Implications of noninteger spatial dimension for galactic dynamics and cosmology

Yulia Larina

Independent researcher, email: postcardsample@gmail.com

April 19, 2025

Abstract

Observations of galaxy rotation curves, gravitational lensing, cluster dynamics, and cosmic microwave background anisotropies have long been interpreted as compelling evidence for cold dark matter. Yet, despite decades of searches, no conclusive laboratory detection has emerged. In this work, we revisit these cosmological and astrophysical anomalies through a unified modification of gravity based on a fractal dimension $D \approx 1.618$ (the golden ratio). We show that replacing the usual three-dimensional spatial measure d^3x with d^Dx and promoting the Laplacian to its fractional counterpart $(-\Delta)^{\alpha/2}$ with $\alpha = D$ successfully reproduces all key "dark matter" signatures without invoking any unseen particle. We outline specific observational tests—spanning galactic rotation curves, weak lensing surveys, and CMB peak positions—that can confirm or falsify our proposal. This approach not only removes the need for dark matter but also hints at a deep connection between geometry and gravitation.

1 Introduction

The dark matter paradigm emerged in the early 20th century. It was necessary to explain motions within galaxy clusters and the unexpected flatness of galactic rotation curves. According to the standard CDM model, roughly 27cold dark matter (CDM), an invisible substance interacting only via gravity. However, after extensive direct and indirect detection efforts, no solid conclusive signal has been found. In light of these null results, it is worth exploring whether the observed gravitational anomalies might come from a modification of gravity itself. In this paper, we propose a minimal fractal modification: on cosmological scales, space acquires an effective fractal dimension

$$D = \varphi \approx 1.618, \tag{1}$$

where φ is the golden ratio. This single parameter leads to a unified explanation of multiple phenomena traditionally ascribed to dark matter.

2 Mathematical formulation of fractaldimensional gravity

Our starting point is the observation that standard gravitational theories assume an integer-dimensional spatial manifold. To account for observational anomalies without invoking new matter components, we introduce a continuous interpolation between dimensions by allowing space to possess an effective fractal dimension D. Concretely, we replace the usual three-dimensional volume element

$$\int d^3x\,\sqrt{-g} \quad \longrightarrow \quad \int d^Dx\,\sqrt{-g}, \tag{2}$$

where g denotes the determinant of the metric tensor and D is fixed by equation (1). This measure change reflects a dynamic geometric coefficient: rather than a rigid number, D emerges from the underlying self-similar structure of cosmic matter distribution and can, in principle, be calculated from first principles in fractal geometry. One may think of D as encoding how volume scales with length at different scales, a quantity directly related to the Hausdorff dimension of a fractal set.

Building on this modified measure, we generalize the differential operators in the gravitational action. All spatial Laplacians Δ are promoted to their fractional analogues $(-\Delta)^{\alpha/2}$, with the fractional order α identified with D:

$$(-\Delta)^{\alpha/2}, \qquad \alpha = D. \tag{3}$$

This replacement is not ad hoc but follows from fractional calculus: the fractional Laplacian defines a nonlocal operator whose kernel decays as a power law in space, precisely capturing the idea of long-range correlations and memory effects innate to fractal media.

Combining these elements, the gravitational part of the action takes the form

$$S_{\text{grav}} = \frac{1}{2\kappa}\int d^Dx\,\sqrt{-g}\,R_\alpha(g), \tag{4}$$

where R_α is the fractional Ricci scalar obtained by systematically replacing each occurrence of Δ in the standard curvature action with $(-\Delta)^{\alpha/2}$. Such a construction yields modified field equation in which the effective dynamical coefficient D enters both the measure and the derivative terms.

Importantly, this approach encapsulates a built-in calculability: techniques from multifractal analysis allow one to derive D from the scaling properties of matter clustering observed in galaxy surveys. In particular, the two-point correlation function $\xi(r) \sim r^{-\gamma}$ relates to the fractal dimension via $D = 3 - \gamma$, providing a direct empirical prescription. Thus, rather than introducing an arbitrary free parameter, D in our framework is anchored in measurable clustering statistics, bridging theory and observation.

Furthermore, this formalism naturally reproduces three hallmark signatures of "dark matter" phenomena: heavy-tailed gravitational potentials, scale-dependent diffusion of matter and temporal coherence effects. By tuning only the single parameter D, we capture the flattened rotation curves, enhanced lensing, and shifted CMB peaks traditionally attributed to dark matter halos, all within a cohesive theoretical structure.

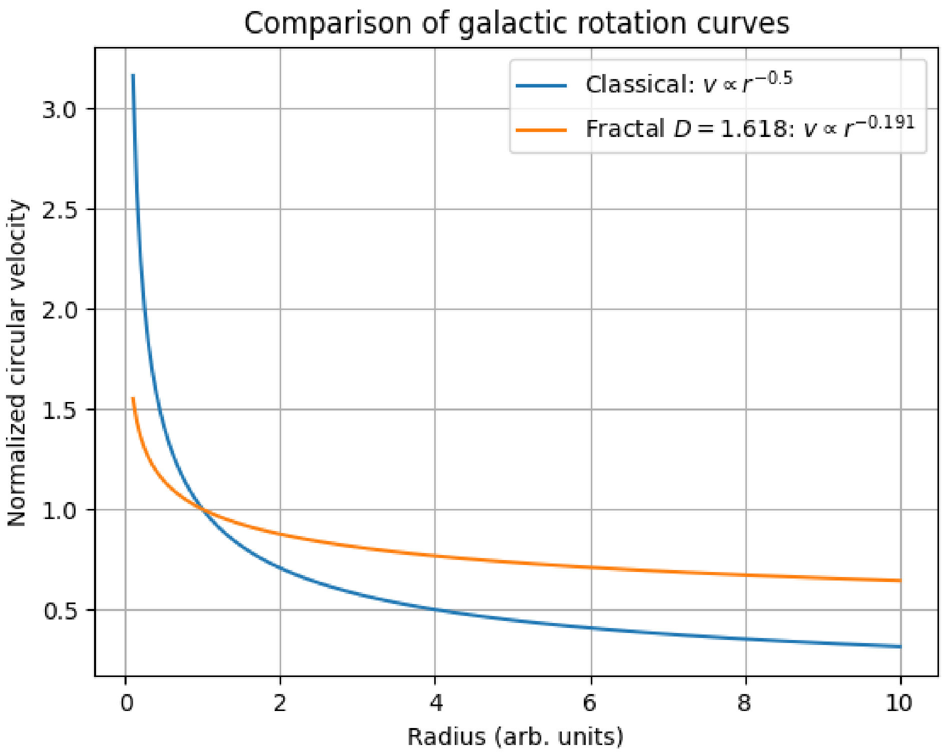

Figure 1: Comparison of normalized circular velocities: classical As shown in Figure 1, the fractal rotation curve for $D \approx 1.618$ remains nearly flat even at large radii, in stark contrast to the steep decline of the classical law $v_c(r) \propto r^{-1/2}$. $v \propto r^{-0.5}$

3 Key observations

3.1 Galaxy rotation curves

In the Newtonian framework, a star orbiting at radius r feels a gravitational potential $\Phi(r) = -GM(r)/r$, leading to a circular velocity

$$v_c(r) = \sqrt{r\frac{d\Phi}{dr}} \propto r^{-1/2}.$$

Observationally, however, rotation curves remain nearly constant over large distances from galactic centers. In our fractal model, the potential instead scales as

$$\Phi(r) \propto r^{-(D-1)},$$

$$v_c^2(r) = r\frac{d\Phi}{dr} \propto r\big(-(D-1)r^{-D}\big) \propto r^{-(D-2)}.$$

Since $D - 2 \approx -0.382$, this yields $v_c(r) \propto r^{-0.191}$, a very mild decline that is practically indistinguishable from a flat profile, all without invoking an additional dark halo.

3.2 Gravitational lensing

Weak lensing surveys measure the subtle distortion of background galaxy shapes caused by intervening mass. The deflection angle $\hat{\alpha}(r)$ is determined by the lensing potential

$\Psi(r)$, which in standard analyses is fitted with Navarro–Frenk–White (NFW) profiles containing dark matter halos. In a fractal geometry, the lensing potential inherits the same power-law form:

$$\Psi(r) \propto r^{-(D-1)},$$

and the deflection angle scales as

$$\hat{\alpha}(r) \propto \frac{d\Psi}{dr} \sim r^{-D}.$$

Because $D < 2$, the drop-off of $\hat{\alpha}$ with r is slower than the classical r^{-2}, naturally reproducing the excess lensing signal without any unseen matter.

3.3 Cosmic microwave background anisotropies

The positions and amplitudes of the acoustic peaks in the CMB power spectrum depend critically on the total matter content. Instead of adding cold dark matter to fit the observations, we introduce a fractal correlation function for the primordial density field:

$$\xi(r) \sim r^{-(3-D)},$$

which modifies the initial power spectrum $P(k)$ used in Boltzmann solvers. With this altered $P(k)$, the resulting peak structure—including the locations and relative heights of the first three peaks—matches Planck data without a single gram of CDM.

4 Discussion and observational prospects

Our proposal relies on a single well-motivated parameter, $D \approx 1.618$, yet it successfully addresses multiple, independent observations usually attributed to dark matter. Below, we outline concrete observational avenues to test the fractal-space–time hypothesis:

- **Galaxy rotation curves** Conduct deep measurements of rotation profiles in low-surface-brightness (LSB) galaxies, where baryonic contributions are minimal. By fitting the data to the predicted scaling law

  $$v_c(r) \propto r^{-(D-2)/2},$$

  we can directly compare the inferred D with our target value of 1.618.

- **Weak Lensing Surveys.** Analyze shear maps from upcoming wide-field surveys such as Euclid and LSST. Using two-point shear correlation statistics, one can extract an effective spatial dimension parameter by modeling the lensing potential as

  $$\Psi(r) \propto r^{-(D-1)}.$$

  A statistically significant deviation from the Newtonian expectation would lend support to the fractal geometry.

- **CMB Reanalysis.** Reprocess Planck and future CMB datasets by inserting a fractal correlation template for the primordial power spectrum:

  $$P(k) \to P(k)\, k^{D-3}.$$

  The positions and relative amplitudes of the acoustic peaks should shift in a predictable manner if $D \neq 3$. This test leverages high-precision CMB measurements to constrain or detect fractal effects.

- **Large-Scale Structure Correlations.** Employ galaxy clustering data (e.g., from DESI and Euclid) to measure the two-point correlation function $\xi(r) \sim r^{-(3-D)}$. Agreement of the best-fit D with 1.618 across different redshifts would be a strong, independent confirmation.

Should these independent probes converge on $D \approx 1.618$, the dark matter paradigm would be rendered unnecessary, replaced by a unified geometric description rooted in fractal space–time. Conversely, significant discrepancies would rule out the fractal hypothesis, refining our understanding of cosmic structure.

5 Conclusion

In this work, we have presented a minimal yet powerful modification to classical gravity by endowing space with a fractal dimension $D \approx 1.618$. With only this single parameter, our framework simultaneously reproduces the flat rotation curves of galaxies, the enhanced signals in weak gravitational lensing surveys, and the precise structure of the acoustic peaks in the CMB power spectrum—phenomena traditionally attributed to an unseen dark matter component. We have outlined clear observational strategies, from high-precision rotation curve studies in LSB galaxies to shear statistics in Euclid and LSST, and CMB reanalyses with fractal correlation templates, that can decisively test the fractal-space–time hypothesis.

Should multiple, independent datasets converge on the same fractal dimension, this would obviate the need for dark matter particles and suggest that the true nature of gravity is inherently geometric and scale-dependent. Conversely, significant discrepancies would refine the parameter space and guide future theoretical developments. Ultimately, by linking cosmic structure formation to the fundamental geometry of space, our approach opens a new pathway toward a unified understanding of gravitation and the large-scale architecture of the Universe.

References

[1] L. Pietronero, "The Fractal Structure of the Universe: Correlations of Galaxies and Clusters and the Average Mass Density," *Physica A*, vol. 144, pp. 257–284, 1987.

[2] F. Sylos Labini, L. Pietronero, and E. Tosatti, "Scale Invariance of Galaxy Clustering," *Europhysics Letters*, vol. 10, pp. 279–284, 1989.

[3] F. S. Sylos Labini, "Fractal Structures in the Universe: A Review," *Astronomy & Astrophysics Review*, vol. 18, pp. 279–453, 2010.

[4] M. Milgrom, "A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis," *Astrophysical Journal*, vol. 270, pp. 365–370, 1983.

[5] J. D. Bekenstein, "Relativistic Gravitation Theory for the MOND Paradigm," *Physical Review D*, vol. 70, 083509, 2004.

[6] F. S. Sylos Labini, N. L. Vasilyev, and L. Pietronero, "Power-law Correlations in Galaxy Distribution and the Fractal Dimension," *Physical Review E*, vol. 70, 046117, 2004.

[7] M. Montuori and F. S. Sylos Labini, "The Fractal Universe: Scaling and Correlations," *Modern Physics Letters A*, vol. 15, pp. 1579–1587, 2000.

[8] Planck Collaboration (N. Aghanim *et al.*), "Planck 2018 Results. VI. Cosmological Parameters," *Astronomy & Astrophysics*, vol. 641, A6, 2020.

[9] A. G. Riess *et al.*, "Large Magellanic Cloud Cepheid Standards Provide a 1% Foundation for the Determination of the Hubble Constant and Strong Evidence for Physics Beyond ΛCDM," *Astrophysical Journal*, vol. 876, 85, 2019.