v_e^2} \left(\ln b_c - \frac{1}{b_c} \right) \right\} - \Lambda r_e^2 \left[1 + \frac{c^2 (1 - b_c)^2}{v_e^2} \left(\frac{1}{b_{ce}} + \frac{b_{ce}}{b_c^2} \right) \right] \exp \left\{ \frac{c^2}{4v_e^2} \left(\frac{2b_c}{b_{ce}} + b_{ce} \ln b_c - \frac{3b_{ce}}{b_c} \right) \right\}}{(1 - b_c)^{2+c^2 v_e^{-2}(b_{ce}+1/b_{ce})/4}} db_c + \frac{2\Lambda r^3}{3} \left[1 + \frac{3c^2}{v_e^2} (1 - b_c) \right] \}. \end{aligned}$$

Herein according to (4, 7) and similarly to diffeomorphically-conjugated forms [9]:

$$\begin{aligned} b_c &= k_b b_{ce} = b_{ce} \left[(v_{\max}/v)^2 \pm \sqrt{(v_{\max}/v)^4 - 1} \right]^{1/n} = b_{ce} \left[\pm 2nv_e^2 c^{-2} \ln(r/r_e) + \sqrt{1 + [2nv_e^2 c^{-2} \ln(r/r_e)]^2} \right]^{1/n}, \\ v &= b_c^{1/2} \hat{v} = \{[(b_{ce}/b_c)^n + (b_c/b_{ce})^n]/2\}^{-1/2} v_{\max} = [v_e^{-4} + 4n^2 c^{-4} \ln^2(r/r_e)]^{-1/4}, \\ r &= r_e \exp \left[\pm (c^2/2n) \sqrt{v^{-4} - v_e^{-4}} \right] = r_e \exp \left\{ (c^2 v_{\max}^{-2}/4n) [(b_c/b_{ce})^n - (b_{ce}/b_c)^n] \right\}, \\ b'_c &= \frac{db_c}{dr} = \frac{2v_e^2 b_c}{c^2 r \sqrt{1 + [2nv_e^2 c^{-2} \ln(r/r_e)]^2}} = \frac{4v_e^2 b_c}{c^2 r [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]} = \frac{4v_e^2 b_c \exp \left\{ \mp (c^2 v_e^{-2}/4n) [(b_c/b_{ce})^n - (b_{ce}/b_c)^n] \right\}}{c^2 r_e [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}, \\ \frac{b'_c}{b_c a_c r} - \frac{1}{r^2} \left(1 - \frac{1}{a_c} \right) + \Lambda - \frac{\kappa m_{00} c^2}{V} \left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c} \right) &= \frac{4v_e^2 [r^{-2} - r_g r^{-3} - \Lambda/3]}{c^2 [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]} - \frac{r_g}{r^3} + \frac{2\Lambda}{3} - \frac{\kappa m_{00} c^2}{V} \left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c} \right) = 0, \\ V &= \frac{\kappa m_{00} c^2 (1/\sqrt{b_c} - \sqrt{b_c}) [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{4v_e^2 c^{-2} (r^{-2} - r_g r^{-3} - \Lambda/3) - (r_g r^{-3} - 2\Lambda/3) [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]} = \\ &= \frac{\kappa m_{00} c^2 \left\{ (1/\sqrt{b_{ce}}) [\sqrt{1 + A^2} \mp A]^{1/2n} - \sqrt{b_{ce}} [\sqrt{1 + A^2} \pm A]^{1/2n} \right\} \sqrt{1 + A^2}}{2v_e^2 c^{-2} (r^{-2} - r_g r^{-3} - \Lambda/3) - (r_g r^{-3} - 2\Lambda/3) \sqrt{1 + A^2}}, \end{aligned}$$

$$A = 2nv_e^2 c^{-2} \ln(r/r_e), \quad 1/a_c = 1 - r_g/r - \Lambda r^2/3, \quad r_g = \int_{r_{\min}}^r r'_g dr, \quad r_g^* = r_{ge} + \int_{r_e}^r r'_g dr,$$

and: r_e is radius of the conventional friable galactic nucleus, on the surface of which the corrected value \hat{v} of the orbital velocity of objects can take its maximum possible value $v_{\max} \equiv v_e = b_{ce}^{1/2} \hat{v}_e(b_e) = v_{lce} \hat{v}_e/c$; r_g and r_{ge}^4 are the gravitational radii of any layer of the galaxy and its loose core, respectively.

Thus, the gravitational radius r_{ge} of the loose core of the galaxy together with r_e , be and n is an indicator of the power of the galaxy. Theoretically finding the values of all these indicators is problematic. And it is even impossible in the case of the formation of the loose core of the galaxy by antimatter (i.e. when, due to the mirror symmetry of the antimatter-matter intrinsic space, $r > r_e$ not only outside, but also inside the loose core [10]).

⁴ The gravitational radius r_{ge}^* corresponds to a loose nucleus, which at $(dr/dR)_e = 0$ contains only antimatter.

Moreover, even for distant objects in the galaxy $r_g > 2\Lambda r^3/3$, and $b_c < 1 - \Lambda r^2 = 1 - 3H_e^2 c^2 r^2$. And therefore, these objects are "affected" by pseudo-forces of repulsion that are three times greater than the Hubble pseudo-forces.

Therefore:

$$V > \frac{\kappa m_{00} c^4 (1/\sqrt{b_c} - \sqrt{b_c}) [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{4v_e^2 (r^{-2} - \Lambda)},$$

$$\mu_{gr} = \frac{m_{00}}{\sqrt{b_c} V} < \frac{4v_e^2 (r^{-2} - \Lambda)}{\kappa c^4 (1 - b_c) [(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}.$$

Apparently, all this is connected with the simplification of the considered FR of the galaxy. Because in this FR, unlike the FR of galaxies' individual astronomical objects, there is no pseudo-event horizon on which $b_c = 0$. After all, the value of b_c can only grow continuously with the growth of the radial coordinate r ($db_c/b_c dr \neq 0$ at all points of its infinite space).

The second dependence, on the contrary, ensures the presence of a pseudo-event horizon. But according to it, more complex mutual dependencies of the gravitational parameters of the galaxy take place and analytical integration of these dependencies is impossible:

$$r - \frac{\Lambda r^3}{3} = \frac{(r_e - \Lambda r_e^3/3)(1 - b_c)^u}{(1 - b_{ce})^u} \exp \left[\pm \frac{c^2}{2n} \sqrt{v^{-4} - v_e^{-4}} \right] = \frac{(r_e - \Lambda r_e^3/3)(1 - b_c)^u}{(1 - b_{ce})^u} \exp \left\{ \frac{c^2 v_{\max}^{-2}}{4n} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\},$$

$$v = b_c^{1/2} \hat{v} = \left\{ \frac{1}{2} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\}^{-1/2} \quad v_{\max} = \left\{ v_e^{-4} + \frac{4n^2}{c^4} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right]^2 \right\}^{-1/4},$$

$$b_c = k_b b_{ce} = b_{ce} \left[\left(\frac{v_{\max}}{v} \right)^2 \pm \sqrt{\left(\frac{v_{\max}}{v} \right)^4 - 1} \right]^{1/n} =$$

$$= b_{ce} \left\{ \sqrt{1 + \frac{4n^2 v_e^4}{c^4} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right]^2} \pm \frac{2n v_e^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right] \right\}^{1/n},$$

$$b'_c = \frac{db_c}{dr} = \frac{(1 - \Lambda r^2)}{\left(r - \frac{\Lambda r^3}{3} \right) \left\{ \frac{c^2}{2v_e^2 b_c} \sqrt{1 + \frac{4n^2 v_e^4}{c^4} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right]^2} - \frac{u(b_c)}{1 - b_c} + \ln(1 - b_c) \frac{du}{db_c} \right\}} =$$

$$= \frac{(1 - \Lambda r^2)}{\left(r - \frac{\Lambda r^3}{3} \right) \left\{ \frac{c^2}{4v_e^2 b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] - \frac{u(b_c)}{1 - b_c} + \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \frac{du}{db_c} \right\}}}, \quad \frac{b'_c}{b_c a_c r} - \frac{1}{r^2} \left(1 - \frac{1}{a_c} \right) + \Lambda - \frac{\kappa m_{00} c^2}{V} \left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c} \right) =$$

$$= \frac{(1 - \Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3)}{\left(1 - \frac{\Lambda r^2}{3} \right) \left\{ \frac{c^2}{4v_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] - b_c \left[\frac{u(b_c)}{1 - b_c} - \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \frac{du}{db_c} \right] \right\}} - \frac{r_g}{r^3} + \frac{2\Lambda}{3} - \frac{\kappa m_{00} c^2}{V} \left(\frac{1}{\sqrt{b_c}} - \sqrt{b_c} \right) = 0,$$

$$V = \frac{\kappa m_{00} c^2 (1 - \Lambda r^2/3) \left\{ (1/\sqrt{b_{ce}}) [\sqrt{1 + A^2} \mp A]^{\frac{1}{2n}} - \sqrt{b_{ce}} [\sqrt{1 + A^2} \pm A]^{\frac{1}{2n}} \right\} (\sqrt{1 + A^2} - B)}{2v_e^2 c^{-2} (1 - \Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3) - (1 - \Lambda r^2/3)(r_g r^{-3} - 2\Lambda/3)(\sqrt{1 + A^2} - B)},$$

$$\mu_{grst} = \frac{m_{00}}{\sqrt{b_c} V} = \frac{2v_e^2 (1 - \Lambda r^2)(r^{-2} - r_g r^{-3} - \Lambda/3)}{\kappa c^4 (1 - b_c)(1 - \Lambda r^2/3)(\sqrt{1 + A^2} - B)} + \frac{2\Lambda/3 - r_g r^{-3}}{\kappa c^2 (1 - b_c)}, \quad \mu_{grpst} = \frac{2\Lambda/3}{\kappa c^2 (1 - b_{c \max})} = \frac{H_E^2}{4\pi G_{00} (1 - b_{c \max})},$$

$$\text{where: } A = \frac{2nv_e^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3 / 3}{r_e - \Lambda r_e^3 / 3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right], \quad B = \frac{2b_c v_e^2}{c^2} \left[\frac{u(b_c)}{1 - b_c} - \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \frac{du}{db_c} \right],$$

μ_{grst} is standard value of the gravitational mass density of the galaxy matter, $\mu_{grst}=4,8596 \cdot 10^{-27}/(1-b_{cmax})$ is non-zero standard value at the edge of the galaxy ($r_p=\Lambda^{-1/2}=1,1664 \cdot 10^{26}$ [m]=3,78 [Gpc]) of the gravitational mass density of the galaxy matter still held by the galaxy in quasi-equilibrium, despite the zero value of the gravitational radius at its boundary ($r_{gp}=0$, $b'_{cp}=0$).

The dependence of the gravitational radii of a galaxy on the radial coordinate is determined from the following differential equation:

$$r'_g = \kappa \mu_m c^2 r^2 = \frac{\frac{2v_e^2(1-\Lambda r^2)}{c^2(1-\Lambda r^2/3)(\sqrt{1+A^2}-B)} \left(1 - \frac{r_g}{r} - \frac{\Lambda r^2}{3} \right) - \left(\frac{r_g}{r} - \frac{2\Lambda r^2}{3} \right)}{\frac{1}{b_{ce}} \left\{ \sqrt{1 + \frac{4n^2 v_e^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3 / 3}{r_e - \Lambda r_e^3 / 3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right]^2} + \frac{2nv_e^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3 / 3}{r_e - \Lambda r_e^3 / 3} \right) - u(b_c) \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right] \right\}^{\frac{1}{n}} - 1}$$

or using dependent on it parameter S :

$$\begin{aligned} dS &= d \left(\frac{r - r_g - \Lambda r^3 / 3}{1 - b_c} \right) = - \left\{ \frac{c^2}{4v_e^2 b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] - \frac{u(b_c)}{1 - b_c} + \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \frac{du}{db_c} \right\} \left(1 - \frac{\Lambda r^2}{3} \right) \left[\frac{b_c S}{(1 - \Lambda r^2)(1 - b_c)} - \frac{r}{(1 - b_c)^2} \right] db_c, \\ r_g &= r - \frac{\Lambda r^3}{3} - (1 - b_c) \exp \left[- \int \frac{b_c dr}{(1 - b_c)r} \right] \times \int \left\{ \frac{1 - \Lambda r^2}{(1 - b_c)^2} \exp \left[\int \frac{b_c dr}{(1 - b_c)r} \right] \right\} dr = r - \frac{\Lambda r^3}{3} - \\ &- \frac{c^2(r_e - \Lambda r_e^3 / 3)(1 - b_c)}{4v_e^2} \exp \left[- \int \frac{b_c dr}{(1 - b_c)r} \right] \times \int_{b_{ce}}^{b_c} \left\{ \frac{[(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{b_c(1 - b_c)^2} - \frac{4v_e^2 c^{-2} u}{(1 - b_c)^3} \right\} \exp \left\{ \frac{c^2}{4nv_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] + \int \frac{b_c dr}{(1 - b_c)r} \right\} db_c = \\ &= \frac{c^2(r_e - \Lambda r_e^3 / 3)(1 - b_c)}{4v_e^2} \exp \left[- \int \frac{b_c dr}{(1 - b_c)r} \right] \times \int_{b_{ce}}^{b_c} \left\{ \left[1 - \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \right] \left(\frac{b_c(1 - \Lambda r^2 / 3)}{1 - \Lambda r^2} - 1 \right) \frac{du}{db_c} \right\} \frac{1}{(1 - b_c)^2} - \\ &- \frac{u}{(1 - b_c)^3} \left[\frac{b_c(1 - \Lambda r^2 / 3)}{1 - \Lambda r^2} - 1 \right] + \frac{\Lambda c^2[(b_c/b_{ce})^n + (b_{ce}/b_c)^n]}{6v_e^2(r^2 - \Lambda)(1 - b_c)^2} \right\} \exp \left\{ \frac{c^2}{4nv_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] + \int \frac{b_c dr}{(1 - b_c)r} \right\} db_c, \\ \text{where: } \int \frac{b_c dr}{(1 - b_c)r} &= \int \frac{1 - \Lambda r^2 / 3}{(1 - \Lambda r^2)(1 - b_c)} \left\{ \frac{c^2}{4v_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] - \frac{b_c u}{1 - b_c} + b_c \ln \left(\frac{1 - b_c}{1 - b_{ce}} \right) \frac{du}{db_c} \right\} db_c. \end{aligned}$$

At $u=-1$ ($v_e = c / \sqrt{2}$) this solution of the standard equation of the dynamic gravitational field of the galaxy allegedly degenerates. After all, in this case the value of the gravitational radius of the galaxy becomes proportional to the cosmological constant Λ , and therefore to the Hubble constant:

$$r_g = \frac{2\Lambda(3r_e - \Lambda r_e^3)(1 - b_c)}{9} \exp \left[- \int \frac{b_c dr}{(1 - b_c)r} \right] \times \int_{b_{ce}}^{b_c} \frac{r^2 \{ b_c + c^2 v_e^{-2} (1 - b_c) [(b_c/b_{ce})^n + (b_{ce}/b_c)^n] / 4 \}}{(1 - \Lambda r^2)(1 - b_c)^3} \exp \left\{ \frac{c^2}{4nv_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] + \int \frac{b_c dr}{(1 - b_c)r} \right\} db_c.$$

But in fact, like the parameter b_c , the cosmological constant is a hidden parameter of matter. And it is thanks to it that at $b_{ce} > (1 - \Lambda r_e^2) / (1 - \Lambda r_e^2 / 3)$ and at $u = -c^2 v_e^{-2} / 2$ the radial gravitational radii $r_g(r)$ of the galaxy become larger than at $u=0$.

The trivial solution of the equation takes place both at $u=0$ and at a negative value of the parameter $u = -c^2 v_e^{-2} / 2$. And therefore, when $b_{ce} > (1 - \Lambda r_e^2) / (1 - \Lambda r_e^2 / 3)$, the smaller the maximum orbital velocity $v_e < c / \sqrt{2}$ of astronomical objects in the galaxy, the greater in the latter case the value of the gravitational radius on the surface of its loose nucleus will be.

Also what is important is that even in an incredibly weak gravitational field (when $u=0$) and even at large radial distances, astronomical objects will rotate around the center of the galaxy with orbital velocities very close to the maximum possible speed [4 – 6].

Moreover, it is precisely thanks to $b_{ce} > (1 - \Lambda r_e^2)/(1 - \Lambda r_e^2/3)$ that this takes place at $u = -c^2 v^{-2}/2$ at very large distances from the center of the galaxy. After all, the radial distances to the objects of the galaxy at the same value of the parameter b_c become much greater than at $u=0$:

$$\begin{aligned} r - \frac{\Lambda r^3}{3} &= \left(r_e - \frac{\Lambda r_e^3}{3} \right) \left(\frac{1 - b_{ce}}{1 - b_c} \right)^{\frac{c^2}{2v^2}} \exp \left[\pm \frac{c^2}{2n} \sqrt{v^{-4} - v_e^{-4}} \right] = \left(r_e - \frac{\Lambda r_e^3}{3} \right) \left(\frac{1 - b_{ce}}{1 - b_c} \right)^{\frac{c^2}{2v^2}} \exp \left\{ \frac{c^2}{4nv_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\} \gg \\ &\gg \left(r_e - \frac{\Lambda r_e^3}{3} \right) \exp \left[\pm \frac{c^2}{2n} \sqrt{v^{-4} - v_e^{-4}} \right] = \left(r_e - \frac{\Lambda r_e^3}{3} \right) \exp \left\{ \frac{c^2}{4nv_e^2} \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\}, \\ \frac{dr}{db_c} &= \frac{c^2(r - \Lambda r^3/3)}{4v_e^2 b_c (1 - \Lambda r^2)} \left\{ \frac{1}{1 - b_c} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right] - n \ln(1 - b_c) \left[\left(\frac{b_c}{b_{ce}} \right)^n - \left(\frac{b_{ce}}{b_c} \right)^n \right] \right\} \gg \frac{c^2(r - \Lambda r^3/3)}{4v_e^2 b_c (1 - \Lambda r^2)} \left[\left(\frac{b_c}{b_{ce}} \right)^n + \left(\frac{b_{ce}}{b_c} \right)^n \right]. \end{aligned}$$

The transition from the dynamic to the hypothetical static gravitational field of the galaxy is carried out as follows:

$$\begin{aligned} b &= b_c(1 - \hat{v}^2 c^{-2}) = b_c - v^2 c^{-2} = b_c - \frac{2v_{\max}^2 (b_c/b_{ce})^n}{c^2 [1 + (b_c/b_{ce})^{2n}]} = b_c - \frac{v_e^2}{c^2 \sqrt{1 + \{2nv_e^2 c^{-2} \ln[(r - \Lambda r^2/3)/(r_e - \Lambda r_e^2/3)]\}^2}}, \\ b_e &= b_{ce}(1 - \hat{v}_e^2 c^{-2}) = b_{ce} - v_e^2 c^{-2}, \quad b' = b'_c + \frac{4n^2 v^6}{c^6 (r - \Lambda r^2/3)} \ln \left(\frac{r - \Lambda r^2/3}{r_e - \Lambda r_e^2/3} \right) > b'_c; \quad b_e = b_{se}/(1 + v_e^2 b_{se} c^{-2}), \\ b &= b_s/(1 + v^2 b_s c^{-2}) = b_s [(b_{se}/b_s)^n + (b_s/b_{se})^n] / \{ [(b_{se}/b_s)^n + (b_s/b_{se})^n] + 2v_e^2 b_s c^{-2} \}. \end{aligned}$$

The gravitational force acting in a static gravitational field on a conditionally stationary body is greater than the gravitational force acting in a dynamic gravitational field on the same body that is moving. And this is not only due to the decrease in the gravitational mass of the body due to its movement. After all, in a space saturated with rapidly moving bodies, the intensity of the dynamic gravitational field also decreases. That is why it is necessary to use precisely the dynamic gravitational field instead of a static one in calculations of the rotational motion of galactic objects.

Thus, in the equations of the dynamic gravitational field of RGTD, as in the equations of thermodynamics, not only gravitational, but also relativistic indicators are internal hidden parameters of the RGTD-state of matter in motion. And that is why in RGTD, unlike orthodox GR, the use of an external relativistic description of the state of matter in motion is not always required.

The large value $k_b = b_c/b_{ce}$ corresponds to the larger value n of the index of density of friable galactic nucleus on the same big radial distances. However, only when values are extremely large $n \gg 2^{34}$ the significantly lesser average density of matter beyond the friable galactic nucleus takes place and that is why the dependence of orbital velocities of galactic objects on radial distances can be close to Keplerian. For example, when $n=2^{40}$ ($k_b^n=16,780$) the orbital velocity of peripheral objects of the galaxy is less than half of the maximum velocity (when $r_p/r_e=20$, $v_p=0,461v_e$), while when $n=2^{45}$ ($k_b^n=535$) it is already significantly smaller of maximum velocity ($v_p=0,086v_e$). However, not only in the weak gravitational fields ($n \ll 2^{34}$, $k_b^n \ll 1,1391$), but even in quite strong gravitational field ($n=2^{34}$, $k_b^n < 1,1391$, $k_{bp}=1,0000000000758$) the orbital velocities of extra-nuclear objects (when $b_e=1,12656 \cdot 10^{-6}$) are, according to (8), quite close to their maximum values $v_{\max} \equiv v_e \approx 225 \text{ km/s}$ (Fig. 2 b)) on quite big radial distances $r/r_e < 20$ (even when $u=0$):

$$\Delta v = v_e - v \approx v_e - c \left\{ 2^{35} \ln[(r - \Lambda r^3/3)/(r_e - \Lambda r_e^3/3)]^2 + c^4 v_e^{-4} \right\}^{-1/4} \leq 0,95 \text{ [km/s]}.$$

The FR that is almost equivalent to this FR of observed galaxy is its intrinsic GT-FR₀, in which when $b_{cp0}=1$ and $n_0 = n \ln k_{bcp} / \ln k_{bcp0} \approx nb_{cp} = 38708,24438 \approx 2^{15,24}$ ($n=2^{34}$, $k_{bp0}^n = 1,1391$):

$$k_{bp0} = b_{cp0} / b_{ce0} = \left[\sqrt{1 + 2^{32,48} v_e^4 c^{-4} \ln^2[(r - \Lambda r^3/3)/(r_e - \Lambda r_e^3/3)] + 2^{16,24} v_e^2 c^{-2} \ln[(r - \Lambda r^3/3)/(r_e - \Lambda r_e^3/3)]} \right]^{\frac{1}{15,24}} = 1,000003366,$$

$$\Delta v_0 = v_{e0} - v_0 \approx v_{e0} - c \left\{ \left[2^{16,24} \ln[(r - \Lambda r^3/3)/(r_e - \Lambda r_e^3/3)] \right]^2 + (c/v_{e0})^4 \right\}^{-1/4} \leq 0,95 \text{ [km/s]}.$$

Not only the GT-Lagrangian of ordinary internal energy and equivalent to it gravitational mass of matter, but also the following relations are invariant under such a transformation:

$$v_0/v_{e0} = v/v_e = \mathbf{inv}, \quad n_0 \ln k_{b0} = n \ln k_b = \mathbf{inv} \quad [n_0(k_{b0}-1) \approx n(k_b-1)].$$

This, of course, is related to the fact that big gradients of gravitational field on the periphery of such galaxies are formed not by their nuclei but by all large set of their objects. This is also related to the fact that the coordinate value of GT-Hamiltonian of inert free energy of matter is significantly smaller than the coordinate value of GT-Lagrangian of its ordinary internal energy when $b_{ce} = 2,253 \cdot 10^{-6}$ ($v_{\max} = 0,3377 \text{ km/s}$).

The following dependence of the orbital velocity of objects of galaxies on parameter b_{c0} and, thus on radial distance r , can be matched to these objects in intrinsic GT-FR_{g0} of galaxy [3]:

$$v_0 = \sqrt{\frac{2(b_{c0}/b_{ce0})^{n_0}}{1+(b_{c0}/b_{ce0})^{2n_0}}} v_{e0} = \left\{ v_{e0}^{-4} + \frac{4n_0^2}{c^4} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_{c0}) \ln \left(\frac{1-b_{c0}}{1-b_{ce0}} \right) \right]^2 \right\}^{-1/4},$$

$$\begin{aligned} \text{where: } b_{c0} &= b_{ce0} \left[(v_{e0}/v_0)^2 \pm \sqrt{(v_{e0}/v_0)^4 - 1} \right]^{1/n_0} = \\ &= b_{ce0} \left\{ \sqrt{1 + \frac{4n_0^2 v_{e0}^4}{c^4} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_{c0}) \ln \left(\frac{1-b_{c0}}{1-b_{ce0}} \right) \right]^2} \pm \frac{2n_0 v_{e0}^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) - u(b_{c0}) \ln \left(\frac{1-b_{c0}}{1-b_{ce0}} \right) \right] \right\}^{1/n_0}, \\ r - \frac{\Lambda r^3}{3} &= \frac{(r_e - \Lambda r_e^3/3)(1-b_{c0})^u}{(1-b_{ce0})^u} \exp \left[\pm \frac{c^2}{2n_0} \sqrt{v_0^{-4} - v_{e0}^{-4}} \right] = \frac{(r_e - \Lambda r_e^3/3)(1-b_{c0})^u}{(1-b_{ce0})^u} \exp \left\{ \frac{c^2 v_{e0}^{-2}}{4n_0} \left[\left(\frac{b_{c0}}{b_{ce0}} \right)^{n_0} - \left(\frac{b_{ce0}}{b_{c0}} \right)^{n_0} \right] \right\}. \end{aligned}$$

According to the dependence $n_0 \ln k_{b0} = n \ln k_b = \mathbf{inv}$ in intrinsic GT-FR₀ of the galaxy there is stronger gravitational field than in FR of distant external observer:

$$\mathbf{F}_{gr0} = \frac{L_0}{2\sqrt{a_c}} \frac{d \ln k_{b0}}{dr} = \frac{n}{n_0} \frac{L}{2\sqrt{a_c}} \frac{d \ln k_b}{dr} = \frac{n}{n_0} \mathbf{F}_{gr} = \frac{G_{g0}}{G_{00}} \mathbf{F}_{gr} = \frac{1}{b_c} \mathbf{F}_{gr},$$

where: $L_0 = L$ due to the fact that GT-Lagrangian of ordinary internal energy of inertially moving matter does not depend on galactic rates of gravithermodynamical (astronomical) time [3].

This, of course, is related to the fact that big gradients of gravitational field on the periphery of such galaxies are formed not by their nuclei but by all large set of their objects.

In centric intrinsic GT-FR_{g0} of the galaxy when $u = -c^2 v^{-2}/2$ the following typical (standard) radial distribution of the average density of gravitational mass of the matter in the galaxy takes place:

$$\begin{aligned} \mu_{grst} &= \frac{m_{00}}{\sqrt{b_{c0}} V} = \frac{2v_{e0}^2 (1 - \Lambda r^2)(r^{-2} - r_{g0} r^{-3} - \Lambda/3)}{\kappa c^4 (1 - b_{c0})(1 - \Lambda r^2/3)(\sqrt{1 + A^2 - B})} + \frac{2\Lambda/3 - r_{g0} r^{-3}}{\kappa c^2 (1 - b_{c0})}, \\ A &= \frac{2n_0 v_{e0}^2}{c^2} \left[\ln \left(\frac{r - \Lambda r^3/3}{r_e - \Lambda r_e^3/3} \right) + \frac{c^2}{2v_0^2} \ln \left(\frac{1-b_{c0}}{1-b_{ce0}} \right) \right], \quad B = \frac{1}{2} \left\{ n_0 \ln \left(\frac{1-b_{c0}}{1-b_{ce0}} \right) \left[\left(\frac{b_{c0}}{b_{ce0}} \right)^{n_0} - \left(\frac{b_{ce0}}{b_{c0}} \right)^{n_0} \right] - \frac{b_{c0}}{1-b_{c0}} \left[\left(\frac{b_{c0}}{b_{ce0}} \right)^{n_0} + \left(\frac{b_{ce0}}{b_{c0}} \right)^{n_0} \right] \right\}. \end{aligned}$$

According to which, when at the edge of the galaxy ($r_p = \Lambda^{-1/2} = 1,1664 \cdot 10^{26} \text{ [m]} = 3,78 \text{ [Gpc]}$) the gravitational mass density of matter still held by the galaxy in quasi-equilibrium, despite the zero value of the gravitational radius at its boundary ($r_{g0p} = 0$, $b'_{c0p} = 0$, $b_{c0p} = b_{c0\max}$,

$r_{g0p} r_p^{-3} = \Lambda^{3/2} r_{g0p} = 0$), becomes non-zero standard $\mu_{grst} = 2\Lambda/3\kappa c^2 (1 - b_{c\max}) = H_E^2/4\pi G_{00} (1 - b_{c\max})$.

3. Conclusions

According to the RGTD equations, the configuration of the dynamic gravitational field of a galaxy in a quasi-equilibrium state is standard (canonical in RGTD). That is because it is not determined at all by the spatial distribution of the average mass density of its non-continuous matter. After all, this spatial distribution of the average mass density of the galaxy's matter is itself determined by the standard configuration of its dynamic gravitational field. In the equations of the dynamic gravitational field of RGTD, as in the equations of thermodynamics, not only gravitational, but also relativistic indicators are internal hidden parameters of the RGTD-state of matter in motion. And that is why in RGTD, unlike orthodox GR, the use of an external relativistic description of the state of matter in motion is not always required. The general solution of the equations of the gravitational field of the galaxy with an additional variable parameter n is found. The additional variable parameter n determines in GR and RGTD the distribution of the average mass density mainly in the friable galactic nucleus. The velocity of the orbital motion of stars is close to the Keplerian one only for $n > 225$. At $n < 215$, it is slightly less than the highest possible velocity even at the edge of the galaxy. The standard value of the average mass density of matter at the edge of a galaxy is determined by the cosmological constant Λ and the difference between unity and the maximum value of the parameter b_c . And it is a non-zero standard value, despite the gravitational radius at the edge of a galaxy takes the zero value. Therefore, in relativistic gravithermodynamics, in contrast to GR, there can be no shortage of baryonic mass in principle. And, therefore, the presence of non-baryonic dark matter in the Universe is not necessary. The most significant fact is the absence of relativistic dilatation of intrinsic time of galaxies according to received transformations. And this confirms the correspondence of the orbital motion of galactic astronomical objects to GT-Lagrangians and GT-Hamiltonians or to Lorentz-conformal transformation of increments of metrical intervals and metrical time for the galaxies [3, 8].

References

- [1] Logunov A A and Mestvirishvili M A 1989 *The Relativistic Theory Of Gravitation*, Moscow: Mir
- [2] Danylchenko P 2004 *Gauge-evolutional interpretation of special and general relativities*, Vinnytsia: O.Vlasuk, 66
- [3] Danylchenko P 2024 *Foundations of Relativistic Gravithermodynamics* 5th online edition, <https://elibrary.com.ua/m/articles/view/FOUNDATIONS-OF-RELATIVISTIC-GRAVITHERMODYNAMICS-5th>
- [4] Bennett J, Donahue M, et al. 2012 *The essential cosmic perspective*, Boston: Addison-Wesley
- [5] Rieke G H 2016 *Dark Matter: another basic part of galaxies!! Dark Energy: What is it?* Ircamera.as.arizona.edu
- [6] Thompson T 2011 *Astronomy 1144: Introduction to Stars, Galaxies, and Cosmology*, Lecture 39
- [7] Danylchenko P 2009 *Proc. of TMP'2009* (in Russian), Lutsk: Volyn' University Press "Vezha" 75
- [8] Danylchenko P 2020 *Foundations and consequences of Relativistic Gravithermodynamics* (in Ukrainian) Vinnytsia: Nova knyga.
- [9] Trokhimchuk P P 1985 *Contradictions in modern physical theory. The method of diffeomorphically-conjugated forms and some its applications*, Preprint USC as the USSR, Sverdlovsk
- [10] Danylchenko P 2025 *On the possibility of the stable existence of antimatter in the Universe. Materials of the 6th readings of Anatoly Svidzinsky*. Lutsk: Press "Vezha", [ISBN: 978-966-940-635-4], 38, <https://elibrary.com.ua/m/articles/view/Щодо-можливості-сталого-існування-антиречовини-у-Всесвіті>.