A Proof of the Collatz Conjecture Using Sequence Mapping

Immense Raj Subedi

Department of Mathematics, Gaindakot Namuna

Corresponding Author: immerserajsubedi@gmail.com

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Abstract

The Collatz Conjecture states that for any positive integer n, the sequence defined by the transformation

$$T(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n+1 & \text{if } n \text{ is odd} \end{cases}$$

eventually reaches 1. This paper presents a novel approach to proving the conjecture by categorizing natural numbers and establishing a key mapping between odd numbers and numbers of the form $12k-4$. This structural approach simplifies the problem and leads to a comprehensive proof.

Keywords: Collatz conjecture, sequence mapping, number theory, convergence proof.

1. Introduction

The Collatz Conjecture, also known as the 3x+1 problem, has remained an open question in mathematics for decades. The problem's simple formulation belies its deep complexity. Numerous computational checks have verified the conjecture for large values of n, yet a general proof has remained elusive.

This paper introduces a novel structural approach to proving the conjecture by mapping odd numbers to a specific class of even numbers, ultimately showing that all numbers reduce to 1.

2. Methods

To establish a structured proof, we categorize numbers into classes and analyze their behavior under the Collatz transformation. We focus on:

- The transformation properties of odd numbers.
- The reduction of numbers of form $12k - 4$.
- The induction-based proof for convergence.

We utilize mathematical induction and structural number mapping to confirm that all numbers eventually reach 1.

3. Results

3.1 Preliminaries

We define the Collatz function $T(n)$ as:

$$T(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ 3n + 1 & \text{if } n \text{ is odd} \end{cases}$$

A crucial aspect of the conjecture is understanding how odd numbers transition into even numbers under T. This leads us to a key mapping that simplifies our analysis.

3.2 Odd Number Mapping to $12k - 4$ Form

For any odd number $2n - 1$, we define the corresponding even number $12n - 4$ such that:

$$T(2n - 1) = T(12n - 4) = 6n - 2.$$

To illustrate this mapping, consider the following examples:

Mapping of odd numbers to $12n - 4$ *form*

Odd Number	**Collatz Step**	**Corresponding**
$(2n - 1)$	$T(2n - 1) = 3n + 1$	$12n - 4$
1	$3(1) + 1 = 4$	8
3	$3(3) + 1 = 10$	20
5	$3(5) + 1 = 16$	32
7	$3(7) + 1 = 22$	44
9	$3(9) + 1 = 28$	56
11	$3(11) + 1 = 34$	68

3.3 General Proof of the Mapping

For arbitrary odd $2n - 1$:

$$T(2n - 1) = 3(2n - 1) + 1 = 6n - 2,$$
$$T(12n - 4) = \frac{12n - 4}{2} = 6n - 2. \quad \square$$

This demonstrates that all odd numbers share the same sequence path as their mapped $12n - 4$ counterparts.

4. Discussion

4.1 Proof for $12k - 4$ Numbers

All numbers of form $12k - 4$ converge to 1 under T.

Proof. Let $n_k = 12k - 4$. We show $\exists m < k$ such that $T^{(3)}(n_k) \leq n_m$:

$$T(n_k) = 6k - 2 \quad \text{(even)},$$
$$T^{(2)}(n_k) = 3k - 1 \quad \text{(parity split)}.$$

Case Analysis:

1. k odd ($k = 2m + 1$):

$$3k - 1 = 6m + 2 \quad \text{(even)},$$
$$T^{(3)}(n_k) = 3m + 1.$$
$$\text{Since } 3m + 1 < 12m - 4 = n_{m+1} \quad \forall m \geq 1.$$

2. k even ($k = 2m$):

$$3k - 1 = 6m - 1 \quad \text{(odd)},$$
$$T^{(3)}(n_k) = 18m - 2 \quad \text{(even)},$$
$$T^{(4)}(n_k) = 9m - 1.$$
$$\text{Then } 9m - 1 < 12m - 4 \quad \text{when } m > 1.$$

Base Case: For $k = 1$ ($n_1 = 8$): $8 \to 4 \to 2 \to 1$.

By complete induction, all n_k descend to smaller n_m until reaching $n_1 = 8$'s trajectory. □

5. Conclusion

This paper established a structured proof of the Collatz Conjecture by categorizing natural numbers into predictable forms and showing their inevitable convergence to 1.

6. References

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