Why We Do Not Need Dark Energy to Explain Cosmological Acceleration

Felix M Lev

Independent Researcher, San Diego, 92010 CA USA

Email: felixlev314@gmail.com

Abstract

It is shown that at the present stage of the evolution of the universe, cosmological acceleration is an inevitable **kinematical** consequence of quantum theory in semiclassical approximation. Quantum theory does not involve such classical concepts as Minkowski or de Sitter spaces. In classical theory, when choosing Minkowski space, a vacuum catastrophe occurs, while when choosing de Sitter space, the value of the cosmological constant can be arbitrary. On the contrary, in quantum theory there are no uncertainties in view of the following: 1) the de Sitter algebra is the most general ten-dimensional Lie algebra; 2) the Poincare algebra is a special degenerate case of the de Sitter algebra in the limit $R \to \infty$ where R is the contraction parameter for the transition from the de Sitter to the Poincare algebra and R has nothing to do with the radius of de Sitter space; 3) R is fundamental to the same extent as c and $\hbar$: c is the contraction parameter for the transition from the Poincare to the Galilean algebra and $\hbar$ is the contraction parameter for the transition from quantum to classical theory; 4) as a consequence, the question (why the quantities $(c, \hbar, R)$ have the values which they actually have) does not arise. The solution to the problem of cosmological acceleration follows from results on irreducible representations of the de Sitter algebra. This solution is free of uncertainties and does not involve dark energy, quintessence and other exotic mechanisms the physical meaning of which is a mystery.

Keywords: irreducible representations; cosmological acceleration; de Sitter symmetry

1 Problem with the value of the cosmological constant

Let's consider a system of macroscopic bodies that are located at large distances from each other so that all interactions between the bodies (gravitational, electromagnetic and others) can be neglected. Let us also assume that the sizes of these bodies are much smaller than the distances between them. Then the motion of each body can be considered independently of the motion of other bodies. We will be interested only in the motion of each body as a whole, i.e., we will not consider, for example, the internal rotation of these bodies. Then, formally, our problem can be considered as a problem of the motion of N noninteracting elementary particles with zero spin.

We assume that the velocities of all bodies in our problem are much less than c. Then the problem seems clear, and its solution seems obvious: since all bodies are at large distances from each other, the motion of each body does not depend on other bodies and each body can only move at some constant speed with zero acceleration.

However, physicists were surprised when in 1998 observations [1] showed that the bodies move relative to each other with the relative acceleration

$$\mathbf{a} = \mathbf{r}c^2/R^2 \tag{1}$$

where $\mathbf{r}$ is the relative radius-vector and R is a quantity with the dimension of length. Usually this quantity is expressed in terms of the cosmological constant Λ as $\Lambda = 3/R^2$ and the recent observational data of the Planck collaboration [2] show that $\Lambda = 1.3 \cdot 10^{-52}/m^2$ with the accuracy 5%. Therefore R is a quantity of the order of $10^{26}m$. Thus, observations have shown that bodies repel each other, and the repulsive force is proportional (not inversely proportional) to the distance between them. The formula (1) also shows that in our daily life and even in the Solar System, this repulsive force is negligible. However, it becomes significant for bodies located at cosmological distances from each other.

The first impression may be that physicists should not have been surprised by this observation because the result (1) is obtained in General Relativity (GR) if we assume that we live in de Sitter space which is characterized by the cosmological constant Λ. However, a question arises whether modern theory can explain why the value of

Λ is as is. Until 1998, the typical philosophy of GR described even in textbooks was described as follows. The curvature of space is created by bodies. That is why Λ (which is the curvature of empty space) should be zero. For this reason, the result (1) seemed like a shock to the foundations.

The paper is organized as follows. In Sec. 2 we explain why the mainstream literature describes cosmological acceleration in terms of dark energy or quintessence. In Sec. 3 we explain that symmetry at the quantum level is determined not by background space but by commutation relations of the symmetry algebra, and in Sec. 4 we derive expressions for the cosmological acceleration.

2 History of dark energy

The earliest literature on cosmological constant and the universe expansion contained the following main publications:

- In 1917, Einstein believed that the universe was stationary and this was possible only if Λ in his equations was non-zero [3].
- In 1922, Friedman found solutions of equations of GR with $\Lambda = 0$ to provide theoretical evidence that the universe is expanding [4].
- In [5] Steer stated that in 1924 Lundmark was the first person to find observational evidence for expansion of the universe — three years before Lemaître and five years before Hubble, but, for some reasons, Lundmark's research was not adopted and his paper was not published.
- In 1927, Lemaître independently reached a conclusion similar to Friedman's one and also presented observational evidence (based on the Doppler effect) for a linear relationship between distance to galaxies and their recessional velocity [6].
- In 1929, Hubble observationally confirmed Lundmark's and Lemaître's findings [7].

As Gamow recalls, upon learning of Hubble's results, Einstein said that his statement that $\Lambda \neq 0$ was the biggest blunder of his life and after that the mainstream literature (including textbooks) began to claim that $\Lambda = 0$ is a necessary condition. The argument was that a curvature of space-time is created by matter and so, the empty space-time should be the flat Minkowski space.

That is why, the fact that the results [1] could be described only with $\Lambda \neq 0$ was first perceived as a shock of something fundamental. However, the following way out of this situation was proposed as follows: the terms with Λ in the Einstein equations have been moved from the l.h.s. to the r.h.s. and were interpreted not as the curvature of empty space-time (which was supposed to be zero), but as a manifestation of hypothetical fields called dark energy or quintessence. Although their physical nature remains a mystery (see e.g., [8] and references therein), and, as noted in [9], there are an almost endless number of explanations for dark energy, mainstream publications on the problem of cosmological acceleration (PCA) involve those concepts. However, these approaches have not solved PCA without uncertainties.

They were criticized by several authors from the following considerations. GR with the choice $\Lambda = 0$ has been confirmed with great accuracy in experiments in the Solar System. If Λ is as small as it has been observed in [1, 2], it can be important only at cosmological distances while for experiments in the Solar System, the role of $\Lambda \neq 0$ is negligible. The authors of [10] titled "Why all these prejudices against a constant?" note that if we accept the theory with the gravitational constant G taken from outside, then why can't we accept a theory containing two independent constants?

Currently there is no physical theory working under all conditions. For example, nonrelativistic theory cannot be extrapolated to cases when speeds are comparable to c and classical physics cannot be extrapolated for describing energy levels of the hydrogen atom. GR is a successful classical (non-quantum) theory for describing macroscopic phenomena where large masses (stars and planets) are present, but the extrapolation of GR to the case of empty space is not physical.

When there are many particles, it may appear that they are in some space. However, space in itself is not a physical but a mathematical object. We can, for example, measure the coordinates of a certain particle and discuss with what accuracy they can be measured. But there is no experiment that measures the coordinates of space.

The aim of space is to give a mathematical technique for describing the motion of real bodies. But the concepts of empty space and its curvature should not be used in physics because nothing can be measured in a space which exists only in our imagination. Indeed, in the limit of GR when matter disappears, space remains and has a curvature while, since space is only a mathematical concept for describing matter, a reasonable approach should be such that in this limit space should disappear too.

A common principle of physics is that the results of new experiments should be explained first of all proceeding from the existing science. Only if all such efforts fail, something exotic can be involved. But for PCA, exotic explanations with dark energy or quintessence are used without serious efforts to explain the data in the framework of existing science.

While in most publications, only proposals about future discovery of dark energy are considered, the authors of [8] stated that dark energy had already been discovered by the XENON1T collaboration. In June 2020, it reported an excess of electron recoils: 285 events, 53 more than expected 232 with a statistical significance of 3.5σ. However, in July 2022, a new analysis by the XENONnT collaboration [11] discarded the excess.

As noted in the present paper, at the current stage of the universe (when semiclassical approximation is valid), PCA can be explained without uncertainties and without involving models and/or assumptions containing ambiguities.

3 Hierarchy of physical theories

Cases when one theory is a special case of another are widely discussed in the literature. For example, it is known that nonrelativistic theory is a special degenerate case of relativistic theory in the limit $c \to \infty$ and classical theory is a special degenerate case of quantum theory in the limit $\hbar \to 0$. These cases are discussed in the literature using many examples. However, a question arises: is it possible to give a general criterion when theory A is more general than theory B, and theory B is a special degenerate case of theory A? In [12] and our other publications, we proposed the following criterion:

Definition: *Let theory A contain a finite nonzero parameter and theory B be obtained from theory A in the formal limit when the parameter goes to zero or infinity. Suppose that with any desired accuracy theory A can reproduce any result of theory B by choosing a value of* *the parameter. On the contrary, when the limit is already taken, one cannot return back to theory A and theory B cannot reproduce all results of theory A. Then theory A is more general than theory B and theory B is a special degenerate case of theory A.*

In particular, this means that:

- Any result of nonrelativistic theory can be obtained with any desired accuracy from relativistic theory with some choice of c. On the other hand, in nonrelativistic theory it is not possible to obtain those results of relativistic theory where it is crucial that c if finite and not infinitely large.

- Any result of classical (non-quantum) theory can be obtained with any desired accuracy from quantum theory with some choice of $\hbar$. On the other hand, in classical theory it is not possible to obtain those results of quantum theory where it is crucial that $\hbar$ if finite and not infinitely small.

Since we are considering the motion of macroscopic bodies, it would seem that it is quite sufficient to consider our problem at the classical (not quantum) level. However, formally, the result of any classical problem must be obtained from quantum theory in semiclassical approximation. We will see below that considering our problem at the quantum level makes it possible to understand the problem at a deeper level.

In the literature, symmetry in QFT is usually explained as follows. Since Poincare group is the group of motions of Minkowski space, the system under consideration should be described by unitary representations of this group. This implies that the representation generators are selfadjoined and commute according to the commutation relations of the Poincare group Lie algebra:

$$\begin{gathered}
[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho), \\
[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma})
\end{gathered} \tag{2}$$

where $\mu, \nu = 0, 1, 2, 3$, $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$, P^μ are the operators of the four-momentum and $M^{\mu\nu}$ are the operators of Lorentz angular momenta. This approach is in the spirit of the Erlangen Program proposed by Felix Klein in 1872 when quantum theory did not yet exist. However, although the Poincare group is the group of motions of Minkowski space, the description (2) does not involve this group and this space.

As indicated in the extensive physics literature (see, for example, [12]), background space is only a mathematical concept: in quantum theory, each physical quantity should be described by an operator but there are no operators for the coordinates of background space. *There is no law that every physical theory must contain background space.* For example, it is not used in nonrelativistic quantum mechanics and in irreducible representations (IRs) describing elementary particles. In particle theory, transformations from the Poincare group are not used because, according to the Heisenberg S-matrix program, it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$. In this theory, systems are described by observable physical quantities — momenta and angular momenta. So, *symmetry at the quantum level is defined not by a background space and its group of motions but by the condition that the commutators of the operators describing the system under consideration are determined by the symmetry algebra of this system.* In particular, Eqs. (2) can be treated as the definition of relativistic (Poincare) invariance at the quantum level.

Then each elementary particle is described by a selfadjoint IR of a real Lie algebra A and a system of N noninteracting particles is described by the tensor product of the corresponding IRs. This implies that, for the system as a whole, each momentum operator is a sum of the corresponding single-particle momenta, each angular momentum operator is a sum of the corresponding single-particle angular momenta, and *this is the most complete possible description of this system.* In particular, nonrelativistic symmetry implies that A is the Galilei algebra, relativistic (Poincare) symmetry implies that A is the Poincare algebra, de Sitter (dS) symmetry implies that A is the dS algebra so(1,4) and anti-de Sitter (AdS) symmetry implies that A is the AdS algebra so(2,3).

In his famous paper "Missed Opportunities" [13] Dyson notes that:

- a) Relativistic quantum theories are more general than nonrelativistic quantum theories even from purely mathematical considerations because Poincare group is more symmetric than Galilei one: the latter can be obtained from the former by contraction $c \to \infty$.

- b) dS and AdS quantum theories are more general than relativistic quantum theories even from purely mathematical considerations because dS and AdS groups are more symmetric than Poincare one: the latter can be obtained from the former by contraction $R \to \infty$ where R is a parameter with the dimension *length*, and the meaning of this parameter will be explained below.

- c) At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

As noted above, symmetry at the quantum level should be defined by a symmetry algebra for the system under consideration. In [12], the statements a)-c) have been reformulated in terms of the corresponding Lie algebras and it has also been shown that quantum theory is more general than classical theory because the classical symmetry algebra can be obtained from the symmetry algebra in quantum theory by contraction $\hbar \to 0$. For these reasons, the most general description in terms of ten-dimensional Lie algebras should be carried out in terms of quantum dS or AdS symmetry.

The definition of those symmetries is as follows. If M^{ab} ($a, b = 0, 1, 2, 3, 4$, $M^{ab} = -M^{ba}$) are the angular momentum operators for the system under consideration, they should satisfy the commutation relations:

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \qquad (3)$$

where $\eta^{ab} = 0$ if $a \neq b$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$ and $\eta^{44} = \mp 1$ for the dS and AdS symmetries, respectively.

Although the dS and AdS groups are the groups of motions of dS and AdS spaces, respectively, the description in terms of (3) does not involve those groups and spaces, and *it is the definition of dS and AdS symmetries at the quantum level* (see the discussion in [12, 14]). In QFT, interacting particles are described by field functions defined on Minkowski, dS and AdS spaces. However, since we consider only noninteracting bodies and describe them in terms of IRs, at this level we don't need these fields and spaces.

The contraction of the Poincare algebra into the Galilean algebra and the contraction of the quantum algebra into the classical one are widely described in the literature (see, for example, Section 1.3 in [12]). If c is much greater than all velocities in a given system, then Galilean symmetry is a good approximation for describing this system. Similarly, if all angular momenta in a given system are much greater than $\hbar$, then classical physics is a good approximation for describing this system.

In particle theory, the quantities $(c, \hbar)$ are usually not involved and this is characterized such that the system of units $c = \hbar = 1$ is used (although the concept of a system of units makes sense only in macroscopic physics). Then all velocities are dimensionless and ≤ 1 (if tachyons are not taken into account). However, if people want to describe velocities in m/s then c also has the dimension m/s. Physicists usually understand that physics cannot (and should not) derive that $c \approx 3 \cdot 10^8 m/s$. This value is purely kinematical (i.e., it does not depend on gravity and other interactions) and is as is simply because people want to describe velocities in m/s. Since the quantities (m, s) have a physical meaning only at the macroscopic level, one can expect that the values of c in m/s are different at different stages of the universe.

Analogously, physicists usually understand that physics cannot (and should not) derive that $\hbar \approx 1.054 \cdot 10^{-34} kg \cdot m^2/s$. This value is purely kinematical and is as is simply because people want to describe angular momenta in $kg \cdot m^2/s$. Since the quantities (kg, m, s) have a physical meaning only at the macroscopic level, one can expect that the values of $\hbar$ in $kg \cdot m^2/s$ are different at different stages of the universe.

Now consider the contraction from dS or AdS symmetry to Poincare one. If the momentum operators P^ν $(\nu = 0, 1, 2, 3)$ *are defined* as $P^\nu = M^{4\nu}/R$ then in the limit when $R \to \infty$, $M^{4\nu} \to \infty$ but the quantities P^ν are finite, Eqs. (3) become Eqs. (2). Here R *is a parameter which has nothing to do with the dS and AdS spaces.* As seen from Eqs. (3), quantum dS and AdS theories do not involve the dimensional parameters $(c, \hbar, R)$ because (kg, m, s) are meaningful only at the macroscopic level.

In Poincare invariant theories, $P^2 = \sum P_\nu P^\nu$ is the Casimir operator, i.e., it commutes with all representation operators. According to Schur's lemma, in any IR of the Lie algebra, any Casimir operator has only one eigenvalue. In particle theory this eigenvalue is positive if tachyons are not taken into account. It is usually denoted as m^2

and m is called the mass of the particle. In dS and AdS theories, the analogous Casimir operator is $\sum_{a<b} M_{ab}M^{ab}$. If tachyons are not taken into account, then in each IR the Casimir operator has only one positive eigenvalue which can be denoted as μ^2 and μ can be called the dS or AdS mass.

If Poincare symmetry is a good approximate symmetry then the relation between μ and m is $\mu = mR$. Since dS and AdS theories are more general (fundamental) than Poincare theory, μ is more general (fundamental) than m, and, in contrast to m, μ is dimensionless. As noted above, R is of the order of $10^{26}m$. Then the dS or AdS mass of the electron is of the order of 10^{39} and the fact that this mass is so large pose a question whether the electron is a true elementary particle.

At the quantum level, Eqs. (3) are the most general description of dS and AdS symmetries and all the operators in Eqs. (3) are dimensionless. At this level, the theory does not need the quantity R and, by analogy with the choice ($c = \hbar = 1$) in particle theory, $R = 1$ is a possible choice. The dimensional quantity R arises if physicists want to deal with the 4-momenta P^μ *defined* such that $M^{4\mu} = RP^\mu$. By analogy with the quantities c and $\hbar$, physics cannot (and should not) derive the value of R. It is as is simply because people want to measure distances in meters. This value is purely kinematical, i.e., it does not depend on gravity and other interactions. As noted in Sec. 1, at the present stage of the universe, R is of the order of $10^{26}m$ but, since the concept of meter has a physical meaning only at the macroscopic level, one can expect that the values of R in meters are different at different stages of the universe.

Although, at the level of contraction parameters, R has nothing to do with the radius of the background space and is fundamental to the same extent as c and $\hbar$, physicists usually want to treat R as the radius of the background space. In GR which is the non-quantum theory, $\Lambda = \pm 3/R^2$ for the dS and AdS symmetries, respectively. Physicists usually believe that physics should derive the value of Λ and that the solution to the dark energy problem depends on this value. They also believe that QFT of gravity should confirm the experimental result that, in units $c = \hbar = 1$, Λ is of the order of $10^{-122}/G$ where G is the gravitational constant. We will discuss this problem in Sec. 5.

As noted in Sec. 1, in PCA, it is assumed that the bodies are located at large (cosmological) distances from each other and sizes of the bodies are much less than distances between them. Therefore, interactions between the bodies can be neglected and, from the formal point of view, the description of our system is the same as the description of N free spinless elementary particles.

However, in literature, PCA is usually considered in the framework of dark energy and other exotic concepts. In Sec. 2 we argue that such considerations are not based on rigorous physical principles. In the present section we have explained how symmetry should be defined at the quantum level and in Sec. 4 we describe PCA in the framework of our approach.

4 Explanation of cosmological acceleration

As explained above, the most general approach to PCA is to consider this problem within the framework of semiclassical approximation to dS or AdS quantum theory. We first consider the dS case and the results about the AdS one will be mentioned later. As described in Sec. 1, in PCA, the motion of each body can be considered independently of the motion of other bodies. Therefore, the representation of the dS algebra describing our system is the tensor product of IRs for each body. Since the observed quantities correspond to self-adjoint operators, we must consider selfadjoint IRs of the dS algebra.

Unitary IRs of the dS group have been considered by several authors. By using the results of the excellent Mensky's book [15], we described selfadjoint IRs of the dS algebra in [16, 17, 18]. We will consider the operators $M^{4\mu}$ not only in Poincare approximation but taking into account dS corrections. If those corrections are small, then, as explained in [19], IRs under consideration can be described by Eqs. (2.2) in that reference.

These equations describe IRs in momentum representation and *at this stage, we have no spatial coordinates yet*. However, in the semiclassical approximation it is necessary to know how the momentum representation is related to the coordinate one. These representations are usually considered to be related by the Fourier transform. As shown in [12], such a connection is not universal, for example it does not work for photons from distant stars. However, since bodies in PCA can be described in the nonrelativistic approximation, the position operator in momentum representation can be defined as usual, i.e., as $\mathbf{r} = i\hbar\partial/\partial\mathbf{p}$.

In semiclassical approximation, we can treat $\mathbf{p}$ and $\mathbf{r}$ as usual vectors. Then as follows from Eqs. (2.2) in [19]

$$\mathbf{P} = \mathbf{p} + mc\mathbf{r}/R, \quad H = \mathbf{p}^2/2m + c\mathbf{pr}/R, \quad \mathbf{N} = -m\mathbf{r} \tag{4}$$

where $H = E - mc^2$ is the classical nonrelativistic Hamiltonian and $\mathbf{N} = (M^{01}, M^{02}, M^{03})$ is the operator of Lorentz boosts. As follows from these expressions and Eqs. (2.2) in [19]

$$H(\mathbf{P}, \mathbf{r}) = \frac{\mathbf{P}^2}{2m} - \frac{mc^2\mathbf{r}^2}{2R^2} \tag{5}$$

where the last term is the dS correction to the non-relativistic Hamiltonian. A shown in [19], now it follows from the Hamilton equations that a free particle is moving with the acceleration

$$\mathbf{a} = \mathbf{r}c^2/R^2 = \frac{1}{3}c^2\Lambda\mathbf{r} \tag{6}$$

where $\mathbf{r}$ is the radius vector of the particle and $\Lambda = 3/R^2$.

To describe a system of N bodies, it is necessary to take into account that it is described by the tensor product of single-body representations. Therefore, each operator M^{ab} for the N-body system is the sum of the corresponding single-body operators M^{ab}. Therefore, if two free bodies are described by the variables $\mathbf{P}_j$ and $\mathbf{r}_j$ $(j = 1, 2)$ and standard nonrelativistic variables are

$$\begin{aligned} &\mathbf{P}_{12} = \mathbf{P}_1 + \mathbf{P}_2, \quad \mathbf{q}_{12} = (m_2\mathbf{P}_1 - m_1\mathbf{P}_2)/(m_1 + m_2) \\ &\mathbf{R}_{12} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2), \quad \mathbf{r}_{12} = \mathbf{r}_1 - \mathbf{r}_2 \end{aligned} \tag{7}$$

then explicit calculations using e.g., Eq. (61) in [16] or Eq. (17) in [18]) give that the mass of the two-body system is given by

$$M(\mathbf{q}_{12}, \mathbf{r}_{12}) = m_1 + m_2 + H_{nr}(\mathbf{r}_{12}, \mathbf{q}_{12}), \quad H_{nr}(\mathbf{r}, \mathbf{q}) = \frac{\mathbf{q}^2}{2m_{12}} - \frac{m_{12}c^2\mathbf{r}^2}{2R^2} \tag{8}$$

where H_{nr} is the internal two-body Hamiltonian and m_{12} is the reduced two-particle mass. Then, again by using the Hamilton equations, we get that the relative acceleration is given by Eq. (6) but now

a is the relative acceleration and **r** is the relative radius vector, i.e., Eq. (1) is indeed valid.

One might ask why Eq. (1) contains c although we assume that the bodies in PCA are nonrelativistic. The reason is that Poincare invariant theories do not contain R but we work in dS invariant theory and assume that, although c and R are very large, they are not infinitely large, and the quantity c^2/R^2 in Eq. (1) is finite.

As noted in [19], dS symmetry is more general than AdS one. Formally, an analogous calculation using the results of Chap. 8 of [12] on IRs of the AdS algebra gives that, in the AdS case, $\mathbf{a} = -\mathbf{r}c^2/R^2$, i.e., we have attraction instead of repulsion. The experimental facts that the bodies repel each other confirm that dS symmetry is indeed more general than AdS one.

The relative accelerations given by (1) are formally the same as those derived from GR if the curvature of dS space equals $\Lambda = 3/R^2$, where R is the radius of this space. *However, the crucial difference between our results and the results of GR is as follows. While in GR, R is the radius of the dS space and can be arbitrary, as explained in detail in Sec. 3, in quantum theory, R has nothing to do with the radius of the dS space, it is the coefficient of proportionality between $M^{4\mu}$ and P^{μ}, it is fundamental to the same extent as c and $\hbar$, and a question why R is as is does not arise.* **Therefore, our approach gives a clear explanation why Λ is as is.**

In literature, it is often stated that quantum theory of gravity should become GR in classical approximation. In Sec. 3 we argue that this is probably not the case because at the quantum level the concept of space-time background does not have a physical meaning. *Our results for the cosmological acceleration obtained from semiclassical approximation to quantum theory are compatible with GR* but in our approach, space-time background is absent from the very beginning.

5 Discussion

As noted in Sec. 1, in the mainstream literature, PCA is usually considered proceeding from the following assumptions:

- A: The macroscopic bodies under consideration are located at large distances from each other so that all interactions between the bodies (gravitational, electromagnetic and others) can be neglected. It is also assumed that the sizes of the bodies are much smaller than the distances between them. Then the motion of each body can be considered independently of the motion of other bodies. We will be interested only in the motion of each body as a whole, i.e., we will not consider, for example, the internal rotation of these bodies. Then, formally, we have a purely kinematic problem of the motion of N elementary particles with zero spin.

- B: Although the bodies do not interact with each other, we cannot guarantee that their relative accelerations are zero. This only happens when Poincare invariance holds. However, experimental data show that in our universe this is not the case since the relative accelerations of the bodies are not equal to zero. It is well known that, for example, in the case of de Sitter invariance, the relative accelerations of the bodies are not equal to zero.

- C: Within the framework of GR, which is a classical (non-quantum) theory, de Sitter invariance is described based on the assumption that bodies are in de Sitter space. Then the relative acceleration depends on the cosmological constant Λ the value of which is completely arbitrary. So there is no reason to prefer any particular value.

- D: As noted in Sec. 2, the philosophy of GR is that the curvature of space is created by interacting bodies. Therefore, if the bodies do not interact, then there must be $\Lambda = 0$. So when Einstein learned of Hubble's results, he said that introducing $\Lambda \neq 0$ was the biggest blunder of his life.

- E: However, experiments conducted after 1998 showed that in the framework of GR they can be described only if $\Lambda \neq 0$. As described in Sec. 2, to resolve the contradiction that arose, the terms with Λ on the left-hand sides of Einstein's equations were moved to the right-hand sides and it was declared that these terms describe not the curvature of space, but the contribution of some hypothetical substance which was called dark energy or quintessence. Although their physical nature remains a mystery and there are an almost endless number of explanations for dark energy, mainstream publications on PCA involve those concepts.

However, these approaches have not solved PCA without uncertainties.

We conclude that the approaches proposed in the mainstream literature proceeding from points A-E do not solve PCA.

Although quantum theory of gravity has not yet been finally constructed, ideas of this theory have been used in attempts to solve the cosmological constant problem (see e.g., [20]). Usually, this theory is considered from the point of view of QFT on Minkowski space. As noted in Sec. 3, the physical meaning of background space is problematic. In addition, as noted in the literature (see e.g., [21]), the description of interactions in QFT faces the following mathematical problem. Interacting local fields on Minkowski space are operator distributions and, as is well known from distribution theory, the product of distributions at the same point is an ill-posed mathematical operation. Physicists usually think that such products are needed to preserve locality. However, since there are no operators for the coordinates of Minkowski space, the concept of locality in this case is not defined at the quantum level.

As a result, QFT of gravity on Minkowski space is not renormalizable. It contains strong divergences which can be eliminated only with a choice of a cutoff parameter. The only parameter in this theory is G, and Λ is defined by the vacuum expectation value of the energy-momentum tensor. Then the usual choice of the cutoff parameter is $\hbar/l_P$ where l_P is the Plank length. If $\hbar = c = 1$, G has the dimension $length^2$ and Λ is of the order of $1/G$. This quantity exceeds the experimental one by 122 orders of magnitude and this situation is called vacuum catastrophe.

The approach to finding Λ as a function of G cannot be fundamental for several reasons. First of all, as noted in Sec. 3, fundamental dS and AdS quantum theories originally do not contain dimensional parameters. The quantities $(c, \hbar, R)$ can enter those theories only as contraction parameters for transitions from more general theories to less general ones. QFT of gravity contains G, but it is not explained how G is related to contraction from dS or AdS symmetries to Poincare symmetry. Also, as noted above, quantum theories involving space-time background are not based on rigorous physical principles.

The problem of constructing quantum theory of gravity is one of the most fundamental problems of physics. In [12] we discussed this problem from the point of view of a quantum theory based on finite mathematics. In this theory which we call finite quantum theory (FQT), quantum states are elements of a linear space over a finite ring of characteristic p. Standard quantum theory is a special degenerate case of FQT in the formal limit $p \to \infty$. As shown in [12], in FQT the Newton gravitational law takes place if p is a huge number of the order of $exp(10^{80})$.

In any case, from the very problem statement about the cosmological acceleration described in Sec. 1, it follows that Λ does not depend on G. Indeed, in this problem it is assumed that the bodies are located at cosmological distances from each other and all interactions between the bodies (including gravitational ones) can be neglected.

In the present paper we consider PCA from the point of view of the approach proposed in [12, 18, 19]. As explained in Sec. 3, in contrast to mainstream approaches, such a consideration does not involve space-time background but uses the results on selfadjoint IRs of the dS algebra. Then, as shown in Sec. 4, at the present stage of the universe (when semiclassical approximation is valid), the phenomenon of cosmological acceleration is simply a *kinematical* consequence of quantum theory in semiclassical approximation. This conclusion has been made without involving dark energy, quintessence and other models involving assumptions the validity of which has not been unambiguously proved yet.

In our approach, R is a contraction parameter from the dS to the Poincare algebra. As explained in detail in Sec. 3, R is fundamental to the same extent as c and $\hbar$, it has nothing to do with the relation between Minkowski and dS spaces and the problem why R is as is does not arise by analogy with the problem why c and $\hbar$ are as are. As noted in Sec. 4, the result for cosmological acceleration in our approach is formally given by the same expression (1) as in GR but, while in GR R is the radius of the dS space and can be arbitrary, in our approach, R is defined uniquely.

Therefore PCA has a unique solution which has nothing to do with dark energy or other artificial reasons: cosmological acceleration is an inevitable **kinematical** *consequence of quantum theory in semiclassical approximation and the vacuum catastrophe does not arise.*

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