

An Indonesian-English Dictionary by John M. Echols and Hassan Shadily and The Graphical Law

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(Dated: April 17, 2025)

Abstract

We study the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, Second Edition, 1963. We draw the natural logarithm of the number of head entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised. We conclude that the Dictionary can be characterised by BP(4, $\beta H = 0.01$), i.e. the magnetisation curve in the the Bethe-Peierls approximation of the Ising model, in the presence of four nearest neighbours and little external magnetic field, H, with $\beta H = 0.01$. β is $\frac{1}{k_B T}$ where, T is temperature and k_B is the tiny Boltzmann constant.

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| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | R | S | T | U | V | W | Z |
|-----|------|----|------|-----|----|-----|-----|-----|----|------|-----|-----|-----|-----|------|-----|------|------|-----|----|-----|----|
| 605 | 1006 | 49 | 1033 | 196 | 86 | 723 | 253 | 264 | 40 | 1651 | 672 | 814 | 271 | 111 | 1211 | 710 | 1578 | 1557 | 228 | 28 | 135 | 30 |

TABLE I. The head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily

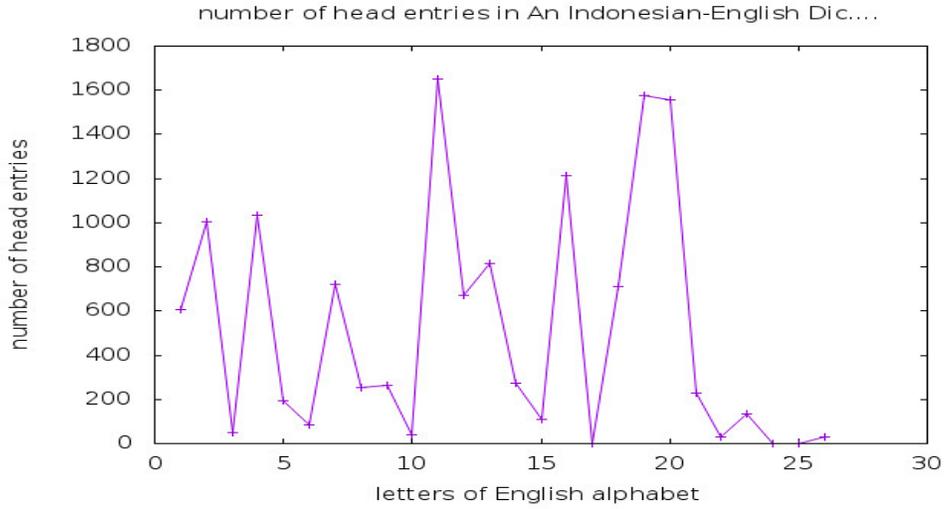


FIG. 1. The vertical axis is number of the the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1]. The horizontal axis is the letters of the English alphabet. Letters are represented by the sequence number in the alphabet.

I. INTRODUCTION

In this paper, we go over to Indonesian language. We study an Indonesian-English Dictionary. We go through the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, Second Edition, 1963, brought out by Cornell University Press, Ithaca, New York, [1]. We count all the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], one by one from the beginning to the end. The result is the table, tableI. To visualise we plot the number of head entries against the respective letters in the dictionary sequence, [1] , in the adjoining figure, fig.1.

Next we look for the graphical law. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found existence of a magnetisation curve under each language. We have termed this

phenomenon as the Graphical Law. Then, we moved on to investigate, [3], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in the references from [4] to [96].

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the head entries of the An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1]. In the section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by f and the respective rank, [97], denoted by k . k is a positive integer starting from one. Moreover, the minimum non-zero number of head entries is twenty eight. Hence, we attach a limiting word number one. The limiting rank is maximum rank plus one, here it is twenty four. As a result both $\frac{\ln f}{\ln f_{max}}$ and $\frac{\ln k}{\ln k_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{\ln f}{\ln f_{max}}$ against $\frac{\ln k}{\ln k_{lim}}$ in the figure fig.2. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table,II,and redo the plot, normalising the $\ln f$ s with $\ln f_{n-max}$, and starting from $k = 2$ in the figure fig.3. Normalising the $\ln f$ s with $\ln f_{2n-max}$, we tabulate in the adjoining table,II, and starting from $k = 3$ we draw in the figure fig.4. Normalising the $\ln f$ s with $\ln f_{3n-max}$ we record in the adjoining table,II, and plot starting from $k = 4$ in the figure fig.5. In this way we obtain figures up to the figure fig.7.

| k | lnk | lnk/ lnk_{lim} | f | lnf | lnf/ lnf_{max} | lnf/ lnf_{nmax} | lnf/ lnf_{2nmax} | lnf/ lnf_{3nmax} | lnf/ lnf_{4nmax} | lnf/ lnf_{5nmax} |
|----|------|------------------|------|-------|------------------|-------------------|--------------------|--------------------|--------------------|--------------------|
| 1 | 0 | 0 | 1651 | 7.409 | 1 | Blank | Blank | Blank | Blank | Blank |
| 2 | 0.69 | 0.217 | 1578 | 7.364 | 0.994 | 1 | Blank | Blank | Blank | Blank |
| 3 | 1.10 | 0.346 | 1557 | 7.351 | 0.992 | 0.998 | 1 | Blank | Blank | Blank |
| 4 | 1.39 | 0.437 | 1211 | 7.099 | 0.958 | 0.964 | 0.966 | 1 | Blank | Blank |
| 5 | 1.61 | 0.506 | 1033 | 6.940 | 0.937 | 0.942 | 0.944 | 0.978 | 1 | Blank |
| 6 | 1.79 | 0.563 | 1006 | 6.914 | 0.933 | 0.939 | 0.941 | 0.974 | 0.996 | 1 |
| 7 | 1.95 | 0.613 | 814 | 6.702 | 0.905 | 0.910 | 0.912 | 0.944 | 0.9666 | 0.969 |
| 8 | 2.08 | 0.654 | 723 | 6.583 | 0.889 | 0.894 | 0.896 | 0.927 | 0.949 | 0.952 |
| 9 | 2.20 | 0.692 | 710 | 6.565 | 0.886 | 0.891 | 0.893 | 0.925 | 0.946 | 0.950 |
| 10 | 2.30 | 0.723 | 672 | 6.510 | 0.879 | 0.884 | 0.886 | 0.917 | 0.938 | 0.942 |
| 11 | 2.40 | 0.755 | 605 | 6.405 | 0.864 | 0.870 | 0.871 | 0.902 | 0.923 | 0.926 |
| 12 | 2.48 | 0.780 | 271 | 5.602 | 0.756 | 0.761 | 0.762 | 0.789 | 0.807 | 0.810 |
| 13 | 2.56 | 0.805 | 264 | 5.576 | 0.753 | 0.757 | 0.759 | 0.785 | 0.803 | 0.806 |
| 14 | 2.64 | 0.830 | 253 | 5.533 | 0.747 | 0.751 | 0.753 | 0.779 | 0.797 | 0.800 |
| 15 | 2.71 | 0.852 | 228 | 5.429 | 0.733 | 0.737 | 0.739 | 0.765 | 0.782 | 0.785 |
| 16 | 2.77 | 0.871 | 196 | 5.278 | 0.712 | 0.717 | 0.718 | 0.743 | 0.761 | 0.763 |
| 17 | 2.83 | 0.890 | 135 | 4.905 | 0.662 | 0.666 | 0.667 | 0.691 | 0.707 | 0.709 |
| 18 | 2.89 | 0.909 | 111 | 4.710 | 0.636 | 0.640 | 0.641 | 0.663 | 0.679 | 0.681 |
| 19 | 2.94 | 0.925 | 86 | 4.454 | 0.601 | 0.605 | 0.606 | 0.627 | 0.642 | 0.644 |
| 20 | 3.00 | 0.943 | 49 | 3.892 | 0.525 | 0.529 | 0.529 | 0.548 | 0.561 | 0.563 |
| 21 | 3.04 | 0.956 | 40 | 3.689 | 0.498 | 0.501 | 0.502 | 0.520 | 0.532 | 0.534 |
| 22 | 3.09 | 0.972 | 30 | 3.401 | 0.459 | 0.462 | 0.463 | 0.479 | 0.490 | 0.492 |
| 23 | 3.14 | 0.987 | 28 | 3.332 | 0.450 | 0.452 | 0.453 | 0.469 | 0.480 | 0.482 |
| 24 | 3.18 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE II. The head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1]: ranking, natural logarithms, normalisations

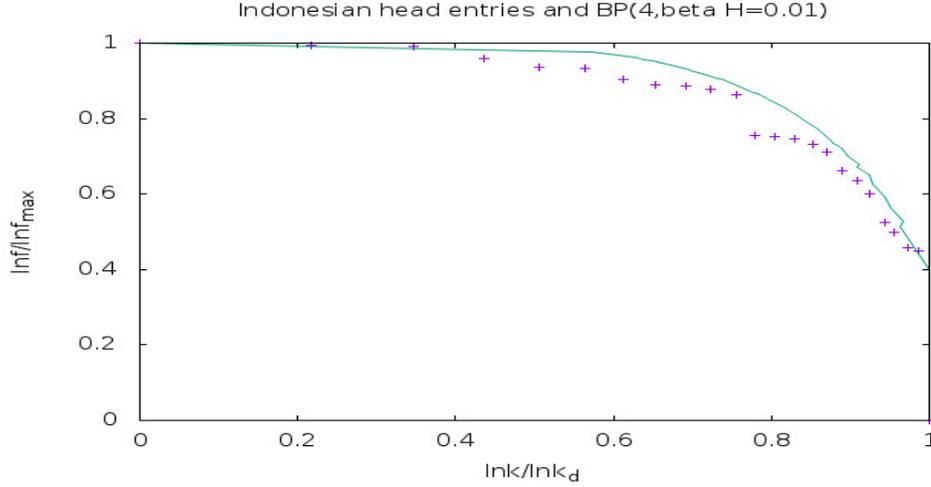


FIG. 2. The vertical axis is $\frac{\ln f}{\ln f_{max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve BP(4, $\beta H = 0.01$), being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H, with $\beta H = 0.01$.

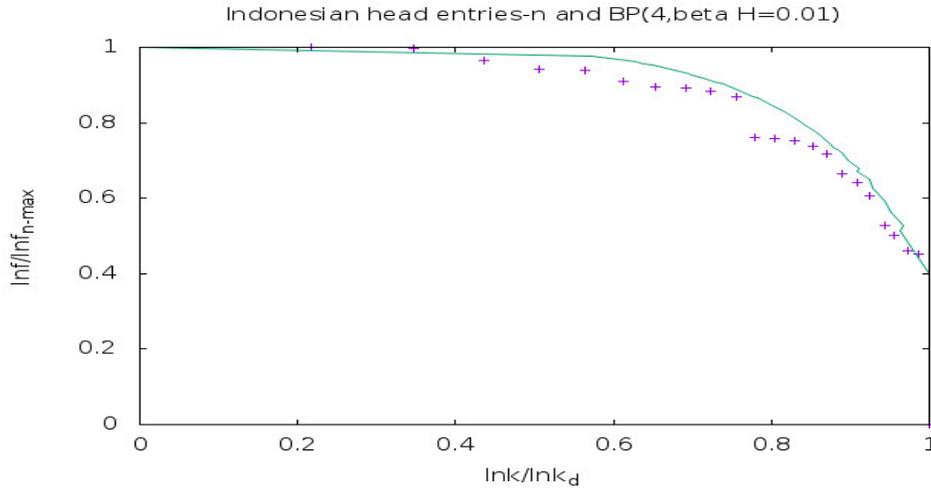


FIG. 3. The vertical axis is $\frac{\ln f}{\ln f_{n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve BP(4, $\beta H = 0.01$), being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H, with $\beta H = 0.01$.

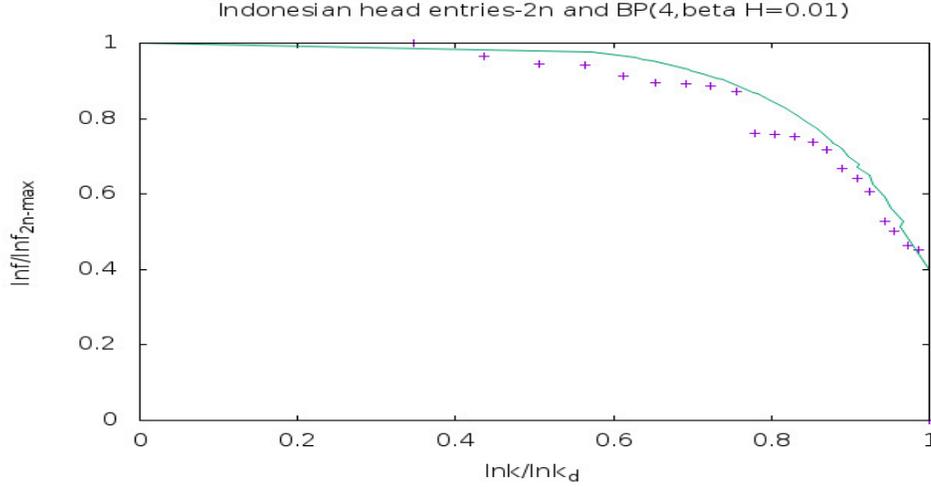


FIG. 4. The vertical axis is $\frac{\ln f}{\ln f_{2n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

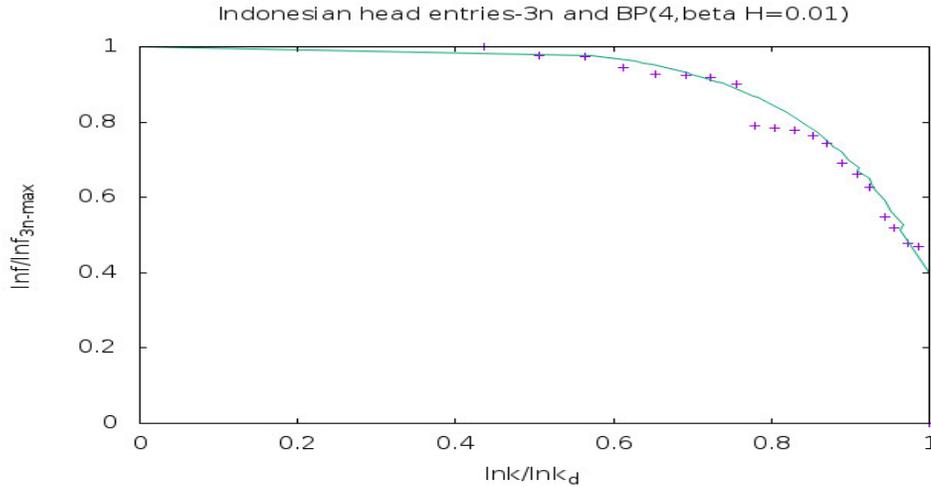


FIG. 5. The vertical axis is $\frac{\ln f}{\ln f_{3n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

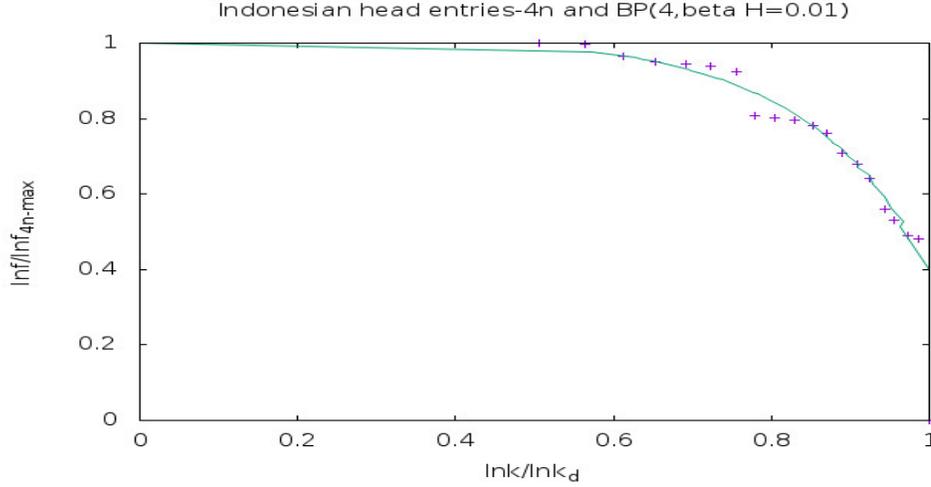


FIG. 6. The vertical axis is $\frac{\ln f}{\ln f_{4n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

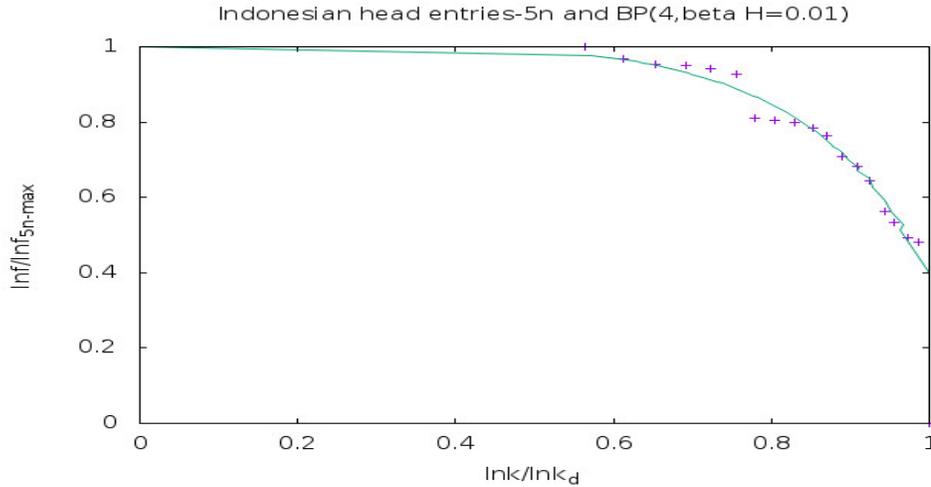


FIG. 7. The vertical axis is $\frac{\ln f}{\ln f_{5n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], with the fit curve, $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

A. conclusion

From the figures (fig.2-fig.7), we observe that there is a curve of magnetisation, behind the head entries of An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1]. This is BP(4, $\beta H = 0.01$). This is the magnetisation curve in the Bethe-Peierls approximation of the Ising Model, in the presence of four nearest neighbours and little external magnetic field, H, with $\beta H = 0.01$. β is $\frac{1}{k_B T}$ where, T is temperature and k_B is the tiny Boltzmann constant.

Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{4n-max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$\ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [104].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\sum_i\sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment , M is $\mu\sum_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL , $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is

referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[98], for the lattice of spins, setting μ to one, is $-\epsilon \sum_{n.n} \sigma_i \sigma_j - H \sum_i \sigma_i$, where n.n refers to nearest neighbour pairs. The difference ΔE of energy if we flip an up spin to down spin is, [99], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $exp(-\frac{\Delta E}{k_B T})$, [100]. In the Bragg-Williams approximation,[101], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [102]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [99]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [98],[99],[100],[101],[102], due to Bethe-Peierls, [103], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}} \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe

| BW | BW($c=0.01$) | BP(4, $\beta H = 0$) | reduced magnetisation |
|-------|----------------|-----------------------|-----------------------|
| 0 | 0 | 0 | 1 |
| 0.435 | 0.439 | 0.563 | 0.978 |
| 0.439 | 0.443 | 0.568 | 0.977 |
| 0.491 | 0.495 | 0.624 | 0.961 |
| 0.501 | 0.507 | 0.630 | 0.957 |
| 0.514 | 0.519 | 0.648 | 0.952 |
| 0.559 | 0.566 | 0.654 | 0.931 |
| 0.566 | 0.573 | 0.7 | 0.927 |
| 0.584 | 0.590 | 0.7 | 0.917 |
| 0.601 | 0.607 | 0.722 | 0.907 |
| 0.607 | 0.613 | 0.729 | 0.903 |
| 0.653 | 0.661 | 0.770 | 0.869 |
| 0.659 | 0.668 | 0.773 | 0.865 |
| 0.669 | 0.676 | 0.784 | 0.856 |
| 0.679 | 0.688 | 0.792 | 0.847 |
| 0.701 | 0.710 | 0.807 | 0.828 |
| 0.723 | 0.731 | 0.828 | 0.805 |
| 0.732 | 0.743 | 0.832 | 0.796 |
| 0.756 | 0.766 | 0.845 | 0.772 |
| 0.779 | 0.788 | 0.864 | 0.740 |
| 0.838 | 0.853 | 0.911 | 0.651 |
| 0.850 | 0.861 | 0.911 | 0.628 |
| 0.870 | 0.885 | 0.923 | 0.592 |
| 0.883 | 0.895 | 0.928 | 0.564 |
| 0.899 | 0.918 | | 0.527 |
| 0.904 | 0.926 | 0.941 | 0.513 |
| 0.946 | 0.968 | 0.965 | 0.400 |
| 0.967 | 0.998 | 0.965 | 0.300 |
| 0.987 | | 1 | 0.200 |
| 0.997 | | 1 | 0.100 |
| 1 | 1 | 1 | 0 |

TABLE III. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours.

data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.8. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

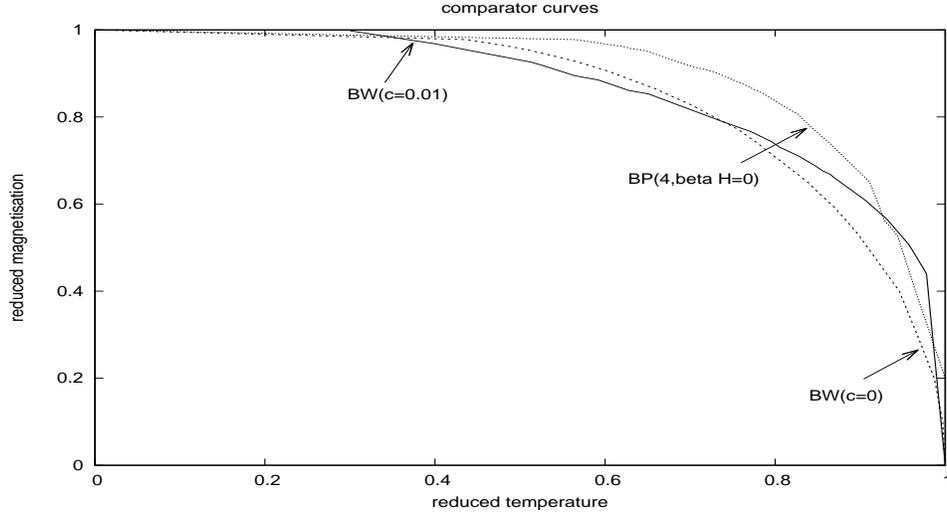


FIG. 8. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [103], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor}^{\frac{\gamma-1}{\gamma}} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula ala [103] is given in the appendix of [7].

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} \text{factor}^{\frac{1}{\gamma}}}{\text{factor}^{\frac{\gamma-1}{\gamma}} - 1}} = \frac{T}{T_c}; \text{factor} = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.9. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

| BP(m=0.03) | BP(m=0.025) | BP(m=0.02) | BP(m=0.01) | BP(m=0.005) | reduced magnetisation |
|------------|-------------|------------|------------|-------------|-----------------------|
| 0 | 0 | 0 | 0 | 0 | 1 |
| 0.583 | 0.580 | 0.577 | 0.572 | 0.569 | 0.978 |
| 0.587 | 0.584 | 0.581 | 0.575 | 0.572 | 0.977 |
| 0.647 | 0.643 | 0.639 | 0.632 | 0.628 | 0.961 |
| 0.657 | 0.653 | 0.649 | 0.641 | 0.637 | 0.957 |
| 0.671 | 0.667 | | 0.654 | 0.650 | 0.952 |
| | 0.716 | | | 0.696 | 0.931 |
| 0.723 | 0.718 | 0.713 | 0.702 | 0.697 | 0.927 |
| 0.743 | 0.737 | 0.731 | 0.720 | 0.714 | 0.917 |
| 0.762 | 0.756 | 0.749 | 0.737 | 0.731 | 0.907 |
| 0.770 | 0.764 | 0.757 | 0.745 | 0.738 | 0.903 |
| 0.816 | 0.808 | 0.800 | 0.785 | 0.778 | 0.869 |
| 0.821 | 0.813 | 0.805 | 0.789 | 0.782 | 0.865 |
| 0.832 | 0.823 | 0.815 | 0.799 | 0.791 | 0.856 |
| 0.841 | 0.833 | 0.824 | 0.807 | 0.799 | 0.847 |
| 0.863 | 0.853 | 0.844 | 0.826 | 0.817 | 0.828 |
| 0.887 | 0.876 | 0.866 | 0.846 | 0.836 | 0.805 |
| 0.895 | 0.884 | 0.873 | 0.852 | 0.842 | 0.796 |
| 0.916 | 0.904 | 0.892 | 0.869 | 0.858 | 0.772 |
| 0.940 | 0.926 | 0.914 | 0.888 | 0.876 | 0.740 |
| | 0.929 | | | 0.877 | 0.735 |
| | 0.936 | | | 0.883 | 0.730 |
| | 0.944 | | | 0.889 | 0.720 |
| | 0.945 | | | | 0.710 |
| | 0.955 | | | 0.897 | 0.700 |
| | 0.963 | | | 0.903 | 0.690 |
| | 0.973 | | | 0.910 | 0.680 |
| | | | | 0.909 | 0.670 |
| | 0.993 | | | 0.925 | 0.650 |
| | | 0.976 | 0.942 | | 0.651 |
| | 1.00 | | | | 0.640 |
| | | 0.983 | 0.946 | 0.928 | 0.628 |
| | | 1.00 | 0.963 | 0.943 | 0.592 |
| | | | 0.972 | 0.951 | 0.564 |
| | | | 0.990 | 0.967 | 0.527 |
| | | | 1.00 | 0.964 | 0.513 |
| | | | | 1.00 | 0.500 |
| | | | | | 0.400 |
| | | | | | 0.300 |
| | | | | | 0.200 |
| | | | | | 0.100 |
| | | | | | 0 |

TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields

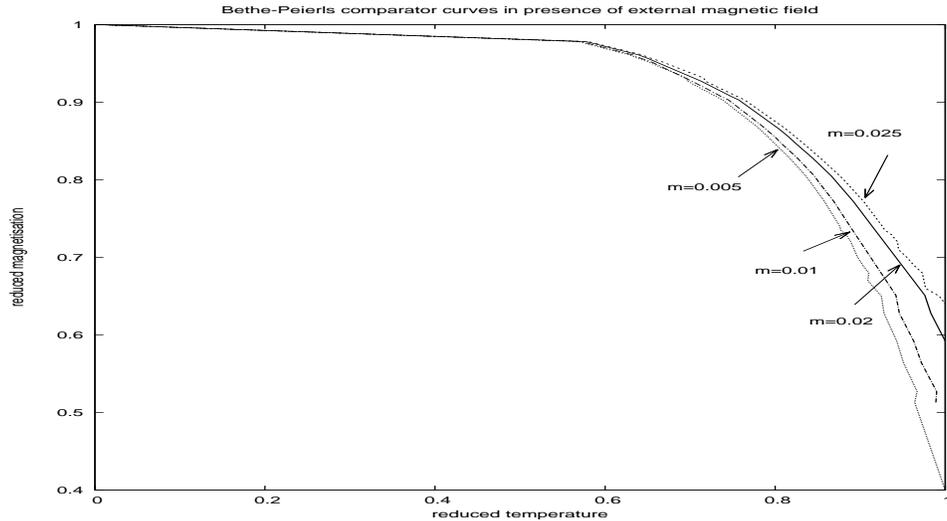


FIG. 9. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

IV. ACKNOWLEDGMENT

The author would like to thank the National Library(Government Of India), Kolkata, to allow us to use An Indonesian-English Dictionary by John M. Echols and Hassan Shadily, [1], in its premises. We have used gnuplot for plotting the figures in this paper.

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