The Geometric Nature

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The spacetime of relativistic physics is forged by the speed of light; that of general relativity, by the additional influence of matter-energy. Nevertheless, the underlying geometry—rooted in a Euclidean conceptual legacy—knows nothing about light and even less about matter. It is an amorphous and unaware spacetime, waiting to be informed by an ad hoc mathematical apparatus to adhere to nature.

The purpose of this paper is to show the existence of a geometry made of light and matter that embodies the universal interaction, where the laws of nature are not imposed, but rather discovered. This geometry does not use coordinates, since these have no direct physical meaning, but only invariant physical quantities which are consistent with the Schwarzschild's differential coordinates, but not with its finite coordinates or with Minkowski's coordinate, since these are affected by human non-natural conventions. It is shown that it is precisely these human conventions that underlie the formal difference between the special and general theories of relativity.

Furthermore, it aims to demonstrate that the spacetime of general relativity is not elementary, rather, the lack of distinction among the three spatial dimensions arises from the superposition of a multitude of elementary physical spaces, where the distinctive characteristics of each axis merge and dissolve into a flat uniformity.

Revealing the nature of the elementary space of interaction unlocks the path to a unified theory of everything.

INTRODUCTION

The element of the geometry of modern physics is the event, in a four-dimensional spacetime consisting of three spatial dimensions and a temporal dimension independent of each other, between which it is possible to define a distance relationship. In this geometry, the relationship between space and time, sanctioned by the constancy of the speed of light, is proposed only as a principle of the theory of relativity confirmed by experimental results. It is in fact guaranteed by the mathematical apparatus, not by geometry itself which, in its fundamental setting, was born to deal with forms in the two-dimensional plane and in three-dimensional space, and only later is it adopted by modern physics through the ad hoc addition of a further temporal dimension. A direct consequence of this setting is the synchronous space $S(t)$ of the observer, i.e. the totality of the space stationary at instant t of his wristwatch, which therefore he could never see or photograph, but which he could observe and photograph if the light were instantaneous.

Einstein, in his famous 1905 article "On the Electrodynamics of Moving Bodies", begins with the problem of defining "the time of the stationary system" which leads back to the determination of the co-ordinates of the synchronous space $S(t)$. The first paragraph, which is titled "Definition of Simultaneity," reads as follows:

«If we wish to describe the motion of a material point, we give the values of its co-ordinates as functions of the time. Now we must bear carefully in mind that a mathematical description of this kind has no physical meaning unless we are quite clear as to what we understand by "time." » (Refs. 5)

«We have to take into account that all our judgments in which time plays a part are always judgments of simultaneous events.»

The question is: how *«to evaluate the times of events occurring at places remote from the watch»* of the observer.

To answer this question, he sets out a definition and a principle:

1. DEFINITION : given, in an inertial system, two clocks located in two places A and B distant from each other, *«we establish by definition that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A.»*

$$t_B - t_A = t'_A - t_B$$

2. PRINCIPLE of the constancy of the speed of light: *«In agreement with experience we further assume the quantity*

$$2AB/(t'_A - t_A) = c,$$

to be a universal constant—the velocity of light in empty space.»

Both points are necessary for the determination of the ("the time of the stationary system"), i.e. the determination of the spatial and temporal coordinates, both of the SR and the GR, obtained by synchronizing all the clocks placed at the vertices of a grid of metersticks, each at a fixed and determined distance with respect to the origin of a reference system. *«The "time" of an event is that which is given simultaneously with the event by a stationary clock located at the place of the event, this clock being synchronous, and indeed synchronous for all time determinations, with a specified stationary clock.»* Once the coordinates have been determined in this way, it is possible to resort to *the methods of Euclidean geometry,* where an n-dimensional spacetime is characterized by the metric:

$$ds^2 = dx_n^2 \qquad \text{or, more generally, for a differential Riemann manifold} \qquad ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu,$$

The first cracks

Once this is done, the first cracks in the building are not long in appearing. For example, the implantation of the representation of knowledge makes it difficult to explain the phenomenon of entanglement and does not help in the explanation of the uncertainty principle, the quantization of phenomena and the unification of forces. The role of external observer, typical of Euclidean geometry, if it is well placed in the sphere of relativity, becomes problematic in that of quantum mechanics. But there's more.

In his book "The meaning of relativity", Einstein writes: *«The assumption of the complete physical equivalence of the systems of co-ordinates, K and K', we call the "principle of equivalence;" this principle is evidently intimately connected with the theorem of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to co-ordinate systems which are in non-uniform motion relatively to each other. In fact, through this conception we arrive at the unity of the nature of inertia and gravitation. For according to our way of looking at it, the same masses may appear to be either under the action of inertia alone (with respect to K) or under the combined action of inertia and gravitation (with respect to K'). The possibility of explaining the numerical equality of inertia and gravitation by the unity of their nature gives to the general theory of relativity, according to my conviction, such a superiority over the conceptions of classical mechanics, that all the difficulties encountered in development must be considered as small in comparison.»* (Refs. 6) The unity of the nature of inertia and gravitation is supported by the experimental equality of gravitational and inertial mass (experiments of Eötvös) and by the experimental observation that, in the event of a free fall, spacetime is locally Euclidean.

Now, the equivalence principle would require a formal identity between the Lorentz angle of rotation $\cosh\zeta = 1/\sqrt{1-v^2}$ and the angle of rotation $\cos\gamma = \sqrt{g_{00}} = \sqrt{1-2V}$ due to the local deformation of spacetime which, however, is not found. This should not be surprising if we consider that, if the deformation of spacetime is the real one and present in an intrinsic geometry such as the Riemannian differential manifold, the Lorentz angle of rotation is instead determined on the basis of a definition (a human convention) within an extrinsic geometry such as the Euclidean one.

Einstein writes: *«We shall be true to the principle of relativity in its broadest sense if we give such a form to the laws that they are valid in every such four-dimensional system of co-ordinates.»* (Refs. 6) Yet, the Doppler redshift, cosmological redshift, and gravitational redshift, just to give an example, have different formulas. And, broadening the discussion, what can be said about the diversity of the mathematical frameworks and conceptual foundations of Special Relativity, General Relativity, and Quantum Mechanics, which make their unification into a general theory highly challenging and still an open problem?

But there is an even more fundamental question. If knowledge is reducible to the network of relations between elementary terms, the system of representation of knowledge, as meta-knowledge, must already know these elements, indeed they must be the constituent part of its structure. More clearly, the elementary terms must constitute the dimensions of the representative space. This structure represents the axioms, the preconceptions, on which knowledge will be based. And yet, despite *«The gravitational field influences and even determines the metrical laws of the space-time continuum.»* (Refs. 6) and although this emerges from the mass, this is not part of the structure of the manifold, it is not a dimension of it. Are not the manifestation of unresolved questions and apparent paradoxes in modern physics the symptoms of the use of a system of representation based on erroneous preconceptions?

The discarded alternative

For the purposes of the discussion that follows, it is important to underline two facts. One is that, contrary to the "principle of the constancy of the speed of light", which is experimentally verifiable, the definition concerning the equality of round trip times is not verifiable, it is precisely a definition dictated only by reasons of symmetry and practicality. The second, much more fundamental, is that everything discussed and established thus far, while following from experimental facts, is nonetheless shaped by the chosen framework—namely, the representation of natural phenomena through events within a spacetime manifold.

The problem with synchronous space defined by Einstein is that it is only relative, not absolute, it is not real, although it can be used for practical purposes, it has, in itself, only a psychological meaning for us humans. The incomprehensibility of the entanglement in current physics is a direct consequence of the non-physical view of its synchronous space. Indeed, the explanation of the entanglement requires that the act, composed of the sending event A and the dual pair of entangled receiving events B and C, is one and instantaneous, the translation into the same act (composed of three events) of the space of potency of the single relationship giver(A)-receiver(B,C), whatever their distance, even across galaxies. From the point of view of the "Synchronous space", instead, the measurement B, which instantly affects the other C, violates Einstein's idea that nothing can travel faster than light.

It is important to note that, in defining "the time of the stationary system", Einstein makes the following consideration: *«We might, of course, content ourselves with time values determined by an observer stationed together with the watch at the origin of the co-ordinates, and co-ordinating the corresponding positions of the (wristwatch) hands with light signals, given out by every event to be timed, and reaching him through empty space. But this co-ordination has the disadvantage that it is not independent of the standpoint of the observer with the watch or clock, as we know from experience.»* (Refs. 5)

In reality, this possible choice, advanced and promptly dismissed by Einstein, is the only truly physical definition. The only time that has a physical meaning, in fact, is proper time.

In light of this, a real space, endowed with a physical meaning, is the space of influence at hand which would follow directly from the discarded option. We define *Space of influence at hand* of an entity as the space actually present at a given time τ marked by its wristwatch, with which can physically interact and which corresponds to its own Minkowski cone of light. From the definition immediately follows the need to distinguish between a passive space and an active space, that is, between a space of receiving and a space of giving, which correspond respectively to the cone of light of the past and that of the future. In summary, we are proposing a paradigm shift:

$$\text{«synchronous space»} \rightarrow \text{«space of influence (give or receive) at hand»}$$

This change involves the modelling of space on light: the relationship between space and time thus becomes intrinsic to geometry and no longer needs to be guaranteed by a mathematical apparatus.

Once this is done, we are immediately faced with seemingly insurmountable problems, as they inexorably clash with our most deep-rooted preconceptions, namely:

1. space and time are two different dimensions and, at least in principle, independent of each other (they are correlated only a posteriori, i.e. by the metric, in Minkowski spacetime)

2. light has a speed that is not infinite. This second preconception is only a particularization of the first, because if space and time are different dimensions, then everything must have a speed, and therefore also light. Now, how can we doubt the finiteness of the speed of light if the time taken by a signal in a round trip is different from zero?

Conversely, in this new scenario of space at hand, light, as it is now that defines synchronous space, is considered instantaneous. More precisely, the new paradigm, since it is focused on interaction, which is why it considers not an abstract space but only the real space at hand of an individual, the one forged by the paths of light, involves the loss of the substantial difference between the spatial dimension and the temporal dimension. The spatial dimension and the temporal dimension are only different connotations of the one-dimensional path of light between an emitter and a receiver. The geometry that has the path of light as its element is therefore one-dimensional, linear. Instant light, therefore, does not mean infinite speed, but without speed. For each of the two individuals in relation: time and space, the whole and its parts, the moment of identity and the moment of difference.

But there is something even more radical. The time interval that opens between the round trip of the signal, for each of the two individuals involved in a reflexive interaction (where reception is immediately followed by reflected sending), since it can no longer represent the continuous time taken by light to travel from one end to the other, must represent the quantum of the interaction, the period of power between one instant in act and another. And this path of light, which interconnects pairs of events in action separated by power, is reflected both on the spatial and temporal dimensions. If the path is not a continuous journey in action, but a space in potential between two events in action, then the absolute is not the path (which is only in potential) but the pair of events in action.

The proposed paradigm shift therefore entails this further change:

$$\text{«constancy of the speed of light»} \rightarrow \text{«invariance of the length of the paths of light between two events»}$$

That is, the linear space of interaction is zero curl and conservative, since simply connected.

THE IRPL SPACE

Before delving into the presentation of the geometry of nature, which is unique to all physical phenomena, it is essential to emphasize its conformity with general relativity, whose foundation it both represents and reveals. Indeed, the manifold of general relativity is not primitive but rather emerges from the amalgamation of the multiplicity of elementary relations. Likewise, the very framework of the geometry of nature is not itself elementary but statistically emerges, as a system of knowledge representation, from an underlying orientation within the multiplicity of free relations, hidden from the observer, at the foundation of reality. In other words, light is everything that appears in that submerged and hidden world of which we are a part.

If, on the one hand, the geometry of the manifold represents the image that emerges from the multiplicity of interactions, i.e., the image provided by classical objects, the geometry of nature, on the other, embodies the interaction. One focuses on the image on the screen, the other on the interaction that lights up each pixel. If, on the one hand, man knows starting from the image, nature, on the other, from its geometry. So, we need both geometries to know. Since the method requires that *"we must advance from what is more obscure by nature, but clearer to us, towards what is more clear and more knowable by nature"*.

The geometry of the absolute

The geometry of modern physics has its origins in Euclidean geometry, from which it evolved into the Minkowski spacetime of special relativity (SR) and the Riemannian differential manifold of general relativity (GTR). The geometry of nature focuses on interaction, of which the space is only an attribute: space is the space of interaction, it does not exist without it. The linear path of interaction, which arises from the Radii (the masses or their electric mirrors), then unfolds by reflecting itself in its moments, among which there is, in addition to space and time, power. *"Power"* and *"Mirroring"* are therefore two fundamental and peculiar aspects of the geometry of nature. Indeed, there can be no quantum without the power that is its content, and mirroring is the nature (the behaviour and form) of power.

While Euclidean geometry and its derivations imagine the unfolding of nature in a manifold all in action, without interruption, which is why the metric is quadratic, the geometry of nature arises from the historical reconstruction, starting from the image received in a here and now. The geometry of nature is therefore a system of knowledge representation of the information present in a timeless image. From this image it is possible to extract the unfolding of nature along a linear path in a space of power, where the same path is made up of points (in act) interspersed with periods of the power. Which is why the metric is linear and the nature quantum. It is the very same path that, in the representation of knowledge, necessitates both difference and identity, reflecting itself in (proposing itself as) space and time, respectively.

If the substance of geometry of nature is the matter-energy (potency-energy), its subject is the give-take relationship: first of all, the hosting relation between the whole and its parts, and therefore, the peer-to-peer relation between the parts with each other. These two relationship always coexist. The main host is the universe, made first of all of undifferentiated matter, then, inside it, the elementary particles that make up the baryonic matter. This relationship is embodied in the geometry of nature whose figure is:

- the mirroring in the period of potency of the relation: everything mirrors everything;
- the exchange of energy in the instant of act, which is the reflective thread that forms the path of light reflected on the axes of space and time.

The give-take relationship requires that the two subjects, placed in front of each other, correspond to each other like two mirrors (see fig.2). That is, the nature of the give-receive relation is the mirroring. The mirroring of the individual therefore pre-exists and is the presupposition of the act, in which the collapse of the wave function of the two individuals involved takes place and in which the relationship manifests itself, and which is the basis of the reflection that emerges from the multiplicity of acts. Reflection would not guarantee information if it were not for the emergence of regularities inherent in mirroring. Between two mirrors, in fact, a recursive mirroring is established which give rise to a geometric progression where the scale factor is the quantum and the common ratio is the energy $dt/d\tau$.

We define "image", a figure carrying some information; "physical object", a multitude of interconnected elementary particles which, as such, has its own power given by the superposition of all the component wave functions. When the power of the object as a whole is dense enough to form a definite image, which manifests itself in its reflected light, then we say that the object is "classical" and that with it knowledge can begin. The existence of elementary particles, i.e. of baryonic matter, without which the universe would be composed only of an undifferentiated matter, is therefore "the conditio sine qua non" of the formation of the structure and of the emergence of its image and therefore of knowledge, and therefore of physics.

Mirroring (that is, correspondence), therefore, is an essential, basic and crucial property of nature and, as such, of the geometry of nature. It is the foundation of knowledge. Historical reconstruction, starting from the present instant, is possible since mirroring is recursive and gives rise to a geometric progression. The IRPL space (Instant Reconstruction of the Path of Light) is only and not other than the reconstruction, starting from the mirror image in the present instant, of the path of the light that takes place between two mirroring individuals which reflect each other recursively.

Because the act is instantaneous, in the geometry of nature, the path of light is instantaneous. It should be noted that instantaneity is not to be confused with infinite speed. Light, in fact, does not have a speed at all but is a path. This path, although made up of stages, that is, of acts of sending-receiving, never stops, but seamlessly composes the fabric of the interactions of the universe that unite the whole with the parts and, with this, the parts with each other.

The historical reconstruction requires the recognition of the identity, represented by a timeline, and of the path of light in its space at hand, given by an orthogonal axis. Since the planes of the act (space-time) of the two individuals are not coplanar, nor are they coplanar with that of the host, the space of interaction is three-dimensional:

- the time axis of identities (baryonic matter), coincident with the axis of the local Minkowski cone;
- the plane of space at hand, coinciding with the surface of the local Minkowski cone, which in turn is divided into:
  - the axis of radiant energy, given by the path of light to and from the other
  - the power axis (of the remaining non-actualized possibilities)

Motion is an inclination of the axis of time on the radiating axis of the other, in the case of receding/approaching, on the axis of the power of the other, in the case of orbital motion.

Each space is within the spherical three-dimensional space of the host universe, which in turn is locally constituted, at each point, by a temporal axis, along the radius of the sphere, and a pair of spatial axes on the surface: one radiant, the ”line of the present in action” of the universe, and the other of power, which divides the hemisphere of receiving from the hemisphere of giving. There is therefore not an absolute space, but the space of the universe, which plays a predominant and decisive role over the rest of matter.

Thanks to the universe, the division between the temporal and spatial nature of the axes is absolute. In particular, an individual that has the time (or baryonic) axis stationary (coincident) with respect to the time axis of the universe, is stationary with respect to the remaining baryonic matter of the universe; similarly, if rotating (spinning) with respect to the time axis of the universe, it is rotating with respect to the remaining baryonic matter of the universe. In particular, the three possible arrangements of the time axis of an elementary particle on the three axes of the space of the universe determine its generation, i.e. its mass and other properties.

If each of the three axes has its own specific character and the arrangement of the space of each interaction with respect to the space of the universe is absolute and determines its characteristics, from the average of the enormous number of three-dimensional spaces of the interactions that form a classical object, emerges the formless manifold of modern physics. In it, the radiant axis is broken down into the three spatial components of the manifold, indistinguishable from each other; the axis of time is added separately, as the fourth dimension of time, while power, the substance of mass, has no place in it, as does its plane of potency, which remains outside the representation.

In Euclidean geometry, the observer is external to the representation, he does not come into play. In the geometry of nature, the observer is the receiver directly involved in the interaction with the sender. The substantial difference between IRPL and Euclidean geometry and its derivations is that while the former represents the absolute space of interaction, the latter represents the space of multiplicity, which emerges as a mediety. In other words, the representation of facts in IRPL geometry is not based on a system of Cartesian co-ordinates existing a priori, independently of individual interactions, physically realized by means of a grid of synchronized and comoving rulers and clocks, through which and with respect to which data are collected progressively over time by means of a meticulous series of measurements and annotations. The geometry of nature, in fact, describes the space of the single interaction. In it, space is only an attribute of interaction. All that is needed is to arrange, on each of the two interacting entities, a wristwatch, suitably synchronized with that of the other, and a mirror that faithfully reflects the image of the watches that are recursively reflected in it. From the single image in act it is now possible to extract, in a single shot, all the data all together, and on the basis of these it is possible to reconstruct the historical space of the interaction. In other words, from a methodological and epistemological point of view, while IRPL physics rests entirely on the information present in the image received in the interaction in which it is directly involved, physics based on Euclidean geometry, which has as its object the observation of phenomena from the outside, must be based on a series of measurements that follow one another over time in an abstract space.

In linear geometry, each receive-"reflected send" event corresponds to two spatial axes (receive-send) and therefore two temporal axes (receive-send), rotated to each other by a γ angle, one of which is real, it is recognized because it is the one in line with the evolution of the being, the other mirror: parallel and opposed to the real axis of the other according to the dualism of giving-receiving. Since physics is based on the received image and therefore on the act of receiving, it follows that, in the reference frames of the geometry of nature, if the axis of time is always the real one (be it of giving or receiving), the spatial axis must always be that of reception (that is, not necessarily the one orthogonal to the axis of time).

Despite the profound differences, a parallel between the two representations is possible (see fig. 4). For a signal from A to B, and from here back to A’, in the phase of approaching, or, vice-versa, from A’ to B, and from here back to A, in the phase of distancing, we have:

Minkowski co-ordinate	distancing (A receiving)	approaching (A' receiving)	(1a)
$t_B = (\tau_{A'} + \tau_A)/2$	$t^\diamond_B = \tau_A$	$t^\diamond_B = \tau_{A'}$	(1b)
$\tau_B = \tau_B$	$\tau^\diamond_B = \tau_B$	$\tau^\diamond_B = \tau_B$	(1c)
$r_B = c(\tau_{A'} - \tau_A)/2$	$r^\diamond_B = \tau^\diamond_B - t^\diamond_B$	$r^\diamond_B = \tau^\diamond_B - t^\diamond_B$	(1d)
$\cosh\zeta = t_B/\tau_B = r_B/\sigma_B$	$t^\diamond_B/\tau^\diamond_B = 1/\cos^\diamond\gamma_e$	$t^\diamond_B/\tau^\diamond_B = \cos^\diamond\gamma_e$	(1e)
	$r^\diamond_B/\sigma^\diamond_B = 1/\cos^\diamond\gamma_i$	$r^\diamond_B/\sigma^\diamond_B = \cos^\diamond\gamma_i$	(1f)

where

$$\cos^\diamond\gamma_e = -\cos^\diamond\gamma_i = K \tag{1g}$$

since, following the path of light, the cosine is positive if both sides enter or exit the node, negative vice versa.

From a geometric standpoint, the space of the historical reconstruction of nature is therefore linear,

$$\cos^\diamond\gamma_e + \sin^\diamond\gamma_e = \cos^\diamond\gamma_i + \sin^\diamond\gamma_i = 1 \quad or \quad \sin^\diamond\gamma_e = 1 - K \quad and \quad \sin^\diamond\gamma_i = 1 + K \tag{1h}$$

being composed of the same path of energy that is reflected both along each of the two real τ time axes and along the common polygonal chain of radiant energy $r^\diamond$. More precisely, the space, given by the paths of light, is conservative or, in other words, the rotor is zero (it is an irrotational (zero curl) or conservative vector field).

We therefore have two geometries and two metrics:

$d\tau^2 = dt^2 - dr^2$	$d\tau^\diamond = dt^\diamond + dr^\diamond$	$d\tau^\diamond = dt^\diamond + dr^\diamond$	(1i)
$1 = cosh^2\zeta - \sinh^2\zeta$	$1 = \cos^\diamond\gamma_e + \sin^\diamond\gamma_e$	$1 = 1/\cos^\diamond\gamma_e - \tan^\diamond\gamma_e$	(1j)
$\left(mc^2\right)^2 = \mathbb{E}^2 - p^2$	$mc^2 = \mathbb{E}^\diamond + p^\diamond$	$mc^2 = \mathbb{E}^\diamond + p^\diamond$	(1k)

one synthetic, emerging on average, the other primitive, conforming to the natural interaction, brick of the whole. It is essential to always keep in mind the essential difference between the two types of co-ordinates: one is relative, the other is always absolute, the same applies to the quantities that derive from it.

Elements

The double-faced sending-receiving act is the building block of the path of light. The IRPL space represents the historical reconstruction of the path of light which is enfolded and unfold from the image (the snapshot) of the world carried by the ray of light received from a sender (both from the other along the space-line and from itself along its own timeline). Each double-faced sending-receiving act unites two dual individuals, and is represented by a pair of parallel and opposite frames facing each other, composed of a common spatial axis, corresponding to the shared horizontal path of the bosons, and of the temporal axes perpendicular to it at the points where the two individuals are located (see fig. 2). For each individual, in each point of space-time intersection (act), two shared horizontal path branch off, the incoming one of the moment of receiving and the outgoing one of the immediately following moment of giving, and therefore two temporal axes or rather two frames rotated between them by an angle γ (fig. 6). For each individual, the sequence of these points of intersection (points in act) composes its time axis, corresponding to the temporal path of its own baryonic matter. As light proceeds from the sender to the receiver, an individual's time axis coincides with the temporal axis of its sending frame, in the moving away; with the temporal axis of its receiving frame, in the approach.

As a result, the historical reconstruction of the series of interactions in the knowledge representation schema reveals the time axes of the individuals in the Linear pseudo-plane of Act and the plane of potency which emerges perpendicular to this. Every individual advances along its timeline by rotating in its plan of potency as a screw whose pitch establishes the quantum of time and therefore the quantum of space. The spin is preparatory to the collapse of the power wave which occurs cyclically in conjunction with the alignment of the giving-receiving spatial axis of the two subjects involved with the radiant axis of universe. The resulting path is made up of segments, whose quantum is the wavelength of light, which come into action only at the crossing points, in the pseudo-plane of the act, where the emission or absorption of the bosons takes place. In this scenario, the three forms of matter-energy, respectively baryonic matter, radiation and potential or momentum, correspond to the three axes of the IRPL spacetime, respectively time, radiation and potency.

The reciprocal rotation angle γ, as well as the mutual distance, is therefore central to the knowledge of the relationship. This knowledge may be gleaned not from a single act, because the two frames are parallelly opposed in it, but solely from the image in which the recursive mirroring of wristwatches is historicized. Knowledge may thus emerge only through interactions involving entities that are complex enough to carry an image through the spatial arrangement of their components.

Also based on the fact that any gravitational boson has not yet been found, we can infer that while electricity is the relationship in act, fulfilled through the exchange of bosons, gravitation is the relationship only in potency, without real exchange of bosons. According to this view, gravitation is the necessary power ground, from which matter in act is born, upon which the electric relationship can arise. That is, the blackboard is gravitational, the pencil electric. In any case, regardless of this, in the IRPL representation, despite their peculiar differences, inertial, gravitational and electric relationships follow the same universal geometric schema and can be treated in a unified way.

In the IRPL representation, the metric does not deal with abstract space and time intervals, but always and only with segments of the real path of light. Light is primitive, it is the pencil that writes on the blackboard. In other words, the space-time dualism is fictitious since light does not have a speed but, with its path, it traces the "time" and the "space" and represents the only meter (wavelength) and clock (period) on which to base the metric ($\Delta\vec{t}^{\diamond} = \sum \vec{r}_i^{\diamond}$). The plane of the Act, therefore, is a pseudo two-dimensional ($\vec{r}^{\diamond}$, $\vec{t}^{\diamond}$) linear vector space defined on the field of rational numbers (the wavelength is the quantum), where the only difference with a Euclidean vector space ($\vec{r}$, $\vec{t}$), truly two-dimensional, is that the length of the sum of the vectors is given by the algebraic sum of their lengths. In other words, the resultant of two vectors in the plane of the Act can be found using the parallelogram or triangle method, just like for Euclidean vectors but, unlike these, its length is the algebraic sum of their lengths:

$$\sin^{\diamond}\gamma + \cos^{\diamond}\gamma = 1 \qquad \text{(zero curl property)} \tag{2a}$$

$$\frac{\mathrm{d}\sin^{\diamond}\gamma}{\mathrm{d}\gamma} = -\frac{\mathrm{d}\cos^{\diamond}\gamma}{\mathrm{d}\gamma} = \cos^{\diamond}\gamma \qquad\qquad \frac{\mathrm{d}\tan^{\diamond}\gamma}{\mathrm{d}\gamma} = \frac{\mathrm{d}\,(1/\cos^{\diamond}\gamma)}{\mathrm{d}\gamma} = \frac{1}{\cos^{\diamond}\gamma} \tag{2b}$$

That is, defining in a natural way:

$$V^{\diamond} = \frac{R}{r^{\diamond}} \qquad\qquad \frac{r^{\diamond}}{\tau^{\diamond}} = \frac{p^{\diamond}}{mc} \qquad\qquad \frac{\mathbb{E}^{\diamond}}{mc^2} = \frac{t^{\diamond}}{\tau^{\diamond}} \tag{2c}$$

we have the metric:

$$\tau^{\diamond} = t^{\diamond} + r^{\diamond} \quad \text{or} \quad 1 = \frac{\mathbb{E}^{\diamond}}{mc^2} + \frac{p^{\diamond}}{mc} \tag{2d}$$

and, in free motion, the following two cases:

$$\frac{\mathbb{E}^{\diamond}}{mc^2} = \cos^{\diamond}\gamma \qquad\qquad \frac{p^{\diamond}}{mc} = \sin^{\diamond}\gamma \qquad \text{approaching} \tag{2e}$$

$$\frac{\mathbb{E}^{\diamond}}{mc^2} = \frac{1}{\cos^{\diamond}\gamma} \qquad\qquad \frac{p^{\diamond}}{mc} = -\tan^{\diamond}\gamma \qquad \text{distancing} \tag{2f}$$

The above equation (2a) derives clearly from a general property of the light path, that is, that the circulation along a closed path is zero (zero curl), where the sign is positive along the direction of the light, and its demonstration rests on evidence. The remaining equations (2) derive from this or are simple definitions.

Given that every relationship (between A and B) develops along the line of the present of the universe (the cosmological proper distance of universe along the radiant axis of the universe) in which it is enclosed, for the geometrization of nature it is necessary and sufficient to define:

1. a Radius R, such that $\sin^{\diamond}\gamma = R/r^{\diamond} = V^{\diamond}$ which, like the other co-ordinates of IRPL spacetime ($\tau^{\diamond}, t^{\diamond}, \sigma^{\diamond}, r^{\diamond}$), is absolute. The Radius appears as diffused in space, as it is composed of parts, and as reflected in time, as it is a whole;

2. and the following fundamental principles :

   Principle 1 ***Mirroring law****: each Radius is mirrored in the other as*

$$R_b^{\circ} = \frac{1}{R_{\bullet a}} \tag{3}$$

where it is to be adopted the (Stoney-like) system dimensions $\ell_{irpl} = 2\sqrt{\alpha}\,\ell_P = 2\ell_S$ and $m_{irpl} = \sqrt{\alpha}\,m_P = m_S$ (where ℓ_P, m_P and ℓ_S, m_S are the Planck and Stoney units respectively and where the factor of 2 in length measurement is the only difference from Stoney units since the IRPL units provide (see fig. 5) the Radius doubled R_2 and the round trip distance s_2). Therefore, each individual exhibits its own gravitational radius and an electric radius borrowed from the other.

Consequentially, the interaction has a double nature based on the nature of its radius which differs in:

(a) a Gravitational Radius, which breaks down into its components along each of the axes of the interaction space:

$$R_p = \frac{p^\diamond}{mc} r^\diamond \qquad \text{momentum, of which the CDM } R_c = V^\diamond r^\diamond \text{ is a particular case, on the time axis} \tag{4a}$$

$$R_\bullet = G/c^2\, M \qquad \text{baryonic rest mass, on the potency axis} \tag{4b}$$

$$R_r = G/c^4 \frac{hc}{\lambda} \qquad \text{radiation energy, on the radiant axis} \tag{4c}$$

(b) an Electrical Radius, mirror of the baryonic gravitational radius of the other (see 17):

$$R_b^\circ = \pm R_{\bullet a}^{-1} \qquad \text{on the radiant axis} \tag{4d}$$

the sign depends on whether it is equidirectional or counterdirectional with respect to the absolute radiant axis of universe.

At last, being the electric radiating axis is out of phase with respect to the gravitational axis of the power by $\pi/2$, for the agreement with the experience it is necessary and sufficient that it is:

$$\cos^\diamond\gamma = 1 - \sin\gamma \qquad \text{for Gravitational fields and inertial systems } (\sin^\diamond\gamma = \sin\gamma) \tag{5a}$$

$$\sin^\diamond\gamma = 1 - \cos\gamma \qquad \text{for Electric fields } (\cos^\diamond\gamma = \cos\gamma) \tag{5b}$$

Principle 2 ***Universality of interaction geometry or Principle of equivalence between inertial - not inertial systems***
In free motion (dynamic metric), every relationship between a sender-receiver pair, at any moment, respects the universal rule "Momentum" = "Potential", that is:

$$V^\diamond \overset{def}{=} \left(\frac{\frac{2(R_a+R_b\mathbb{E}^\diamond)}{(1+\mathbb{E}^\diamond)}}{r_a^\diamond} = \frac{\frac{2(R_b+R_a\mathbb{E}^\diamond)}{(1+\mathbb{E}^\diamond)}}{r_b^\diamond} = \frac{R_a + R_b}{r^\diamond} \right) \mathbf{=} \left(\frac{r_a^\diamond}{\tau_a^\diamond} = \frac{r_b^\diamond}{\tau_b^\diamond} = \frac{r^\diamond}{\tau^\diamond} \right) \overset{def}{=} p^\diamond/(mc) \tag{6}$$

where R is the global Radius, sum of the above mentioned. Both potential and momentum depend on the Lorenz$^\diamond$'s rotation angle γ between the emitter-receiver time axis pair, which are measured starting from the intersection point $t^\diamond = \tau^\diamond = 0$. The (6) is the linearized version of the total energy of a system:

$$\left(1 - V^\diamond\right) + \frac{p^\diamond}{mc} = 1 \qquad \textit{that is} \qquad \mathbb{E}^\diamond + c\,p^\diamond = mc^2 \quad \textit{or} \quad \cos^\diamond\gamma + \sin^\diamond\gamma = 1 \tag{7}$$

As evident, the interaction scheme is presented in two closely related dual versions, depending on the point of view of the involved individual, each of which develops from a geometric progression that has the energy $\mathbb{E}^\diamond$ as common ratio and respectively $R_{Ab} = (R_a + R_b\mathbb{E}^\diamond)$ and $R_{Ba} = (R_b + R_a\mathbb{E}^\diamond)$ as scale factor. Henceforth, we use the average of the two versions ($r^\diamond = (r_a^\diamond + r_b^\diamond)/2, t^\diamond = (t_a^\diamond + t_b^\diamond)/2, \tau^\diamond = \cdots, R_{tot} = (R_{Ab} + R_{Ba})/(1 + \mathbb{E}^\diamond)$) which represents the point of view of a far away observer.

3. the Left-Hand Side and Right-Hand Side of the (6) correspond, respectively, to the arrangement of two spaces, both three-dimensional, strictly correlated to each other (see fig. 6 and 7 and its evolutions 9). That is:

   (a) an internal, primitive, real one (see fig. 5), which is the space of potential ($R_a, r_a^\diamond, R_b, r_b^\diamond, \gamma, \theta$).
   Indeed, the interaction takes place in the original Internal space of Potential, in which the Radius and the path of the outgoing light constitute the two orthogonal axes of the potential frame of each individual. In this space, the light path connects the head of the sending Radius with the tail of the opposite receiving Radius and then crosses it (see fig. 5). In other words, in the internal space of the potential the event of sending does not immediately follow the event of reception seamless along the spatial axes but, between the two events, the light crosses the Radius.

(b) an external, derived, phenomenal one (see fig. 4), which is the familiar momentum spacetime ($t_a^\diamond, r_a^\diamond, t_b^\diamond, r_b^\diamond, \gamma, \theta$). The act of sending-receiving A'B is immediately followed by the reflex act of return BA and so on recursively forming a seamless concatenation. Along the path of light, each act consists of a pair of events send-receive occurring simultaneously, which bound the intermediate spatial segment $r^\diamond$ forming a geometric progression. Along time line of each individual, an instant in act follows a period of potency and so on forming a geometric progression. More precisely, the reception event is concatenated to the sending event as they co-belong to their respective dual spaces at hand in the common instant B; vice-versa, between the sending event A and the reflected receiving event A', the period AA' of the potency of relation opens;

From a geometric point of view, the IRPL scheme, which derives entirely from the Principle of equivalence 2, consists of the alternation of an external frame or Momentum and an internal one or Potential, where:

1. each frame, both momentum and potential, is made of an alternating succession of two dual zero curl triangle types (see fig. 4): one internal $\mathbb{G}\left(\triangle_i^\diamond\right)$, and the other external $\mathbb{G}\left(\triangle_e^\diamond\right)$ such that:

$$\mathbb{G}\left(\triangle_e^\diamond\right) = \left(\cos\gamma_e^\diamond + \sin\gamma_e^\diamond\right) = 1 \tag{8a}$$

$$\mathbb{G}\left(\triangle_i^\diamond\right) = \left(\cos\gamma_i^\diamond + \sin\gamma_i^\diamond\right) = 1 \tag{8b}$$

   where $\cos\gamma_e^\diamond = \cos\gamma^\diamond = -\cos\gamma_i^\diamond$. In particular, an internal triangle $\mathbb{G}\left(\triangle_i^\diamond\right)$ represents the potential frame while an external triangle $\mathbb{G}\left(\triangle_e^\diamond\right)$ represents the homologue dual momentum frame.

2. the length of each segment is a multiple of the relationship wavelength $f_a(R_a) = f_b(R_b) = f_{ab}(R_a + R_b)$,

3. each segment develops from a geometric progression that has the energy $\mathbb{E}^\diamond$ as common ratio (both versions $\mathbb{E}_e^\diamond$ and $\mathbb{E}_i^\diamond$ alternating) and a segment of a more primitive nature as scale factor, and so backwards up to the primitive elements which are the Radii of the involved individuals (see fig. 6). Indeed, $\tau^\diamond$ comes from a geometric progression (see 18) that has $r^\diamond$ as scale factor which, in turn, comes from a geometric progression that has $s^\diamond = b(\gamma)r^\diamond$ as scale factor which, at last, comes from a geometric progression that has $b(\gamma)R$ as scale factor.

We thus have two equivalent metrics: a linear pair and a quadratic one.

Note that the distance between A and B (also the round trip distance) is different when observed by A, if observed by B, or when observed by a third party. Not only because of a problem of synchronism between observations, since sending and receiving, for an elementary individual, take place at different times, but above all because of a problem of asymmetry of distances. In fact, if we imagine an entity as a sphere with a radius equal to its Radius, the light, on the round trip, goes from the surface of the observer (outside the Radius) to the center of the observed (inside the Radius), and back. Indeed $s_{2Ab} - s_{2Ba} = R_a - R_b$. The formula $r^\diamond = \tau_A - \tau_B$ is therefore ambiguous because it does not specify who is the observer, i.e. the receiver, and who is the observed, i.e. the giver. For this reason, by notational convention, which aligns with that of the literature, we always indicate with $t^\diamond$ the $\tau_{receiving}$ (the observer) and with τ the $\tau_{donating}$ (the observed).

About the Radii, the sign of the axes of the recipient and that of the donor are inverted with respect to each other; as a result, the potential is negative in the relationship between a receiver and a giver, since $R_{giving}R_{receiving} < 0$, positive vice-versa. Since τ, and therefore momentum, is positive in distancing and negative vice versa, it always concords with the potential which is negative in attraction and positive vice versa.

Regarding R_p, it represents, at any given moment, the Radius which, added to the Radius of the force field present, returns the global Radius conforming to the geometric configuration in place. Since any interaction always takes place within a Host Radius, momentum is never null. In other words, the momentum $p^\diamond = r^\diamond/\tau$ between two bodies separated by an arbitrary distance $r^\diamond$, while decreasing as τ increases, stops at a minimum when τ reaches its maximum limit which is equal to the Radius of the universal R_h, host of the interaction. That is, the radius of the universe R_ω or of an elementary particle. It therefore acquires a fundamental meaning in the interactions involving matter within a Host Radius. That is, in the relationship between the whole and its parts. In this case we have $\tau = R_h$ and R_p takes the name of Cold Dark Matter $R_c = r^{\diamond 2}/R_h$. In particular, within the universe ($\tau_{max} = R_\omega$), $A^\diamond = 1/R_\omega$ is the minimum local acceleration, $p^\diamond = r^\diamond/R_\omega$ is the minimum local momentum, i.e. the Hubble recession velocity, and $R_c = r^{\diamond 2}/R_\omega$ is the **C**old**D**ark**M**atter (CDM). The radius R_c of CDM is therefore always positive, and since the baryonic mass of an elementary particle $R_{\bullet_b}$ arises within the universe as a condensate of CDM (the interior of baryonic particles is in turn made of CDM, precisely the CDM inside a sphere of radius R_b°), it is always positive. If the CDM is the presupposition of the baryonic matter, the individual made of baryonic matter is the subject of the relation which is the object of physics, its "conditio sine qua non", the subject of gravitational radius, of the momentum radius, of the electric radius and the cause of the radiation.

The electric Radius, being the mirror of the other (see appendix §), is such that: $U_a^\diamond = m_a V_b^\diamond = R_{\bullet a} R_b^\circ / r^\diamond = 1/r^\diamond$. Since inside the host R_b°, that is, in weak and strong interactions, is $R = r^{\diamond 2}/R_b^\circ \leq R_b^\circ$, we have that inside it is $V^\diamond = r^\diamond / R_b^\circ$ (weak and strong interactions), outside it is $V^\diamond = R_b^\circ / r^\diamond$ (Coulomb interactions). At last, the radius of radiant energy, generally negligible, plays a key role only in cosmology.

Linear, Minkowski, and Schwarzschild coordinate mapping

In the space-time manifold of modern physics, the element is the point, i.e. the event. This can be measured either by a frame attached to the body that generated it, or by any frame external to it. In the first case proper coordinates, i.e. wristwatch time or proper time τ and proper distance σ, are employed, which are denoted by Greek letters. All observers agree on the value of the wristwatch time or of the proper distance between two events. In the second case, however, frame coordinates r and t denoted by Latin letters are used, and these are different from frame to frame.

In the linear representation, vice versa, there are no external observers, nor does proper frame and coordinate frame dualism exist but there is only the equal relationship between two individuals within the relationship with their universal host, since observer and observed are only two roles of the relationship. In the linear representation, in fact, the only measuring instruments are wristwatches synchronized at the point of contact. The wavelength/period is the quantum and the number of wavelengths/periods measures space and time in a homogeneous way. The sole measurement is therefore that of the proper time, which is measured by a wristwatch attached to the transmitting body and relayed along the path of the light to the receiver, who compares it to the time of his own wristwatch to determine the distance $r^\diamond = c\left(t^\diamond - \tau^\diamond\right) \equiv c\left(\tau_{receiving} - \tau_{sending}\right)$. Thus, while the coordinate system of modern physics is dependent on the system of representation used, that of the geometry of nature is absolute.

Although the finite coordinates of both theories of relativity are affected by the conventions adopted, i.e. the definitions of synchronism in a geometry where space and time are distinct dimensions, the Schwarzschild differential coordinates gets rid of it, conforming to those of the geometry of nature, albeit, in the absence of matter, the former are trivially reduced to the latter. This difference in behavior is due to the different nature of the relative radii that mean that while the relative velocity of SR, and therefore the Lorentz rotation that represents it, continues to be affected by the conventions adopted for co-coordinates, the local deformation of the space-time of the GTR, being caused solely by the mass that is real and absolute, must be equally real and absolute.

In the linear representation, vice versa, there are no space-time deformation, and not even a space-time in itself. The linear representation, which recognizes no other absolute outside of the relationship, leads the local deformation of the space-time predicted by the GTR to a local rotation of the two frames, according to the universal schema of fig.(6), thus allowing a true and complete unification of the two representations.

In it, the coordinate-proper dualism is replaced by the send-receive dualism.

Of course

$$\tau = \tau^\diamond \tag{9a}$$

About frame coordinates, let A' be the point where a signal is sent by the observer, B the arrival point on the observed body, and A the return point. In Minkowski spacetime the segment $A'B$ and the segment BA are of equal length and symmetrical with respect to the spatial axis of the observer and $r = (A'B + BA)/2 = A'B(1+1)/2$ at the instant $t = (\tau_{A'} + \tau_A)/2$. In linear coordinates, on the other hand, the same measurement sees the segment $A'B$ perpendicular to the temporal axis of the observer and the segment BA ($A'B \neq BA$) perpendicular to the temporal axis of the observed rotated by an angle γ with respect to the former. Expressed in formulas:

$$r = \frac{A'B + BA}{2} = \frac{s_2^\diamond}{2} = \frac{1}{2}\frac{V_i^\diamond}{V_i^\diamond - 1} r^\diamond = \frac{r^\diamond + r^\diamond \cos^\diamond \gamma}{2} = r^\diamond\left(1 - \frac{\sin^\diamond \gamma}{2}\right) = r^\diamond\left(1 + \frac{1}{2}\frac{p^\diamond}{E^\diamond}\right) = r^\diamond - \frac{1}{2}\frac{R}{E^\diamond} \qquad \text{distancing} \tag{9b}$$

$$r = \frac{AB + BA'}{2} = \frac{s_2^\diamond}{2} = \frac{1}{2}\frac{V_i^\diamond}{V_i^\diamond - 1} r^\diamond = \frac{r^\diamond + r^\diamond / \cos^\diamond \gamma}{2} = r^\diamond\left(1 + \frac{\tan^\diamond \gamma}{2}\right) = r^\diamond\left(1 + \frac{1}{2}\frac{p^\diamond}{E^\diamond}\right) = r^\diamond + \frac{1}{2}\frac{R}{E^\diamond} \qquad \text{approaching} \tag{9c}$$

(9d)

and

$$t_b^\diamond = \tau_A^\diamond = \tau_B^\diamond / \cos^\diamond \gamma = t_B + r = \tau_B \cosh\zeta + \tau_B \sinh\zeta \tag{9e}$$

$$t_{b'}^\diamond = \tau_{A'}^\diamond = \tau_B^\diamond \cos^\diamond \gamma = t_B - r = \tau_B \cosh\zeta - \tau_B \sinh\zeta \tag{9f}$$

from which it follows that:

$$e^{-\zeta} \quad \equiv \quad \cos^\diamond \gamma \tag{9g}$$

In the (9b), the term

$$b_{\Gamma/2} = \left(1 + \frac{1}{2}\frac{p^\diamond}{E^\diamond}\right) \tag{9h}$$

is the conversion factor between linear and Minkowski coordinates. In other words, $2r = t^\diamond_{receiving} - t^\diamond_{donating} = \frac{V_i^\diamond}{E^\diamond} r^\diamond$ where:

$$\frac{V_i^\diamond}{E^\diamond} = 2b_{\Gamma/2} \tag{9i}$$

At last, from the (6) and the (9) we have:

$$r = b_{\Gamma/2} r^\diamond = r^\diamond \pm \frac{1}{2}\frac{R}{E^\diamond} \tag{9j}$$

$$t = t^\diamond \pm r \tag{9k}$$

Summing up, in free motion,:

$$\frac{\mathbb{E}_-{}^\diamond}{mc^2} = \cos^\diamond \gamma \qquad \frac{p_-^\diamond}{mc} = \sin^\diamond \gamma \qquad b_{-\Gamma/2} = \left(1 + \frac{1}{2}\frac{p_-^\diamond}{E_-^\diamond}\right) = \left(1 + \frac{1}{2}\tan^\diamond \gamma\right) \qquad \text{approaching} \tag{10a}$$

$$\frac{\mathbb{E}_+{}^\diamond}{mc^2} = \frac{1}{\cos^\diamond \gamma} \qquad \frac{p_+^\diamond}{mc} = -\tan^\diamond \gamma \qquad b_{+\Gamma/2} = \left(1 + \frac{1}{2}\frac{p_+^\diamond}{E_+^\diamond}\right) = \left(1 - \frac{1}{2}\sin^\diamond \gamma\right) \qquad \text{distancing} \tag{10b}$$

and of course

$$\frac{p}{mc} = \sinh \zeta = -b_{+\Gamma/2}\frac{p_+^\diamond}{mc} = b_{-\Gamma/2}\frac{p_-^\diamond}{mc} \tag{10c}$$

$$\frac{\mathbb{E}}{mc^2} = \cosh \zeta = \frac{\mathbb{E}_+{}^\diamond}{mc^2} + b_{+\Gamma/2}\frac{p_+^\diamond}{mc} = \frac{\mathbb{E}_-{}^\diamond}{mc^2} + b_{-\Gamma/2}\frac{p_-^\diamond}{mc} = \sqrt{1 + b^2_{\Gamma/2}\,(p^\diamond/mc)^2} = \mathbb{E}^\diamond \sqrt{\frac{1}{\mathbb{E}^{\diamond 2}} + b^2_{\Gamma/2}\frac{(p^\diamond/mc)^2}{\mathbb{E}^{\diamond 2}}} \tag{10d}$$

In the (10d, 10c) appear the energy and momentum conversion factors in free motion:

$$E = \frac{\mathbb{E}}{\mathbb{E}^\diamond} = mc^2 \sqrt{\frac{1}{\mathbb{E}^{\diamond 2}} + b^2_{\Gamma/2}\frac{(p^\diamond/mc)^2}{\mathbb{E}^{\diamond 2}}} \qquad P = \frac{p}{p^\diamond} = \pm b_{\Gamma/2} \tag{10e}$$

where

$$\tau_+ = \cos^{\diamond\, 2}\gamma\, \tau_- \quad \left(\frac{\tau_+}{t} = \frac{t}{\tau_-} = \cos^\diamond \gamma\right) \qquad \frac{E_+}{mc^2} = \cos^{\diamond\, 2}\gamma \frac{E_-}{mc^2} \qquad b_{+\Gamma/2} = -\cos\gamma\, b_{-\Gamma/2} \tag{10f}$$

that is:

$$R = -\tan^\diamond \gamma\, r_+^\diamond = \tan^{\diamond\, 2}\gamma\, \tau_+ \qquad \frac{E_+}{mc^2} = \frac{\mathbb{E}}{\mathbb{E}_+^\diamond} = \sqrt{\cos^{\diamond\, 2}\gamma + b^2_{+\Gamma/2}\sin^{\diamond\, 2}\gamma} \qquad \frac{p}{mc} = -b_{+\Gamma/2}\frac{p_+^\diamond}{mc} = -\frac{b_{+\Gamma/2}}{\cos^\diamond \gamma}\frac{dr_+^\diamond}{dt_+^\diamond} \tag{10g}$$

$$R = \sin^\diamond \gamma\, r_-^\diamond = \sin^{\diamond\, 2}\gamma\, \tau_- \qquad \frac{E_-}{mc^2} = \frac{\mathbb{E}}{\mathbb{E}_-^\diamond} = \sqrt{\frac{1}{\cos^{\diamond\, 2}\gamma} + b^2_{-\Gamma/2}\frac{\sin^{\diamond\, 2}\gamma}{\cos^{\diamond\, 2}\gamma}} \qquad \frac{p}{mc} = b_{-\Gamma/2}\frac{p_-^\diamond}{mc} = \frac{b_{+\Gamma/2}}{\cos^\diamond \gamma}\frac{dr_-^\diamond}{d\tau} \tag{10h}$$

Usually, the version $b_{+\Gamma/2}$ and E_+ will be used in both cases, that is, $b_{+\Gamma/2}$ and E_+ in the distancing, $b_{+\Gamma/2}/\cos\gamma$ and $E_+/\cos^2\gamma$ in the approaching.

The Lorentz transformation

In the instant there is no difference between an inertial system travelling at a constant speed and a system immersed in a force field, since the metric is determined, point by point, solely by the local rotation angle γ and this in turn depends solely on the nature of the Radii of the bodies involved.

Whether a spaceship is traveling at a constant speed in absence of mass, or is standing still near a massive body, in both cases it will be observed aboard the spaceship:

- Time dilation: Time slows down
- Length contraction: length contracts in the radial direction, i.e., along the direction of motion or toward or away from the mass

The geometry of nature, unlike relativity, traces both cases to the same rotation in the same linear space.

In the most general case, in addition to the γ angle, there is a nutation angle ϑ of the time-power plane around the axis of the nodes $r^\diamond$ (see fig. 7 and equations 21), which gives rise to the rotational motion. When $\vartheta = 0$, we have:

$$\begin{cases} \text{Minkowski s.t.} \\ \sigma = x\cos\zeta - ict\sin\zeta \\ ic\tau = x\sin\zeta + ict\cos\zeta \end{cases} \leftrightarrow \begin{cases} \text{Linear Momentum Plane -approaching-} \\ x^\diamond = \sigma^\diamond \cos^\diamond\gamma_i + t^\diamond \sin^\diamond\gamma_e \\ \tau^\diamond = \sigma^\diamond \sin^\diamond\gamma_i + t^\diamond \cos^\diamond\gamma_e \end{cases} \tag{11a}$$

$$\begin{cases} \sigma = \dfrac{x - vt}{\sqrt{1-v^2}} \\ \tau = \dfrac{t - vx}{\sqrt{1-v^2}} = \sqrt{1-v^2}\,t - v\sigma \end{cases} \leftrightarrow \begin{cases} \sigma^\diamond = \dfrac{x^\diamond - V_e^\diamond t^\diamond}{1 - V_i^\diamond} \\ \tau^\diamond = \dfrac{-t^\diamond + V_i^\diamond x^\diamond}{1 - V_i^\diamond} = (1 - V_e^\diamond)t^\diamond + V_i^\diamond\sigma^\diamond \end{cases} \tag{11b}$$

At last, since integrating $dx/dt = v$ we have:

$$x = vt + \delta s \qquad \leftrightarrow \qquad x^\diamond = V_e^\diamond t^\diamond + \delta s^\diamond; \tag{11c}$$

substituting the (11c) in the (11b) and substituting $V_i^\diamond/(V_i^\diamond - 1)$ with $2b_{\Gamma/2}$ we have at last:

$$\begin{cases} \sigma = \dfrac{x - vt}{\sqrt{1-v^2}} \qquad \tau = \dfrac{t}{\sqrt{1-v^2}} - \dfrac{v\,x}{\sqrt{1-v^2}} \\ \delta\sigma = \dfrac{\delta s}{\sqrt{1-v^2}} \qquad \tau = \sqrt{1-v^2}\,t - \dfrac{v\,\delta s}{\sqrt{1-v^2}} \end{cases} \leftrightarrow \begin{cases} \sigma^\diamond = \dfrac{x^\diamond - V_e^\diamond t^\diamond}{1 - V_i^\diamond} \qquad \tau^\diamond = \dfrac{t^\diamond}{1 - V_e^\diamond} - 2b_{\Gamma/2}x^\diamond \\ \delta\sigma^\diamond = -\dfrac{\delta s^\diamond}{1 - V_e^\diamond} \qquad \tau^\diamond = (1 - V_e^\diamond)t^\diamond - 2b_{\Gamma/2}\delta s^\diamond \end{cases} \tag{11d}$$

As is immediately evident, the first row of the (11d), which uses the coordinate $x(t)$, called "vector radius" or "position vector" (the line joining observer-observed), represents the position of the observed in its receding or approaching, it is therefore a dynamic point of view, suitable for the equations of motion. The second equation, on the other hand, which uses the local coordinate s ($s \ll x$), aims to measure the length of a spatial segment present and stationary in the frame of the observed, and is therefore the static point of view, suitable for the equations of the metric. In other words, although both the coordinates x and s are usually denoted by the same symbol r, because both denote intervals of the same spatial axis, while $dx/dt = v$ is a velocity, s is a constant segment and $ds/dt = 0$ cancels out.

In the linear geometry of nature, between the static and the dynamic point of view an exchange of places occurs between $\tau^\diamond$ and $t^\diamond$ with the same spatial axis.

To sum up, an exchange of places between $\tau^\diamond$ and $t^\diamond$ occurs whenever:

- occurs the transit between static and dynamic point of view;
- occurs the transit between approaching and distancing.

The meaning of this exchange is that while in the approach the observer observes the other, in the distance he observes himself reflected in the other.

Inertial system ($\mathrm{d}R_p \neq 0$; $\mathrm{d}\gamma = 0$)

This is the case when Radius is due to momentum, or CDM, on the time axis.

What differentiates an inertial system from a force field is the fact that in the former the radius R_p varies proportionally to the variation of the distance $r^\diamond$ and this implies that the angle γ remains constant. In other words, during motion, we have $\mathrm{d}p^\diamond = 0$ and $\mathrm{d}\gamma = 0$ and we have the metric:

$$\frac{1}{\tau^2}\left(\tau^2 = t^2 - r^2\right) \quad \equiv \quad 1 = \left(\frac{\mathbb{E}}{mc^2}\right)^2 - p^2 = \tag{12a}$$

$$= 1 = \cosh^2\zeta - \sinh^2\zeta = \tag{12b}$$

$$= 1 = \frac{\left(E_+/mc^2\right)^2}{\cos^{\diamond\,2}\gamma} - b^2_{+\Gamma/2}\frac{\sin^{\diamond\,2}\gamma}{\cos^{\diamond\,2}\gamma} \tag{12c}$$

$$c^2\mathrm{d}t_+^{\diamond\,2} = \left(E_+/mc^2\right)^2\frac{c^2\mathrm{d}t_+^{\diamond\,2}}{\cos^{\diamond\,2}\gamma} - b^2_{+\Gamma/2}\frac{\mathrm{d}r_+^{\diamond\,2}}{\cos^{\diamond\,2}\gamma} - \left[b^2_{+\Gamma/2}r_+^{\diamond\,2}\mathrm{d}\Omega_+^2\right] \qquad \text{distancing} \tag{12d}$$

$$c^2\mathrm{d}\tau^{\diamond 2} = \left(E_+/mc^2\right)^2\frac{c^2\mathrm{d}\tau^{\diamond 2}}{\cos^{\diamond\,2}\gamma} - b^2_{+\Gamma/2}\frac{\mathrm{d}r_-^{\diamond\,2}}{\cos^{\diamond\,2}\gamma} - \left[b^2_{-\Gamma/2}r_-^{\diamond\,2}\mathrm{d}\Omega^2\right] \qquad \text{approaching} \tag{12e}$$

Centrally symmetric Force Field ($\mathrm{d}R_\bullet = 0$**;** $\mathrm{d}\gamma \neq 0$))

This is the case when the Radius is due to the baryonic mass, or electric charge, on the spatial axis (the radiating one for the electric field, the power axis for the gravitational field).

From the point of view of an observer falling freely under the influence of that object, unlike what happens in an inertial system, it is the Radius that is constant and not the γ angle. In an attractive force field, from the (10h) we have that:

$$\frac{p}{mc} = b_{+\Gamma/2}\frac{p_+^\diamond}{mc} = \frac{b_{+\Gamma/2}}{\cos^\diamond\gamma}\frac{dr_+^\diamond}{dt_+^\diamond} = \left(\frac{1}{\cos^\diamond\gamma}\right)\frac{dr_+^\diamond}{dt_+^\diamond} + \frac{1}{2}\frac{\mathrm{d}R_\bullet}{\mathrm{d}t_+^\diamond} \tag{13a}$$

$$\frac{p}{mc} = b_{-\Gamma/2}\frac{p_-^\diamond}{mc} = \frac{b_{+\Gamma/2}}{\cos^\diamond\gamma}\frac{dr_-^\diamond}{d\tau} = \left(\frac{1}{\cos^\diamond\gamma}\right)\frac{dr_-^\diamond}{d\tau} + \frac{1}{2}\frac{\mathrm{d}R_\bullet}{\mathrm{d}\tau} \tag{13b}$$

That is, for $\mathrm{d}R_\bullet = 0$,

$$dr^\diamond = dr_{Schwarz.} \qquad dt^\diamond = dt_{Schwarz.} \qquad d\sigma^\diamond = d\sigma_{Schwarz.} \tag{13c}$$

and

$$c^2\mathrm{d}t^{\diamond 2} = (E_+/mc^2)^2\frac{c^2\mathrm{d}t^{\diamond 2}}{\cos^{\diamond\,2}\gamma} - \frac{\mathrm{d}r^{\diamond 2}}{\cos^{\diamond\,2}\gamma} - \left[r^{\diamond 2}\mathrm{d}\Omega^2\right] \qquad \text{repulsive} \tag{13d}$$

$$c^2\mathrm{d}\tau^{\diamond 2} = (E_+/mc^2)^2\frac{c^2\mathrm{d}\tau^{\diamond 2}}{\cos^{\diamond\,2}\gamma} - \frac{\mathrm{d}r^{\diamond 2}}{\cos^{\diamond\,2}\gamma} - \left[r^{\diamond 2}\mathrm{d}\Omega^2\right] \qquad \text{attractive} \tag{13e}$$

In the circular motion, the time axes of the two involved individuals, in addition to the rotation angle γ in the pseudo-plane of the act, rotate by an angle ϑ in the plane of potency. Since $\cos^\diamond\gamma = \left(1 - V^\diamond\right)$ and given the constant of motion $r^\diamond\mathrm{d}\Omega/\mathrm{d}\tau = L/(mc\,r^\diamond)$ we have, at last, the Schwarzschild metric :

$$\frac{1}{2}mc^2\left(1 - \frac{E_+^2}{m^2c^4}\right) = mc^2\left(V^\diamond - \frac{1}{2}V^{\diamond 2} - \frac{1}{2}\frac{\mathrm{d}r^{\diamond 2}}{c^2\mathrm{d}\tau^2} - \frac{1}{2}\frac{L^2}{m^2c^2r^{\diamond 2}}\left(1 - V^\diamond\right)^2\right) \tag{13f}$$

Derivation of the Schwarzschild metric

To equal the Schwarzschild metric to the linear one, it is necessary that:

$$g_{00} = 1/g_{rr} = 1 - 2V = \left(1 - V^\diamond\right)^2 = \left(\frac{\mathbb{E}^\diamond}{mc^2}\right)^2 \tag{14a}$$

which implies the following relation between the Schwarzschild potential V and the linear potential $V^\diamond$:

$$V = V^\diamond\left(1 - \frac{V^\diamond}{2}\right) \tag{14b}$$

Therefore we have :

$$R = \frac{G}{c^2}(m_a + m_b) \qquad V^\diamond = \frac{R}{r^\diamond} \qquad \text{linear co-ordinates} \tag{14c}$$

$$R_2 = 2R\frac{1+\cos^\diamond\gamma}{2} = 2R\,b_{\Gamma/2} = 2R\left(1 - \frac{V^\diamond}{2}\right) \qquad V = \frac{\frac{1}{2}R_2}{r^\diamond} = V^\diamond\left(1 - \frac{V^\diamond}{2}\right) \qquad \text{Schwarzschild co-ordinates} \tag{14d}$$

This means that, unlike the linear potential $V^\diamond$, the Schwarzschild potential arises not from the rest mass of the two individuals involved, but from the masses immersed in their resulting field $\mathbb{E}^\diamond = \cos^\diamond\gamma$. That is, for a far away observer, the global Radius R_2 that creates the field, and employed by the Schwarzschild potential, is given by half of the sum of the Radius of A in the field of B, i.e. $R_{2_{Ba}} = 2\left(R_b + R_a\cos^\diamond\gamma\right)$, and of the Radius of B in the field of A, i.e. $R_{2_{Ab}} = 2\left(R_a + R_b\cos^\diamond\gamma\right)$.

Our objective is now to demonstrate that, while employing the same equations of general relativity, but acknowledging that—contrary to the standard postulate of general relativity—the matter density does not vanish in vacuum but instead conforms to the geometrical structure of nature, we obtain $g_{00} = \left(1 - V^\diamond\right)^2$.

According to literature [see 9, pag. 283], to find the metric in a centrally symmetric gravitational field, it is convenient to start from:

$$ds^2 = e^\nu c^2 dt^{\diamond 2} - r^{\diamond 2}\left(d\theta^2 + \sin^2\theta d\phi^2\right) - e^{-\lambda}dr^{\diamond 2} \tag{15a}$$

To get the equations of gravitation we must calculate the components of the tensor R^i_k, which gives:

$$\begin{cases} e^{-\lambda}\left(\dfrac{\nu'}{r^\diamond} + \dfrac{1}{r^{\diamond 2}}\right) - \dfrac{1}{r^{\diamond 2}} = \dfrac{8\pi G}{c^4}T^1_1 \\ e^{-\lambda}\left(\dfrac{\lambda'}{r^\diamond} - \dfrac{1}{r^{\diamond 2}}\right) + \dfrac{1}{r^{\diamond 2}} = \dfrac{8\pi G}{c^4}T^0_0 \\ \dot{\lambda} = 0 \end{cases} \tag{15b}$$

The components of the energy momentum tensor, in an arbitrary reference system, can be expressed in terms of the energy density ϵ of the matter, its pressure p, and the four-velocity u^i which, given the central symmetry, must be radial

$$T^i_k = (\epsilon + p)u_i u^k - p\delta^k_i \tag{15c}$$

According to literature [see 9, pag. 283]: *The equations (15b) can be integrated exactly in the very important case of a centrally symmetric field in vacuum, that is, outside of the masses producing the field, that is, setting the energy-momentum tensor equal to zero. In this special case, we get $g_{00} = r_s/r$.*

However, according to the geometry of nature, as is also confirmed by cosmology, in the universe the energy density of matter never goes to zero. Indeed, between two individuals A and B very far away in the universe, τ reaches its maximum $\tau_{max} = R_\omega = c/H_0$ and dark matter becomes prevalent over baryonic mass and from the (6) , $R_a = R_b = R = r^{\diamond 2}/\tau^\diamond$ where $\tau^\diamond = R_\omega$, and therefore the energy of matter is $R_{Ab} = R_{Ba} = R_2$ and :

$$\int 4\pi r^{\diamond 2}\epsilon\left(r^\diamond\right)dr^\diamond = \frac{c^2}{G}\frac{R_2}{2} = \frac{c^2}{G}\frac{r^{\diamond 2}}{\tau^\diamond}\left(1 - \frac{\sin^\diamond\gamma}{2}\right) = \frac{c^2}{G}\frac{r^{\diamond 2}}{\tau^\diamond} - \frac{1}{2}\frac{c^2}{G}\frac{r^{\diamond 3}}{\tau^{\diamond 2}} \tag{15d}$$

and at last

$$T^0_0 = \epsilon\left(r^\diamond\right) = \frac{c^2}{8\pi G}\frac{4}{r^\diamond\tau^\diamond} - \frac{c^2}{8\pi G}\frac{3}{\tau^{\diamond 2}} \tag{15e}$$

and since $\lambda = -\nu$ and $T^0_0 = -T^1_1$, we reduce to the only equation:

$$e^{-\lambda}\left(\frac{\lambda'}{r^\diamond} - \frac{1}{r^{\diamond 2}}\right) + \frac{1}{r^{\diamond 2}} = \frac{4}{r^\diamond\tau^\diamond} - \frac{3}{\tau^{\diamond 2}} \tag{15f}$$

which admits one solution

$$e^{-\lambda} = \left(1 - \frac{r^\diamond}{\tau^\diamond}\right)^2 \tag{15g}$$

Therefore, the metric of universe in the usual general relativity coordinate system (τ, σ, t, r), observer dependent, which correspond to an "accelerated" frame, like that of an observer held at a fixed spatial point in the surrounding spacetime, is:

$$dl^2 = \left(1 - \frac{r^\diamond}{\tau^\diamond}\right)^2 c^2 dt^{\diamond 2} - \frac{dr^{\diamond 2}}{\left(1 - \frac{r^\diamond}{\tau^\diamond}\right)^2} - r^{\diamond 2} d\theta^2 - r^{\diamond 2} \sin^2\theta \, d\phi^2 \tag{15h}$$

Or, since $R/r^\diamond = r^\diamond/\tau^\diamond$, that is $V^\diamond = p^\diamond/mc$

$$dl^2 = \left(1 - \frac{R}{r^\diamond}\right)^2 c^2 dt^{\diamond 2} - \frac{dr^{\diamond 2}}{\left(1 - \frac{R}{r^\diamond}\right)^2} - r^{\diamond 2} d\theta^2 - r^{\diamond 2} \sin^2\theta \, d\phi^2 \tag{15i}$$

Denoted by R_h the Radius of the host entity within which the interaction takes place, we will use the eq. (15h) inside R_h, where $\tau^\diamond = R_h$ is constant and the cold dark matter gives place to $R_c = r^{\diamond 2}/\tau^\diamond \leq R_h$; the eq. (15i) outside R_h, where $R = R_h$ is now constant (as long as the cdm of the universe is still negligible) and $\tau^\diamond = r^{\diamond 2}/R_h \leq c/H_0$.

THE DECOMPOSITION OF DISTANCES

It is possible and it is useful to decompose the distance according to the axes, that is, according to the types of energy. Indeed, given that $\Omega_x = R_x/R$, where $x \in \{b, c, r\}$, the (6) specializes in its components:

$$\frac{R_x}{r} = \frac{r}{\tau_x} = \Omega_x \sin^\diamond \gamma \tag{16a}$$

$$R = \sum R_x \qquad \frac{1}{\tau} = \sum \frac{1}{\tau_x} \tag{16b}$$

In the transition from $\gamma \leq \pi/2$ to $\gamma \geq \pi/2$ the interaction goes from outside to inside the Radius. It is the transition from a Newtonian interaction to an interaction within the Radius, or from a Coulomb interaction to a strong and weak interaction. Basically, the radius $R = \tau_h$, the threshold between inside and outside, is the gravitational Radius of host Universe R_ω for gravitation, the electrical Radius of host particle $R_e^\circ = R_e^{-1}$ for electrical interactions. Depending on whether the angle γ is less than or greater then $\pi/2$:

$$\frac{R_x}{r_x} = \frac{r_x}{\tau} = \sqrt{\Omega_x} \sin^\diamond \gamma = V_x \qquad \text{for external interactions } (\gamma \leq \pi/2) \tag{16c}$$

$$\frac{R}{r_x} = \frac{r_x}{\tau_{h_x}} = \sqrt{\Omega_x} \sin^\diamond \gamma = V_x \qquad \text{for internal interactions } (\gamma \geq \pi/2) \tag{16d}$$

In the first case R is constant while $\tau(\gamma)$ varies; in the second, having τ reached its minimum limit $\tau_h = r_h = R$ at the point $\gamma = \pi/2$, the vice-versa is true, that is, thanks to the cdm, a new Radius arises $R_c(\gamma)$ and varies while $\tau = \tau_h = R$ is constant. The decomposition of electrical $\tau_h = R_e^\circ$ into its three components $\tau_{h_c}, \tau_{h_b}, \tau_{h_r}$ corresponds to the three generations of elementary particles which, in turn, depend on the density of the three components of matter in the universe.

Inside τ_h, a Cold Dark Matter Radius R_c arises which, from the (6), grows as the distance increases :

$$R_c(\gamma) = \frac{r^{\diamond 2}}{\tau_h} \tag{16e}$$

Therefore, the potential $V^\diamond$ reverse from inside to outside:

$$V^\diamond = \frac{R}{r^\diamond} \qquad \text{outside Radius } R \tag{16f}$$

$$V^\diamond = \frac{R_c(\gamma)}{r^\diamond} = \frac{r^\diamond}{R} \qquad \text{inside Radius } R \tag{16g}$$

Analogously, inside the Radius we have:

$$\frac{1}{r^2} = \frac{1}{r_r^2} + \frac{1}{r_b^2} + \frac{1}{r_c^2} = \frac{\Omega_r}{r^2} + \frac{\Omega_b}{r^2} + \frac{\Omega_c}{r^2} \tag{16h}$$

while, on the other hand, outside the radius we have:

$$r^2 = r_b^2 + r_r^2 + r_c^2 = \Omega_b\, r^2 + \Omega_r\, r^2 + \Omega_c\, r^2 \tag{16i}$$

Outside the radius, the centrifugal acceleration only affects the baryonic component

$$A_{centrifugal} = \frac{v^2_{centrifugal}}{r_b} \tag{16j}$$

For gravitation, since $R \simeq R_\bullet + R_c = R_\bullet + r^{\diamond 2}/R_\omega$, where $R_\omega = c/H_0$ on radial orbits, i.e. stars plunging in and out of the galactic center, $R_\omega = 2\pi\, c/H_0$ on circular orbits, we have:

$$A_x = \frac{R_x}{r_x^2} = A = \frac{R}{r^2} = \frac{R_\bullet}{r^2} + \frac{1}{R_\omega} = \frac{1}{\tau} \tag{16k}$$

and from (16j , 16k and 16i), equaling centrifugal acceleration to the baryonic component $A_b = V_b/r_b$ of acceleration:

$$v_{centrifugal} = \sqrt{V_b} = \sqrt[4]{\frac{R}{r^2} R_\bullet} = \sqrt[4]{\frac{R_\bullet^2}{r^2} + \frac{R_\bullet}{R_\omega}} \tag{16l}$$

and the limits

$$r_{b_\infty} = \lim_{r \to R_\omega} \sqrt{\frac{R_\bullet}{R}}\, r = \sqrt{R_\bullet R_\omega} \tag{16m}$$

$$v_\infty = \lim_{r \to R_\omega} v_{centrifugal} = \sqrt[4]{\frac{R_\bullet}{R_\omega}} \tag{16n}$$

At last, the part of relationship

$$R_{part} : R_{whole} = R_{whole} : R_\omega \tag{16o}$$

requires that every relation finds its place inside an individual more complex of which it is a part of, providing all the mirroring universe scale: baryons, stars, galaxies, clusters and so on.

The decomposition of distances (see 16b) hints:

Comp.	ℓ_ω	r_ω	Ω	R_ω	τ_ω
c	ℓ_{ω_c}	$r_{\omega_c} = \ell_{\omega_c} R^*_\omega$	$\Omega_c = \ell^2_{\omega_c}/\ell^2_\omega$	$R_{\omega_c} = \Omega_c\, R_\omega$	$\tau_{\omega_c} = \Omega_c^{-1}\, R_\omega$
b	ℓ_{ω_b}	$r_{\omega_b} = \ell_{\omega_b} R^*_\omega$	$\Omega_b = \ell^2_{\omega_b}/\ell^2_\omega$	$R_{\omega_b} = \Omega_b\, R_\omega$	$\tau_{\omega_b} = \Omega_b^{-1}\, R_\omega$
r	ℓ_{ω_r}	$r_{\omega_r} = \ell_{\omega_r} R^*_\omega$	$\Omega_r = \ell^2_{\omega_r}/\ell^2_\omega$	$R_{\omega_r} = \Omega_r\, R_\omega$	$\tau_{\omega_r} = \Omega_r^{-1}\, R_\omega$
Tot	$\ell_\omega = \sqrt{\sum \ell^2_{\omega_x}}$	$r_\omega = \sqrt{\sum r^2_{\omega_x}} = \ell_\omega R^*_\omega$	$\Omega = \sum \Omega_x = 1$	$R_\omega = \sum R_{\omega_x}$	$\tau_\omega = \left(\sum \frac{1}{\tau_{\omega_x}}\right)^{-1} = R_\omega$

where $R_\omega = \ell_\omega R^*_\omega = c/H_0$ and ℓ_ω is the Universe scale factor and R^*_ω the common ratio.

CONCLUSIONS

This paper advances a primary thesis alongside a complementary thesis. The central claim is that physics emerges entirely from the geometry of nature. The secondary claim is that its boundary limits are established not by chance or contingency, but

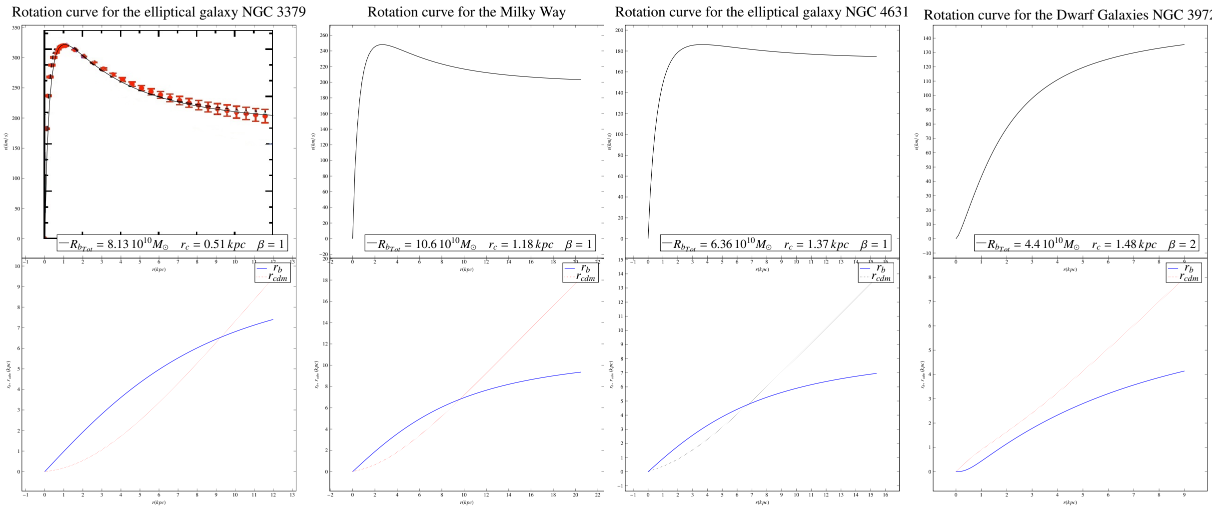

Figure 1. On the top panel, the rotation curves for galaxies. We adopted the mass distribution model $R_b(r) = R_{b_{Tot}} \left(\frac{r}{r_c + r} \right)^{3\beta}$ of a spherically symmetric galaxy, where r_c is the inner core and $\beta = 1$ for HSB galaxies and 2 for LSB and Dwarf galaxies. On the bottom panel, the trend of $r_b = \sqrt{\frac{R_b(r)}{R_b(r) + r^2/(2\pi c/H_0)}}\, r$ and $r_{cdm} = \sqrt{\frac{r^2/(2\pi c/H_0)}{R_b(r) + r^2/(2\pi c/H_0)}}\, r$ with $H_0 = 73.22$. The rotation curves correspond to Newton's velocities once replaced the total distance r with its baryonic component r_b.

are themselves governed by fixed geometric relations. Supporting the secondary thesis is the constancy of the electric Radius in cosmic time, and the geometric nature of the cosmological parameters.

To assess the implications of the geometry of nature, which manifests itself in its three dimensions, time, power and act, as inertia, gravitation and electricity, an overview of its implications in the fields of standard model (electricity) and cosmology is provided in the appendix.

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The Electric Radius

Mirroring is the key to nature's geometry: everything mirrors everything else. From this comes the thesis:

1 *Coexistence and relationship between electromagnetism and gravitation: in the relationship, each of the two individuals has, in addition to its own "proper" gravitational Radius $R_\bullet$, the electric "mirror" R° of the gravitational Radius of its other. The electric radius, as it mirrors it, is the inverse of the gravitational radius of the other:*

$$R_a^\circ = R_{\bullet b}^{-1} \tag{17a}$$

Indeed, the geometry of nature requires that the global angle γ splits in two angles $\gamma = \phi +^\diamond \psi$ (see fig. 5) such that:

$$\lambda_a = 2\pi \frac{R^\circ{}_b}{\sin^\diamond \varphi} = 2\pi \frac{R^\circ{}_b}{V_a^\diamond} \qquad = \lambda_b = 2\pi \frac{R^\circ{}_a}{\sin^\diamond \psi} = 2\pi \frac{R^\circ{}_a}{V_b^\diamond} \qquad = 2\pi r^\diamond \tag{17b}$$

Now, since from the De Broglie relation $\lambda = h/p$, we have:

$$\lambda_a = 2\pi \frac{\hbar/m_a}{p_a/m_a} = 2\pi \frac{R_{\bullet a}^{-1}}{p_a/m_a} \qquad = \lambda_b = 2\pi \frac{\hbar/m_b}{p_b/m_b} = 2\pi \frac{R_{\bullet b}^{-1}}{p_b/m_b} \qquad = 2\pi r^\diamond \tag{17c}$$

and since the Principle (2) demands that $V^\diamond = p^\diamond/mc$, it follows the thesis (1).

The discrete one-dimensional geometry of the Act

The scheme underlying the mirroring

The IRPL (Instant Reconstruction of the Path of Light) is only and not other than the reconstruction, starting from the present instant, of the path of the intermediaries of the interaction (i.e. the bosons) that takes place between two individuals in relationship. This is the same path as the light between two mirroring individuals: each one reflects and is reflected by the other recursively.

In fact, if we place a clock on each of the two individuals involved in the interaction (see fig. 3), we can historically reconstruct distances and time intervals from the sequence of times that appears in the mirror image. If we denote by $s_n^\diamond = t_n^\diamond - t_{n-1}^\diamond$ the distance between the two individuals at time t_n, we discover (see fig. 4) that the historical reconstruction of the distance series forms a geometric progression

$$t^\diamond = s_0^\diamond + s_1^\diamond + s_2^\diamond + s_3^\diamond + \ldots = s_0^\diamond \left(1 + K^\diamond + K^{\diamond 2} + K^{\diamond 3} + \ldots\right) = \frac{s_0^\diamond}{1 - K^\diamond}$$

where $s_0^\diamond$ is the scale factor and $k = \cos^\diamond \gamma$ is the common ratio. Therefore

$$\Delta\lambda^\diamond = t^\diamond - t_{-1}^\diamond = s_0^\diamond \quad \text{and} \quad V^\diamond = \frac{\Delta\lambda^\diamond}{t^\diamond} = \frac{\overline{AB}}{\overline{\overline{0A}}} = 1 - K^\diamond$$

Figure (4) compares the representation of the progression of events $A, B, A', B', \ldots$ in Minkowski's spacetime with that in IRPL.

In a IRPL diagram, each segment arises from a geometric progression which has as its common ratio $\cos^\diamond \gamma$ and as scale factor a segment of a more primitive nature. Below the genesis of the spacetime (see fig.5):

The core of a IRPL diagram consists of the radius of the two interacting individuals linked by the path of light during their interaction. In the interaction, the light path cyclically connects the head of each radius with the tail of the opposite radius, crossing the same radii.

starting from the above schema, indicating with:

$$R_a = \frac{G}{c^2} M_a \qquad\qquad R_{tot} = R_a + R_b \tag{18a}$$

Since for each observer A, its proper mass at rest is opposed to the remaining masses B placed in their centre of gravity and subjected to the total gravitational field, the global energy-momentum Radius of A and B is

$$R_{2_{Ab}}^\diamond = 2\left(R_a + R_b \cos^\diamond \gamma\right) \qquad\qquad R_{2_{Ba}}^\diamond = 2\left(R_b + R_a \cos^\diamond \gamma\right) \tag{18b}$$

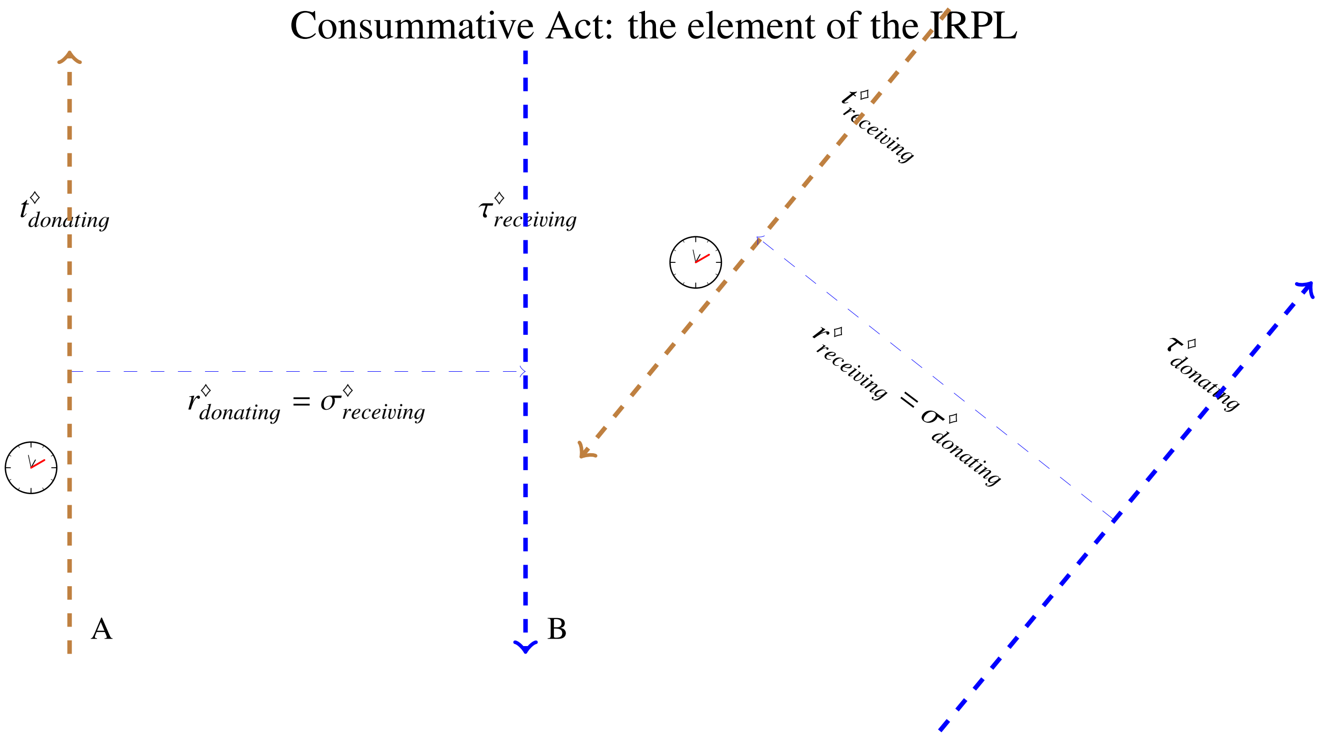

Figure 2. Consummative Act (not the event) is the element of the IRPL: light does not have a speed, each segment of the path of light itself constitutes the space axis and determines the time axis, orthogonal to it, constituting the frames of the two individuals who oppose each other in the interaction. Consequently, for each individual, one frame corresponds to the act of giving and another frame corresponds to the act of receiving. The two frames are rotated to each other by a real γ angle. The determination of the γ angle is subject to the Uncertainty principle. Indeed, in a measurement, while the measuring instrument A is necessarily classic and therefore reflective, so we know $P^{\diamond} = t^{\diamond}_{A_i} - t^{\diamond}_{A_{i-1}}$, the measured B could be non-classic, therefore we would not know the proper time $t^{\diamond}_{B_i}$ and therefore we would not know $\cos\gamma^{\diamond} = \left(t^{\diamond}_{B_i} - t^{\diamond}_{A_{i-1}}\right) / \left(t^{\diamond}_{A_i} - t^{\diamond}_{B_i}\right)$ and vice-versa.

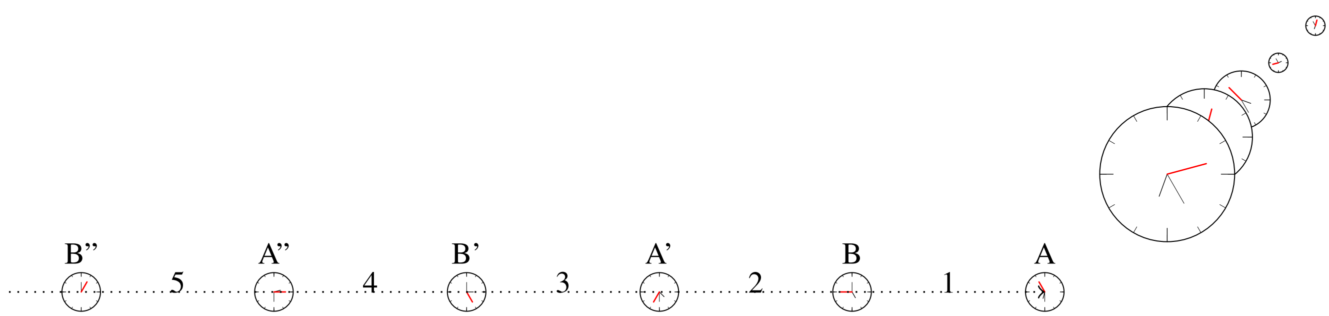

Figure 3. Recursive mirroring: two mirrors facing each other are reflected recursively. If there is a clock on each of them, in the reflected image present in every instant it is possible to reconstruct distances historically and therefore the velocities and accelerations over time, as far as the reflection allows.

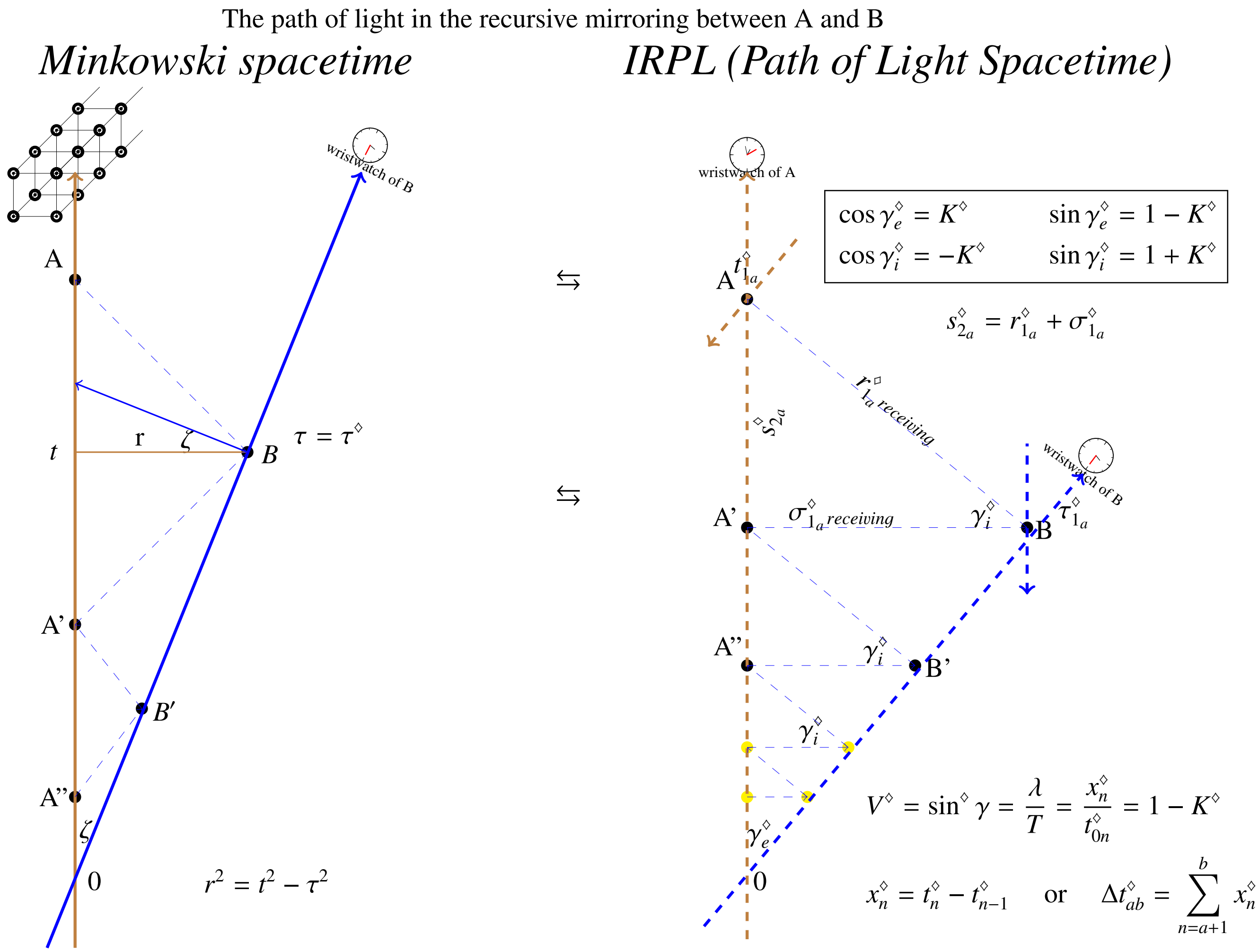

Figure 4. isomorphism: in comparison the representations of the geometric progression A, B, A', B', A'', B'', ... with $K^{\diamond}(\gamma)$ as the common ratio, deriving from the recursive mirroring of individuals A and B (see fig. 3). The IRPL diagram emerges from the historical reconstruction that connects the act of giving with the previous act of receiving and so on. Consequently, In the IRPL diagram the homologous frames, and therefore the homologous axes, face each other forming an angle γ (the heterologous frames, and therefore the heterologous axes give-riceive are in fact always parallel to each other).

and since a round trip route passes through both A and B, it descends that space and time proceed from mass-energy as follows:

$$R_2^{\diamond} = \frac{R_{2_{Ab}}^{\diamond} + R_{2_{Ba}}^{\diamond}}{2} = R_{tot}(1 + \cos^{\diamond}\gamma) = R_{tot}\sin^{\diamond}\gamma_i \tag{18c}$$

$$s_2^{\diamond} = \sum_{-\infty}^{0} R_{2_i}^{\diamond} = R_2^{\diamond}\left(1 + \cos^{\diamond}\gamma + \cos^{\diamond 2}\gamma + \cdots\right) = \frac{R_2^{\diamond}}{\sin^{\diamond}\gamma_e} \tag{18d}$$

$$r^{\diamond} = \sum_{-\infty}^{0} s_{2_i}^{\diamond} = s_2^{\diamond}\left(1 - \cos^{\diamond}\gamma + \cos^{\diamond 2}\gamma - \cdots\right) = \frac{s_2^{\diamond}}{\sin^{\diamond}\gamma_i} \tag{18e}$$

$$\tau^{\diamond} = \sum_{-\infty}^{0} r_i^{\diamond} = r^{\diamond}\left(1 + \cos^{\diamond}\gamma + \cos^{\diamond 2}\gamma + \cdots\right) = \frac{r^{\diamond}}{\sin^{\diamond}\gamma_e} \tag{18f}$$

where

$$s_2^{\diamond} = \frac{s_{2_a}^{\diamond} + s_{2_b}^{\diamond}}{2} \qquad r^{\diamond} = \frac{r_2^{\diamond}}{2} = \frac{r_a^{\diamond} + r_b^{\diamond}}{2} \qquad \tau^{\diamond} = \frac{\tau_a^{\diamond} + \tau_b^{\diamond}}{2} \tag{18g}$$

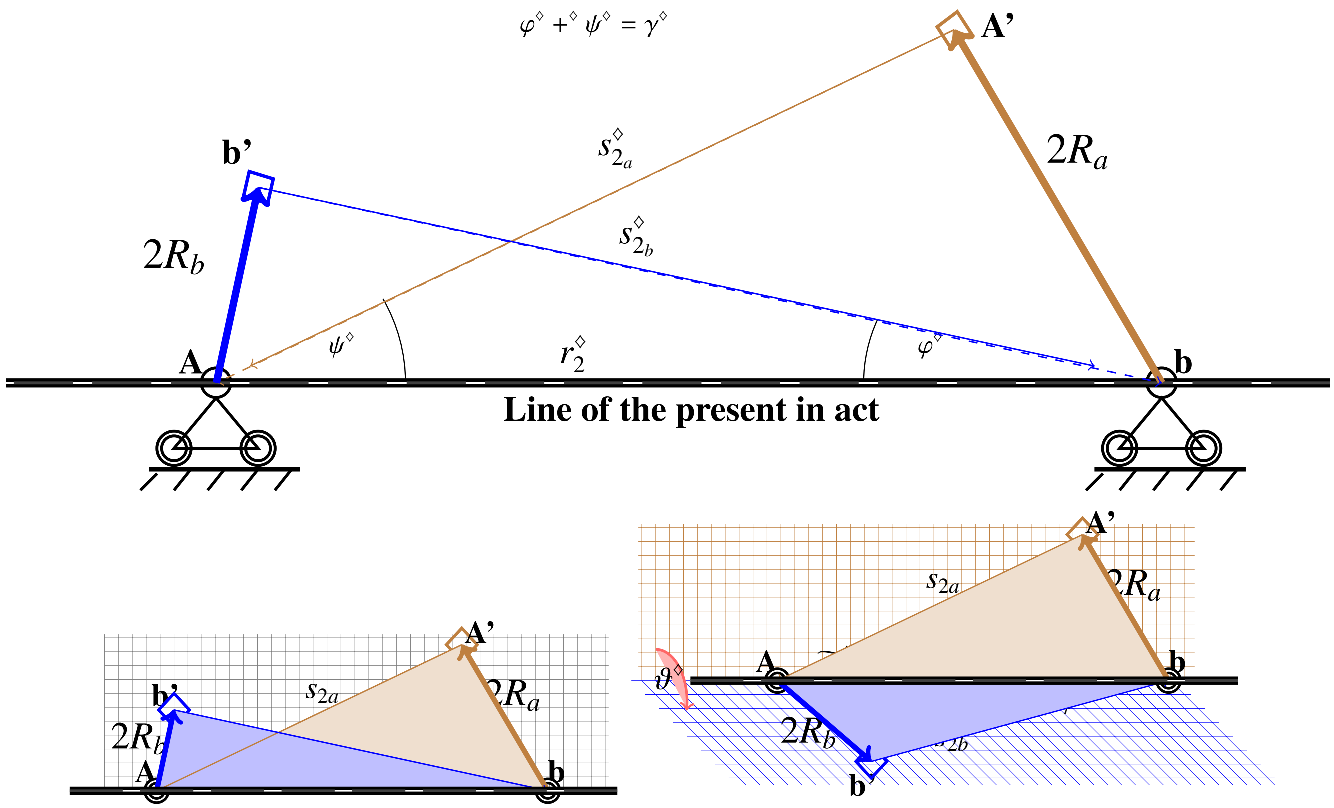

Figure 5. the path of light: In the interaction, the light path cyclically connects the head of each radius with the tail of the opposite radius, crossing the same radii. The path between an emitter and a receiver is therefore equal to $r_2^\diamond = 2r^\diamond = r_a^\diamond + r_b^\diamond = \vec{Ab} = 2R_a + s_{2_a}^\diamond = 2R_b + s_{2_b}^\diamond = 2(R_a + R_b)/\sin\gamma = 2R_a/\sin\psi = 2R_b/\sin\phi = 2(R_a + R_b)/(\sin\psi + \sin\phi)$.

from the eq. (18c, 18d, 18e, 18f) descends the fundamental relation:

$$V^\diamond = \quad R_{tot} : r^\diamond = r^\diamond : \tau^\diamond \quad = p^\diamond/m \tag{18h}$$

which expresses the "principle of equivalence, in the instant, between inertial and not inertial systems", see fig. (fig. 6).

Since the linear operators ($\sin^\diamond$, $\cos^\diamond$) are defined as the same ratios of the sides of a right triangle as the corresponding trigonometric functions, the rules for adding angles do not change. Indeed, denoting by $+^\diamond$ the reflective sum of two angles, we have $\gamma = (\varphi +^\diamond \psi) \neq (\varphi + \psi)$

$$\sin\left(\varphi \pm^\diamond \psi\right) = \sin^\diamond \varphi \pm \sin^\diamond \psi \qquad\qquad \cos\left(\varphi \pm^\diamond \psi\right) = \cos^\diamond \varphi \mp \sin^\diamond \psi \tag{19a}$$

At last, it is easy to verify (see fig. 5) that:

$$r^\diamond = \frac{R_a + R_b}{\sin\gamma} = \frac{R_a}{\sin\psi} = \frac{R_b}{\sin\phi} = \frac{R_a + R_b}{\sin\psi + \sin\phi} = \frac{R_a + R_b}{\sin(\psi +^\diamond \phi)} \tag{20}$$

At last, all systems, whether gravitational or electric (or inertial), share the universal metric illustrated in the next section.

The universal metric

According to the standard convention of intrinsic (mobile axis) Euler rotations, three consecutive elemental rotations are always sufficient to describe the orientation of any target frame B with respect to a fixed coordinate system A. However, since the Euler angle representation involves three frames, the initial fixed reference frame, an intermediate frame, the final target frame, we use it to represent the disposition of the target frame B with respect to the (initial) frame A inside the fixed (intermediate) frame of the Universe, or of the barycentre. That is:

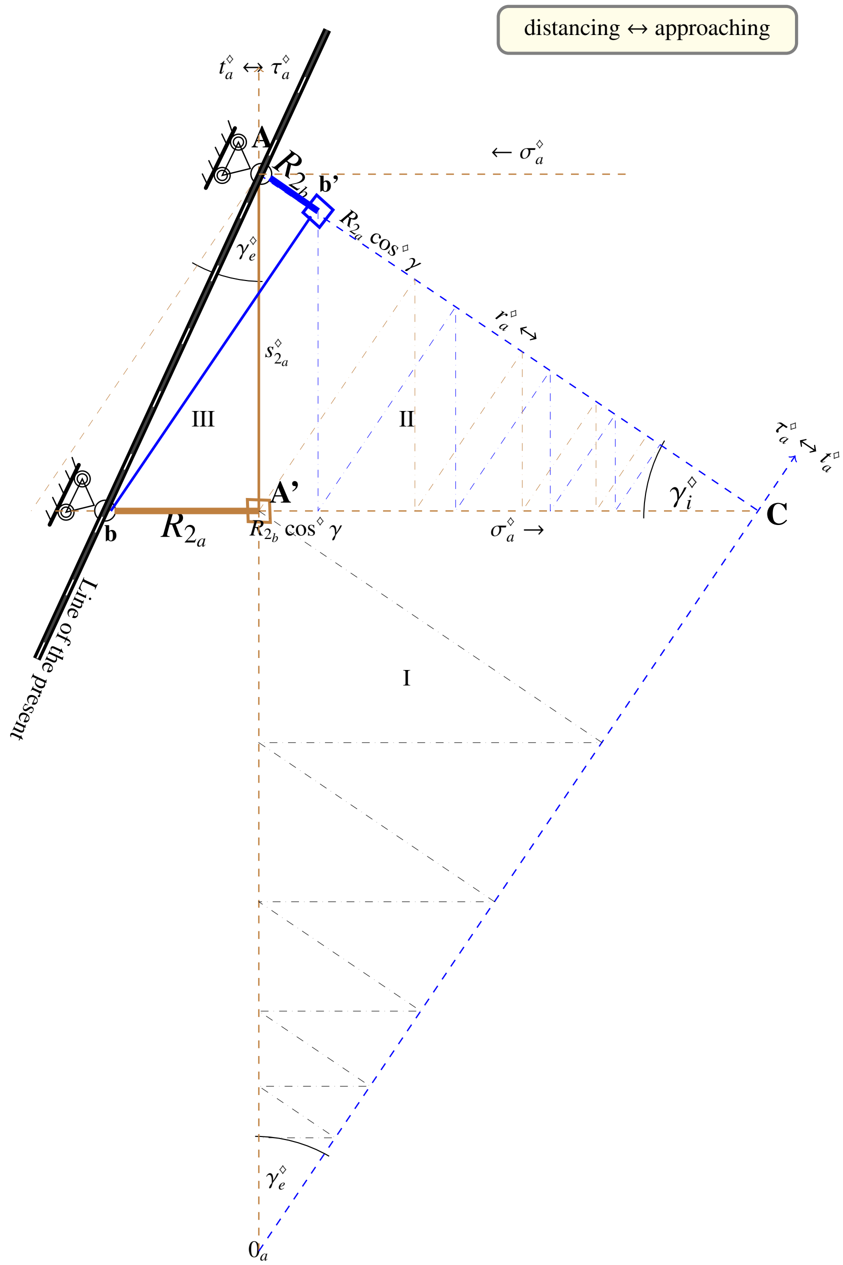

Figure 6. The whole relation is enfolded and unfolds from the Radii of the two conjoined individuals with the dual angles γ_e and γ_i alternating each other. It is governed by the relation $V^\diamond = R_{tot} : r^\diamond = r^\diamond : \tau^\diamond = p^\diamond$. That is, $R_{tot} : CA = CA : OC$ or $R_{tot} : A'C = A'C : OA'$ in distancing and $R_{tot} : AC = AC : OA$ in approaching. Indeed the three quadrants represent time, space and Radius and recursively follow one another. In particular the III-II quadrants represent the internal energy-space plane, while the II-I quadrants the external space-time plane.
The diagram represents the historical reconstruction of the relationship starting from the current instant. It coincides with real history only when γ is constant.

1. $\varphi \rightarrow$ Rotation of the frame A around the initial *Zaxis* of the universe (the potency axis),
2. $\vartheta \rightarrow$ Rotation of the frame B around the nutation N axis of the universe (the radiant axis),
3. $\psi \rightarrow$ Rotation of the frame B around its final z'' axis.

$$\begin{bmatrix} dx^\diamond \\ id\tau^\diamond \\ \sigma^\diamond d\phi \end{bmatrix} = \begin{bmatrix} \cos^\diamond\psi & \sin^\diamond\psi & 0 \\ -\sin^\diamond\psi & +\cos^\diamond\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta & +\sin\vartheta \\ 0 & -\sin\vartheta & \cos\vartheta \end{bmatrix} \begin{bmatrix} \cos^\diamond\varphi & \sin^\diamond\varphi & 0 \\ -\sin^\diamond\varphi & +\cos^\diamond\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d\sigma^\diamond \\ idt^\diamond \\ r^\diamond d\phi \end{bmatrix} \tag{21a}$$

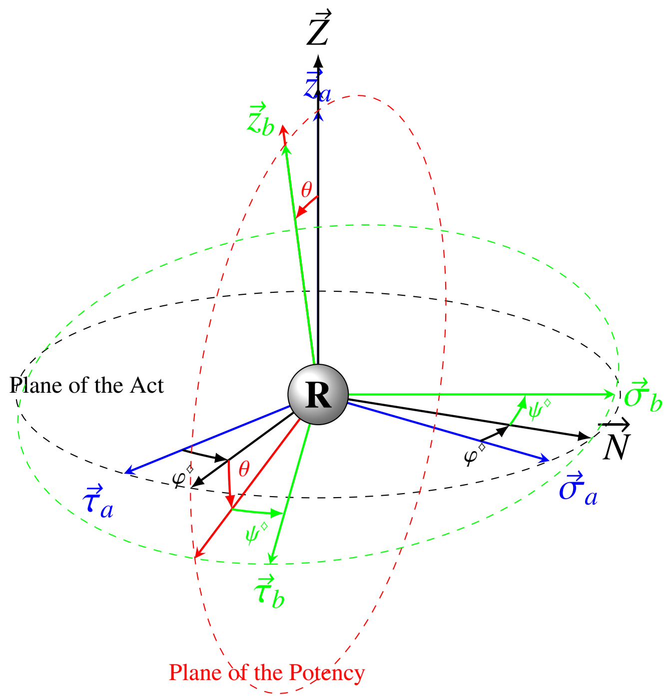

Figure 7. The two reference frames weave around the axis of the nodes $r^\diamond$ decomposing the γ angle according to its component frames $\gamma = \psi +^\diamond \varphi$.

$$\begin{bmatrix} dx^\diamond \\ id\tau^\diamond \\ \sigma^\diamond d\phi \end{bmatrix} = Z_1X_2Z_3 = \begin{bmatrix} c_1c_3 - c_2s_1s_3 & -c_1s_3 - c_2c_3s_1 & s_1s_2 \\ c_3s_1 + c_1c_2s_3 & c_1c_2c_3 - s_1s_3 & -c_1s_2 \\ s_2s_3 & c_3s_2 & c_2 \end{bmatrix} \begin{bmatrix} d\sigma^\diamond \\ idt^\diamond \\ r^\diamond d\phi \end{bmatrix} \tag{21b}$$

where s and c represent sine and cosine (e.g., s_1 represents the sine of ψ) and

$$\cos\xi = \cos(\psi + \varphi) = \frac{1}{1 - V^\diamond} \tag{21c}$$

$$\cos\vartheta_r = \sqrt{\frac{r^2}{r^2 + a^2}} \qquad \tan\vartheta_r = i\frac{(L+J)/m}{r} = i\frac{a}{r} \tag{21d}$$

$$\cos\vartheta_\sigma = \sqrt{\frac{r^2(1-V^\diamond)^2}{r^2(1-V^\diamond)^2 + a^2}} = \sqrt{\frac{(1-V^\diamond)^2}{(1-V^\diamond)^2 + a^2/r^2}} \qquad \frac{1}{\cos\vartheta_\sigma} = \sqrt{1 + \frac{a^2/r^2}{(1-V^\diamond)^2}} \tag{21e}$$

Since

$$dx^\diamond = \left(v^\diamond + r^\diamond d\phi/dt^\diamond\right) i\, dt^\diamond + dr^\diamond = \left(a_{12} i dt^\diamond + a_{13} r^\diamond d\phi\right) + dr^\diamond_r \hat{\mathbf{e}}_r + dr^\diamond_\phi \hat{\mathbf{e}}_\phi \tag{21f}$$

we have:

$$d\sigma^\diamond = \frac{dr^\diamond}{a_{11}} \tag{21g}$$

$$\mathrm{d}\left(\frac{\left(a_{21}\hat{\mathbf{e}}_t + a_{31}\hat{\mathbf{e}}_\phi\right)}{a_{11}} r^\diamond\right) = \mathrm{d}\left(V^\diamond r^\diamond\right) = \left[d\vec{R}\right] = 0 \qquad \text{in a force field} \tag{21h}$$

we have:

$$\left[d\vec{l}^\diamond\right] = \left(\begin{bmatrix} 1/a_{11} & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}\right) \begin{bmatrix} dr^\diamond \\ idt^\diamond \\ r^\diamond d\phi \end{bmatrix} \tag{21i}$$

At last, it results:

$$\left[d\vec{l}^{\diamond}\right] = \begin{bmatrix} p_{rr} & 0 & 0 \\ 0 & \mathbb{E}_{tt} & p_{t\vartheta} \\ 0 & p_{\vartheta t} & p_{\vartheta\vartheta} \end{bmatrix} \begin{bmatrix} dr^{\diamond} \\ idt^{\diamond} \\ r^{\diamond} d\phi \end{bmatrix} \tag{21j}$$

This general metric can be expressed in the general form:

$$\pm i\vec{\mathrm{k}}\mathbb{E} \pm \vec{\mathrm{i}}p + \vec{\mathrm{j}}m = 0 \tag{21k}$$

or specialized for electric and gravitational interactions:

$$\left(\gamma^{\mu}\partial_{\mu} + im\right)\varphi = 0 \qquad\qquad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} \tag{21l}$$

When $\vartheta = 0$, we trivially have that $\gamma = \phi + \psi$. Since the spatial axis of the electrical relation, which is the radiant one, is orthogonal to the gravitational one, which is that of power, and therefore out of phase by $\pi/2$, even in accordance with experimental evidence, it must be:

$$\cos^{\diamond}\gamma = 1 - \sin\gamma \qquad \text{for Gravitational fields and inertial systems } (\sin^{\diamond}\gamma = \sin\gamma) \tag{22a}$$

$$\sin^{\diamond}\gamma = 1 - \cos\gamma \qquad \text{for Electric fields } (\cos^{\diamond}\gamma = \cos\gamma) \tag{22b}$$

When $\vartheta \neq 0$, for a probe in a force field we have two different cases.

The Kerr Metric

In particular, when the angle $\varphi^{\diamond} = 0$, it is $c_3 = 1,\ s_3 = 0$ and

$$\left[i\,d\vec{l}^{\diamond}\right] = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_1 c_2 & -c_1 s_2 \\ 0 & s_2 & c_2 \end{bmatrix} \begin{bmatrix} d\sigma^{\diamond} \\ i\,dt^{\diamond} \\ r^{\diamond} d\phi \end{bmatrix} \tag{23a}$$

By dividing $d\sigma^{\diamond}$ and $i\,d\tau^{\diamond}$ by $\cos\vartheta_{\sigma}$ and $i\,dt^{\diamond}$ and $r^{\diamond}d\phi$ by $\cos\vartheta_r$, we obtain

$$\frac{id\vec{l}}{\cos\vartheta_{\sigma}} \equiv \cos\vartheta_{\sigma}\frac{dr_r}{(V_i - 1)}\hat{\mathbf{e}}_r + \{idt\,(1 - V_e) - rd\phi\,(1 - V_e)\tan\vartheta_r\}\,\hat{\mathbf{e}}_t + \{idt\tan\vartheta_r + rd\phi\}\,\hat{\mathbf{e}}_{\phi} \tag{23b}$$

and, given the (21d) and (21e), and since the (14a and 14b), we have:

$$dl^2 = (1 - 2V)\,dt^2 + 4\frac{a}{r}Vrd\phi dt - \frac{dr^2}{(1 - 2V) + \frac{a^2}{r^2}} - \left(1 + \frac{a^2}{r^2} - 2V\frac{a^2}{r^2}\right)r^2 d\phi^2 \tag{23c}$$

The Schwarzchild Metric

On the other hand, when the angle $\psi = 0$, it is $c_1 = 1,\ s_1 = 0$ and

$$\left[i\,d\vec{l}^{\diamond}\right] = \begin{bmatrix} 1/c_3 & 0 & 0 \\ 0 & c_2 c_3 & -s_2 \\ 0 & c_3 s_2 & c_2 \end{bmatrix} \begin{bmatrix} dr^{\diamond} \\ i\,dt^{\diamond} \\ r^{\diamond} d\phi \end{bmatrix} \tag{24a}$$

$$\begin{aligned} idl\left(\cos\vartheta\,\hat{\mathbf{e}}_t + \sin\vartheta\,\hat{\mathbf{e}}_{\phi}\right) \equiv \frac{dr^{\diamond}}{1 - V^{\diamond}}\hat{\mathbf{e}}_r &+ \left\{idt^{\diamond}(1 - V^{\diamond})\cos\vartheta - r^{\diamond}d\phi\sin\vartheta\right\}\hat{\mathbf{e}}_t + \\ &+ \left\{idt^{\diamond}(1 - V^{\diamond})\sin\vartheta + r^{\diamond}d\phi\cos\vartheta\right\}\hat{\mathbf{e}}_{\phi} \end{aligned} \tag{24b}$$

and squaring:

$$dl^2 = c^2 dt^{\diamond 2}(1 - V^{\diamond})^2 - \frac{dr^{\diamond 2}}{(1 - V^{\diamond})^2} - r^{\diamond 2}d\phi^2. \tag{24c}$$

At last, substituting the two constants of motion $r^{\diamond 2} d\phi/d\tau = L/m$ and $dt^{\diamond} = E/(mc^2)\, d\tau/(1-V^{\diamond})^2$ we have:

$$\mathbf{U} = \frac{1}{2}mc^2\left[-2V^{\diamond} + V^{\diamond 2} + \left(\frac{dr^{\diamond}}{d\tau}\right)^2 + \frac{L^2 V_{\vartheta}^{\diamond 2}}{m^2R^2c^2}\left(1-V^{\diamond}\right)^2\right] \tag{24d}$$

where

- the potential $V^{\diamond} = \sin\gamma^{\diamond} \leq 1$, reverses from outside $V^{\diamond} = R_h/r^{\diamond}$ to inside $V^{\diamond} = r^{\diamond}/R_h$ when the distance $r^{\diamond}$, overflowing its seat, crosses the threshold R_h;
- the pseudo potential $V_{\vartheta}^{\diamond}$ term is equal to $V_{\vartheta}^{\diamond} = R_h/r^{\diamond}$ when the native seat of the relationship is outside R_h, to $V_{\vartheta}^{\diamond} = r^{\diamond}/R_h$ otherwise. But, contrarily to the potential $V^{\diamond}$, its formula does not reverse but continues to grow when the distance $r^{\diamond}$, overflowing its seat, crosses the threshold R_h.

  It is the conservation of angular momentum, therefore, that determines the confinement of the relationship on one side or the other of Radius R_h in the strong interaction.
- for electrical interactions, it holds $mR = R_{\bullet}R^{\circ} = 1$;
- all electrical interactions share $L/c = n/\alpha$

Of particular importance is the case when $\gamma = \phi +^{\diamond} \varphi = \pi/2$ and $\vartheta = \pi/2$, where we have:

Kerr Metric	$\varphi = 0 \quad \vartheta = \pi/2 \quad \psi = \pi/2$	Schwarzchild Metric	$\varphi = \pi/2 \quad \vartheta = \pi/2 \quad \psi = 0$
	$\hat{r} \rightarrow \hat{t}$		$\hat{r} \rightarrow -\hat{p}$,
	$\hat{t} \rightarrow -\hat{p}$		$\hat{t} \rightarrow \hat{r}$,
	$\hat{p} \rightarrow -\hat{r}$		$\hat{p} \rightarrow \hat{t}$.

While both transformations interchange space and time, it is worth noting that the difference between the Schwarzschild and the Kerr configuration, as well as that between the gravitational and the electric relation, involves a swapping of radial axis and potency axis.

That is, the Kerr metric is to the gravitational interaction what the Schwarzchild metric is to the electrical one. More explicitly, the metric of extreme spin elemental electric particles is the Schwarzchild metric.

The relations of the three axes

The first relationship is the "part of" relation between the universe $R_{\bullet} = c/H_0 = R_{\Omega}$ and the element of cold dark matter that we call Amorone $R^{\circ} = R_{\Omega}^{-1} = H_0/c = R_{\alpha}$. For both universe and its element, we have $R_{\bullet} = R^{\circ}$, that is coincidence between gravitation and electromagnetism, potency and act, crushed on a single axis.

The universe would be composed solely of cold dark matter, uniformly distributed and subject to gravitation-electromagnetism, hitherto indistinguishable from each other, were it not for the fact that, for the very reason of being endowed with a finite Radius, it gives rise to a new special element, on which electromagnetism is much stronger than gravitation, and to radiation and therefore to all baryonic matter. At last

2 *The birth of the Electric Universal: There is, and is unique, a special individual within the universe such that its gravitational Radius $R_{\bullet}$ is exactly equal to the mass of the CDM contained in its electric Radius R°:*

$$R_{\bullet} : R^{\circ} = R^{\circ} : R_{\Omega_b} \tag{25a}$$

where $R^{\circ} = R_{\bullet}^{-1}$ and $R_{\Omega_b} = R_{\Omega}/\Omega_b$ and where $\ell_{irpl} = 2\sqrt{\alpha}\,\ell_P = 2\ell_S$ and $m_{irpl} = \sqrt{\alpha}\, m_P = m_S$ (where ℓ_P, m_P and ℓ_S, m_S are the Planck and Stoney units respectively and where the factor of 2 in length measurement is the only difference from Stoney units since the IRPL units provide (see fig. 5) the Radius doubled R_2 and the round trip distance s_2). This special individual which, in the various forms it takes on the basis of position and combination, gives rise to all baryonic matter, is the electron:

$$\left(\frac{2m_e}{m_{irpl}}\right)^{-3} \ell_{irpl} = \frac{R_{\Omega}}{\Omega_b}. \tag{25b}$$

That is, the composite (gravitationally) elementary (electrically) individual R_e, is the sole individual that is in equilibrium with universe. It is the stage of weak and strong interactions between elementary forms of electron that, with their three generations, give rise to all baryonic matter (neutrinos, quarks, hadrons, bosons).

With the birth of the electric universal and of the consequent baryonic matter, the three axes of the Universe are no longer almost overlapping. At last, starting from the baryon, the part of relationship

$$R_{part} : R_{whole} = R_{whole} : R_{\omega_b} \tag{25c}$$

requires that every relation finds its place inside an individual more complex of which it is a part of, providing all the mirroring universe scale: baryons, stars, galaxies, clusters and so on.

3 *universality of quantum angular speed* $\omega = \Delta\vartheta/\Delta t = R_{\bullet e}$ *and then of electric spin:*

The universality of the spin of fermions guarantees the conservation of the sign of the charge, which corresponds to the alignment of the radial axis with that of the give or receive of the universe.

4 *The three axes correspond to the three fundamental symmetry operations in particle physics:*

Potency	***P**arity*	*reverses signs of space coordinates*
Time	***T**ime reversal*	*reverses sign of time coordinate*
Radiation	***C**harge conjugation*	*exchanges particle and antiparticle*

$$CP \equiv T \qquad CT \equiv P \qquad PT \equiv C \qquad CPT \equiv 1$$

5 *The three different generations of matter come from the three possible alignments of the individual's temporal axis on the three axes of the universe:*

$$\Omega_c : \Omega_b : \Omega_r \sim m_e^{-1} : m_\mu^{-1} : m_\tau^{-1} \sim \|V_{\mathrm{ud}}\|^2 : \|V_{\mathrm{cd}}\|^2 : \|V_{\mathrm{td}}\|^2 \sim \ldots \tag{25d}$$

Each of the three axes, $\hat{p}, \hat{r}, \hat{t}$, hosts the Radii of their own specific relationships, i.e. gravitational, electrical, inertial.
An elementary individual, the electric lepton $R_h = R_e^\circ$ or the gravitational black hole $R_h = R_\bullet$, occupies an area of phenomenal spacetime given by its Radius R_h. Since in the transition from outside to inside R_h, space and time are interchanged, it follows a transition from a field force to an inertial regime. That is, elementary individuals, from the outside are the quantum of their space inside the universe, from the inside they are part of the Universe. An elementary individual cannot be composed of anything other than the CDM, i.e. $R_h = r^2/\tau$ where, when the threshold R_h is crossed, the variable τ, having reached its minimum R_h, remains constant. In summary, the following points apply to hosts (Universe, electric individuals and black holes) seen in their entirety as a whole:

- the whole has a curvature radius R_0 equal to its Radius R_h
- the horizon of the present in act has constant surface acceleration (gravitational o electric) equal to $1/\tau_0 = 1/R_h$
- the change of energy is related to change of area A, angular momentum J, and electric charge Q by

$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ \tag{25e}$$

  for $dJ = dQ = 0$, being the surface gravity $\kappa = 1/R_h$ and where A is the horizon area.

Unlike an elementary electric particle, whose radius is fixed by the constraint of equilibrium with the universe, a black hole can increase its radius by wrapping baryonic matter. Unlike a black hole, which has the Radius on the power axis, an elementary electric particle, which has the Radius on the radiating axis, can emit, as well as absorb radiant energy, whenever the rotation period of its spin aligns its radiant axis with the radiant axis of the universe.

The three areas of electrical interaction

Depending on the angle $\gamma = \gamma_A \pm \alpha/n$, we have all kinds of interactions, that is:

$\gamma_A = 0$	in the external area (Newton/Coulomb),
$\gamma_A = \pi/2$	in the border area (strong force),
$\gamma_A = \pi$	in the internal area (weak force).

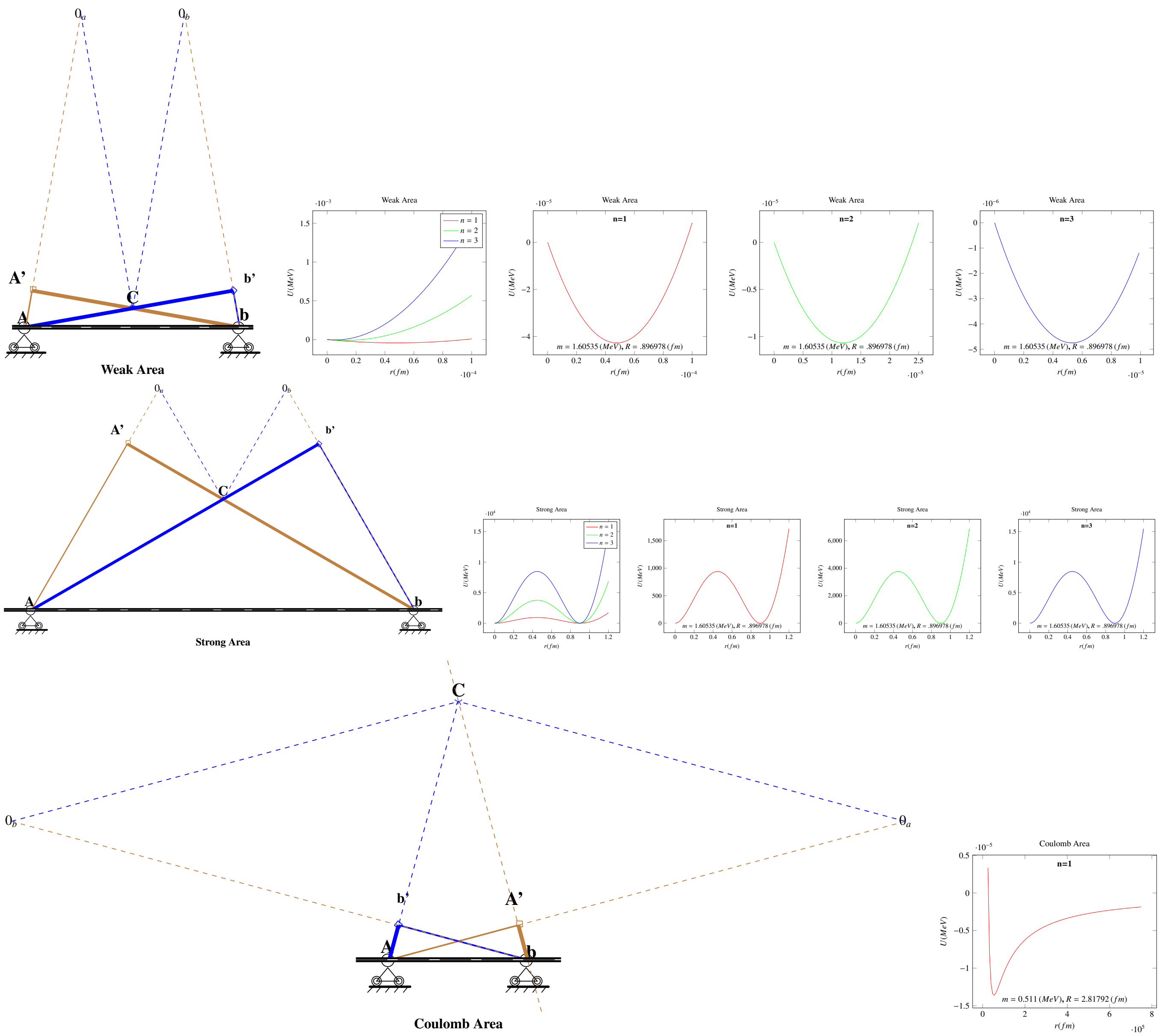

Figure 8. From top to bottom the electrical potential (24d) respectively for weak, strong and Coulomb interactions.

The whole range of the relationship is covered by the only equation (24d) (see fig. 10), where, since for electrical interactions it holds $mR = R_{\bullet}R^{\circ} = 1$ and $L/c = n/\alpha$, the (24d) becomes:

$$\mathbf{U} = Q\frac{1}{2}mc^2\left[-2V^{\diamond} + V^{\diamond 2} + \left(\frac{dr^{\diamond}}{d\tau}\right)^2 + n^2\alpha^{-2}V_{\vartheta}^{\diamond 2}\left(1 - V^{\diamond}\right)^2\right] \tag{25f}$$

where it is recalled that, while the potential $V^{\diamond} = \sin\gamma^{\diamond} \leq 1$ reverses when $r^{\diamond}$ crosses the threshold of the Radius R° passing from $V^{\diamond} = R^{\circ}/r^{\diamond}$ on the outside to $V^{\diamond} = r^{\diamond}/R^{\circ}$ on the inside and vice-versa, the pseudo potential $V_{\vartheta}^{\diamond}$, on the contrary, is always equal to $V_{\vartheta}^{\diamond} = R^{\circ}/r^{\diamond}$ when the native seat of the relationship is outside R°, to $V_{\vartheta}^{\diamond} = r^{\diamond}/R^{\circ}$ otherwise, and its formula does not reverse but it continues to grow when the distance $r^{\diamond}$, overflowing its seat, crosses the threshold R°.

From the behaviour of the $V_{\vartheta}^{\diamond}$ *in the (25f) it follows that the confinement of quarks and gluons into hadrons (and of the components of neutrinos into neutrinos) depends on the conservation of angular momentum.*

It has:

three real roots at $V \simeq \{0,\ 2L^{-2},\ 1 - L^{-1}\}$ i.e. γ or $\pi - \gamma \simeq \{0,\ 2L^{-1},\ \pi/2 - L^{-1}\}$;

a global minimum $U = -1/2\,mc^2$ at $V = 1$ i.e. γ or $\pi - \gamma \simeq \pi/2$ (on the center panel -mesons zone-);

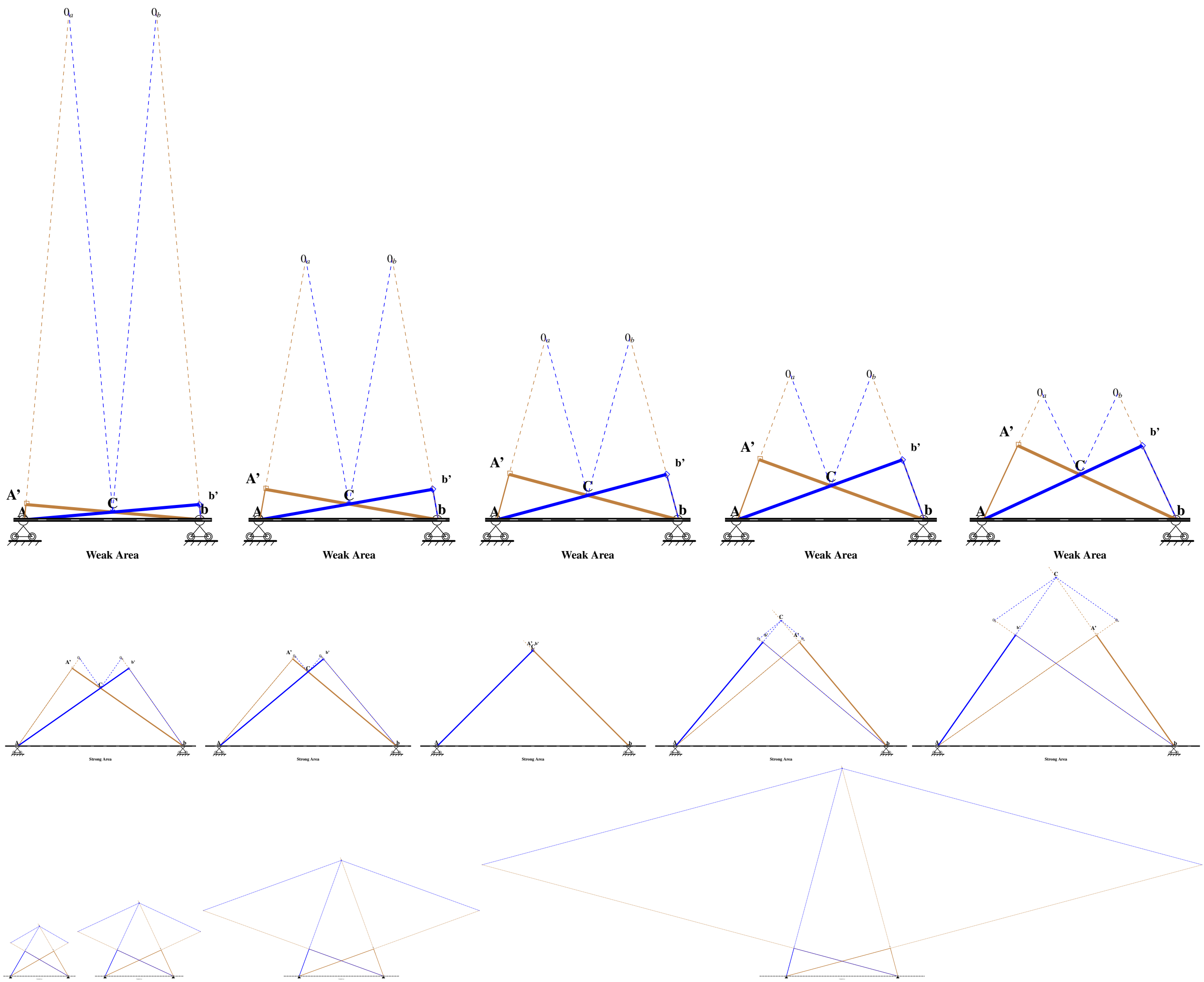

Figure 9. The evolution of the universal diagram of the relationship from interactions within the Radius, above, through the surface, in the center, and finally outside it, below.

a local minimum $U \simeq -1/2\,L^{-2}mc^2$ at $V \simeq L^{-2}$ i.e. γ or $\pi - \gamma \simeq \sqrt{2}L^{-1}$ (on the right and left panel -weak and Coulomb zone-); a local maximum $U \simeq \left(-3/8 + (1/2)^5 L^2\right) mc^2$ at $V = 1/2 - L^{-2}$ i.e. γ or $\pi - \gamma \simeq \pi/3 - L^{-2}$ (on the center panel -baryon/Higgs zone-).

In the passage between outside R and inside R the axes are reversed as follows:

Inside Radius	$\leftrightarrow$	Outside Radius
t	$\leftrightarrow$	r
p	$\leftrightarrow$	t
r	$\leftrightarrow$	$-p$

6 *the mass/energy present within the electric Radius ($r^\diamond = \sin^\diamond \gamma\, R_e^\circ \leq R_e^\circ$) is that of cold dark matter $R_\bullet = r^{\diamond 2}/R_{\Omega_b} = \sin^{\diamond\, 2}\gamma\, R_{\bullet e}$. This constraints the mass of neutrinos and of quarks in which CDM is condensed. Thus, neutrinos and quarks are present only within the electric Radius of a Host of which they constitute the content.*

that is the energy, i.e. Balmer's radiation, bosons $W^{\pm}, Z_0$, mesons, X, γ radiation, depends on the cosine

$$\Delta E_{n1}^{n2} = [\pi]^a\, m_e \left(\Delta \cos\left(j\,\pi \pm \frac{\alpha}{n}\right)\right)^{\pm 1} \tag{25g}$$

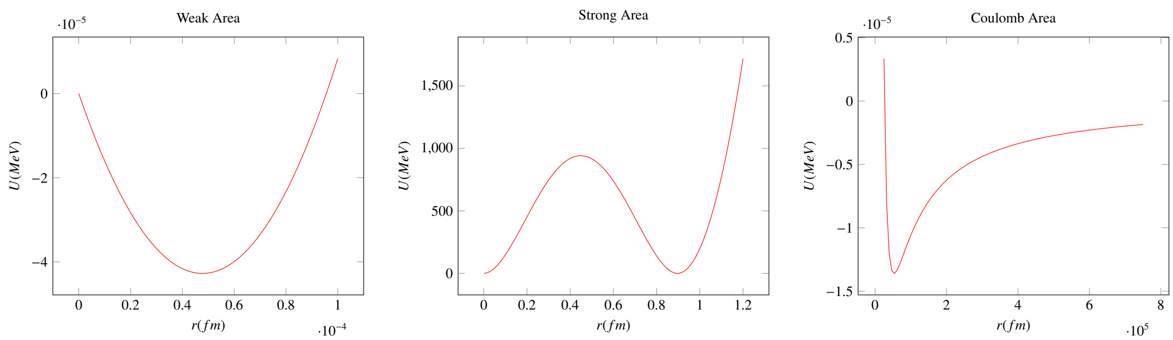

Figure 10.   The graph of eq. (24d) in its three areas. The weak and the Coulomb areas are symmetrical with respect to the t axis but with a very different r scale.

where m_e is the mass of the electron, $j = 0,\ 1/2,\ 1$ respectively in the Coulomb, strong and weak interactions and where the exponent is positive outside the Radius (Coulomb interactions), negative otherwise and, at last, the exponent "a" is equal to 0 or 1 depending on whether the orbital motion is radial or circular.

Quarks, which exist as such only in the strong interaction, where each individual component, each arranged on one of the three axes of space converging at the point $\gamma = \pi/2$, has a charge equal to $\pm 1/3$, because it is free to interact only one time out of three, in accordance with the cyclical alternation of its three moments (PotencyEnergyAct). Each moment corresponds to a colour of chromodynamics. Furthermore, it is easy to verify that in the linear geometry of the act it turns out

7 *The sum of two angles $\pi/3$ gives rise to a right angle: $\pi/3 +^{\diamond} \pi/3 = \pi/2$. Indeed:*

$$\sin^{\diamond} \pi/3 + \sin^{\diamond} \pi/3 = 2(1 - \cos \pi/3) = 1 = \sin^{\diamond}(\pi/3 +^{\diamond} \pi/3) = \sin^{\diamond} \pi/2 \tag{25h}$$

This property establishes the constitution of baryons from a ternary relation in the strong interaction area, where the charge for each individual is $-1/3$ and the total angle γ between each pair is $\pi/3 +^{\diamond} \pi/3 = \pi/2$. In other words, while the weak interaction is the relationship between an individual and its anti on opposite sides of the same axis, the strong interaction is the triadic relationship that takes place at the crossing point of the three axes of time, potency and radiation, between of them perpendicular, in which the three pairs, arranged at $(\pi/3 +^{\diamond} \pi/3) + (\pi/3 +^{\diamond} \pi/3) + (\pi/3 +^{\diamond} \pi/3)$, relate.

From these assumptions it follows that neutrinos, as they are electrically neutral, are constituted by a couple matter-antimatter $(-1, +1)$ linked via weak interaction. Analogously, while the quark Down is the electron in the strong area with charge $-1/3$, the quark Up is supposed to be constituted by a couple of individuals matter-antimatter $(-1/3, +1)$ where only one is engaged in the strong interaction, the one with charge $-1/3$, while the other is linked to this via weak interaction, far away, and therefore does not interfere with the strong interaction and has charge $+1$.

As a result, Coulomb interactions, which take place outside the Radius, typically occur between an electron of mass $m_e \simeq 0.511 MeV$ and its reflection in the nucleus which assumes electric Radius $R_e^{\circ} \simeq 2.81794 fm$ (the vice-versa is negligible). On the other hand, weak and strong interactions, which take place inside the Radius between leptons, typically occur between an electron of augmented mass $m_\epsilon = \pi m_e \simeq 1.60535 MeV$ and its reflection in the other lepton conjoined in the relationship, which assumes electric Radius $R_\epsilon = R_e^{\circ}/(\pi) \simeq 0.896978 fm$ (see fig.8). All electrical interactions share $L/c = n/\alpha$.

Coulomb and Weak area $(\gamma \to [\pi] \pm \alpha/n)$

The Coulomb interaction (see fig. 10) takes place in the external area where $\gamma \to \pm\alpha/n$ and the angle between spatial axes is γ_i. Its inverse, the weak interaction, vice-versa, takes place in the internal area where $\gamma \to \pi \pm \alpha/n$ and the angle between spatial axes is γ_e.
In the Weak zone the radiant axis passes through the potential axis. Therefore the Weak interaction violates both charge conjugation and parity in-variance. However, the weak interaction leaves systems invariant under the combination CP.
The weak relationship involves a couple of individuals matter-antimatter and takes place where $\gamma = \pi - \dfrac{\alpha}{n}$ where n in $\{1,2,3\}$ and $V^{\diamond} = (1 - \cos \gamma) = \dfrac{1}{2} \dfrac{\alpha^2}{n^2}$.
The result of the weak relationship between an electron and a positron are the three generations of *Neutrinos*, one for each of the three levels n of the weak interaction.

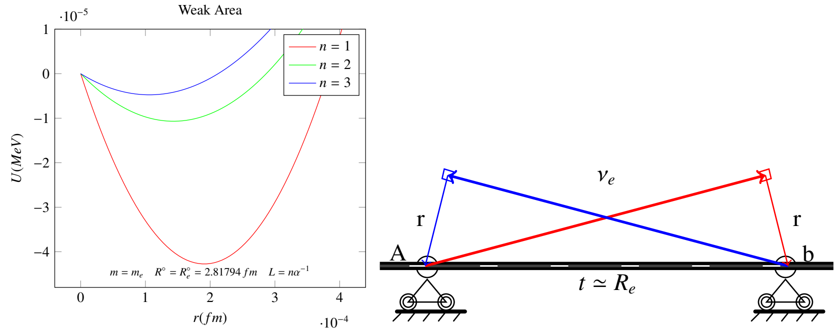

Figure 11. Neutrino: $\pi m_e\left[-V + 1/2V^2 + 1/2(nL)^2V^2(1-V)^2\right]$ where $R^\circ = .896978$ and $L = n\alpha^{-1}$. There is one generation for each of the three angular momentum levels predicted by the weak interaction.

Their mass emerges from the dark matter (CDM) contained within their space r_ν . Therefore we must have:

$$r_\nu = V^\diamond R^\circ_\epsilon = \frac{\alpha^2}{n^2} R^\circ_\epsilon$$

$$m_\nu = \frac{r^2}{R^{\circ 2}_\epsilon}\frac{R^{\circ 2}_\epsilon}{R_\omega} = V^{\diamond 2} m_\epsilon = \frac{\alpha^4}{n^4}\pi m_e$$

Table I.

neutrino	n	V[a] $4(2n-1)/(n\alpha^{-1})^2$	R_h R°_ϵ[b]	r (m) VR_h	cross section (m^2) $8/3\cdot\pi r^2$	m (eV/c^2) r^2/R_ω[c]
ν_e	1	0.000213005	.896981 fm	1.9106^{-19}	3.058204^{-37}	0.072837
ν_μ	2	0.000159754	.896981 fm	1.4330^{-19}	1.720240^{-37}	0.040971
ν_τ	3	0.000118336	.896981 fm	1.0615^{-19}	9.438901^{-38}	0.022481

[a] the $4(2n-1)$ factor is suggested by the cross section measures of neutrino: *"The average electroweak characteristic size is $r^2 = n \times 10^{-33}\ cm^2$ ($n \times 1$ nanobarn), where $n = 3.2$ for electron neutrino, $n = 1.7$ for muon neutrino and $n = 1.0$ for tau neutrino" [21]* . It should be due to an oscillation around the equilibrium point.
[b] $R^\circ_\epsilon = R^\circ_e/\pi = 8.969810^{-16}m$
[c] $1/R_\omega = m_\epsilon/(R^\circ_\epsilon)^2$ where $m_\epsilon = m_e\pi = 1.60535\ MeV/c^2$

Table II. Coulomb-Weak Areas

area	γ	$\Delta\mathbb{E}^{\diamond n2}_{n1} = \left(\Delta\left[\cos\gamma\right]^{n2}_{n1}\right)^{\pm 1} mc^2$ [a]	products
outside	$+\frac{\alpha}{n}$	$= \frac{1}{2}\Delta\frac{1}{r^\diamond} = \frac{1}{2}\left(\frac{1}{n_1^2\alpha^{-2}R^\circ_{Tot}} - \frac{1}{n_2^2\alpha^{-2}R^\circ_{Tot}}\right)$	***Balmer's radiation***
inside	$\pi - \frac{\alpha}{n}$	$= \frac{2}{\Delta r^\diamond} = \left(\frac{1}{2}\left(\frac{1}{n_1^2\alpha^{-2}R_{\bullet Tot}} - \frac{1}{n_2^2\alpha^{-2}R_{\bullet Tot}}\right)\right)^{-1}[\pi]$ [b]	***bosons***

[a] the exponent is equal to 1 or -1 depending on the area of relation: Coulomb or Weak respectively.
[b] the π exponent is equal to 1 or 0 depending on the kind of motion: circular or radial.

A jump between different levels ΔU, corresponding to different angular momentum L (see tab. II), gives place:

- electromagnetic waves in the electromagnetic Interaction, where $\Delta U \to \Delta E$.
  We arrive at the Balmer's formula considering that $R^\circ_{Tot} = (R^\circ{}_e + R^\circ{}_{nucleus})$ where $R^\circ{}_e >> R^\circ{}_{nucleus}$ and therefore $R^\circ_{Tot} \simeq R^\circ{}_e$.
- $W^\pm$ and Z_0 bosons in the weak interaction where $\Delta U \to \Delta M$ and where the levels are only three(1,2 and 3).
  In particular, in the beta decay, if $R^\circ{}_a$ and $R^\circ{}_b$ are the heterologous individuals of a quark Down and anti-Up, jumping

from n=2 to n=1, we have $\Delta M \simeq 2(1-1/4)^{-1}\alpha^{-2}\pi m_{0e} = 80.3915$ GeV which is equal to the mass of $W^{\pm}$.
Analogously, if $R^{\circ}{}_a$ and $R^{\circ}{}_b$ are the heterologous individuals of a quark Up and anti-Up on n=2 and n=3, and both these individuals jump on n=1, then we have $\Delta M = W^{\pm} + (1-1/9)^{-1}\alpha^{-2} m_{0e} = 91.1871$ GeV which is equal to the mass of Z_0.
More generally, a change from $n = i$ to $n = j$ is never direct since it requires less energy to change from $n = i$ to $n = 1$ and then from $n = 1$ to $n = j$.

$$\mu^- + \bar{\nu}_\mu \rightarrow W^- \rightarrow e^- + \bar{\nu}_e$$

$$q_d + \bar{q}_u \rightarrow W^- \rightarrow e^- + \bar{\nu}_e$$

STRONG area ($\gamma{\rightarrow}\pi/2{\pm}\alpha/n$)

The strong relation (see fig.14) takes place on the border where $\gamma{\rightarrow}\pi/2{\pm}\alpha/n$
It has two main symmetrical configurations :

- a pair matter anti-matter with vertex on the border of the Radius, with $L = n$ and $V^{\diamond} = 1$ since $V^{\diamond} = \sin^{\diamond}(\gamma) = \sin^{\diamond}(\varphi{+}^{\diamond}\psi) = \sin^{\diamond}(\pi/3 +^{\diamond} \pi/3) = \sin^{\diamond}(\pi/2)$
- a triad *crb* with vertex on the middle of the Radius, with $L = n\alpha^{-1}$ and $V^{\diamond} = 1/2$ since $V^{\diamond} = \sin^{\diamond}(\varphi) = \sin^{\diamond}(\psi) = \sin^{\diamond}(\pi/3)$ for each of the three pairs

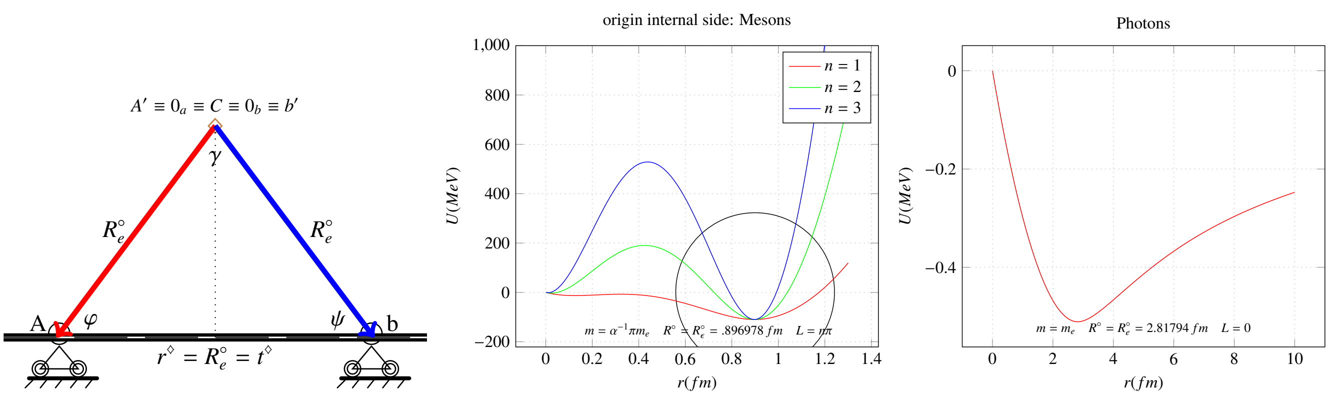

Figure 12. Photon: the photon is characterized by $\gamma = \pi/2$, $Spin = \hbar$, $E = h\nu$, $v = c$ and $r = \overline{aC} + \overline{bC} = 2R^{\circ}_e$

Table III. Strong Area

area	γ	$\Delta\mathbb{E}^{\diamond n2}_{n1} = \left(\Delta\,[\cos\gamma]^{n2}_{n1}\right)^{\pm 1} mc^2$ [a]	products
outside	$\frac{\pi}{2} - \frac{\alpha}{n}$	$= \frac{1}{2}\Delta\frac{1}{r} = \left(\frac{1}{n_1\alpha^{-1}R^{\circ}_{Tot}} - \frac{1}{n_2\alpha^{-1}R^{\circ}_{Tot}}\right)$	**X,** γ ***radiation***
inside	$\frac{\pi}{2} + \frac{\alpha}{n}$	$= \frac{2}{\Delta r} = \left(\frac{1}{n_1\alpha^{-1}R_{\bullet Tot}} - \frac{1}{n_2\alpha^{-1}R_{\bullet Tot}}\right)^{-1} [\pi]$ [b]	***mesons***

[a] the exponent is equal to 1 or -1 depending on the side of the relation: external or internal respectively.
[b] the π exponent is equal to 1 or 0 depending on the kind of motion: circular or radial.

A jump between different levels ΔU, corresponding to different angular momentum L (see tab. III), gives place:

- **X,** γ ***radiation*** outside the radius, where $\Delta U \rightarrow \Delta E$.

- **Mesons** inside the radius, where $\Delta U \rightarrow \Delta M$.
  Mesons are constituted by a couple quark-antiquark which links two individuals of equal and opposite charge 1/3. The presence of both matter and antimatter in the quarks UP doesn't change the structure of interaction, since only one of them $(\pm 1/3)$ is engaged in the strong interaction while the conjoined (∓ 1) is linked to this via weak interaction, therefore far outside the range of strong interaction. Mesons can decay or via electromagnetic interaction in presence of a couple of quarks of the same type, or via weak interaction otherwise. In the neighbourhood of $V^{\diamond} = 1$ or $\gamma = \pi/2$ the eq. (24d) becomes compatible with the behaviour of a quantum harmonic oscillator.

All the electrical relationships, inside, on the border or ouside the radius R, having to be linked by an attractive force, always involve a matter-antimatter pair.
The only exception to this rule is the interaction between three quarks, which links three individual homologues with charge -1/3 (the eventual charge + 1, in the UP quarks, is drawn via weak interaction and does not participate in the strong interaction) which forms the baryons. (see fig. 13). The three quarks constituent, having the same charge -1/3, repel each other but, since each one occupies one of the three possible states, for the Pauli exclusion principle they cannot escape since whatever change implies to invade the place of the other.

In its simplest configuration, as we expect the case of the proton to be, we expect a symmetrical arrangement of three quarks $Q = -1/3$ (see 13) at $\varphi = \psi = \pi/3$ between them and $L = \alpha^{-1}$.

The (24d) presents a local maximum at $\varphi = \psi = \pi/3$ where $V = 1/2$ and therefore $U_{2\pi/3} \simeq 3/2 \cdot Q \cdot mc^2 \left[-1 + 1/4 + (1/2)^4 L^2\right]$.
Therefore, for the simplest baryon, where $m = m_{\epsilon} = \pi m_e$ and $L = \alpha^{-1}$ and $Q = -1/3$ we have $U_{2\pi/3} = 941.48$ MeV.
For the proton, we must subtract the mass of the two leptons (positrons with $m_{\epsilon} = \pi m_e$), originally belonging to the two UP quarks, that are now linked via weak interaction as two neutrinos, $U_{2\pi/3} - 2m_{\epsilon} = 938.2704 MeV \simeq m_p$.
Analogously, for the neutron, we must subtract the mass of the single lepton (positron with $m_{\epsilon} = \pi m_e$): $U_{2\pi/3} - m_{\epsilon} = 939.876 MeV \simeq m_n$

Nevertheless, like the mesons, the baryon's quarks too oscillate in the neighbourhood of $V^{\diamond} = 1/2$ or $\widetilde{\varphi} \simeq 2\pi/3$ since $\gamma = \widetilde{\varphi} +^{\diamond} \widetilde{\psi} = \pi/2 + \alpha/n$. The eq. (24d) for baryons, therefore, becomes compatible with the behaviour of a three-dimensional harmonic oscillator, for each quark being $Q = -1/3$ and $V^{\diamond} = 1/2$.

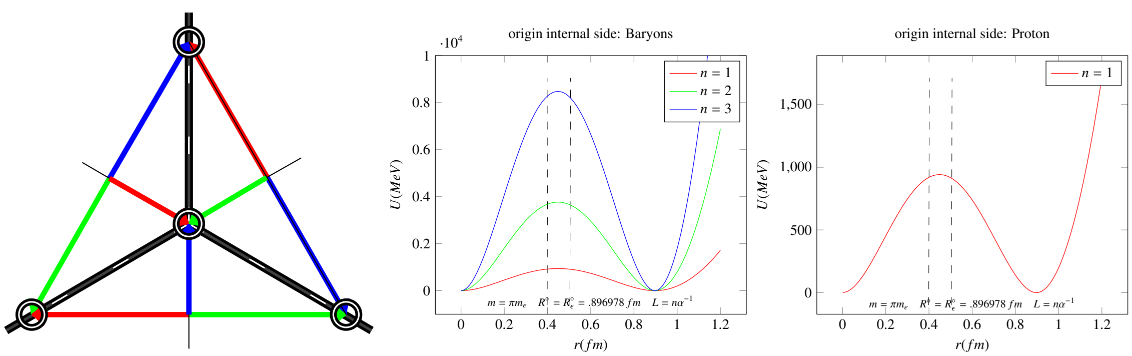

Figure 13. Baryon: the baryon is characterized by $\gamma = \pi/2$, $\varphi = \psi = \pi/3$
the plot of the universal equation (24d). Strong Area repulsive force between three negative charge -1/3 : $m = \pi 0.511\, MeV$; $L = n\alpha^{-1}$; $R^{\circ} = .896978 fm$.

The internucleon potential takes place on the external side of the strong area where $R^{\circ} = R^{\circ}_{\epsilon} = R^{\circ}_{e}/(2\pi) = 0.896978$ fm.

Cosmology

Gravity is thought to be the dominant force in modern cosmology, shaping an expanding (or, otherwise, contracting) universe which is Homogeneous and Isotropic on large scale. Modern Cosmology is therefore based on Einstein's field equations, Friedmann equations and FLRW metric.

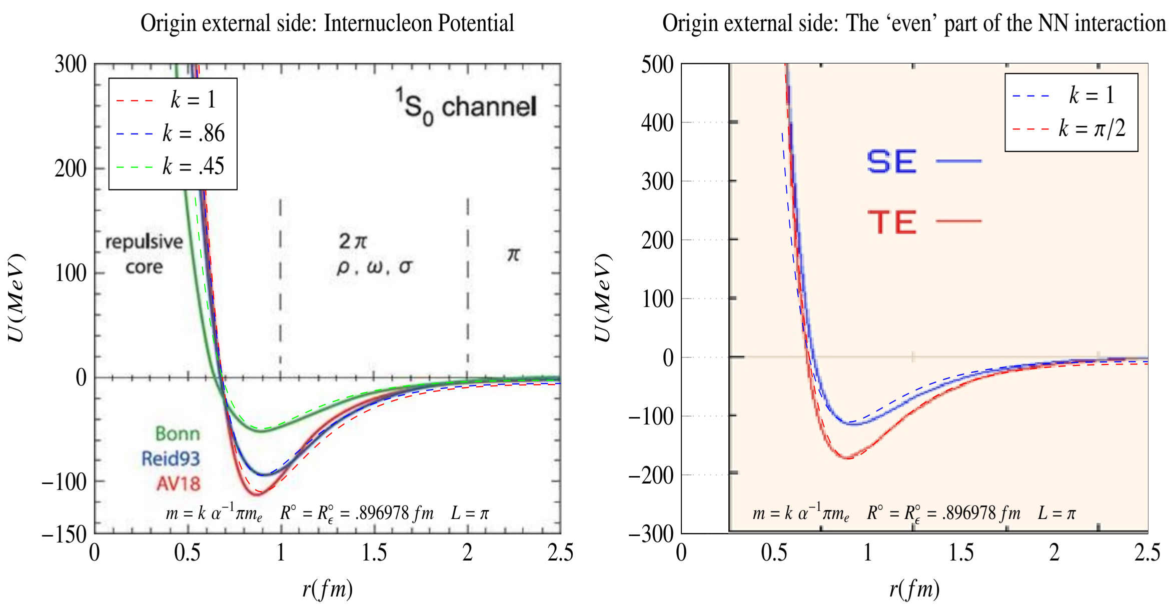

Figure 14. The internucleon potential comes from the external side of the strong area, where $R^\circ = R^\circ_\epsilon = R^\circ_e/\pi = 0.896978$ fm and the eq. (24d) is particularized as: $U = mc^2\left(-V^\diamond + \frac{1}{2}V^{\diamond 2} + \frac{1}{2}\frac{L^2}{c^2}\left(\frac{R^\circ}{r^\diamond}\right)^2(1-V^\diamond)^2\right)$. The potential $V^\diamond = \frac{R^\circ}{r^\diamond}$ for $r^\diamond \geq R^\circ$, $V^\diamond = \frac{r^\diamond}{R^\circ}$ for $r^\diamond \leq R^\circ$. The results are in good agreement with the experience, though neglecting spin-orbit and hyperfine structure terms.

On the left, the three plots represent respectively the comparison with the AV18, Reid93 and Bonn potential.
On the right, the "even" part of the nucleon-nucleon interaction. This is the part of the interaction that applies when the relative wavefunction is even (symmetric) under exchange of two particles. Notice that it is attractive as described above, with a short range repulsion. The red curve on the right that is labeled TE is the Triplet-Even (S=1, T=0) case that applies to the deuteron (neutron and proton with parallel spins) in its J=1+ ground state. It is just barely negative enough to bind a neutron and proton into a stable nucleus. Two neutrons cannot be in this configuration because it would violate the Pauli principle for identical particles. The blue curve on the right that is labeled SE is the Singlet-Even (S=0 T=1) case that applies to the deuteron (as well as the di-neutron or di-proton) with anti-parallel spins (in a J=0+ state). You can see that it is not as negative as the TE interaction, and this small difference is enough that it cannot produce a bound state.

The General relativity's mathematical description of universe

The Friedmann–Lemaître–Robertson–Walker (FLRW) solution represents the dynamic point of view of the Universe and its comoving coordinates (including the cosmic time) are universal and play the same roles as those of an observer falling freely under the influence of that object.

Although The FLRW metric is an exact solution of Einstein's field equations of general relativity, it doesn't derive from Einstein's field equations: it follows from the geometric properties of homogeneity and isotropy, that is from the symmetry properties in the case of complete isotropy. In this special case of an isotropic space, the curvature properties are determined by just one constant which is the scalar curvature.
"To investigate the metric it is convenient to start from geometrical analogy, by considering the geometry of isotropic three-dimensional space as the geometry on a hypersurface known to be isotropic, in a fictitious four-dimensional space (This four-space is understood to have nothing to do with four-dimensional space-time). Such a space is a hypersphere; the three-dimensional space corresponding to this has a positive constant curvature." [9, pag 334]

It is possible to establish a spherical coordinate system, with inclination γ, on the spherical surface of Radius R_0 (R_0 is the "radius of curvature" of the Universe). Usually, these Spherical coordinates (R_0, $\gamma^\diamond$) are converted into cylindrical coordinates ($r^\diamond$, $t^\diamond$) which correspond to the cosmic coordinates (d_M, t). The resulting metric, that is the FLRW metric:

$$-\,ds^2 = -c^2 dt^{\diamond 2} + a(t^\diamond)^2\left(\frac{dr^2}{1 - r^2/R_0^2} + r^2 \mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2\right) \tag{26a}$$

or equivalently:

$$-\,ds^2 = -c^2 dt^{\diamond 2} + a(t^\diamond)^2 R_0^2\left(\mathrm{d}\gamma^2 + \sin^2\gamma\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right) \tag{26b}$$

introduces a scale factor varying with time:

$$a(t^\diamond) = \frac{\lambda_{emitted}}{\lambda_{received}} = \frac{1}{1+z} \tag{26c}$$

However, since it evolves according to Einstein's field equations, the metric has an analytic solution to Einstein's field equations given by the Friedmann equations when the energy-momentum tensor is similarly assumed to be isotropic and homogeneous.

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3} \tag{27a}$$

$$\dot{H} + H^2 \equiv \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{27b}$$

The IRPL Universe

Nevertheless, Einstein's early model of universe was a closed, static, spherical Universe with a finite but unbounded structure. To achieve a static state, he introduced the cosmological constant Λ to counteract gravitational collapse. The balance between gravity and cosmological constant gives rise to a constant universe's radius:

$$R_E = \sqrt{\frac{c^2}{4\pi G\rho}} = \frac{c}{H_0}\sqrt{\frac{2}{3}\frac{\rho_c}{\rho}} \tag{28}$$

Einstein later abandoned this static universe model after Hubble's discovery of the universe's expansion in 1929.

Einstein, based on observational evidences, did not believe that the universe was a black hole. The main reasons are that a black hole is considered a region of space, where everything falls inward, and therefore has a center and an event horizon. Furthermore, Einstein did not believe that physical black holes could exist at all as real astrophysical objects, thinking that angular momentum and other forces would prevent the collapse.

In the geometry of nature, instead, the Radius is not located in the external, derived, phenomenal momentum spacetime, that is in a region of space, but in the internal, primitive, space of potential that reverses space with time. Indeed the universe, if it does not have a spatial center, it has a temporal beginning.

It is the relation between the parts and the whole, that is, between the universe and its matter. While, in the phenomenal momentum space, the Radius gives rise to the potential $V^\diamond = r^\diamond/\tau$ of CDM, which is attractive, and therefore $R_c = r^{\diamond 2}/\tau$ with $\tau^\diamond = \tau^\diamond_{max} = R_0 = R_\Omega$, in the potential space of cosmology it gives rise to the Hubble recession velocity $v^\diamond = r^\diamond/t^\diamond$ with $t^\diamond = t^\diamond_{max} = R_0 = R_\Omega$.

Since any interaction always takes place within a Host Radius $\tau \leq R_h$, momentum $R_p = r^{\diamond 2}/\tau$ is never null. In other words, the momentum $p^\diamond = r^\diamond/\tau$ between two bodies separated by an arbitrary distance $r^\diamond$, while decreasing as τ increases, stops at a minimum when τ reaches its maximum limit which is equal to the Radius of the universal R_h, host of the interaction. That is, the radius of the universe R_ω or of a black hole or of an elementary particle. It therefore acquires a fundamental meaning in the interactions involving matter within a Host Radius. That is, in the relationship between the whole and its parts. In this case we have $\tau = R_h$ and R_p takes the name of Cold Dark Matter $R_c = r^{\diamond 2}/R_h$. In particular, within the universe ($\tau_{max} = R_\omega$), $A^\diamond = 1/R_\omega$ is the minimum local acceleration, $p^\diamond = r^\diamond/R_\omega$ is the minimum local momentum, i.e. the Hubble recession velocity, and $R_c = r^{\diamond 2}/R_\omega$ is the **C**old**D**ark**M**atter (CDM).

The radius R_c of CDM is therefore always positive, and since the baryonic mass of an elementary particle $R_{\bullet_b}$ arises within the universe as a condensate of CDM (the interior of baryonic particles is in turn made of CDM, precisely the CDM inside a sphere of radius R°_b), it is always positive.

In the scenario of Nature's geometry, then, we expect an inertial universe, apparently expanding, with a constant and finite Radius, and determined not by chance but by geometric proportions.

Indeed, the flatness of the universe is supported by multiple lines of observational evidence. Here some of the main key proofs:

1. *Cosmic Microwave Background (CMB) Anisotropies* : both the *Wilkinson Microwave Anisotropy Probe (WMAP)* and *Planck satellite* show that the angular size of the first peak in the CMB power spectrum is consistent with a flat universe. If the universe were curved, light from the early universe would be either focused (positive curvature) or spread out (negative curvature), shifting the peak's location. However, data aligns with a flat geometry.

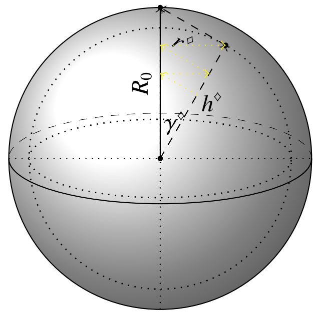

Figure 15. hypersphere with positive curvature R_0 and universal FLRW co-moving coordinates (observer falling freely).

2. *Friedmann Equations and Critical Density*: According to the *Friedmann equations*, derived from General Relativity, which describe the universe's expansion, the flatness condition occurs when the total energy density (Ω) equals 1. Observations suggest that the total energy density is very close to 1.

3. *Large-Scale Structure of the Universe*: The distribution of galaxies and cosmic voids follows patterns expected in a flat universe. Moreover, Baryon Acoustic Oscillations (BAOs), which are imprints of sound waves in the early universe, also match predictions for a flat cosmos.

These pieces of evidence collectively indicate that the universe is very close to flat, with only tiny deviations within measurement uncertainties.

Definitions, Notation and conventions

We indicate with:

- R_Ω the constant gravitational Radius of the Universe, i.e. the constant total amount of matter-energy of the universe
- $R_{(t)} = c/H_{(t)}$ the Radius of curvature of the observable Universe, i.e. the positive curvature of the hypersphere corresponding to the observable universe

  (as usual, it is indicated with t_0 , H_0 and $R_0 = c/H_0$ respectively the cosmic time, the Hubble constant and the curvature Radius at the present time).

Finally it is assumed, on the basis of the evidence of observations, $R_\Omega = R_0 = c/H_0$.

Cosmology on the path of light

About the fig. (15)

- the vector $r^\diamond$ is the radial distance in the linear coordinates system $r^\diamond = c\left(t^\diamond_{receiving} - \tau^\diamond_{sending}\right)$, where $\tau^\diamond$ and $t^\diamond$ are the proper time starting (with zero) from the point of minimum distance;
- the vectors R_0 represents the Hubble time $R_0 = c/H_0$ now, at the instant of reception. The vector $h^\diamond$ represents the Hubble time at the instant of emitting.

Therefore:

$$\vec{r}^\diamond + \vec{h}^\diamond = \vec{R}_0 \qquad \sin\gamma + \cos^\diamond\gamma = 1 \tag{29a}$$
$$r^\diamond = \sin\gamma\, R_0 \qquad h^\diamond = \cos^\diamond\gamma\, R_0 = (1-\sin\gamma)\, R_0 \tag{29b}$$

and

$$\mathrm{d}h^\diamond = R_0(1-\sin\gamma)\mathrm{d}\gamma \qquad \mathrm{d}r^\diamond = -R_0(1-\sin\gamma)\mathrm{d}\gamma \qquad \mathrm{d}h^\diamond = -\mathrm{d}r^\diamond \tag{29c}$$

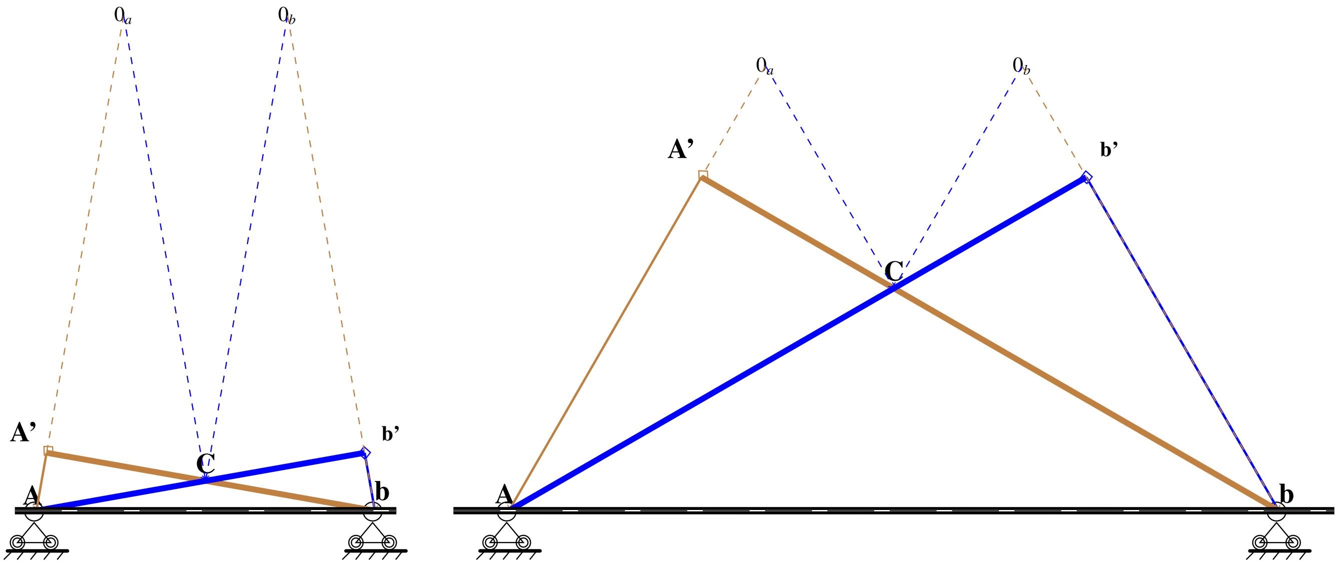

Figure 16. The cosmology investigates the area $\pi > \gamma > \pi/2$. The segment $CO_b b$ (or $CO_a A$) represent the radius of Universe $t^{\diamond}_{max} = R_\omega$. The segment $b'A$ (or $A'b$) represents the radial distance $r^{\diamond} = \sin\gamma R_\omega$. The segments $b'b$ and $A'A$ represent the momentum Radius $R_m = b'b + A'A = 2 * r^{\diamond} \tan^{\diamond}\gamma/2 = r^{\diamond}\tan^{\diamond}\gamma$. The line of the present Ab is the arc of circumference $b(\gamma)R_\omega\,\gamma$ of radius R_ω and corresponds to the proper distance (or comoving distance) of universe.

and

$$a^{\diamond}(\gamma) = \frac{\lambda_{emitted}}{\lambda_{received}} = \frac{h^{\diamond}}{R_0} = (1 - \sin\gamma) \quad \equiv \quad a(t) = \frac{1}{1+z} \qquad \textit{the scale factor} \tag{29d}$$

and, from the eq (29d)

$$\gamma = \arcsin\frac{z}{z+1} \qquad\qquad z = \frac{1}{1-\sin\gamma} - 1 \tag{29e}$$

and from the (9j,9k) and the equivalence between inertial and force fields, the equivalence of the three redshifts:

$$\text{Gravitational redshift} \quad \frac{1}{\sqrt{g_{00}}} = \frac{1}{\sqrt{2M/r}} \qquad = \frac{1}{1-\sin\gamma} \tag{30a}$$

$$\text{Doppler redshift} \quad \sqrt{\frac{1+v}{1-v}} = \sqrt{\frac{1+\tanh\zeta}{1-\tanh\zeta}} \qquad = \frac{1}{1-\sin\gamma} \tag{30b}$$

$$\text{FLRW redshift} \quad \frac{R_0}{h^{\diamond}} = \frac{1}{a} = 1+z \qquad = \frac{1}{1-\sin\gamma} \tag{30c}$$

The cosmological theses

Therefore

$$-ds^2 = -c^2dt^{\diamond 2} + a(t^{\diamond})^2\, b^2_{\Gamma/2}\left(\frac{dr^{\diamond 2}}{1 - b^2_{\Gamma/2} r^{\diamond 2}/R_0^2} + r^{\diamond 2}\mathrm{d}\theta^2 + r^{\diamond 2}\sin^2\theta\,\mathrm{d}\phi^2\right) \tag{31}$$

$$-ds^2 = -c^2dt^{\diamond 2} + a(t^{\diamond})^2\, b^2_{\Gamma/2}\, R_0^2\left(\mathrm{d}\gamma^2 + \sin^2\gamma\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)\right) \tag{32}$$

where

$$\frac{b^2_{\Gamma/2}\, dr^{\diamond 2}}{1 - b^2_{\Gamma/2} r^{\diamond 2}/R_0^2} = \frac{dr^2}{1 - r^2/R_0^2} \tag{33}$$

In reality, in the linear geometry of the act there are no curves, there are no differentials or integrals. Nevertheless, along the lines of quadratic geometry, it is possible to define derivatives and integrals to trace the power.

$$d\sin^{\diamond}\gamma = \cos^{\diamond}\gamma\ \mathrm{d}\gamma \tag{34}$$

$$dr^{\diamond} = R_{\omega}d\sin^{\diamond}\gamma = R_{\omega}\cos^{\diamond}\gamma\ \mathrm{d}\gamma \tag{35}$$

$$\frac{dr^{\diamond}}{1-r^{\diamond}/R_0} = \frac{dr^{\diamond}}{\cos^{\diamond}\gamma} = R_{\omega}d\chi \overset{linear-quadratic}{\Longleftrightarrow} R_{\omega}d\chi = \frac{\mathrm{d}r}{\cos\gamma} = \frac{\mathrm{d}r}{\sqrt{1-r^2/R_0^2}} \tag{36}$$

Note that $b_{\Gamma/2}$ depends only on the global angle γ and is therefore constant along the integration line of r.
Since the Universe is expanding, the (6) asserts that:

$$V^{\diamond} = \frac{R_m}{r^{\diamond}} = \tan^{\diamond}\gamma = \frac{r^{\diamond}}{\tau} = p^{\diamond}/mc \tag{37a}$$

and therefore (see fig.16):

$$\frac{R_m}{r^{\diamond}/\cos^{\diamond}\gamma} = \sin\gamma = \frac{r^{\diamond}}{R_0} = v^{\diamond}/c \tag{37b}$$

Since from the (2b) it is $d\sin^{\diamond}\gamma = \cos^{\diamond}\gamma\ \mathrm{d}\gamma$, we have:

$$dr^{\diamond} = R_{\omega}d\sin^{\diamond}\gamma = R_{\omega}\cos^{\diamond}\gamma\ \mathrm{d}\gamma \tag{37c}$$

$$dR_m = \tan\gamma\, dr^{\diamond} = \frac{\sin^{\diamond}\gamma}{\cos^{\diamond}\gamma}R_{\omega}d\sin^{\diamond}\gamma = R_{\omega}\sin^{\diamond}\gamma\ \mathrm{d}\gamma \tag{37d}$$

The (37d) conforms to, and support, the holographic principle and the first law of black hole mechanics [10–12].
To get the linear metric of the universe actually seen by an observer inside the universe, it is necessary to take into account the extra path in the Radius of CDM

$$c\Delta t^{\diamond} = b(\gamma)\ r^{\diamond} \quad \Rightarrow \quad c\mathrm{d}t^{\diamond} = b(\gamma)\,\mathrm{d}r^{\diamond} \tag{37e}$$

and, dividing by $\cos^{\diamond}\gamma = a\left(t^{\diamond}\right)$, we get:

$$\mathrm{d}\,d_M = \frac{cdt^{\diamond}}{a(t^{\diamond})} = b(\gamma)\ R_0 d\chi \tag{37f}$$

At last, based on considerations of a geometric nature on the scheme of the universal relationship (see fig.16), we introduce the cosmological thesis:

Thesis 1 *the light path between an emitter and a receiver has an extra path in the total Radius of the observer and observed, so that, in a free falling frame, we have:*

$$\mathrm{d}\,d_{M\,observed} = \frac{\mathrm{d}r^{\diamond}}{\cos^{\diamond}\gamma} + \mathrm{d}R_m = (1+v^{\diamond})R_0\,\mathrm{d}\chi = b(\gamma)\,R_0\,\mathrm{d}\chi \tag{37g}$$

where $v^{\diamond}(\gamma)$ is the Hubble velocity between the sender and the receiver and $b_\Gamma = \left(1+v^{\diamond}\right)$ the Rpath factor.

Note that the Rpath factor $b(\gamma)$ is constant along the path of integration.

The metric of the universe actually seen by the observer

Since radiation density is crucial in the early universe, its imprint remains in the CMB and early structure formation, but becomes negligible in the later stages compared to matter, and since the IRPL model does not contemplate dark energy $\Omega = \Omega_c + \Omega_b + \Omega_r \approx \Omega_c + \Omega_b \approx 1$, we can provisionally compute the cosmological distances since the matter dominated era, with good precision, neglecting the radiation density:

$$d_M = b(\gamma)\int_0^{\gamma} R_{\omega}d\chi = b(\gamma)\cdot\frac{c}{H_0}\gamma = \frac{c}{H_0}\gamma + R_m(\gamma) \tag{38a}$$

That is:

$$d_M = b(\gamma)\int_0^\gamma R_\omega d\chi = \frac{c}{H_0}\left(1+\sin\gamma\right)\gamma \qquad = \frac{c}{H_0}\left(1+\frac{z}{z+1}\right)\arcsin\left(\frac{z}{z+1}\right) \tag{38b}$$

$$d_A = a d_M = \frac{c}{H_0}\left(1-\sin^2\gamma\right)\gamma \qquad = \frac{c}{H_0}\frac{(2z+1)}{(z+1)^2}\arcsin\left(\frac{z}{z+1}\right) \tag{38c}$$

$$d_L = \frac{d_A}{a^2} = \frac{c}{H_0}\frac{1+\sin\gamma}{1-\sin\gamma}\gamma \qquad = \frac{c}{H_0}\,(2z+1)\arcsin\left(\frac{z}{z+1}\right) \tag{38d}$$

For a distance determination that also takes radiation into account, it is necessary to resort to the FLRW metric and to the Friedmann equations revised in light of the b_Γ factor. That is, the (26b) revised by multiplying the spatial terms by b_Γ:

$$\frac{c^2 dt^{\diamond 2}}{a\left(t^\diamond\right)^2} = b^2\left(\gamma\right)\left(R_0^2 d\chi^2 + r^{\diamond 2} d\theta^2 + r^{\diamond 2}\sin^2\theta d\phi^2\right) \tag{39a}$$

and similarly the Friedmann equations (27a) considering that:

1. the increase $\mathrm{d}d_M$ (see eq. 38a) represents the progressive advancement in the path $\mathrm{d}\chi$ between a predetermined sender-receiver pair whose distance Γ remains fixed. In other words, the extra path $\mathrm{d}R_m \propto \mathrm{d}r^\diamond$, where the multiplicative coefficient $V^\diamond = \sin\gamma$ is constant and depends on the angle γ of the path between the sender-receiver pair. In the FLRW metric, on the contrary, the increment of the path coincides with the increment of the distance $\mathrm{d}\chi \equiv \mathrm{d}\gamma$.

   Therefore, in the quadratic FLRW metric, when $\mathrm{d}\chi \equiv \mathrm{d}\gamma$, the increases $\mathrm{d}d_M$ and $\mathrm{d}z$ along the Line Of Sight become:

$$\mathrm{d}d_M = \frac{c}{H_0}(1+\sin\gamma+\gamma\cos\gamma)\mathrm{d}\gamma \tag{39b}$$

$$dz = \frac{\cos\gamma}{(1-\sin\gamma)^2}d\gamma = \sqrt{(1+\sin\gamma)\,a^{-3}}\,d\gamma = \sqrt{2a^{-3}-a^{-2}}\,d\gamma \tag{39c}$$

2. the Friedmann equation (27a) is expressed as a function of t. In other words, $\dot{a}$ stands for $\mathrm{d}a/\mathrm{d}t = \mathrm{d}a/\mathrm{d}t^\diamond \cdot \mathrm{d}t^\diamond/\mathrm{d}t$. Now, since $\mathrm{d}t^\diamond = a\,\mathrm{d}d_M = b(\gamma)\,a\,R_0\,\mathrm{d}\gamma$ while $\mathrm{d}t = a\,R_0\,\mathrm{d}\gamma$, we have that

$$H_{(t^\diamond)} = \frac{\mathrm{d}t}{\mathrm{d}t^\diamond}\cdot H_{(t)} = \frac{R_0}{d'_{M(\gamma)}}\cdot H_{(t)} = \frac{H_{(t)}}{(1+\sin\gamma+\gamma\cos\gamma)} \tag{39d}$$

therefore the Friedmann equation (27a) becomes:

$$H^2_{(t^\diamond)} = \left(\frac{\dot{a}_{(t^\diamond)}}{a_{(t^\diamond)}}\right)^2 = \frac{8\pi/3\,G\rho - kc^2 a^{-2}_{(t^\diamond)}}{(1+\sin\gamma+\gamma\cos\gamma)^2} \tag{39e}$$

and at last, provisionally neglecting the density of radiation Ω_r, we have again the (38b, 38c, 38d) in the form:

$$H_{(t^\diamond)} = \frac{\mathrm{d}z}{\mathrm{d}d_M} = \frac{\mathrm{d}\gamma}{\mathrm{d}d_M}\frac{\mathrm{d}z}{\mathrm{d}\gamma} = H_0\sqrt{\Omega_{0_m}\frac{(1+\sin\gamma)}{(1+\sin\gamma+\gamma\cos\gamma)^2}\,a^{-3}} = H_0\sqrt{\frac{\Omega_{ms}(\gamma)}{a^3}} \tag{39f}$$

$$d_M = \int_0^z \frac{c\,dz}{H(z)} \qquad t^\diamond = \int_0^z \frac{a}{H(z)}\mathrm{d}z \tag{39g}$$

where $\Omega_{ms}(\gamma)$ is the sum of the two components:

$$\Omega_m(\gamma) = \frac{\Omega_{0_m}}{(1+\sin\gamma+\gamma\cos\gamma)^2} \qquad \Omega_s(\gamma) = \frac{\Omega_{0_m}\,\sin\gamma}{(1+\sin\gamma+\gamma\cos\gamma)^2} \tag{39h}$$

We will indicate this second term, i.e. Ω_s, with the name of "shadow matter". It is assumed to be a fictitious mass of a different nature from proper mass. Indeed the universe is finite and has a positive spatial curvature $R_0 = \dfrac{c/H_0}{\sqrt{2-\Omega_0}}$.

Already with this first approximation, we obtain results almost identical to those of the ΛCDM model but not in the radiation dominated era.

This divergence is closed when the last ingredient of the Universe, i.e. radiation, is also taken into account so that $\Omega_0 = \Omega_{0_m} + \Omega_{0_r} = 1$. For this purpose, given the relationship between space and mass-energy (see eq. 16h), it is possible to break down the elements of the metric according to the type of energy.

In details:

$$\mathrm{d}d_{M_r} = \mathrm{d}\left(b_r(\gamma)\int\frac{\mathrm{d}r_r^\diamond}{A_r}\right) = \mathrm{d}\,r_r^\diamond = R_{0_r}\,\mathrm{d}\sin\gamma = -R_{0_r}\,\mathrm{d}a \tag{40a}$$

$$\mathrm{d}d_{M_b} = \mathrm{d}\left(b_b(\gamma)\int\frac{\mathrm{d}r_b^\diamond}{A_b}\right) = R_{0_b}\,(1+\sin\gamma+\gamma\cos\gamma)\,\mathrm{d}\gamma \tag{40b}$$

$$\mathrm{d}d_{M_c} = \mathrm{d}\left(b_c(\gamma)\int\frac{\mathrm{d}r_b^\diamond}{A_c}\right) = R_{0_c}\,(1+\sin\gamma+\gamma\cos\gamma)\,\mathrm{d}\gamma \tag{40c}$$

where $A_r = 1$ $\quad A_b = A_c = \sqrt{1-r^2/R_0^2} = \cos\gamma$ $\quad b_r(\gamma) = 1$ $\quad b_b(\gamma) = b_c(\gamma) = (1+\sin\gamma)$

$$\frac{1}{R_0^2} = \frac{1}{R_{0_r}^2} + \frac{1}{R_{0_b}^2} + \frac{1}{R_{0_c}^2} = \frac{\Omega_r}{R_0^2} + \frac{\Omega_b}{R_0^2} + \frac{\Omega_c}{R_0^2} \tag{40d}$$

$$\frac{1}{\mathrm{d}r^2} = \frac{1}{\mathrm{d}r_r^2} + \frac{1}{\mathrm{d}r_b^2} + \frac{1}{\mathrm{d}r_c^2} = \left(\frac{1}{R_{0_r}^2} + \frac{1}{R_{0_b}^2} + \frac{1}{R_{0_c}^2}\right)\frac{1}{(\mathrm{d}\sin\gamma)^2} \tag{40e}$$

$$\frac{1}{\mathrm{d}d_M^2(z)} = \frac{1}{\mathrm{d}d_{M_r}^2(z)} + \frac{1}{\mathrm{d}d_{M_b}^2(z)} + \frac{1}{\mathrm{d}d_{M_c}^2(z)} = \frac{H^2(z)}{c^2\mathrm{d}z^2} \tag{40f}$$

$$H^2(z) = H_r^2(z) + H_b^2(z) + H_c^2(z) \tag{40g}$$

and at last

$$H(\gamma) = H_0\sqrt{\frac{\Omega_{0_r}}{a^4(\gamma)} + \frac{\Omega_{ms}(\gamma)}{a^3(\gamma)}} = H_0\sqrt{\rho_r+\rho_m+\rho_s} \tag{41a}$$

$$d_M = \int_0^z \frac{c\,dz}{H_0\sqrt{\frac{\Omega_{0_r}}{a^4(t)} + \frac{1+\sin\gamma}{(1+\sin\gamma+\gamma\cos\gamma)^2}\frac{\Omega_{0_m}}{a^3(t)}}} \tag{41b}$$

The cosmological factor b_Γ has important consequences on the metric and constitutes the original difference compared to the ΛCDM model. In fact, it implies that, although the total amount of energy and matter in the Universe remains constant, space varies instead with a law different from the simple cube of distance.

Nevertheless the extra distance and the extra mass given by the cosmological factor b_Γ are fictitious and give rise to a fictitious curvature. Consequently, although the universe is locally flat $\Omega_r + \Omega_m = 1$, it is finite and has a positive spatial curvature (spherical). Indeed, the "shadow matter" breaks down into two components:

$$\Omega_s(\gamma) = \frac{\mathbf{1}-\mathbf{1}+\sin\gamma}{(1+\sin\gamma+\gamma\cos\gamma)^2}\,\Omega_m = \Omega_h(\gamma) + \Omega_k(\gamma)\,a(\gamma) \tag{42a}$$

We can therefore rewrite the eq. (41a) as follows:

$$H(a) = H_0\left[\frac{\Omega_r}{a^4(t)} + \frac{\Omega_m(\gamma)}{a^3(t)} + \frac{\Omega_h(\gamma)}{a^3(t)} + \frac{\Omega_k(\gamma)}{a^2(t)}\,\frac{a(\gamma)}{a(t)}\right] \tag{42b}$$

where

$$\Omega_m(\gamma) = \frac{1}{(1+\sin\gamma+\gamma\cos\gamma)^2}\,\Omega_m \tag{42c}$$

$$\Omega_h(\gamma) = \frac{1}{(1+\sin\gamma+\gamma\cos\gamma)^2}\,\Omega_m \tag{42d}$$

$$\Omega_k(\gamma) = \frac{-1}{(1+\sin\gamma+\gamma\cos\gamma)^2}\,\Omega_m \tag{42e}$$

Since along the line of sight $a(\gamma) = a(t)$, the universe appears endowed with a fictitious positive curvature. Nevertheless, the universe is flat in itself as cosmological observations reveal.

Furthermore, the present model does not need end therefore does not contemplate dark energy. Indeed the cosmological factor b_Γ gives rise to a fictitious matter pressure which, alone, gives reason for all the acceleration in the expansion of the universe.

Indeed, starting from the above density formulas, since, using the first equation of Friedmann equations (27a), the second equation (27b) can be re-expressed as:

$$\dot{\rho} = -3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) \qquad \text{or} \qquad \frac{p}{c^2} = -\left(\frac{1}{3}\frac{a}{\dot{a}}\dot{\rho} + \rho\right) \tag{43}$$

where the ρ includes the shadow matter, i.e. $\rho = \rho_r + \rho_m + \rho_s$, and the pressure, consequently, the shadow pressure. Since, from the (31), $dt = ad'_M(\gamma)d\gamma$ and $da = -\cos\gamma d\gamma$, we find the three components of pressure $p(t) = p_r(t) + p_m(t) + p_s(t)$:

$$\frac{p_r(t)}{c^2} = \rho_{crit}\,\frac{1}{3}\Omega_r a_{(t)}^{-4} = \frac{\rho_r(t)}{3} \tag{44a}$$

$$\frac{p_m(t)}{c^2} = -\frac{2}{3}\,\frac{2\cos\gamma - \gamma\sin\gamma}{1 + \sin\gamma + \gamma\cos\gamma}\,\frac{1}{\cos\gamma}\cdot a_{(t)}\,\rho_m(t) \tag{44b}$$

$$\frac{p_s(t)}{c^2} = \sin\gamma\cdot\frac{p_m(t)}{c^2} + \frac{1}{3}\cdot a_{(t)}\rho_m(t) \tag{44c}$$

When $t \to t_0$, that is $\gamma \to 0$ or $a(t) = a(\gamma) \to 1$, we have that the proper matter pressure becomes negative and equal in magnitude to its positive energy density:

$$\lim_{t\to t_0}\frac{p_{ms}(t)}{c^2} = -\rho_m(t_0) \tag{45a}$$

and, from the (27b), the acceleration in the expansion of the universe becomes positive:

$$\lim_{t\to t_0}\frac{\ddot{a}_{(t)}}{a_{(t)}} = -\frac{4\pi G}{3}\left(\rho_r(t_0) + \rho_m(t_0) + 3\left(\frac{\rho_r(t_0)}{3} - \rho_m(t_0)\right)\right) \simeq \frac{R_\Omega}{R_0^3} = (cH_0)^2 \tag{45b}$$

In summary, the fundamental difference with respect to the standard model is given by the factor b_Γ. The cosmological factor b_Γ has dramatic impact on the metric. In fact, it implies that, even if the total amount of energy and matter in the Universe remains constant and equal to R_ω, space varies instead with a law different from the simple cube of distance. As well as in the ΛCDM model, also in the present model $\Omega_{0_m} = \Omega_{0_b} + \Omega_{0_c}$ where Ω_{0_b} represents the baryonic density now and Ω_{0_c} the Cold Dark Matter (CDM) density now. However, they are not constant but vary with redshift z.

Impacts of the IRPL hypothesis on cosmology

Since $\Omega_c = 1 - \Omega_r - \Omega_b$, the IRPL Model is determined by only five of the six parameters of the ΛCDM model:

$$\omega_{b_0},\, h,\, n_s,\, \tau,\, N_{eff} \tag{46a}$$

Both the scalar spectral index n_s, which describes how the density fluctuations from inflation vary with scale ($n_s = 1$ corresponding to scale invariant fluctuations) [15], and the reionization optical depth τ [15], which describes the fraction of CMB photons that were scattered by free electrons after the universe became ionized and which lead to a more or less damping of the small scale CMB temperature anisotropies, are crucial for determining the initial conditions of the universe and influence the CMB power spectrum. According to Standard cosmology, the CMB constraints give respectively:

$$n_s = 0.9649 \pm 0.0042 \qquad \tau = 0.0544 \pm 0.0073 \qquad \text{(for the } \Lambda\text{CDM model)} \tag{46b}$$

Conversely, regardless of the cosmological model used, the number of effective relativistic degrees of freedom N_{eff}, which affects the expansion rate of the universe especially during the early universe and Big Bang Nucleosynthesis (BBN), the CMB temperature T_{cmb}, which constraints the photon density, and the baryon density, are precisely determined according the physics of the standard model, to the CMB temperature measurement and to the IRPL physics respectively as:

$$N_{eff} = 3.044 \pm 2\times 10^{-5} \qquad T_{cmb} = 2.7255 \pm 0.0006 \qquad \Omega_b = \frac{c/H_0}{\left(\frac{2m_e}{m_{irpl}}\right)^{-3}\ell_{irpl}} \tag{46c}$$

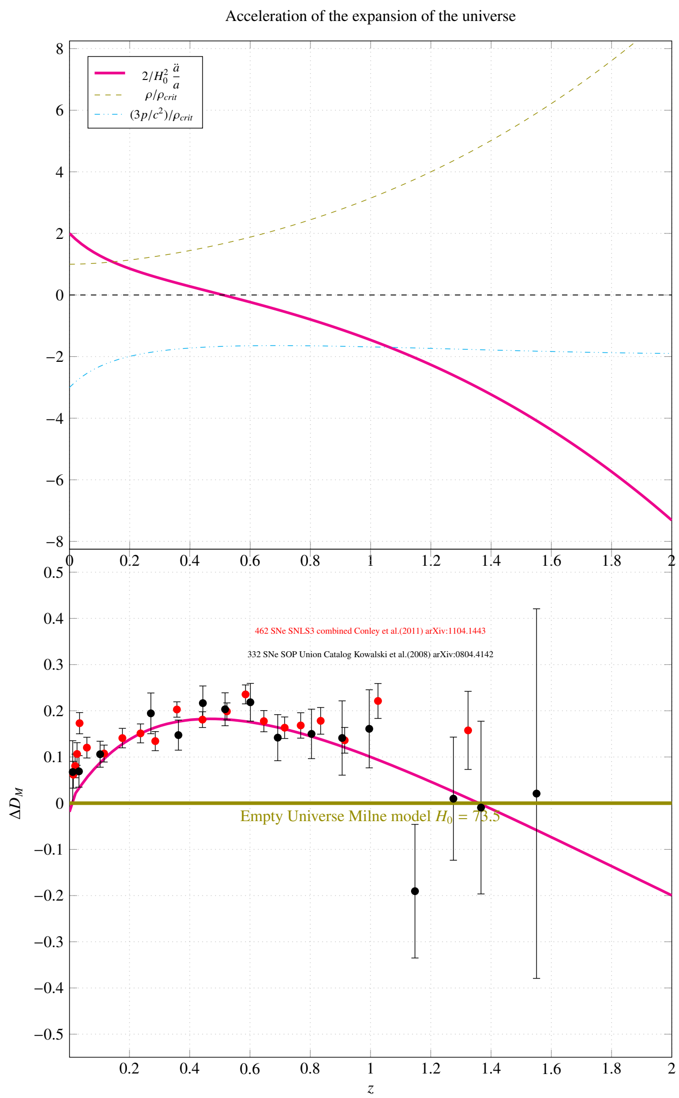

Figure 17. The top panel shows the pressure $3p/c^2 = \rho_r + \rho_m - \rho_s - 2\cos\gamma \frac{d''_M(\gamma)}{d'_M(\gamma)} \rho_m$, the density $\rho = \rho_r + \rho_m + \rho_s$ and the acceleration $\frac{2}{H_0^2}\frac{\ddot{a}}{a} = \frac{-1}{\rho_{crit}}\left(\rho + 3p/c^2\right)$ in the expansion of the universe.
On the bottom panel, the brightness or faintness of distant supernovae relative to the empty Universe model $\Omega = 0$ (the green curve) is plotted vs redshift. The blue-red curve, $\Delta(d_M) = 5\log_{10}\left(\frac{d_L}{R_\omega z\left(1+\frac{z}{2}\right)}\right)$ is the difference between the distance modulus determined from the computed flux $d_L = d_M(1+z)$ and the distance modulus computed from the redshift in the empty Universe model. The Hubble constant used in computing the empty Universe Milne model which is subtracted off is 73.5 km/sec/Mpc, and not 63.8 as in Riess et al. (2007).

and therefore it holds:

$$\Omega_\gamma = 2.469 \times 10^{-5} h^{-2} \qquad \Omega_r = \Omega_\gamma \left(1 + 0.2271\, N_{eff}\right) \qquad \Omega_b = 3.15225\,\frac{c/H_0}{\text{Mpc}} \tag{46d}$$

$$\Omega_c = 1 - \Omega_r - \Omega_b \qquad \Omega_m = 1 - \Omega_r \tag{46e}$$

it follows that the metric of the IRPL Model (eq. (41) is determined by a single parameter:

$$\mathscr{M}(h) \tag{46f}$$

where h is the dimensionless reduced Hubble constant $h = H_0/100$. About the history of the universe, both models basically share the same phases.

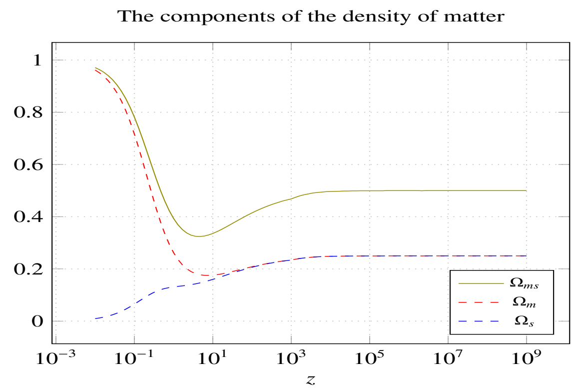

Figure 18. On the panel, the density of matter and its two components.

In the Radiation-dominated age, the expansion rate: $H(a) \simeq H_0\sqrt{\frac{\Omega_r}{a^4}}$ is identical for both models.
The Radiation-Matter transition happened when $H_r = H_m$, or $d'_{M_m} = d'_{M_r}$, that is $\Omega_m(z) = \Omega_r(1+z)$ or:

$$\frac{1-\sin\gamma\left(z_{eq_m}\right)}{\left(1+\sin\gamma(z_{eq_m})+\gamma(z_{eq_m})\cos\gamma(z_{eq_m})\right)^2} = \frac{\Omega_r}{1-\Omega_r} \tag{46g}$$

Contrary to what happens in the radiation dominate era, in the matter dominated era the expansion rate of the universe is quite different.

As densities vary with redshift, it is important to bear in mind that, unlike the ΛCDM model, in the IRPL model we must use the appropriate value of the density of matter according to the cosmological context. For this reason, although in the Radiation-dominated age, the expansion rate is identical for both models, the nucleosynthesis and the dynamics of the acoustic oscillation are nevertheless different.
Indeed, about acoustic waves dynamic, CMB temperature and polarization anisotropies are determined not only by the metric but also by the speed of acoustic wave and by the Baryon drag which depend only on the matter component which varies according to the causal region as:

$$c_s(z) \equiv c\sqrt{\frac{\dot{P}_\gamma+\dot{P}_{m_b}}{\dot{\rho}_\gamma+\dot{\rho}_{m_b}}} \simeq \frac{c}{\sqrt{3}}\frac{1}{\sqrt{1+\frac{3\Omega_{b(z)}}{4\Omega_\gamma(1+z)}}} \qquad r_{s_{(z)}} = \int_z^\infty \frac{c_s(z)}{H(z)}dz \tag{46h}$$

Nevertheless, the exact value of the baryon density is critical for a whole series of constraints that the cosmological model must satisfy in order to conform to the experimental evidence.

Likewise, the model predicts a density of matter that varies over time and this, together with the values of H_0, must satisfy a series of constraints such as:

1. Decoupling of the photons at last scattering (CMB release) and of the baryons at the end of the Compton drag epoch (BAO release). As the universe expanded and cooled, temperature of the Universe fell, the free electrons combined with naked protons to produce neutral hydrogen, and, at redshifts z^*, the Universe became transparent (CMB release). Shortly after, at redshift z_{drag}, baryons were no longer dominated by photon pressure (BAO release) and could move under gravity's influence setting the stage for galaxy formation. The respective redshifts z^* and z_{drag}, which depend on the ionization history and the atomic physics of recombination, can be determined by using the accurate recombination fitting formulae [8]. Based on the universe metric $\mathscr{M}(h)$, these redshifts, in turn, determine the comoving sound horizons and the comoving angular diameter distances which must satisfy, in each of the two epochs :

   - the acoustic angular scale constraint:

$$\frac{\pi}{\ell_a} = \theta_* = \frac{r_s(h,z^*)}{d_M(h,z^*)} = 0.0104109 \pm 0.00030 \quad (68\%,\ \text{TT,TE,EE+lowE}) \tag{46i}$$

   where ℓ_a is the multipole moment corresponding to acoustic angular scale θ_* seen in the CMB power spectra,

- the BAO measurement constraint:

$$\left(\frac{r_s(h, z_{drag})h}{\mathrm{Mpc}}\right) \simeq 101 \tag{46j}$$

measured from galaxy surveys that appears to be $(101.056 \pm 0.036)\left(\frac{0.3}{\Omega_m}\right)^{-0.4}$ for the ΛCDM $\mathscr{M}$etric [15], 101 ± 1 for the $\mathscr{M}$etric of the present model (fig. 19).

The acoustic angular scale constraint and the transverse baryon acoustic oscillation scale constraint are consistent with (see fig. 21) a value of:

$$H_0 = 73.48 \pm 0.5 \tag{46k}$$

which solves the Hubble Tension (at 5σ) while having one less parameter to play with.

2. "Late universe" H_0 measurements constraint: "Late universe" H_0 measurements using calibrated distance ladder techniques have converged on a value of approximately $H_0 \simeq 73.4$ km/s/Mpc. In particular, 73.4 ± 1.4 km/s/Mpc [16] from standard distance ladder, 73.3 ± 1.7 km/s/Mpc [23] from strong gravitational lensing effects on quasar systems.

3. the angular power spectrum of the CMB, which provides precise measurements of the baryon density and dark matter density of the universe at recombination [7]. In particular, 2nd/1st peak ratio allows to determine the baryon density, one of the most robust and best-determined CMB outputs, since it controls the relative amplitudes of the alternating odd and even peaks, which correspond to modes undergoing maximal compression and rarefactions at the time of recombination. Analogously, CDM density affects the overall shape and the amplitudes and locations of the peaks. Planck 2028 measures:

$$\Omega_b h^2 \simeq 0.0224 \qquad \Omega_c h^2 \simeq 0.120 \tag{46l}$$

In the IRPL model, the causal region of the momentum density $m_{eff} = 1 + R(\theta_*) = 1 + \frac{3\,\Omega_b(\theta_*)}{4\,\Omega_\gamma(1+z^*)}$, which provides extra inertia in the joint Euler equation, is the distance between baryons with respect to their barycentre given by the acoustic angular scale $\theta_* = \arcsin\left(r_s(z^*)/D_M(z^*)\right)$, while the causal region for the CDM density is given by the angle γ^* corresponding to the redshift z^* for $H_0 = 73.225$. That is:

$$\omega_b(\theta_*) = \frac{1+\sin\theta_*}{(1+\sin\theta_* + \theta_*\cos\theta_*)^2}\omega_{b_0} = 0.022381(1) \qquad \omega_c(\gamma^*) = \frac{1}{(1+\sin\gamma^* + \gamma^*\cos\gamma^*)^2}\omega_{c_0} = 0.1205(3) \tag{46m}$$

4. BBN primordial element abundances measurement:

$\mathrm{Yp}\cdot 10^{-01} = 2.453\pm0.034(a)$ $\quad$ $\mathrm{D/H}\cdot 10^{-05} = 2.527\pm0.030(b)$ $\quad$ $^3\mathrm{He/H}\cdot 10^{-05} = 1.1\pm0.2(c)$ $\quad$ $^7\mathrm{Li/H}\cdot 10^{-10} = 1.58^{+0.35}_{-0.28}(d)$

(a) Aver et al. [1], (b) Cooke et al. [3] , (c) Bania et al. [2], (d) Sbordone et al. [18]. The predictions of the standard BBN theory rest on balance between expansion rate and on the astrophysical nuclear reaction rates and on three additional parameters: the number of light neutrino flavours (N_ν), the neutron lifetime (τ_n) and the baryon-to-photon ratio ($\eta = n_B/n_\gamma$). While both the expansion rate of the universe during the BBN and the baryon-to-photon ratio η are almost the same, the baryon density of IRPL is almost one half with respect to ΛCDM. Indeed from the (39h)

$$\Omega_b(BBN) = \Omega_{b_0}\frac{1+\sin\gamma}{(1+\sin\gamma+\gamma\cos\gamma)^2} \approx \frac{1}{2}\Omega_{b_0} \tag{46n}$$

consequently, the nuclear reaction rate of the IRPL model is half that of the ΛCDM model. At last, IRPL BBN (table IV and fig. 20) solves the lithium problem but, in its place, raises a deuterium problem.

5. the growth of the cosmological perturbations by looking at the large-scale structure of the Universe and how it has changed across the cosmic epochs. The most recent cosmic shear data release [20] from KiDS+VIKING-450 [19] and from both KiDS-1000 and DESY3 [22], confirms a tension with the Standard Model

$$S_8 = \sqrt{\frac{\Omega_m}{0.3}}\sigma_8 = 0.737^{+0.040}_{-0.036} \quad \text{from KiDS+VIKING-450.} \tag{46o}$$

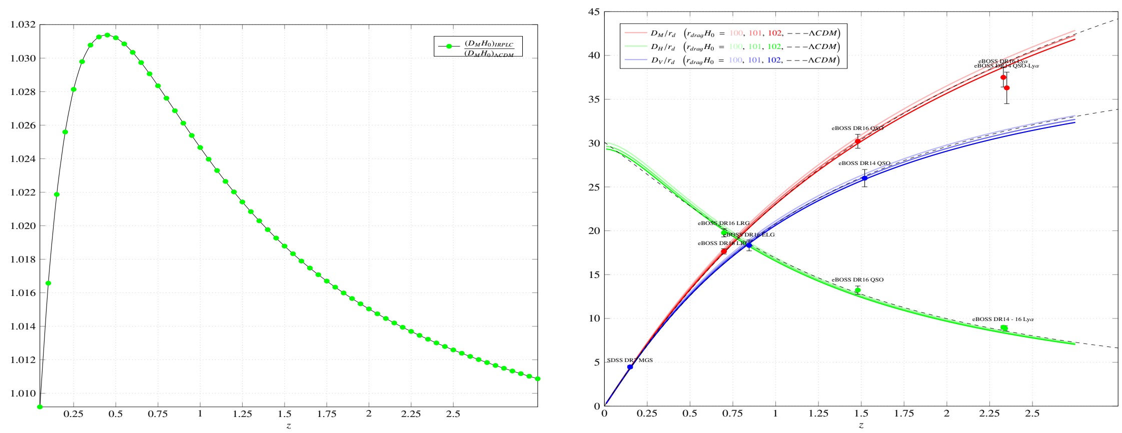

Figure 19. The top panel shows the rate between the normalized comoving distance $D_M H_0$ of fiducial IRPL model ($\omega_b = 0.02325, H_0 = 73.48$) and that of fiducial ΛCDM mode ($\omega_b = 0.02242, \omega_m = 0.3111, H_0 = 67.66$).
The bottom panel shows BAO "Hubble diagram" : Black dashed lines represent the fiducial ΛCDM model, coloured solid lines represent the fiducial IRPL model for $H_0 r_{drag} = 100, 101, 102$.

Table IV. Primordial abundances of elements in the big-bang nucleosynthesis (BBN)
For the measured values see: (a) Aver et al. [1], (b) Cooke et al. [3] , (c) Bania et al. [2], (d) Sbordone et al. [18]
For the ΛCDM calculated values see: Pitrou et al. [14]
The IRPL calculated values were produced by the software AlterBBN halving all the rates of nuclear reactions and with $\eta = 6.38158 \times 10^{-10}$, $\tau_n = 879.4$, $N_{eff} = 3.044$

	Yp (10^{-01})	D/H (10^{-05})	^{3}He/H (10^{-05})	^{7}Li/H (10^{-10})
Observations:	2.453 ± 0.034 (a)	2.527 ± 0.030 (b)	1.1 ± 0.2 (c)	$1.58\ ^{+0.35}_{-0.28}$ (d)
ΛCDM ($\eta_{10} = 6.13792$):	2.4721 ± 0.00014	2.439 ± 0.037	1.039 ± 0.014	**5.464 ± 0.220**
IRPL ($\eta_{10} = 6.325$):	2.447 ± 0.0032	**6.621 ± 0.063**	1.511 ± 0.016	1.545 ± 0.11

6. the acceleration in the expansion of the universe determined by comparing the brightness or faintness of distant supernovae relative to the empty Universe model [17]. In particular, this constraint is satisfied by the IRPL model without dark energy (see fig.17.
   From the (44) it follows that the accelerated expansion of the universe (fig. 17) has begun since $z \simeq 0.5099$ when the universe was 7.996 billion years old, roughly almost 5 billion years ago, since the age of the universe is 12.826 billion years.

Furthermore, since there is no Dark Energy in the IRPL model, the matter-dominated era extends to the present and thus encompasses the final era of accelerated expansion of the universe.

The measure of the Universe

At last,

Comp.	Universe scale factor	$r_\omega = \ell_\omega$ (common ratio)	Ω
c	$\ell_{\omega_c} = 1$	$r_{\omega_c} = \ell_{\omega_c}\left(\alpha^{-1} e^{\alpha^{-1}}\right)\ell_{irpl}$	$\Omega_c = (r_{\omega_c}/R_\omega)^2 = 0.95686$
b	$\ell_{\omega_b} = \left(\dfrac{1}{3/2\,\pi}\right)$	$r_{\omega_b} = \ell_{\omega_b}\left(\alpha^{-1} e^{\alpha^{-1}}\right)\ell_{irpl}$	$\Omega_b = (r_{\omega_b}/R_\omega)^2 = 0.04309$
r	$\ell_{\omega_r} = \alpha$	$r_{\omega_r} = \ell_{\omega_r}\left(\alpha^{-1} e^{\alpha^{-1}}\right)\ell_{irpl}$	$\Omega_r = (r_{\omega_r}/R_\omega)^2 = 5.095 \times 10^{-5}$
Tot		$R_\omega = \sqrt{r_{\omega_c}^2 + r_{\omega_b}^2 + r_{\omega_r}^2} = 1.26331 \times 10^{26} mt.$	$\Omega = \Omega_c + \Omega_b + \Omega_r = 1$

Table V. Parameters for the base ΛCDM and IRPL models compared

Parameter	ΛCDM TT,TE,EE+LowE+lensing 68% limits	IRPL	IRPL z context
ω_{b_0}	0.02237 ± 0.00015	0.02310 ± 0.00035	
ω_{c_0}	0.1200 ± 0.0012	0.513 ± 0.001	
$\omega_b(\theta^*)$.....		0.02238 ± 0.000033	(CMB release)
$\omega_c(z^*)$.....		0.1206 ± 0.00005	(CMB release)
$100\theta_{MC}$	1.04092 ± 0.00031	idem	
τ	0.0544 ± 0.0073	idem	
$ln\left(10^{10}A_s\right)$ ..	3.044 ± 0.014	idem	
n_s	0.9649 ± 0.0042	idem	
$H_0[kms^{-1}Mpc^{-1}]$	67.36 ± 0.54	73.225 ± 0.04	
Ω_Λ	0.6847 ± 0.0073		
Ω_m	0.3153 ± 0.0073	$0.999923 \pm 8 \times 10^{-8}$	
$\Omega_m(z_{eq})$		$0.240540 \pm 6 \times 10^{-6}$	
Age[Gyr] ...	13.797 ± 0.023	12.860 ± 0.07	
z^*	1089.92 ± 0.25	1132.3 ± 0.05	
$r^*[Mpc]$....	144.43 ± 0.26	129.89 ± 0.05	
$100\theta^*$	1.04110 ± 0.00031	1.04101	
z_{drag}	1059.94 ± 0.30	1034.6 ± 0.7	
$r_{drag}[Mpc]$...	147.09 ± 0.26	138.02 ± 0.7	
z_{eq}	3402 ± 26	3108.37 ± 4.3	
σ_8	0.8111 ± 0.0060	0.8123 ± 0.007	
S_8	0.832 ± 0.013	0.727 ± 0.007	

we have the impressive correspondence with the value of Ω_b given by the (25b) :

$$\Omega_b = \frac{c/H_0}{\left(\frac{2m_e}{m_{irpl}}\right)^{-3}\ell_{irpl}} = \frac{R_\omega}{(R_e^\circ/\ell_{irpl})^3\ell_{irpl}} = 0.04305 \qquad \Omega_b = \left(\frac{r_{\omega_b}}{R_\omega}\right)^2 = 0.04309 \tag{47}$$

where $r_{\omega_b} = \frac{1}{3/2\,\pi}\left(\alpha^{-1}\,e^{\alpha^{-1}}\right)\ell_{irpl}$ and $R_\omega = \sqrt{1^2 + \left(\frac{1}{3/2\,\pi}\right)^2 + \alpha^2}\left(\alpha^{-1}\,e^{\alpha^{-1}}\right)\ell_{irpl} = 1.26331 \times 10^{26}mt = c/H_0 = c/73.225\,Mpc.$

The causal horizon problem

The constancy of electric radius and the (47) support the secondary claim of this paper, according to which the boundary limits of universe are established not by chance or contingency, but are themselves governed by fixed geometric relations. This evidence points to a different solution to the causal horizon problem. In truth, the ad hoc inflation hypothesis might be replaced with a more natural and general hypothesis.

In other words, for each individual, the present, which comes from the continuous Big Bang (as source) as an approaching future (matter and increasing entropy), as soon as it surfaces, it submerges as past (antimatter and decreasing entropy) that move away to go towards the continuous Big Crunch (as well), and in this descent informs of itself the future that ascend in the opposite direction. The past that is moving away is also the future that is approaching, and it is the possibility of the present. The present is the realization of a possible history of the past, among the totality of physically possible histories in accordance with quantum mechanics.

The mechanism that places the same initial conditions everywhere, therefore, is not to be found in a causal contact occurred in the past of a linear time, but in the dialogue, with a period P_ω equal to the apparent age of the universe, between the big bang and the present, in a cyclical time: each time, the new present in act is the result of the big bang that took place P_ω years before and is the foundation of the big bang that will take place P_ω years later.

This hypothesis, compared to the correspondent of standard cosmology, radically changes the meaning but leaves the entire phenomenology and physics of the universe unchanged.

The surface is the determined act, the interior is the power that explains it, the surface is electrical, the interior gravitational. In a panpsychistic vision, the surface is the consciousness, the interior is the soul.

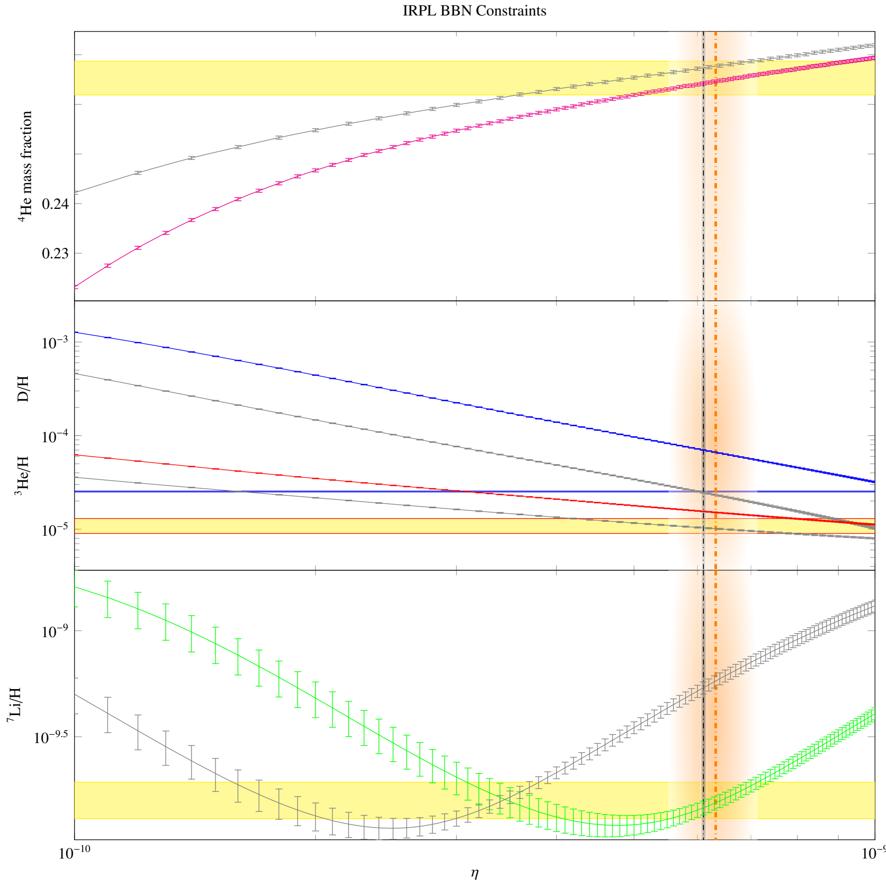

Figure 20. Comparison between the primordial abundances expected for the light nuclei according to the ΛCDM model, grey lines, and the IRPL model, coloured lines. Yellow horizontal rectangles show range of the uncertainties in the primordial abundances measured values. The orange vertical line indicates the value of $\eta = 6.325 \pm 0.8 \times 10^{-10}$ deduced in the present IRPL analysis, the grey one $\eta = 6.105 \pm 0.055 \times 10^{-10}$ for the ΛCDM model [13]. The values were calculated using the version 2 of *AlterBBN* software, an open public code for the calculation of the abundance of the elements from Big-Bang nucleosynthesis. For the purpose of the IRPL model, the *bbnrate.c* file was modified by adding the instruction " **f[ie]=0.5*f[ie];** " at the end of the loop of the *rate all* function in order to halve all the reaction rates.

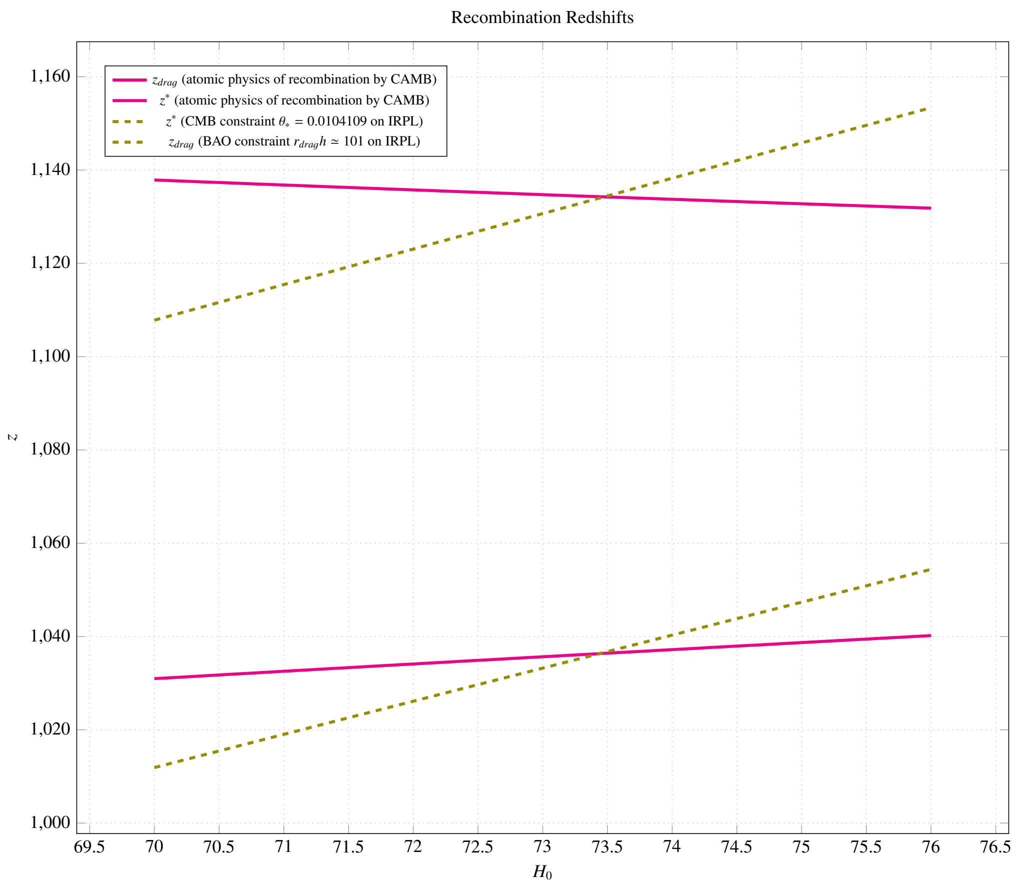

Figure 21. The panel shows the redshift z^* and $z_{\rm drag}$ for the parameters $\Omega_b = 0.04305$ and $\Omega_c = 1 - \Omega_r - \Omega_b - \Omega_\nu$, where:
- The dashed olive lines represent the solution in the IRPL model, which satisfies the constraints $\theta_* = 0.0104109$ and $r_{\rm drag}h \approx 101$ Mpc.
- The solid magenta lines represent the results of the CAMB code where Ω_b and Ω_c have been brought back to the historical context of recombination by multiplying them by $f(z) = (1 + \sin\gamma_z)/(1 + \sin\gamma_z + \gamma_z \cos\gamma_z)^2$, with $f(z) \simeq 0.4696$ for $z^* = 1135 \pm 3$ and $f(z) \simeq 0.4683$ for $z_{\rm drag} = 1035 \pm 5$.