# INVERSE N-BODY PROBLEM - THE KEY TO SOLVING GRAVITY PROBLEMS

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***Abstract.*** *Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious physical phenomenon in physics and cosmology. For a long time, the force of gravitational interaction was represented by only one physical law: Newton's law of gravitation $F = GMm/r^2$. Newton's law of gravitation is not enough to fully describe gravity. Newton's law shows the force of gravitational interaction of two bodies out of all N bodies in the Universe. The formula of Newton's law describes gravity only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies in the Universe. In gravity, at least three more laws of gravity have not been discovered. The secrets of gravity are revealed by the inverse N-body problem. The solutions to the inverse N-body problem are at least 4 laws of gravity. The solution of the inverse N-body problem for N = 2 yields two laws of gravity: Newton's law $F = GMm/r^2$ and a new law of gravity $F = mR^3/T^2r^2$. We also show that the solution of the inverse N-body problem for $N \to \infty$ yields the third and fourth laws of gravity: $F = (mc^2)\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$. Newton's law $F = GMm/r^2$ and the new law of gravity $F = mR^3/T^2r^2$ give the gravitational force of two bodies. The third and fourth laws of gravity $F = (mc^2)\sqrt{\Lambda}$, $F = mGme/\alpha re^2$ give an additional gravitational force of N bodies in the Universe. Three new physical laws appeared in gravity in addition to Newton's law. They were missing in Newtonian dynamics. The new laws of gravity complement Newtonian dynamics and give a complete and consistent description of gravity.*

***Keywords:*** *Newton's law; N-body problem; law of universal gravitation; задачи Бертрана; parameters of the observable universe; galaxy rotation curve; cosmological constant Λ.*

## 1. Introduction

Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious and intriguing physical phenomenon in physics and cosmology. The opinion has taken root that the law of gravitation discovered by Newton is the only law of gravitation that shows the force of gravitational interaction. The revealed gravitational anomalies in the dynamics of stars show that at large distances Newton's law does not hold and has significant discrepancies with observations [1, 2].

For large distances and large masses in the Universe, the gravitational force dominates, which Newton's law "does not see". Newton's law $F = GMm/r^2$ shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula of the law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies in the Universe. Newton's law does not give a complete description of gravity. Newton's law of gravity does not take into account that all bodies in the Universe participate in gravitational interaction simultaneously.

The formula $F = GMm/r^2$ is quite accurate on the scale of the Solar System. But it is not applicable on the scale of the Universe. It is obvious that in addition to Newton's law, there are still undiscovered laws of gravity. In gravity, the most important physical law remains undiscovered, which shows the strength of the gravitational interaction of all N bodies in the Universe. In physics, the opinion has taken root that it is impossible to obtain a law of gravity for many bodies. This opinion is largely supported by the N-body problem. It is known that the N-body problem has no analytical solution for $N \geq 3$. In the Copernican Scholium, Newton also pointed out the impossibility of taking into account all the causes of motion and the impossibility of describing them by exact laws [3]: "...*the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

Here we show how to obtain the law of gravity, which shows the strength of the gravitational interaction of all N bodies in the Universe. This mystery of gravity is revealed by the inverse N-body problem. We also show that the solution of the inverse N-body problem gives three new laws of gravity.

## 2. Bertrand's Problem

In the late 1870s, J. Bertrand formulated the problem of finding the law of gravitational force from known properties of the trajectory of bodies [4]. The Bertrand problem is the inverse of the two-body problem. The first and second Bertrand gravitational problems are known. The first Bertrand problem was formulated for trajectories that are conic sections. The second Bertrand problem was formulated for trajectories that are closed curves. In the general case, for trajectories represented by algebraic curves, this problem is known as the Koenigs problem [4].

It is believed that the solution to the Bertrand and Koenigs problems yields Hooke's law, or Newton's law of gravitation [4]. In fact, the solution to these problems only establishes that the force is inversely proportional to the square of the distance. Proportionality to masses does not follow from the solution to the Bertrand and Koenigs problems. The solution of the Bertrand and Koenigs problems does not give a complete formula for Newton's law of gravitation, but only the inverse-square law. The inverse-square law is not enough to conclude that the solution to the problems is Newton's law of gravitation. It is well known that the inverse-square law is included not only in Newton's law of gravitation. The solutions of the Bertrand and Koenigs problems do not lead to unambiguous conclusions. The third solution to the Bertrand problem was missed, which is the new law of gravitation $F = mR^3/T^2r^2$ [5]. It also includes the inverse-square law, but it is not Newton's law. The new law of gravitation instead of mass includes the parameters of an elliptical orbit: the semimajor axis R and the period T.

The initial data in the Bertrand problems are the positions of a moving point on a trajectory. This entails the use of differential equations (trajectory equations), which do not take into account the integral parameters of bodies. The Bertrand problem does not use integral parameters, such as mass or orbital parameters, as input data. For this reason, the solution to the Bertrand problem does not provide a complete Newtonian law, but only a part of it, namely, only the inverse square law.

An elliptical orbit can be described in two ways. The differential method of description uses the positions of the moving point on the trajectory. The differential method is used in the Bertrand problem. The second way to describe an elliptical orbit is to use integral parameters of the orbit. Based on Kepler's laws, such an integral parameter is the Kepler constant $R^3/T^2$. This integral parameter of the orbit allows us to obtain the law of gravity without using the trajectory equation.

The inverse problem of two bodies in Bertrand's formulation only indirectly indicates Newton's law of gravity. To obtain a complete formula for the law of gravity, a different formulation of the inverse problem of two bodies is needed. The emphasis in the new formulation should be placed not on the differential description of the trajectory, but on the integral parameters of the orbit. This makes it possible to confirm not only the inverse proportionality to the square of the distance, but also to obtain a complete formula for the law of gravitational force.

## 3. The first inverse N-body problem

In [6], the problem of finding the law of gravitational force based on the integral parameters of the N-body system is formulated as follows:

"*Knowing the integral characteristics of the N-body system, find the law of gravitational force with which N bodies act on a body of mass m*."

If the N-body system is the Universe, then the problem has several solutions depending on the choice of the integral parameter of the Universe. Unlike the Bertrand problem, all solutions of the inverse N-body problem give complete formulas for the laws of gravitation. A practically significant solution to the inverse N-body problem is the law of gravitation $F = mc^2\sqrt{\Lambda}$ [6]. This law of gravitation includes the cosmological constant $\Lambda$.

## 4. The Second Inverse N-Body Problem

The inverse gravitational N-body problem can be presented in an extended formulation. The extended formulation of the inverse N-body problem is as follows:

"*Knowing the integral characteristics of the N-body system, or the integral parameters of the central body, or the integral parameters of the orbit of a body of mass m, find the law of the gravitational force that acts on a body of mass m*."

The extended formulation of the inverse N-body problem combines the inverse two-body problem and the inverse problem for the N-body system of the Universe. The second inverse N-body problem has several solutions, of which at least four solutions are of practical interest. For N = 2, this is the inverse two-body problem. Unlike the Bertrand problem, the second inverse N-body problem uses either the integral parameters of the orbit or the integral parameter of the central body as input data. For N = 2, the problem has two solutions, which yield two laws of two-body gravity: Newton's law $F = GMm/r^2$ and the new law of gravity $F = mR^3/T^2r^2$.

For $N \to \infty$, this is an inverse problem for a system of N bodies in the Universe. For $N \to \infty$, the problem has a third and fourth solution, which yield the laws of N-body gravity in the Universe $F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$. The solutions to the inverse N-body problem show that gravitational interaction is represented not by one Newton's law, but by at least four laws of gravity.

## 5. Newton's law of gravitation $F = GMm/r^2$ as a solution to the second inverse N-body problem for N = 2.

For N = 2, the second inverse N-body problem is a two-body inverse problem. If the mass of the central body is used as an integral parameter, then the solution to the second inverse N-body problem for N = 2 yields Newton's law of gravitation $F = GMm/r^2$.

## 6. The new law of gravitation $F = mR^3/T^2r^2$ as the second solution to the second inverse N-body problem for N = 2.

If the integral parameters of an elliptical orbit are used, then the solution to the second inverse N-body problem for N = 2 yields a new law of gravitation. The new law of gravitation includes the characteristics of an elliptical orbit as parameters: the semi-major axis R and the period of revolution T in the form of an integral parameter - the Kepler constant $R^3/T^2$. The acceleration is represented by the formula: $a = R^3/T^2r^2$. Accordingly, the law of gravity has the form [5]:

$$\mathbf{F = mR^3/T^2r^2} \qquad (1)$$

Where: **F** is the force; **m** is the mass of the body; **R** is the semi-major axis of the elliptical orbit; **T** is the period of revolution; **r** is the distance.

This unknown law of gravitational interaction of two bodies was pointed out by Robert Hooke in his correspondence with Newton in 1679 even before the discovery of Newton's laws [5]. The law of gravitation $F = mR^3/T^2r^2$ is a more accurate and perfect law of gravitational interaction of two bodies than Newton's law $F = GMm/r^2$, since distances and periods are known from observations with greater accuracy than mass. I call this physical law the Hooke-Kepler law of gravitation.

## 7. New laws of gravitation $F = mc^2\sqrt{\Lambda}$ and $F = mGm_e/\alpha r_e^2$ as the third and fourth solutions of the second inverse problem of N bodies at $N \to \infty$.

The law of gravitation of two bodies does not give a complete description of gravity. The reason is that in reality all bodies in the Universe participate in gravitational interaction. It is known that the N-body problem has no analytical solution for $N \geq 3$. Here we set the goal of obtaining a solution to the inverse N-body problem. The N-body inverse problem has solutions for N = 2 and for $N \to \infty$. The inverse problem for N bodies in the formulation proposed above has not been studied in physics. Here we present a new method for finding the law of gravitation for N bodies. The method is based on reducing the inverse N-body problem to the inverse two-body problem, where the central body is a system of N bodies [6].

If the N-body system is the Universe, then the problem has several solutions depending on the choice of the integral characteristic of the Universe. If we consider either the mass of the Universe Mu, or the radius Ru, or the cosmological constant Λ, or the time Tu, or the gravitational constant G as the integral parameter of the Universe, then the solutions to the inverse N-body problem will be the following laws of gravitation: F = (mc^2)/Ru, $F = GmMu/Ru^2$, $F = mc^2\sqrt{\Lambda}$, F = mc/Tu, $F = Gmm_e/\alpha r_e^2$. Of all the parameters of the Universe, the cosmological constant Λ and the gravitational constant G are accessible for measurement. Accordingly, of all the solutions to the inverse N-body problem for $N \to \infty$, the two new laws of gravitation are practically applicable solutions:

$$\mathbf{F = mc^2\sqrt{\Lambda}} \qquad (2)$$

$$\mathbf{F = mGm_e/\alpha r_e^2} \qquad (3)$$

Where: **F** is the force; **m** is the mass of the body; **c** is the speed of light; **Λ** is the cosmological constant; **G** is the gravitational constant; $\mathbf{m_e}$ is the mass of the electron; $\mathbf{r_e}$ is the classical radius of the electron; **α** is the fine structure constant.

The new laws of gravitation differ from Newton's law. The cosmological gravitational force has a linear dependence on mass and does not obey the inverse square law. These are two equivalent laws of the cosmological force. The new laws of gravitation include the cosmological constant Λ and the gravitational constant G. The new laws of gravitation $F = mc^2\sqrt{\Lambda}$ and $F = mGm_e/\alpha r_e^2$ allow us to overcome the limitations inherent in Newton's law of gravitation $F = GMm/r^2$.

## 8. Useful solutions to the second inverse N-body problem.

Fig.1 shows solutions of the second inverse N-body problem using integral parameters from the group including G, M, R, T, Λ, Mu, Ru, Tu. Of all the solutions shown, four solutions of the second inverse N-body problem have practical application. These are the solutions that include integral parameters available for measurement. Such parameters include the orbital parameters R, T, the mass of the central body M, the cosmological constant Λ, and the gravitational constant G.

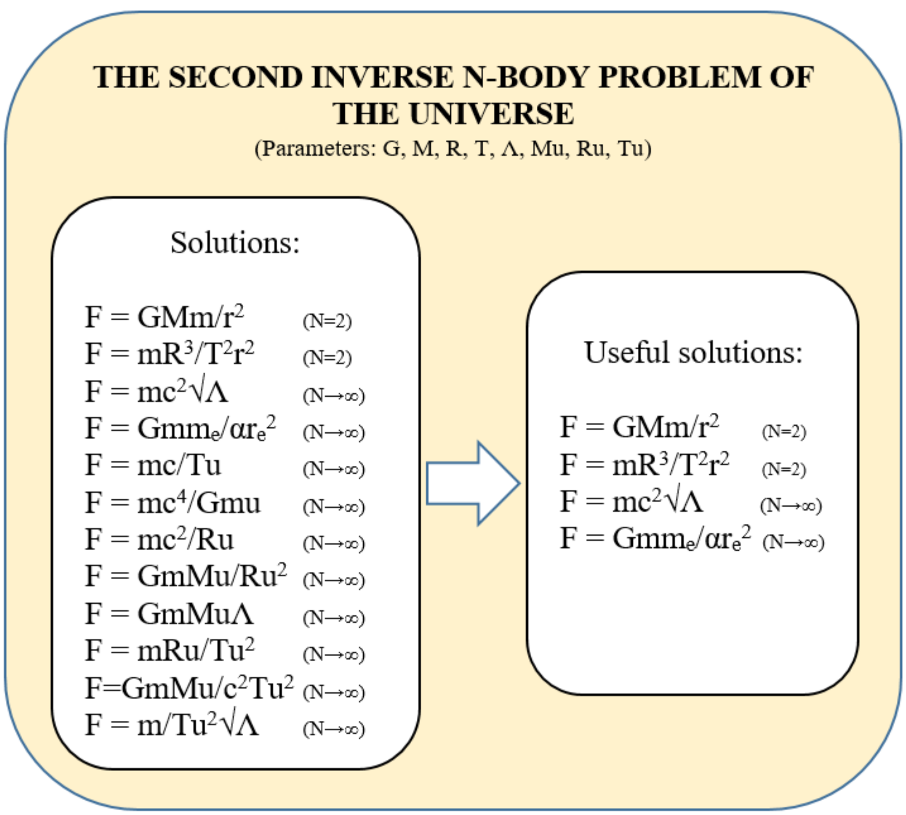


Fig. 1. Solutions of the second inverse N-body problem. Where: F is the force; m is the mass of the body; M is the mass of the central body; c is the speed of light; Λ is the cosmological constant; G is the gravitational constant; R is the semi-major axis of the elliptical orbit; T is the orbital period; r is the distance; Mu is the mass of the Universe; Ru is the radius of the Universe; Tu is the time; $m_e$ is the electron mass; $r_e$ is the classical radius of the electron; α is the fine structure constant.

The Hubble parameter H is not used here. This is due to the unsolved problem of the Hubble voltage. Using the Hubble parameter H can give some more useful solutions to the second inverse N-body problem as $N \to \infty$. In this case, the expected solutions are: $F = mH\alpha h/m_e r_e$, $F = mcH$, $F = mR_u H^2$, $F=GmM_u H^2/c^2$, $F = mH^2/\sqrt{\Lambda}$.

## 9. The four laws of gravitation as the basis for the new law of universal gravitation.

Newton's law of gravitation $F = GMm/r^2$ is only one of the four laws of gravitation. It is the law of gravitation of two bodies. Newton's law of gravitation "sees" only a part of the force of universal gravitation. The second law of gravitation of two bodies is the new law of gravitation $F = mR^3/T^2r^2$. This law of gravitation also "sees" only a part of the force of universal gravitation. The third law of gravitation is the new law of gravitation $F = mc^2\sqrt{\Lambda}$. This is the law of gravitation of N bodies in the Universe. The fourth law of gravitation $F = mGme/\alpha re2$ is also a law of gravitation of N bodies in the Universe. All four laws of gravitation are the solution to the second inverse problem of N bodies (Fig. 2).

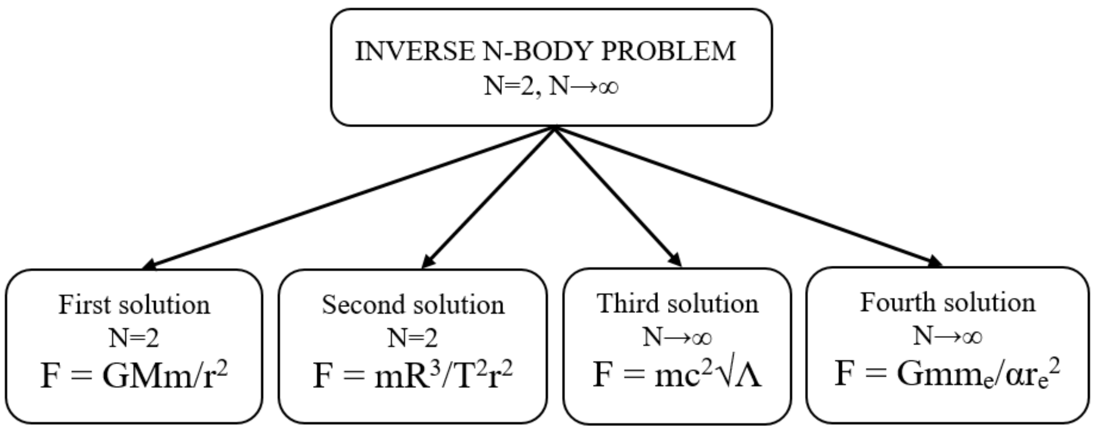


Fig. 2. Four solutions of the second inverse N-body problem.

Each of the four laws of gravitation taken separately does not give the value of the total force of universal gravitation. Newton's law $F = GMm/r^2$, and the second law of gravitation $F = mR^3/T^2r^2$, and the third law of gravitation $F = mc^2\sqrt{\Lambda}$, and the fourth law of gravitation $F = mGm_e/\alpha r_e^2$ separately give only a part of the total force of gravitational interaction. Newton's law of gravitation $F = GMm/r^2$ and the law of gravitation $F = mR^3/T^2r^2$ "do not see" the cosmological force and do not take into account the influence of all bodies in the Universe. The laws of cosmological force ($F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$) show an additional force to the force of gravitational interaction of two bodies. They complement the gravitational interaction of two bodies. Only the combination of the two laws gives the value of the total force of universal gravitation. These are the following combinations of the laws of gravitation:

- Newton's law of gravitation $F = GMm/r^2$ + the law of cosmological force $F = mc^2\sqrt{\Lambda}$;
- Newton's law of gravitation $F = GMm/r^2$ + the law of cosmological force $F = mGm_e/\alpha r_e^2$;
- Hooke-Kepler law $F = mR^3/T^2r^2$ + the law of cosmological force $F = mc^2\sqrt{\Lambda}$;
- Hooke-Kepler law $F = mR^3/T^2r^2$ + the law of cosmological force $F = mGm_e/\alpha r_e^2$.

The law of universal gravitation has four equivalent forms of representation:

$$\mathbf{F_U = GMm/r^2+mc^2\sqrt{\Lambda}} \qquad (4)$$

$$\mathbf{F_U = mR^3/T^2r^2 + mc^2\sqrt{\Lambda}} \qquad (5)$$

$$\mathbf{F_U = GMm/r^2 + Gmm_e/\alpha r_e^2} \qquad (6)$$

$$\mathbf{F_U = mR^3/T^2r^2 + Gmm_e/\alpha r_e^2} \qquad (7)$$

The law of universal gravitation is presented as a unification of the law of gravitation of two bodies with the law of cosmological force (Fig.3).

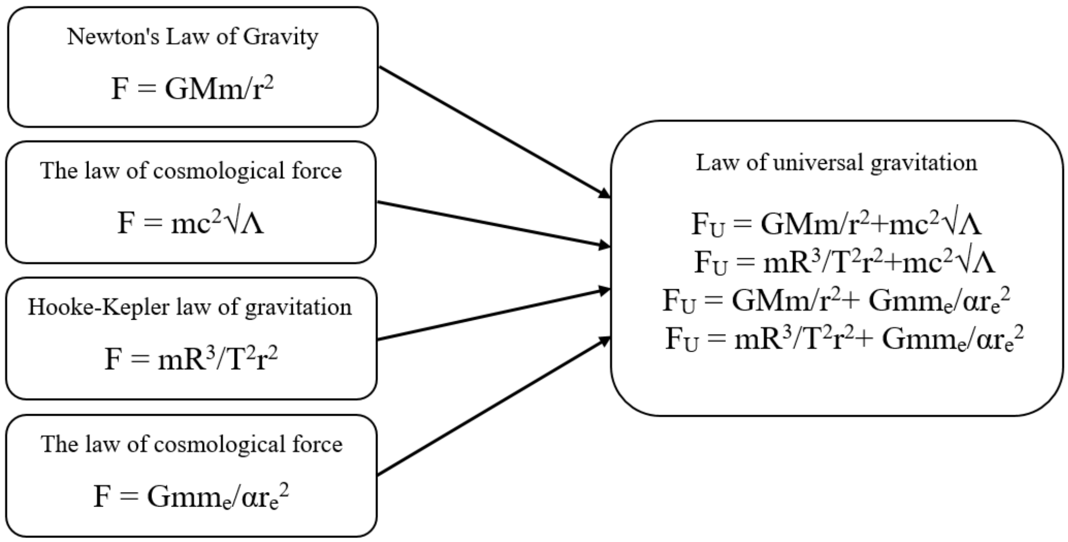


Fig. 3. Four equivalent formulas of the law of universal gravitation as a unification of the law of gravitation of two bodies and the law of cosmological force.

The law of universal gravitation turned out to be much more complex than Newton thought. Newton's law is included as a component in the law of universal gravitation. The missing value of the force of universal gravitation is represented by the laws of cosmological force $F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$.

## 10. The cosmological constant Λ as a fundamental constant of gravitation.

Two solutions of the inverse N-body problem as $N \to \infty$ yield two equivalent laws of gravitation: $F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$. The new law of gravitation F=(mc^2)√Λ includes the cosmological constant Λ. The cosmological constant Λ is an integral characteristic of the N-body system, where the N-body system is represented by all the bodies in the Universe. The cosmological constant Λ is a fundamental constant in the new law of gravitation.

The new law of gravitation $F = mGm_e/\alpha r_e^2$ includes the gravitational constant G. The constant G is also an integral characteristic of the N-body system, where the N-body system is represented by all the bodies in the Universe. The gravitational constant G is a fundamental constant in the new law of gravitation.

From these two equivalent laws of gravitation follows a relation that shows the connection between the constants Λ and G:

$$c^2\sqrt{\Lambda} = \frac{Gm_e}{\alpha\, r_e^2} \qquad (8)$$

Where: G is the gravitational constant; c is the speed of light; Λ is the cosmological constant; $m_e$ is the electron mass; $r_e$ is the classical electron radius; α is the fine structure constant.

Equation (8) predicts the expected value of the cosmological constant Λ with a high accuracy, close to the accuracy of the constant G:

$$\Lambda = \frac{G^2 m_e^2}{c^4 \alpha^2 \ r_e^4} = 1.36285... \bullet 10^{-52} \ m^{-2} \qquad (9)$$

The appearance of the constant Λ in the gravitational equation F=(mc^2)√Λ and its inclusion in the law of universal gravitation $F_U$ = GmM/r^2 + (mc^2)√Λ) together with the gravitational constant G is an unusual fact in gravitation. In the law of universal gravitation, the cosmological constant Λ manifests itself as a gravitational constant no less significant than the gravitational constant G. This phenomenon requires a more in-depth study. The revealed connection between the constants Λ and G poses the problem of re-evaluating their role in gravitation. In particular, the unique status of the gravitational constant G in gravitation is questioned. On the other hand, the cosmological constant Λ acquires the status of a fundamental constant of gravitation.

## 11. Cosmological acceleration Ao = 10.4922... • $10^{-10}$ m/s^2 - integral parameter of the Universe

In the new laws of gravity (2) and (3), combinations of constants give a new constant: cosmological acceleration Ao = 10.4922... • 10^(-10) m/s^2. This is a fundamental parameter of the Universe. The cosmological acceleration constant Ao has the following relationship with other parameters of the Universe:

$$A_0 = \frac{GM_U}{R_U^2} = \frac{R_U}{T_U^2} = c^2\sqrt{\Lambda} = \frac{c^4}{GM_U} = GM_U\Lambda = \frac{c}{T_U} = ... = \frac{Gm_e}{\alpha \ r_e^2} = 10.4922... \bullet 10^{-10} m/s^2 \quad (10)$$

Where: G is the gravitational constant; Mu is the mass of the Universe; Ru is the radius of the Universe; Tu is time; c is the speed of light; Λ is the cosmological constant; $m_e$ is the mass of the electron; $r_e$ is the classical radius of the electron; α is the fine structure constant.

The value of the acceleration (Ao = 10.4922... • 10^(-10) m/s^2) associated with the cosmological force turned out to be very close to the MOND prediction [7] and to the pioneer anomaly [8].

Equation (10) allows us to predict not only the value of the cosmological constant Λ and the constant Ao, but also other parameters of the Universe (mass Mu, radius Ru, age Tu) with an accuracy close to the accuracy of the constant G:

$$\mathbf{Mu = c^4\alpha r_e^2/G^2 m_e = 1.15348... \bullet 10^{53}\ kg} \qquad (11)$$

$$\mathbf{Ru = c^2\alpha r_e^2/Gm_e = 0.856594... \bullet 10^{26}\ m} \qquad (12)$$

$$\mathbf{Tu = c\alpha r_e^2/Gm_e = 2.85729... \bullet 10^{17}\ s} \qquad (13)$$

$$\mathbf{A_0 = Gm_e/\alpha r_e^2 = 10.4922... \bullet 10^{-10}\ m/s^2} \qquad (14)$$

$$\mathbf{\Lambda = G^2 m_e^2/c^4\alpha^2 r_e^4 = 1.36285... \bullet 10^{-52}\ m^{-2}} \qquad (15)$$

Equations (11) - (15) are not the only ones for calculating the parameters of the Universe. The parameters of the Universe can be represented using Planck units.

## 12. Parameters of the Universe as scale replicas of Planck units

The parameters of the Universe are related to Planck units by very beautiful equations (Fig.4).

$$M_U = m_P\sqrt{\alpha^3 D_0^3} = 1.15348...\bullet 10^{53}\,kg$$

$$R_U = l_P\sqrt{\alpha^3 D_0^3} = 0.856594...\bullet 10^{26}\,m$$

$$T_U = t_P\sqrt{\alpha^3 D_0^3} = 2.85729...\bullet 10^{17}\,s$$

$$A_0 = \frac{a_P}{\sqrt{\alpha^3 D_0^3}} = 10.4922...\bullet 10^{-10}\,m/s^2$$

$$\Lambda = \frac{1}{l_P^2\alpha^3 D_0^3} = 1.36285...\bullet 10^{-52}\,m^{-2}$$

Fig. 4. Parameters of the Universe represented by Planck units. Where: Mu is the mass of the Universe; Ru is the radius of the Universe; Tu is time; $A_0$ is the cosmological acceleration; $\Lambda$ is the cosmological constant; $m_p$ is the Planck mass; $l_P$ is the Planck length; $t_P$ is the Planck time; $a_P$ is the Planck acceleration; $\alpha$ is the fine structure constant; $D_0$ is the large Weyl number ($D_0$=4.16561... • $10^{42}$).

Using Planck units [9], the equations for calculating the parameters of the Universe are significantly simplified, their entry is freed from unnecessary coefficients. It is evident that all parameters of the Universe are scaled Planck units. The scaling factor for the parameters of the Universe is the large number $D_{120} = \alpha^3 D_0^3 = 28.088 \bullet 10^{120}$ [10]. The equations (Fig. 4) demonstrate the amazing connection between the parameters of the microcosm and the megacosm.

## 13. Conclusion.

In 1687, Newton discovered only one of the four laws of gravitation. The other three laws of gravitation remained undiscovered for more than 300 years. The solution of the inverse N-body problem yields three unknown laws of gravitation $F = mc^2\sqrt{\Lambda}$, $F = mR^3/T^2r^2$, $F = mGm_e/\alpha r_e^2$. These three new laws of gravitation complement Newtonian dynamics. This allows us to overcome the shortcomings and limitations of Newtonian dynamics. Newtonian dynamics, supplemented by the three laws of gravity, becomes a complete gravitational dynamics. For the first time, it becomes possible to give an adequate description of gravity that takes into account the force of gravitational interaction of all N bodies in the Universe. Fig. 5 shows all the laws of the complete gravitational dynamics.

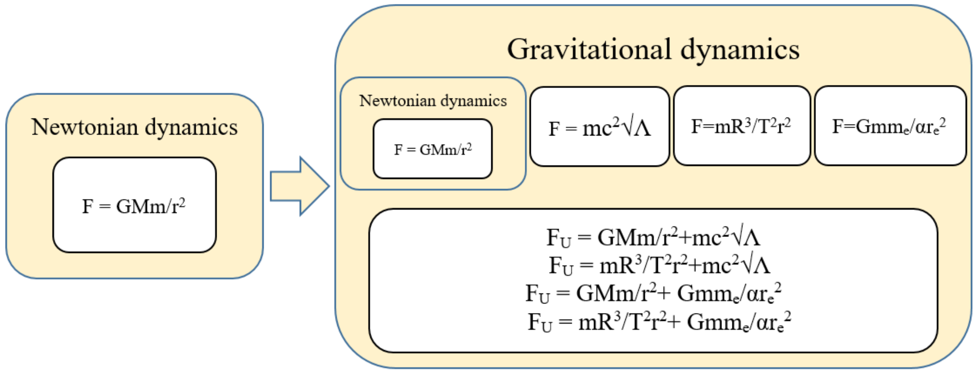


Fig. 5. The law of Newtonian dynamics and the laws of complete gravitational dynamics.

Newtonian dynamics gives very little information about gravity. It is only a small part of gravitational dynamics. For a complete description of gravity, at least four fundamental laws were missing.

In the new laws of gravity, the cosmological constant Λ acts as a gravitational constant. A close connection between the cosmological constant Λ and the gravitational constant G is visible. This unexpected connection between the fundamental constants G and Λ is not accidental. This phenomenon requires deeper study.

## 14. Conclusions

1. For over 300 years, the gravitational force was represented by a single law of gravitation: Newton's law.

2. Newton's law describes the gravity of two bodies. It "sees" only a part of the gravitational force and does not take into account the gravitational force of N bodies in the Universe.

3. Newton's law of gravitation is insufficient to represent the full force of universal gravitation.

4. Not all laws of gravitation have been discovered. Newton's law is one of the four laws of gravitation. At least three laws of gravitation remain undiscovered.

5. Solving the second inverse N-body problem for $N = 2$ and for $N \to \infty$ yields three new laws of gravitation: $F = mR^3/T^2r^2$ , $F = (mc\verb|^|2)\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$.

6. The new law of gravitation $F = mR^3/T^2r^2$ is a solution to the second inverse N-body problem for $N = 2$. This law of gravitation gives the gravitational force of two bodies, just like Newton's law. The formula for the new law of gravitation contains the inverse square law. Robert Hooke was the first to give a verbal formula close to the law of gravitation $F = mR^3/T^2r^2$. This happened back in 1679, 7 years before the discovery of Newton's laws [5].

7. Two new laws of gravitation $F = (mc\verb|^|2)\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$ are a solution to the second inverse N-body problem for $N \to \infty$. These laws of gravitation give an additional gravitational force of N bodies in the Universe. The additional gravitational force has a linear dependence on mass and does not obey the inverse square law.

8. Three new physical laws appeared in gravitational dynamics in addition to Newton's law. They were absent in Newtonian dynamics. The four laws of gravitation provide a complete and adequate description of gravitational interaction.

9. Two new laws of gravitation $F = (mc^2)\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$ lead to a new law of universal gravitation, which takes into account not only the gravity of two bodies, but also the additional gravity of all N bodies in the Universe.

10. The cosmological constant Λ is a gravitational constant no less significant than the gravitational constant G. A relationship was found between the constants Λ and G. The found relationship between the constants Λ and G predicts the expected value of the cosmological constant Λ ($\Lambda = 1.36285 \cdot 10^{-52}\ m^{-2}$).

11. The discovered relationship between the constants Λ and G raises the issue of re-evaluating their role in gravity. In particular, the unique status of the gravitational constant G in gravity is questioned. There are two fundamental constants in gravity: Λ and G.

12. A new parameter of the Universe is proposed — cosmological acceleration. The acceleration value $Ao = 10.4922... \cdot 10^{-10}\ m/s^2$ turned out to be very close to the MOND prediction and to the Pioneer anomaly.

13. The new laws of gravity provide an explanation for the dark matter problem, galaxy rotation curves, Pioneer anomalies, and the problem of the cosmological constant Λ.

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