Considerations on the stopping time of the 3n+1 problem

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Abstract
This paper presents some considerations on the stopping time of the 3n+1 problem. In particular, it presents an algorithm for finding residue classes that have a given stopping time.

3n + 1 problem (or conjecture)

In the $3\cdot n+1$ problem[1] it is possible to define the function:

$$f(n)=\begin{cases}3\cdot n+1 & if\ n\equiv 1(mod\,2)\\ \dfrac{n}{2} & if\ n\equiv 0(mod\,2)\end{cases}$$

the sequence obtained using the function $f(n)$ is as follows:

$$a_i=\begin{cases}n & for\ i=0\\ f(a_{i-1}) & for\ i>0\end{cases}$$

the smallest $i=k$ such that $a_i<a_0$ is called the stopping time of n .

If n is even $n\equiv 0(mod\,2)$ then the stopping time $k=1$ and $a_1=\dfrac{n}{2}$.

If $n\equiv 1(mod\,2^2)$ then the stopping time $k=3$ and $a_3=\dfrac{3\cdot n+1}{2^2}$.

Fixed an integer x with $x>1$ if n odd and $n\equiv r(mod\,2^{k-x})$ then for some residue classes with $r>1$ it is possible to observe that

$$a_k=\frac{3^x\cdot n+s}{2^{k-x}}$$

with $s\geq 3^x-2^x$ and $s=3^{x-1}+\sum_{j=0}^{x-2}3^j\cdot 2^{\sum_{i=1}^{x-1-j}c_i}$ and with $y=k-x=\lfloor 1+x\cdot\log_2(3)\rfloor$.

As it is possible to analyze also in OEIS A177789[2] and in OEIS A020914[3] the following results can be obtained:

- stopping time 1 :

 numbers 0 (mod 2)

- stopping time 3 :

 numbers 1 (mod 2^2)

- stopping time 6 :

numbers 3 (mod 2^4)

- stopping time 8 :

numbers $11,23$ (mod 2^5)

- stopping time 11 :

numbers $7,15,59$ (mod 2^7)

- stopping time 13 :

numbers $39,79,95,123,175,199,219$ (mod 2^8)

From an analysis of the classes of remainder represented above it is possible to observe that the numbers $3,23,15,95,575$ have values of $s=3^x-2^x$ or $s=5,19,65,211,665$ these values of s can also be obtained using the sequence $s_i=3*s_{i-1}+2^{x_i-1}$ with x_i the value of x associated with s_i ;

and the numbers $11,59,219$ have values of $s=23,85,287$ these values of s can also be obtained using the sequence $s_i=3*s_{i-1}+2^{x_i}$ with x_i the value of x associated with s_i .
Knowing the values of s generated by the sequences described above, it is possible to use the following algorithm to list the residue classes based on the value of x and therefore on the equivalent stopping time.

Below is the algorithm to generate the residue classes for $x>2$:

```
xmax=8;N=100000;Ix=vector(xmax+1);R=vector(N);S=vector(N);I1=vector(N);
log2_3=log(3)/log(2);R[1]=1;S[1]=1;I1[1]=0;Ix[1]=1;R[2]=3;S[2]=5;Ix[2]=2;I1[2]=1;
for(x=2,xmax,y=floor(1+x*log2_3);x1=x+1; y1=floor(1+x1*log2_3);print("x = ",x1,"    -
stopping time: ",y1+x1,"\n");c=0; if(Ix[x]<=N,for(i=Ix[x-1]+1,Ix[x], for(i1=I1[i],y-x,s=S[i];
s=3*s+2^(x+i1-1);c=c+1;n1=((2^y1-s)*lift(Mod(1/3^x1,2^y1)))%2^y1;if(c+Ix[x]<=N,
R[c+Ix[x]]=n1;S[c+Ix[x]]=s; I1[c+Ix[x]]=i1);print1(" - ",n1)));Ix[x+1]=Ix[x]+c;print("\n\ntotal
residue classes (modulo 2^",y1,"):  ",c,"\n\n")));
```

PARI/GP code of the algorithm

x = 3   -   stopping time: 8

- 23 - 11

total residue classes (modulo 2^5): 2

x = 4   -   stopping time: 11

- 15 - 7 - 59

total residue classes (modulo 2^7): 3

x = 5   -   stopping time: 13

- 95 - 175 - 79 - 39 - 199 - 219 - 123

total residue classes (modulo 2^8): 7

x = 6   -   stopping time: 16

- 575 - 287 - 735 - 367 - 815 - 975 - 999 - 423 - 583 - 923 - 347 - 507

total residue classes (modulo 2^10): 12

x = 7   -   stopping time: 19

- 383 - 2239 - 1855 - 1087 - 2975 - 2591 - 1823 - 4063 - 3295 - 2031 - 1647 - 879 - 3119 - 2351 - 1231 - 463 - 615 - 231 - 3559 - 1703 - 935 - 3911 - 3143 - 2587 - 2203 - 1435 - 3675 - 2907 - 1787 - 1019

total residue classes (modulo 2^12): 30

x = 8   -   stopping time: 21

- 255 - 4223 - 3967 - 3455 - 2431 - 1983 - 1727 - 1215 - 191 - 5439 - 4927 - 3903 - 4159 - 3135 - 6815 - 6559 - 6047 - 5023 - 2079 - 1567 - 543 - 799 - 7967 - 3551 - 3039 - 2015 - 2271 - 1247 - 5871 - 5615 - 5103 - 4079 - 1135 - 623 - 7791 - 8047 - 7023 - 2607 - 2095 - 1071 - 1327 - 303 - 719 - 207 - 7375 - 7631 - 6607 - 4455 - 4199 - 3687 - 2663 - 7911 - 7399 - 6375 - 6631 - 5607 - 1191 - 679 - 7847 - 8103 - 7079 - 7495 - 6983 - 5959 - 6215 - 5191 - 2331 - 2075 - 1563 - 539 - 5787 - 5275 - 4251 - 4507 - 3483 - 7259 - 6747 - 5723 - 5979 - 4955 - 5371 - 4859 - 3835 - 4091 - 3067

total residue classes (modulo 2^13): 85

x = 9   -   stopping time: 24

- 5631 - 19199 - 13567 - 2303 - 12543 - 6783 - 1151 - 22655 - 127 - 9087 - 30591 - 8063 - 13695 - 23935 - 22911 - 20927 - 15295 - 4031 - 14271 - 23231 - 11967 - 22207 - 27839 - 5311 - 4287 - 18751 - 7487 - 17727 - 23359 - 831 - 32575 - 14399 - 24639 - 23615 - 9375 - 3743 - 25247 - 2719 - 11679 - 415 - 10655 - 16287 - 26527 - 25503 - 7199 - 28703 - 6175 - 11807 - 22047 - 21023 - 2847 - 13087 - 12063 - 16863 - 5599 - 15839 - 21471 - 31711 - 30687 - 12511 - 22751 - 21727 - 8431 - 2799 - 24303 - 1775 - 10735 - 32239 - 9711 - 15343 - 25583 - 24559 - 6255 - 27759 - 5231 - 10863 - 21103 - 20079 - 1903 - 12143 - 11119 - 15919 - 4655 - 14895 - 20527 - 30767 - 29743 - 11567 - 21807 - 20783 - 14031 - 2767 - 13007 - 18639 - 28879 - 27855 - 9679 - 19919 - 18895 - 23399 - 17767 - 6503 - 16743 - 25703 - 14439 - 24679 - 30311 - 7783 - 6759 - 21223 - 9959 - 20199 - 25831 - 3303 - 2279 - 16871 - 27111 - 26087 - 30887 - 19623 - 29863 - 2727 - 12967 - 11943 - 26535 - 4007 - 2983 - 28999 - 17735 - 27975 - 839 - 11079 - 10055 - 24647 - 2119 - 1095 - 29467 - 23835 - 12571 - 22811 - 31771 - 20507 - 30747 - 3611 - 13851 - 12827 - 27291 - 16027 - 26267 - 31899 - 9371 - 8347 - 22939 - 411 - 32155 - 4187 - 25691 - 3163 - 8795 - 19035 - 18011 - 32603 - 10075 - 9051 - 2299 - 23803 - 1275 - 6907 - 17147 - 16123 - 30715 - 8187 - 7163

total residue classes (modulo 2^15): 173

References

[1] https://en.wikipedia.org/wiki/Collatz_conjecture

[2] https://oeis.org/A177789

[3] https://oeis.org/A020914