# NEAR LIGHT SPEED INTERSTELLAR TRAVEL

**Time Dilation and Velocity Verses Time for Relativistic Star Travel**

Program *StarTravel* Technical Note 2025-1

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## Abstract

Using the concepts of general relativity relativistic interstellar travel to the stars approaching the speed of light can be computed for the general case of accelerated motion for half the journey and decelerated motion for the remaining half of the journey. According to general relativity a rocket and its passengers feel the effects of gravity when accelerating in flat spacetime where motion is described relative to a proper reference frame. Because of **time dilation** an astronaut can travel stellar distances, that is many light years within his/her own lifetime while many thousands of years will have elapsed on the planet of departure. This analysis determines the proper time dilation or elapsed time on a spaceship whose speed approaches the light speed and the instantaneous velocity or 4-velocity of a spaceship relative to the speed of light at some constant acceleration. As explained in general relativity the 4-velocity is the rate of change of the 4-vector with respect to the moving frames' proper time. The results presented here are independent of the propulsion systems used. These results are equally valid for a spaceship powered by an advanced antimatter propelled **photon** rocket**,** or a spaceship powered by a **VASIMR** (Variable Specific Impulse Magnetoplasma Rocket) plasma rocket motor.


## Nomenclature

| | | |
|---|---|---|
| c | = | Speed of light in the vacuum of space |
| g | = | Acceleration of the rocket or spaceship in g's |
| t | = | Stationary reference frame (Earth for example) |
| $\tau$ | = | Proper time (moving reference frame) of the accelerated observer |
| $x$ | = | Distance traveled in the stationary reference frame (Earth for example) |
| $v_e$ | = | Exhaust velocity for the classical and relativistic equations |
| MR | = | Mass ratio, ratio of the initial rest mass $m_0$ and the final rest mass $m_1$ |
| $\Delta V$ | = | Mission change in velocity |
| ly | = | Distance light travels in one year |

## GENERAL RELATIVITY AND ACCELERATED MOTION

Using the concepts of general relativity relativistic interstellar travel to the stars approaching the speed of light can be computed for the general case of accelerated motion. According to general relativity[1,2] a rocket and its passengers feel the effects of gravity when accelerating in flat spacetime where motion is described relative to an inertial proper reference frame. Velocity and acceleration of the observer is relative to this inertial reference frame. Where, the spaceship's 4-velocity is the derivative with respect to proper time, $\tau$ of the spaceship event-displacement vector. Also, the passenger's 4-velocity satisfies the condition, $u^2 = -1$ where the 4-velocity is fixed in magnitude implying the 4-acceleration[1] is the following.

$$\mathbf{a} = \frac{du}{d\tau} \tag{1}$$

The 4-accelaration is orthogonal to the 4-velocity because.

$$0 = \left({d}/{d\tau}\right)\left(\frac{1}{2} u \cdot u\right) = a \cdot u \tag{2}$$

This equation implies that $a^0 = 0$ in the passenger's frame of reference where at the instant in question $u = e_0$. In this frame the space components of $a^\mu$ reduce to the ordinary definition of acceleration, $a^i = d^2x^i/d^2t$. From the components $a^\mu = \left(0; a^i\right)$ in the rest frame the magnitude of the acceleration can be computed as a invariant.

$$a^2 = a^\mu a_\mu = (d^2x/d^2t)^2 \tag{3}$$

This analysis uses **general relativity**[1] to describe the nonlinear hyperbolic relationship between change in velocity, proper time and rest frame time for an observer at constant acceleration, g. This contrasts with **special relativity**[2] where velocity is constant and the relationship between proper time and rest frame time is linear. In addition, the observer's acceleration occurs in the $x^1$ direction of the selected inertial frame and the motion in the $x^2$and $x^3$ directions are exactly zero. The equations for the motion of an observer in the inertial frame become.

$$\frac{dt}{d\tau} = u^0 \qquad \frac{dx}{d\tau} = u^1 \qquad \frac{du^0}{d\tau} = a^0 \qquad \frac{du^1}{d\tau} = a^1 \tag{4}$$

The three algebraic equations for an accelerated observer are.

$$u^\mu u_\mu = -1 \tag{5}$$
$$u^\mu a_\mu = -u^0 a^0 + u^1 a^1 = 0 \tag{6}$$
$$a^\mu a_\mu = g^2 \tag{7}$$

Solving for the observer acceleration the linear differential equations become.

$$a^0 = \frac{du^0}{d\tau} = g u^1 \qquad a^1 = \frac{du^1}{d\tau} = g u^0 \tag{8}$$

These linear differential equations are solved using a suitable choice for the origin.

$$t = \frac{g^{-1}}{c} \sinh g\tau \qquad x = \frac{g^{-1}}{c} \cosh g\tau \tag{9}$$

After some algebra the equations for relativistic accelerated motion are the following.

$$x = \frac{c^2}{g}\left[\sqrt{1+\left(\frac{gt}{c}\right)^2} - 1\right] \quad \Rightarrow \quad t = \frac{c}{g}\sqrt{\left(\frac{gx}{c^2}+1\right)^2 - 1} \tag{10}$$

$$\tau = \frac{c}{g}\sinh^{-1}\frac{gt}{c} \qquad \Delta V = c\tanh\frac{g\tau}{c} \tag{11}$$

## RELATIVISTIC ROCKET EQUATION

The classical Tsiolkovsky rocket equation calculates the change in velocity that a rocket can experience as a function of exhaust speed $v_e$ and mass ratio MR, i.e. the ratio of the initial rest mass $m_0$ and the final rest mass $m_1$ after the propellant is expended. However, as rocket velocity increases it becomes increasingly important to consider the relativistic effects that time dilation $\tau = \gamma t$ and relativistic mass $m = \gamma m_0$ have on the energy requirements for speeds beyond 0.3c.

In this context, StarTravel uses the relativistic rocket equation[3] to determines change of velocity $\Delta V$, proper time t of photon rocket operation and rest frame time t of photon rocket operation using these three variables. The following hyperbolic motion equations relate velocity and time.

$$\Delta V = c\ tanh\left(\frac{v_e}{c}\, ln(MR)\right) \tag{12}$$

$$t = \frac{v_e}{g}\, ln(MR) \qquad \tau = \frac{c}{g}\, sinh\left(\frac{g\,t}{c}\right) \tag{13}$$

The non-relativistic rocket equation for $\Delta V \ll c$, is displayed below for comparison with the relativistic rocket equation, Equation 12. Notice in Equation 14, $v_e$is the exhaust velocity and MR is the same mass ratio used for the relativistic case. Please notice there is no dependency on speed of light, c in Equation 14 for computing rocket change in velocity.

$$\Delta V = v_e\ ln(MR) \tag{14}$$

## APPLICATION OF THE THEORY

According to general relativity[4,5,6], interstellar travel approaching the speed of light is only possible using propulsion systems far exceeding the power and specific impulse of any propulsion system available today. It is impossible to exceed the speed of light because as an object approaches the speed of light the inertial mass of an object and therefore its mass approach infinity. ***Therefore, it would take infinite power to accelerate an object beyond the light barrier, c or Einstein limit.*** However, because of **time dilation**[2] as predicted by Einstein's theory of Relativity, an astronaut can travel stellar distances, that is many light years within his/her own lifetime while many thousands of years will have elapsed on the planet of departure or Earth in our case if a speed greater than 0.3c can be achieved. Where, time dilation is defined as measurements taken in the same location in a moving inertial reference frame. The time interval between two events measured in that frame is called the **proper time**[2] interval and measurements of the same time interval in any other reference frame, i.e. on Earth are always greater than the proper time. To illustrate these concepts, program StarTravel[7,8] was used to model a starship traveling near the speed of light at constant acceleration, g for half its journey and then decelerating at the same rate for the remaining half of its journey to a star located 1,000 light years (ly) away.

StarTravel input data displayed in the upper left-hand box of Figure-1 assume the travelers leave Earth with a constant acceleration of 0.999998 g's toward a star located 1,000 light-years from Earth and then decelerate at the same rate. StarTravel determined (a) the elapsed time on Earth when the starship reaches the star and (b) the proper time on the ship, relative to Earth clocks.

Using Figure-1, the StarTravel program predicts that a) the elapsed time on Earth during the flight is 1,002.65 years and b) the elapsed time or proper time on the starship is 13.46 years. These results are identical to the case presented by Dr. Wernher von Braun in his Popular Science Magazine article, *Can We Ever Go to the Stars*[8]. Figure-2 illustrates a free-body thrust diagram of the spaceship as it accelerates during the thrusting and braking phases of flight. Using $\sum F = mg$ it is easy to imagine how a constant acceleration of 1g can be maintained during the thrusting and braking phases of flight. Passengers aboard the spaceship think they are experiencing earth-level gravity during the flight because of the relativistic equivalence of gravity and acceleration. Finally, Figure-3 presents Mathcad generated results for proper time verses time, velocity verses proper time and velocity verses time for the starship flight displayed in Figure-1.

**PROGRAM STARTRAVEL RESULTS**

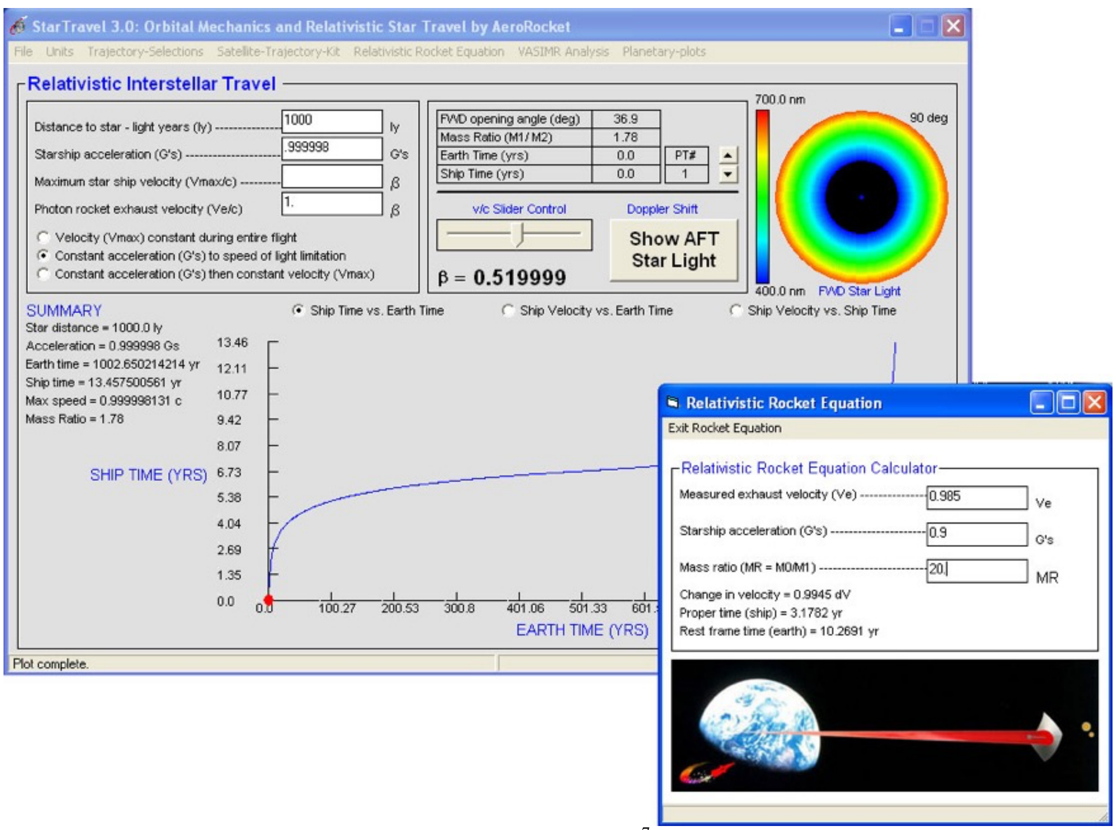


Figure-1, Time Dilation and Doppler light shift analysis[7] showing forward (FWD) star colors.

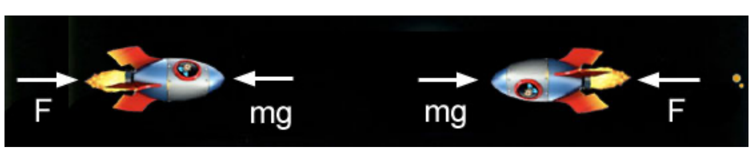


Figure-2, **Acceleration** leaving Earth then **deacceleration** at half-way point

## MATHCAD PLOTS OF TIME AND PROPER TIME VERSES VELOCITY

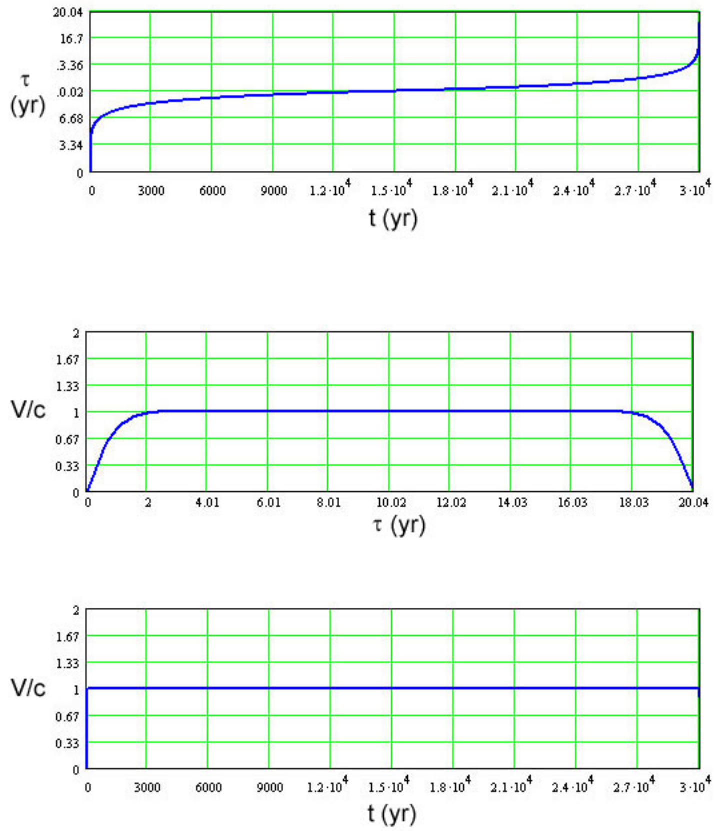


Figure-3, Proper time verses time, velocity verses proper time and velocity verses time


## REFERENCES

[1]C. W. Misner, K. S. Thorn, J. A. Wheeler, *GRAVITATION* (Freeman, San Francisco, 1971)
[2]D. Halliday, R. Resnick, *Fundamentals of Physics-7th Edition* (John Wiley & Sons, New Jersey, 2005)
[3]Wikipedia article that may be useful, https://en.wikipedia.org/wiki/Relativistic_rocket
[4]Humble, Henry and Larson, *Space Propulsion Analysis and Design*, (McGraw-Hill, New York, 1995)
[5]S. M. Carroll*, SPACETIME AND GEOMETRY* (Addison Wesley, San Francisco, 2004)
[6]Ray Skinner*, RELATIVITY FOR SCIENTISTS AND ENGINEERS* (Dover, New York, 1982)
[7]John Cipolla, StarTravel™, Two-Body Astrodynamics and Relativistic Star Travel Program, 2012 to 2025
[8]Wernher von Braun, *Can We Ever Go to the Stars*, Popular Science Magazine, July 1963