INVERSE N-BODY PROBLEM - THE KEY TO SOLVING GRAVITY PROBLEMS

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Abstract. *Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious physical phenomenon in physics and cosmology. For a long time, the force of gravitational interaction was represented by only one physical law: Newton's law of gravitation F = GMm/r^2. Newton's law of gravitation is not enough to fully describe gravity. Newton's law shows the force of gravitational interaction of two bodies out of all N bodies in the Universe. The formula of Newton's law describes gravity only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies in the Universe. In gravity, at least three more laws of gravity have not been discovered. The secrets of gravity are revealed by the inverse N-body problem. We show that the laws of gravity are a solution to the inverse N-body problem. The solution of the inverse N-body problem for N = 2 yields two laws of gravity: Newton's law F = GMm/r^2 and a new law of gravity F = mR^3/T^2r^2. We also show that the solution of the inverse N-body problem for N → ∞ yields the third and fourth laws of gravity: F = (mc^2)√Λ, F = mGm$_e$/αr$_e$^2. In gravity, at least three laws of gravity remained undiscovered.*

Keywords: *Newton's law; N-body problem; law of universal gravitation; задачи Бертрана; parameters of the observable universe; galaxy rotation curve; cosmological constant Λ.*

1. Introduction

Despite the fact that the law of gravitation was discovered more than 300 years ago, gravity remains the most mysterious and intriguing physical phenomenon in physics and cosmology. The opinion has taken root that the law of gravitation discovered by Newton is the only law of gravitation that shows the force of gravitational interaction. The revealed gravitational anomalies in the dynamics of stars show that at large distances Newton's law does not hold and has significant discrepancies with observations [1, 2].

For large distances and large masses in the Universe, the gravitational force dominates, which Newton's law "does not see". Newton's law $F = GMm/r^2$ shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula of the law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies in the Universe. Newton's law does not give a complete description of gravity. Newton's law of gravity does not take into account that all bodies in the Universe participate in gravitational interaction simultaneously.

The formula $F = GMm/r^2$ is quite accurate on the scale of the Solar System. But it is not applicable on the scale of the Universe. It is obvious that in addition to Newton's law, there are still undiscovered laws of gravity. In gravity, the most important physical law remains undiscovered, which shows the strength of the gravitational interaction of all N bodies in the Universe. In physics, the opinion has taken root that it is impossible to obtain a law of gravity for many bodies. This opinion is largely supported by the N-body problem. It is known that the N-body problem has no analytical solution for $N \geq 3$. In the Copernican Scholium, Newton also pointed out the impossibility of taking into account all the causes of motion and the impossibility of describing them by exact laws [3]: "...*the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined* *motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

Here we show how to obtain the law of gravity, which shows the strength of the gravitational interaction of all N bodies in the Universe. This mystery of gravity is revealed by the inverse N-body problem. We also show that the solution of the inverse N-body problem gives three new laws of gravity.

2. Bertrand's Problem

In the late 1870s, J. Bertrand formulated the problem of finding the law of gravitational force from known properties of the trajectory of bodies [4]. The Bertrand problem is the inverse of the two-body problem. The first and second Bertrand gravitational problems are known. The first Bertrand problem was formulated for trajectories that are conic sections. The second Bertrand problem was formulated for trajectories that are closed curves. In the general case, for trajectories represented by algebraic curves, this problem is known as the Koenigs problem [4].

It is believed that the solution to the Bertrand and Koenigs problems yields Hooke's law, or Newton's law of gravitation [4]. In fact, the solution to these problems only establishes that the force is inversely proportional to the square of the distance. Proportionality to masses does not follow from the solution to the Bertrand and Koenigs problems. The solution of the Bertrand and Koenigs problems does not give a complete formula for Newton's law of gravitation, but only the inverse-square law. The inverse-square law is not enough to conclude that the solution to the problems is Newton's law of gravitation. It is well known that the inverse-square law is included not only in Newton's law of gravitation. The solutions of the Bertrand and Koenigs problems do not lead to unambiguous conclusions. The third solution to the Bertrand problem was missed, which is the new law of gravitation $F = mR^3/T^2r^2$ [5]. It also includes the inverse-square law, but it is not Newton's law. The new law of gravitation instead of mass includes the parameters of an elliptical orbit: the semimajor axis R and the period T.

The initial data in the Bertrand problems are the positions of a moving point on a trajectory. This entails the use of differential equations (trajectory equations), which do not take into account the integral parameters of bodies. The Bertrand problem does not use integral parameters, such as mass or orbital parameters, as input data. For this reason, the solution to the Bertrand problem does not provide a complete Newtonian law, but only a part of it, namely, only the inverse square law.

An elliptical orbit can be described in two ways. The differential method of description uses the positions of the moving point on the trajectory. The differential method is used in the Bertrand problem. The second way to describe an elliptical orbit is to use integral parameters of the orbit. Based on Kepler's laws, such an integral parameter is the Kepler constant R^3/T^2. This integral parameter of the orbit allows us to obtain the law of gravity without using the trajectory equation.

The inverse problem of two bodies in Bertrand's formulation only indirectly indicates Newton's law of gravity. To obtain a complete formula for the law of gravity, a different formulation of the inverse problem of two bodies is needed. The emphasis in the new formulation should be placed not on the differential description of the trajectory, but on the integral parameters of the orbit. This makes it possible to confirm not only the inverse proportionality to the square of the distance, but also to obtain a complete formula for the law of gravitational force.

3. The first inverse N-body problem

In [6], the problem of finding the law of gravitational force based on the integral parameters of the N-body system is formulated as follows: "*Knowing the integral characteristics of the N-body system, find the law of gravitational force with which N bodies act on a body of mass m*."

If the N-body system is the Universe, then the problem has several solutions depending on the choice of the integral parameter of the Universe. Unlike the Bertrand problem, all solutions of the inverse N-body problem give complete formulas for the laws of gravitation. A practically significant solution to the inverse N-body problem is the law of gravitation $F = mc^2\sqrt{\Lambda}$ [6]. This law of gravitation includes the cosmological constant Λ.

4. The Second Inverse N-Body Problem

The inverse gravitational N-body problem can be presented in an extended formulation. The extended formulation of the inverse N-body problem is as follows: "*Knowing the integral characteristics of the N-body system, or the integral parameters of the central body, or the integral parameters of the orbit of a body of mass m, find the law of the gravitational force that acts on a body of mass m*."

The extended formulation of the inverse N-body problem combines the inverse two-body problem and the inverse problem for the N-body system of the Universe. The second inverse N-body problem has several solutions, of which at least four solutions are of practical interest. For N = 2, this is the inverse two-body problem. Unlike the Bertrand problem, the second inverse N-body problem uses either the integral parameters of the orbit or the integral parameter of the central body as input data. For N = 2, the problem has two solutions, which yield two laws of two-body gravity: Newton's law $F = GMm/r^2$ and the new law of gravity $F = mR^3/T^2r^2$.

For $N \to \infty$, this is an inverse problem for a system of N bodies in the Universe. For $N \to \infty$, the problem has a third and fourth solution, which yield the laws of N-body gravity in the Universe $F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$. The solutions to the inverse N-body problem show that gravitational interaction is represented not by one Newton's law, but by at least four laws of gravity.

5. Newton's law of gravitation $F = GMm/r^2$ as a solution to the second inverse N-body problem for N = 2.

For N = 2, the second inverse N-body problem is a two-body inverse problem. If the mass of the central body is used as an integral parameter, then the solution to the second inverse N-body problem for N = 2 yields Newton's law of gravitation $F = GMm/r^2$.

6. The new law of gravitation $F = mR^3/T^2r^2$ as the second solution to the second inverse N-body problem for N = 2.

If the integral parameters of an elliptical orbit are used, then the solution to the second inverse N-body problem for N = 2 yields a new law of gravitation. The new law of gravitation includes the characteristics of an elliptical orbit as parameters: the semi-major axis R and the period of revolution T in the form of an integral parameter - the Kepler constant R^3/T^2. The acceleration is represented by the formula: $a = R^3/T^2r^2$. Accordingly, the law of gravity has the form [5]:

$$\mathbf{F = mR^3/T^2r^2} \qquad (1)$$

Where: **F** is the force; **m** is the mass of the body; **R** is the semi-major axis of the elliptical orbit; **T** is the period of revolution; **r** is the distance.

This unknown law of gravitational interaction of two bodies was pointed out by Robert Hooke in his correspondence with Newton in 1679 even before the discovery of Newton's laws [5]. The law of gravitation $F = mR^3/T^2r^2$ is a more accurate and perfect law of gravitational interaction of two bodies than Newton's law $F = GMm/r^2$, since distances and periods are known from observations with greater accuracy than mass. I call this physical law the Hooke-Kepler law of gravitation.

7. New laws of gravitation $F = mc^2\sqrt{\Lambda}$ and $F = mGm_e/\alpha r_e^2$ as the third and fourth solutions of the second inverse problem of N bodies at $N \to \infty$.

The law of gravitation of two bodies does not give a complete description of gravity. The reason is that in reality all bodies in the Universe participate in gravitational interaction. It is known that the N-body problem has no analytical solution for $N \geq 3$. Here we set the goal of obtaining a solution to the inverse N-body problem. The N-body inverse problem has solutions for $N = 2$ and for $N \to \infty$. The inverse problem for N bodies in the formulation proposed above has not been studied in physics. Here we present a new method for finding the law of gravitation for N bodies. The method is based on reducing the inverse N-body problem to the inverse two-body problem, where the central body is a system of N bodies [6].

If the N-body system is the Universe, then the problem has several solutions depending on the choice of the integral characteristic of the Universe. If we consider either the mass of the Universe Mu, or the radius Ru, or the cosmological constant Λ, or the time Tu, or the gravitational constant G as the integral parameter of the Universe, then the solutions to the inverse N-body problem will be the following laws of gravitation: F = (mc^2)/Ru, $F = GMum/r^2$, $F = mc^2\sqrt{\Lambda}$, F = mc/Tu, $F = Gmm_e/\alpha r_e^2$. Of all the parameters of the Universe, the cosmological constant Λ and the gravitational constant G are accessible for measurement. Accordingly, of all the solutions to the inverse N-body problem for $N \to \infty$, the two new laws of gravitation are practically applicable solutions:

$$\mathbf{F = mc^2\sqrt{\Lambda}} \qquad (2)$$

$$\mathbf{F = mGm_e/\alpha r_e^2} \qquad (3)$$

Where: **F** is the force; **m** is the mass of the body; **c** is the speed of light; **Λ** is the cosmological constant; **G** is the gravitational constant; $\mathbf{m_e}$ is the mass of the electron; $\mathbf{r_e}$ is the classical radius of the electron; **α** is the fine structure constant.

The new laws of gravitation differ from Newton's law. The cosmological gravitational force has a linear dependence on mass and does not obey the inverse square law. These are two equivalent laws of the cosmological force. The new laws of gravitation include the cosmological constant Λ and the gravitational constant G. The new laws of gravitation $F = mc^2\sqrt{\Lambda}$ and $F = mGm_e/\alpha r_e^2$ allow us to overcome the limitations inherent in Newton's law of gravitation $F = GMm/r^2$.

8. The four laws of gravitation as the basis for the new law of universal gravitation.

Newton's law of gravitation $F = GMm/r^2$ is only one of the four laws of gravitation. It is the law of gravitation of two bodies. Newton's law of gravitation "sees" only a part of the force of universal gravitation. The second law of gravitation of two bodies is the new law of gravitation $F = mR^3/T^2r^2$. This law of gravitation also "sees" only a part of the force of universal gravitation. The third law of gravitation is the new law of gravitation $F = mc^2\sqrt{\Lambda}$. This is the law of gravitation of N bodies in the Universe. The fourth law of gravitation F = mGme/αre2 is also a law of gravitation of N bodies in the Universe. All four laws of gravitation are the solution to the second inverse problem of N bodies (Fig. 1).

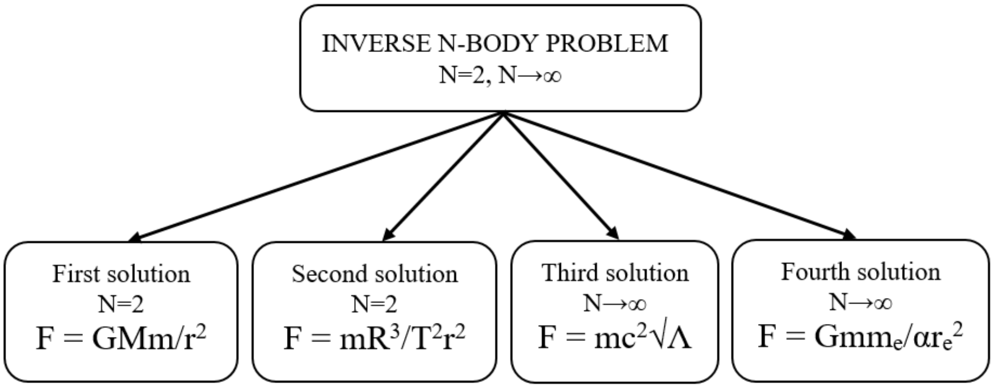

Fig. 1. Four solutions of the second inverse N-body problem.

Each of the four laws of gravitation taken separately does not give the value of the total force of universal gravitation. Newton's law $F = GMm/r^2$, and the second law of gravitation $F = mR^3/T^2r^2$, and the third law of gravitation $F = mc^2\sqrt{\Lambda}$, and the fourth law of gravitation $F = mGm_e/\alpha r_e^2$ separately give only a part of the total force of gravitational interaction. Newton's law of gravitation $F = GMm/r^2$ and the law of gravitation $F = mR^3/T^2r^2$ "do not see" the cosmological force and do not take into account the influence of all bodies in the Universe. The laws of cosmological force ($F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$) show an additional force to the force of gravitational interaction of two bodies. They complement the gravitational interaction of two bodies. Only the combination of the two laws gives the value of the total force of universal gravitation. These are the following combinations of the laws of gravitation:

- Newton's law of gravitation $F = GMm/r^2$ + the law of cosmological force $F = mc^2\sqrt{\Lambda}$;
- Newton's law of gravitation $F = GMm/r^2$ + the law of cosmological force $F = mGm_e/\alpha r_e^2$;
- Hooke-Kepler law $F = mR^3/T^2r^2$ + the law of cosmological force $F = mc^2\sqrt{\Lambda}$;
- Hooke-Kepler law $F = mR^3/T^2r^2$ + the law of cosmological force $F = mGm_e/\alpha r_e^2$.

The law of universal gravitation has four equivalent forms of representation:

$$\mathbf{F_U = GMm/r^2+mc^2\sqrt{\Lambda}} \qquad (4)$$

$$\mathbf{F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}} \qquad (5)$$

$$\mathbf{F_U = GMm/r^2+ Gmm_e/\alpha r_e^2} \qquad (6)$$

$$\mathbf{F_U = mR^3/T^2r^2+ Gmm_e/\alpha r_e^2} \qquad (7)$$

The law of universal gravitation is presented as a unification of the law of gravitation of two bodies with the law of cosmological force (Fig. 2).

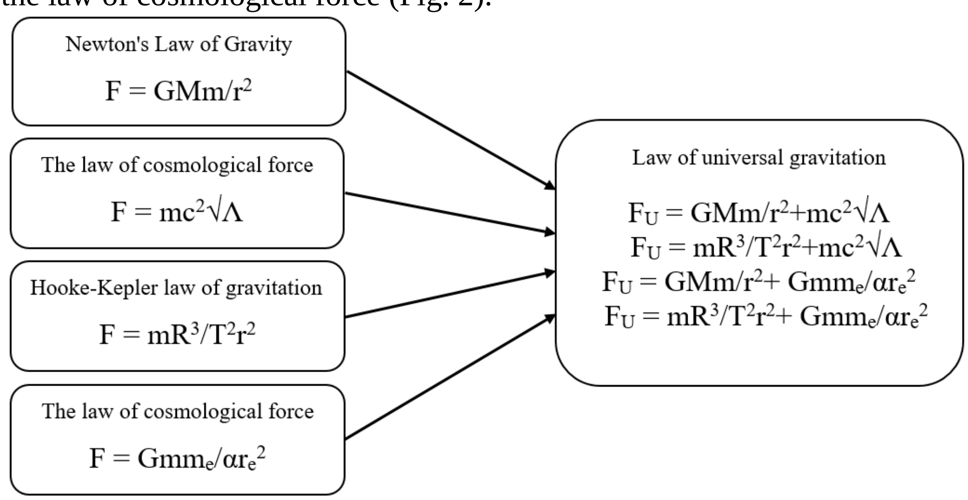

Fig. 2. Four equivalent formulas of the law of universal gravitation as a unification of the law of gravitation of two bodies and the law of cosmological force.

The law of universal gravitation turned out to be much more complex than Newton thought. Newton's law is included as a component in the law of universal gravitation. The missing value of the force of universal gravitation is represented by the laws of cosmological force $F = mc^2\sqrt{\Lambda}$, $F = mGm_e/\alpha r_e^2$.

9. Conclusion.

In 1687, Newton discovered only one of the four laws of gravitation. The other three laws of gravitation remained undiscovered for more than 300 years. The solution of the inverse N-body problem yields three unknown laws of gravitation $F = mc^2\sqrt{\Lambda}$, $F = mR^3/T^2r^2$, $F = mGm_e/\alpha r_e^2$. These three new laws of gravitation complement Newtonian dynamics. This allows us to overcome the shortcomings and limitations of Newtonian dynamics. Newtonian dynamics, supplemented by the three laws of gravity, becomes a complete gravitational dynamics. For the first time, it becomes possible to give an adequate description of gravity that takes into account the force of gravitational interaction of all N bodies in the Universe. Fig. 3 shows all the laws of the complete gravitational dynamics.

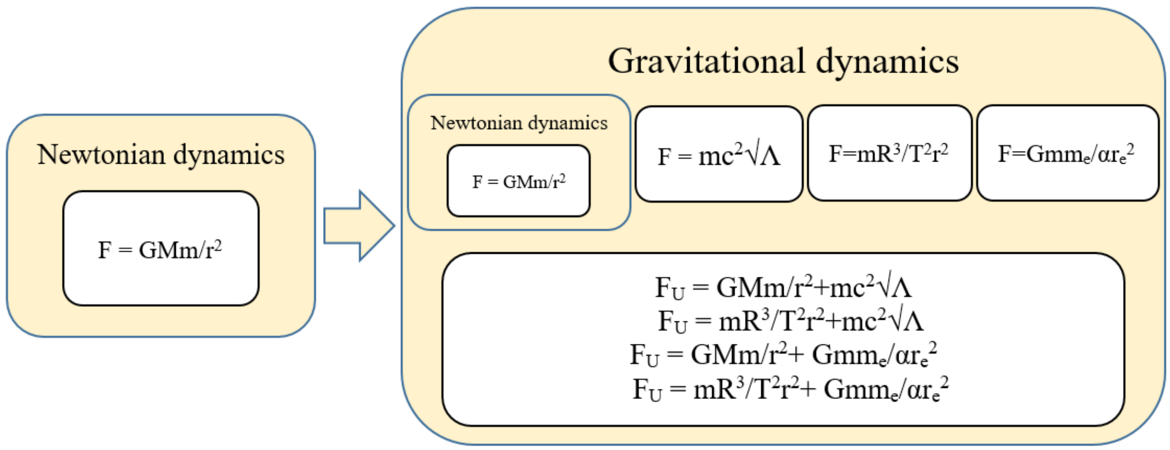

Fig. 3. The law of Newtonian dynamics and the laws of complete gravitational dynamics.

Newtonian dynamics gives very little information about gravity. It is only a small part of gravitational dynamics. For a complete description of gravity, at least four fundamental laws were missing.

In the new laws of gravity, the cosmological constant Λ acts as a gravitational constant. A close connection between the cosmological constant Λ and the gravitational constant G is visible. This unexpected connection between the fundamental constants G and Λ is not accidental. This phenomenon requires deeper study.

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