Reformulations in Classical Electromagnetic Theory and a Proposal for the "Luminiferous Ether".

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Abstract.

We present in this article fundamental elements that can guide us in the process of developing a possible classical electromagnetic theory. Maxwell's equations, which involve line integrals of the electric and magnetic fields, are reformulated into expressions that are almost completely symmetrical. We show that the continuity equation for current naturally arises from the reformulated Maxwell-Ampère law. Ampère's law is generalized and applied to calculate the magnetic field due to some common current distributions found in the literature, including a moving charged particle and a finite current segment. The expression for magnetic force is modified, and the cyclotron frequency of a charged particle in a uniform magnetic field is obtained in accordance with classical mechanics. The magnetic field in the spin-orbit interaction within an atom is determined from the reference frame at rest with the nucleus, which cannot be achieved with electromagnetic theory in its usual form. The force between two long current-carrying wires and a brief discussion on the interaction between two arbitrary current circuits are addressed using the reformulated laws. Next, we present a special non-material medium to correspond to the luminiferous ether, that is, a medium through which the electromagnetic wave propagates. Then, the electromagnetic interaction between two charged particles is examined. Thus, in this work, we propose basic elements to complete the classical electromagnetic theory.

Keywords:
Maxwell's equations; Ampère's law; Magnetic force; Doppler effect; Luminiferous Ether.

1. Introduction.

Since Maxwell modified Ampère's law [1,2,3] in 1865, the pursuit of symmetries within the fundamental laws of a theory have become a subject of general scientific interest. Among the goals of this work, one is to present a new formulation of the Maxwell-Ampère and Faraday laws [3,4,5], a reformulation that exhibits an almost complete symmetry between these two laws upon interchanging **E** and **B**, symbols representing the electric and magnetic fields, respectively, as used throughout this study.

In the development of the electromagnetic theory presented here, we will use Gaussian units, for which $\varepsilon_0 = {}^1/4\pi$ and $\mu_0 = 4\pi/c^2$. Therefore, Maxwell's equations [3,6] take the following forms:

Gauss's Law $$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad (1.1)$$
Gauss's law for the magnetic field $$\nabla \cdot \mathbf{B} = 0, \quad (1.2)$$
Faraday's Law $$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{B}, \quad (1.3)$$
Maxwell and Ampère`s Law $$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}. \quad (1.4)$$

The symbols have their usual meanings as found in literature. The last two equations, (1.3) and (1.4), will be rewritten in more generalized and nearly symmetric forms.

To achieve our objectives, we will consider Faraday's law of induction for a closed-circuit *C*, with velocity $\mathbf{v}' = \frac{d\mathbf{r}'}{dt}$, as shown in Fig. (1.1), expressed in integral form [3,4]

$$\oint_C \mathbf{E} \cdot d\boldsymbol{l} = -\frac{1}{c}\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a}. \quad (1.5)$$

where **E** is the electric field induced at the position of the element d***l*** in *C*, due to the variation in magnetic flux,

$$\varphi(t) \equiv \int_S \mathbf{B} \cdot da, \quad (1.6)$$

across a surface *S* with boundary *C*. Since the circuit *C* is moving with velocity **v**´, the total time derivative in (1.5) must account for this motion. Given that the divergence of the magnetic field is zero, $\nabla \cdot \mathbf{B} = 0$, the total time derivative of the magnetic flux φ(t), through the moving circuit *C* [3] can be expressed as

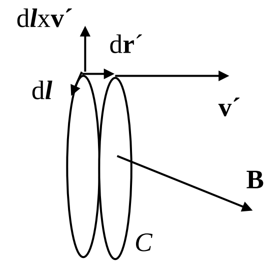

Figure (1.1). Circuit *C* with velocity **v**' in the magnetic field **B**.

$$\frac{d}{dt}\varphi(t) = \frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{a} = -\oint_C \mathbf{B} \cdot (d\boldsymbol{l} \times \mathbf{v}') + \int_S \frac{\partial}{\partial t}\mathbf{B} \cdot d\mathbf{a}. \quad (1.7)$$

Thus, Faraday's law (1.5) can be written in the form

$$\oint_C \mathbf{E} \cdot d\boldsymbol{l} = -\frac{1}{c}\oint_C(-\mathbf{v}' \times \mathbf{B}) \cdot d\boldsymbol{l} - \frac{1}{c}\int_S \frac{\partial}{\partial t}\mathbf{B} \cdot d\mathbf{a}, \quad (1.8)$$

where the line integral in the first term on the right-hand side represents the rate of change of magnetic flux through *S* due to the motion of the circuit *C*; it also corresponds to the rate at which the magnetic flux or the magnetic field lines are swept by the curve *C*. Furthermore, regarding the first term on the right-hand side of relation (1.8), we can state that it represents the circulation of the magnetic force per unit charge, or the induced electromotive force, which arises due to the motion of *C* [4,5]. In fact, the magnetic force on a particle with charge q in the circuit *C*, moving with velocity **v**', is

$$\mathbf{F} = \frac{q}{c}\,\mathbf{v}' \text{x}\, \mathbf{B}. \quad (1.9)$$

We will use these interpretations, referring to the first term on the right side of expression (1.8), to develop a reformulation of Faraday's and Maxwell-Ampère's laws in the following sections.

In the literature, one finds the integral form of Faraday's law (1.8) generalized for an arbitrary movement of circuit *C*, denoting $\mathbf{v}' = \frac{d\mathbf{r}'}{dt}$ as the velocity of the infinitesimal element d***l*** of the circuit, as shown in Fig. (1.1), and considering the circuit as a closed imaginary line *C* [3,4,5].

With a new formulation in mind, we will consider the velocity **v** = **u** - **v**´ as the velocity of the **E** and **B** field lines relative to the d***l*** element of curve *C*. Here, we introduce the concept of field line velocity, where **u** represents the velocity of the **E** and **B** field lines at the position **r** of the element d***l*** respect to the laboratory reference frame, the observation frame (see Fig. (1.2)).

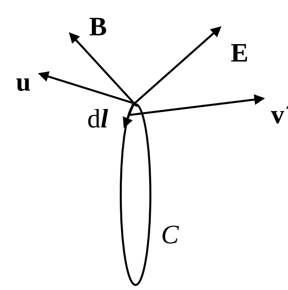

Figure (1.2). Curve *C*, the element d***l*** with velocity **v**'. **E** and **B** represent the field lines at d***l*** with velocity **u**.

2. Reformulation of Faraday's Law and Maxwell-Ampère's Law.

We will begin the present development by considering the situation where the temporal variations of the electric and magnetic fields at the observation point **r** occur due to the movement of the field lines relative to **r**, as in the case, for example, of the dragging of fields lines configurations by a charged particle or a magnet due to their motion, say with velocity **u**.

Let **B**(**r**, t) and **E**(**r**, t) be, respectively, the magnetic field and the electric field at the position **r** of the element d$\boldsymbol{l}$ of curve *C*, and

$$\mathbf{v} = \mathbf{u} - \mathbf{v}' \tag{2.1}$$

the velocity of the magnetic and/or electric field lines relative to *C* at the position **r** of the element d$\boldsymbol{l}$, as shown in Fig. (1.2), which is determined by the motion of the sources generating **B** and **E** (such as a moving electromagnet or a moving charged particle) with velocity **u** and by the movement of curve *C* with velocity **v**' at **r**. So, keeping in mind the restriction imposed on the variation of the fields, we write the modified form of Faraday's law

$$\oint_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{1}{\mathrm{c}} \oint_C (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\,\boldsymbol{l} = -\frac{1}{\mathrm{c}} \oint_C (\mathbf{u} - \mathbf{v}') \times \mathbf{B} \cdot \mathrm{d}\boldsymbol{l}, \tag{2.2}$$

which results directly from expression (1.5), and which, when compared with relation (1.8), leads us to identify the equality

$$\oint_C (\mathbf{u} \times \mathbf{B}) \cdot \mathrm{d}\boldsymbol{l} = \int_S \frac{\partial^{\mathbf{u}}}{\partial \mathrm{t}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}. \tag{2.3}$$

Here, we observe that the variation of the magnetic field across surface *S*, with boundary on curve *C*, considered momentarily at rest, is due to the movement of the magnetic field lines **B** inward (or outward) through the curve. This variation is quantified in relation (2.3), which relates the flux swept by the curve per unit time with the flux variation through *S* due to the movements of the **B** field lines. The index u in the derivative operation within the integral on the right-hand side indicates that we are considering only variations in **B** due to the movement of the field lines. The negative factor on the right side of (2.2), as in (1.5), is required by Lenz's experimental law [3,4,5,6,8], which we can state here as follows: the induced electric field **E** along curve *C* is such that it opposes the flux of the magnetic field **B** swept by curve *C*.

The differential form corresponding to (2.3) is

$$(\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{B} = -\frac{\partial^{\mathbf{u}}}{\partial \mathrm{t}} \mathbf{B}, \tag{2.4}$$

which can be obtained using Stokes' theorem [3,4,5] in relation (2.3), Maxwell's equation (1.2), $\boldsymbol{\nabla} \cdot \mathbf{B} = 0$, and assuming that the velocity **u** is independent of the position vector **r** of the observation point. Such a relation can be used to determine **u** in situations where we cannot directly relate **u** to the velocity of the source of the magnetic field, as in the case of a magnet's velocity.

Let us consider law (2.2) for the case **v**' = 0, with the curve *C* at rest, and use equality (2.3) to express the relations

$$\oint_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{1}{\mathrm{c}} \oint_C \mathbf{u} \times \mathbf{B} \cdot \mathrm{d}\boldsymbol{l} = -\frac{1}{\mathrm{c}} \oint_S \frac{\partial^{\mathbf{u}}}{\partial \mathrm{t}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}, \tag{2.5}$$

where we identify the integral form of Faraday's law, which here involves only the flux variation over *S* due to the movement of the field lines. The flux variation of the magnetic field due to the creation of field lines or the increase of field lines in the study region because of changes in field sources, such as the variation in current in an electromagnet, was not considered. In the above development, only the motion of the electromagnet, with its current fixed, was addressed. Thus, we propose the following reformulation of Faraday's law (1.3) in the integral form for a general situation:

$$\oint_C \mathbf{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{1}{\mathrm{c}} \oint_C (\mathbf{v} \times \mathbf{B}) \cdot \mathrm{d}\,\boldsymbol{l} - \frac{1}{\mathrm{c}} \int_S \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{B} \cdot \mathrm{d}\mathbf{a}, \tag{2.6}$$

where the flux variation per unit time across *S*, due to the movement of the **B** field lines relative to curve *C*, is represented by the first integral on the right side, and the integral in the last term denotes the flux variation of the magnetic field **B** per unit time due to changes in the field source, such as variations in the current in an electromagnet. The superscript f in the last integral of (2.6) is and will be used to specify this type of variation of magnetic field **B** and the electric field **E** in the following developments.

Keeping in mind the importance and application of symmetry in physics [2,3,4,7] and the reformulation of Faraday's law (2.6), we propose the new form for the Maxwell-Ampère law:

$$\oint_C \mathbf{B} \cdot \mathrm{d}\boldsymbol{l} = \frac{1}{\mathrm{c}} \oint_C (\mathbf{v} \times \mathbf{E}) \cdot \mathrm{d}\boldsymbol{l} + \frac{1}{\mathrm{c}} \int_S \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{E} \cdot \mathrm{d}\mathbf{a}, \tag{2.7}$$

where the first integral on the right side represents the flux of the electric field **E** swept by curve *C* per unit time due to the movement of the **E** field lines relative to *C*, and the integral in the last term represents the flux variation of the electric field **E** over *S* per unit time, resulting from changes in the source of the **E** field, such as variations in the magnetic field in the region. Here, we observe an almost complete symmetry between laws (2.6) and (2.7), with symmetry broken by the negative factor in (2.6), a consequence of Lenz's law [3,4,5,7] as previously mentioned.

Using Stokes' theorem [3,4,5], we then write the differential form for the reformulated Faraday's law (2.6):

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{\mathrm{c}} \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B}) - \frac{1}{\mathrm{c}} \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{B}, \tag{2.8}$$

and for the Maxwell-Ampère law (2.7):

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{\mathrm{c}} \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{E}) + \frac{1}{\mathrm{c}} \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{E}. \tag{2.9}$$

We should note, by taking the divergence of expressions (2.8) and (2.9), that

$$\nabla \cdot \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{B} = 0 \quad \mathrm{e} \quad \nabla \cdot \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{E} = 0, \tag{2.10}$$

In other words, the increments in the electric and magnetic fields per unit time, due solely to variations in the sources (such as the acceleration of charged particles, current variations, or variations in the **E** and **B** fields, excluding variations in **E** and **B** due to the movement of field lines at point **r**) have zero divergence.

At this point, by accepting relations (2.6) and (2.7) as valid, we can assert that at an observation point **r** moving relative to magnetic (electric) field lines, an electric (magnetic) field exists, and likewise, variations in the magnetic (electric) field due to source changes imply the existence of an electric (magnetic) field at **r**. The integral forms (2.6) and (2.7) and the differential forms (2.8) and (2.9) are nearly symmetric with respect to the exchange of **E** for **B**, which is not the case with the conventional forms, where the Maxwell-Ampère law explicitly contains the term $4\pi\mathbf{J}/c$, involving the current density **J**.

To ensure the acceptance of relations (2.8) and (2.9), along with their integral forms, as generalizations of Faraday's and Maxwell-Ampère's laws, respectively, we will derive from them the well-known forms of Faraday's and Maxwell-Ampère's laws [3,4,5,6]. For this, we consider curve *C* at rest, i.e., $\mathbf{v}' = 0$, and assume that **u**, the velocity of the field lines, is independent of the observation position **r**. So, we can write that [4] $\boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{E}) = \boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{E}) = \mathbf{u}(\boldsymbol{\nabla} \cdot \mathbf{B}) - (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{B}$, and since $\nabla \cdot \mathbf{B} = 0$, it follows from (2.8) that

$$\boldsymbol{\nabla} \times \mathbf{E} = \frac{1}{\mathrm{c}} (\mathbf{u} \cdot \boldsymbol{\nabla})\mathbf{B} - \frac{1}{\mathrm{c}} \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{B}. \tag{2.11}$$

And then, by using relation (2.4), the usual form of Faraday's law (1.3) follows immediately

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{\mathrm{c}} \frac{\partial^{\mathbf{u}}}{\partial \mathrm{t}} \mathbf{B} - \frac{1}{\mathrm{c}} \frac{\partial^{\mathrm{f}}}{\partial \mathrm{t}} \mathbf{B} = -\frac{1}{\mathrm{c}} \frac{\partial}{\partial \mathrm{t}} \mathbf{B}, \tag{2.12}$$

that is

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{\mathrm{c}} \frac{\partial}{\partial \mathrm{t}} \mathbf{B}, \tag{2.13}$$

where we introduce the expression

$$\frac{\partial}{\partial t}\mathbf{B} = \frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{B} + \frac{\partial^{f}}{\partial t}\mathbf{B}, \quad (2.14)$$

for the total time variation of $\mathbf{B}(\mathbf{r}, t)$ at $\mathbf{r}$.

Now let us consider the generalized Maxwell-Ampère law (2.9), and write [4]

$$\nabla \times \mathbf{B} = \frac{1}{c}[\mathbf{u}(\nabla \cdot \mathbf{E}) - (\mathbf{u} \cdot \nabla)\mathbf{E}] + \frac{1}{c}\frac{\partial^{f}}{\partial t}\mathbf{E}. \quad (2.15)$$

The spatial differential operations are associated with the displacement d**r** of the field lines relative to the observation point **r**. Keeping in mind Gauss's law [3,4,5] $\nabla \cdot \mathbf{E} = 4\pi\rho$, where ρ is the charge density at point **r**, and using relation (2.4) for the electric field, which becomes $(\mathbf{u} \cdot \nabla)\mathbf{E} = -\frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{E}$, we can derive from relation (2.15) the expression

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\rho\mathbf{u} + \frac{1}{c}\frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{E} + \frac{1}{c}\frac{\partial^{f}}{\partial t}\mathbf{E}. \quad (2.16)$$

And, as **u**, the speed of the electric field line, is the velocity of the charge carrier at **r**, it follows that $\mathbf{u}\rho(\mathbf{r}, t) = \mathbf{J}(\mathbf{r}, t) = \mathbf{J}$ is the current density, and thus from (2.16) we write the Maxwell-Ampère law (1.4),

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E}, \quad (2.17)$$

where we use the relation

$$\frac{\partial}{\partial t}\mathbf{E} = \frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{E} + \frac{\partial^{f}}{\partial t}\mathbf{E}, \quad (2.18)$$

for the total time variation of the electric field $\mathbf{E}(\mathbf{r}, t)$ at point **r**.

It is useful to take a moment here to introduce the important continuity equation for current [3,4,5], which expresses the law of charge conservation. To do so, let us take the divergence of relation (2.17), use relation (2.18), and apply the relation $\nabla \cdot \frac{\partial^{f}}{\partial t}\mathbf{E} = 0$ from (2.10), which allows us to write

$$\frac{4\pi}{c}\ \nabla \cdot \mathbf{J} + \frac{1}{c}\nabla \cdot \frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{E} = \frac{4\pi}{c}\ \nabla \cdot \mathbf{J} + \frac{1}{c}\frac{\partial^{\mathbf{u}}}{\partial t}\nabla \cdot \mathbf{E} = 0,$$

where the permutation between the operators $\frac{\partial^{\mathbf{u}}}{\partial t}$ and ∇ was used. And, finally, using Gauss's law (1.1), it results in

$$\nabla \cdot \mathbf{J} + \frac{\partial^{\mathbf{u}}}{\partial t}\rho = 0, \quad (2.19)$$

the continuity equation of the current, which tells us that the variation of charge within a closed surface per unit of time is solely due to the displacement of charges across the surface per unit of time. Thus, the presence of the index **u** in relation (2.19) can now be omitted, and we write

$$\nabla \cdot \mathbf{J} + \frac{\partial}{\partial t}\rho = 0, \quad (2.20)$$

the continuity equation of the current as presented in the literature [3,4,5]. In the present formulation, we can state that the relations

$$\frac{\partial}{\partial t}\rho = \frac{\partial^{\mathbf{u}}}{\partial t}\rho \quad \text{ou} \quad \frac{\partial^{f}}{\partial t}\rho = 0$$

are also expressions of charge conservation; in other words, we can state that the temporal variation of the charge density $\rho(\mathbf{r}, t)$ occurs exclusively due to the movement of charge between point **r** and its vicinity, a movement here remembered by the index u.

Let's briefly address in this paragraph the concept of displacement current density $\mathbf{J}_D$ [3,4,5,7,8]. For this, let us consider again expression (2.16) and write the expressions

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{4\pi}{c}\epsilon_0\frac{\partial^{\mathbf{u}}}{\partial t}\mathbf{E} + \frac{1}{c}\frac{\partial^{f}}{\partial t}\mathbf{E}, \quad (2.21)$$

and

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}(\mathbf{J} + \mathbf{J}_D) + \frac{1}{c}\frac{\partial^f}{\partial t}\mathbf{E}, \tag{2.22}$$

after we use the relation $\epsilon_0 = {}^1/_{4\pi}$ and that

$$\mathbf{J}_D = \epsilon_0 \frac{\partial^u}{\partial t}\mathbf{E}. \tag{2.23}$$

Where a new definition for the displacement current density $\mathbf{J}_D$ is presented in terms of $\frac{\partial^u}{\partial t}\mathbf{E}$. We also recall that, in the last term on the right side of (2.22), the time derivative with index f is taken by considering the electric field lines momentarily at rest.

Thus, considering the developments above, we can accept that relations (2.8) and (2.9) are, respectively, generalizations of Faraday's law (1.3) and Maxwell-Ampère's law (1.4). We rewrite them below in their integral forms (2.6) and (2.7) without using the indices u and f, noting that the line integrals on the right-hand sides represent the fluxes of the fields swept per unit time by the curve *C*, due to the motion of the field lines relative to *C*. The surface integrals represent the flux variations of the fields on surface *S*, per unit time, due to variations in the sources of the fields.

Faraday´s Law:
$$\oint_C \mathbf{E}\cdot d\boldsymbol{l} = -\frac{1}{c}\oint_C(\mathbf{v}\times\mathbf{B})\cdot d\,\boldsymbol{l} - \frac{1}{c}\int_S \frac{\partial}{\partial t}\mathbf{B}\cdot d\mathbf{a}, \tag{2.24}$$

Maxwell -Ampère´s Law:
$$\oint_C \mathbf{B}\cdot d\boldsymbol{l} = \frac{1}{c}\oint_C(\mathbf{v}\times\mathbf{E})\cdot d\,\boldsymbol{l} + \frac{1}{c}\int_S \frac{\partial}{\partial t}\mathbf{E}\cdot d\mathbf{a}. \tag{2.25}$$

In the next section, we will address some simple problems in electromagnetism using the integral form of Maxwell-Ampère's law, (2.25). We will select the observer's reference frame such that **v**' = 0 and thus **v** = **u**.

3. The Generalized Ampère's Law and Applications.

In this section, the chosen curve *C* is at rest, so **v**' = 0 and **v** = **u**. We will apply the generalized law (2.25) in low-velocity regimes, u<<c, so that we can use classical expressions for the electric field **E** in determining the magnetic field **B** due to certain current distributions, problems thoroughly covered in electromagnetism texts using Ampère's law and/or the Biot-Savart law [3,4,5,7,8]. In other words, these are problems where $\int_S \frac{\partial}{\partial t}\mathbf{E}\cdot d\mathbf{a} = 0$. Thus, the relation to be used is

$$\oint_C \mathbf{B}\cdot d\boldsymbol{l} = \frac{1}{c}\oint_C(\mathbf{u}\times\mathbf{E})\cdot d\,\boldsymbol{l}. \tag{3.1}$$

Which we will call Ampère's generalized law. The integral on the right side of (3.1), to the current distributions here addressed, corresponds to $4\pi(\mathbf{I} + \mathbf{I}_D)$, where **I** is the conduction current and $\mathbf{I}_D$ is the displacement current associated with the displacement current density $\mathbf{J}_D$ defined in equation (2.23).

i) **Infinitely long current-carrying wire**.

In this first example, we observe that the resulting electric field $\mathbf{E}_r$ due to the long current-carrying wire is zero. However, at any position **r**, Fig. (3.1), there are moving electric field lines, associated with the electric field **E** of the charge carriers moving at velocity **u**, generating the current I. Let **λ** be the linear charge density of these carriers; by integrating Coulomb's law or applying Gauss's law [3,4,5,7,8] over the infinite linear charge distribution with density **λ**, it follows the expression

$$\mathbf{E} = \frac{2\lambda}{r}\hat{\mathbf{r}} \tag{3.2}$$

for the electric field **E**, whose field lines are in motion.

Then, using Ampère's generalized law (3.1), expression (3.2), and the symmetry shown in Fig. (3.1), we obtain the magnetic field,

$$\mathbf{B} = \frac{1}{c}\mathbf{u} \times \mathbf{E} = \frac{2\lambda}{cr}\mathbf{u} \times \hat{\mathbf{r}}, \qquad (3.3)$$

and since $\mathbf{u} = u\hat{\mathbf{z}}$ and $\lambda u = I$, it is the current in the wire, the expression results as

$$\mathbf{B} = \frac{2I}{cr}\hat{\mathbf{z}} \times \hat{\mathbf{r}}, \qquad (3.4)$$

which is the expected result [7,8].

We observe in this example that there is no variation in the electric field flux through a surface *S* with boundary *C*, and the flux through such a surface is zero. Consequently, the integral on the right side of Ampère's generalized law (3.1) is exclusively the flux swept by the curve *C* per unit of time. It can also be shown here that

$$\oint_C \mathbf{B} \cdot d\boldsymbol{l} = \frac{4\pi}{c} I \qquad (3.5)$$

for any curve *C*, regardless of its shape. That is, one can obtain from expression (3.1) the Ampère's law, highlighting the generality of law (3.1).

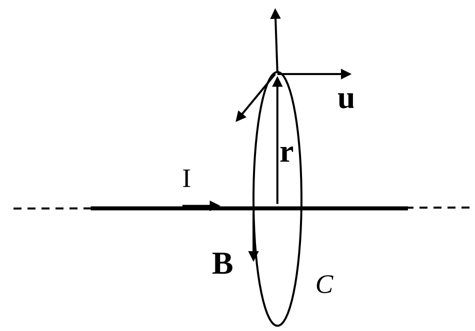

Figure (3.1) Speed **u** of the electric field line **E** at point **r**, due to the charge carriers in the infinite straight wire. The field lines **u**x**E** and **B**, coinciding with curve *C*, are also shown.

ii) **Particle with charge q and constant velocity u**.

In this example sketched in Fig. (3.2), the velocity **u** of the electric field line, at position **r** relative to the particle, is equal to the velocity of this. The electric field lines **E** are radial, emanating from the charge, and the lines of the vector field **u**x**E** and the magnetic field **B** are circular lines, perpendicular and coaxial to the direction of **u**, consequences of the symmetry of this problem, which leads us to choose the curve *C* coaxial and perpendicular to z. Then it follows from the modified Ampère's law (3.1) that

$$\mathbf{B} = \frac{1}{c}\,\mathbf{u} \times \mathbf{E}. \qquad 3.6)$$

Where for **E**, because of the low-velocity regime, we consider the Coulomb field [4,5,7,8],

$$\mathbf{E} = \frac{q\mathbf{r}}{r^3} \qquad (3.7)$$

And it follows, for the magnetic field, the result:

$$\mathbf{B} = \frac{q\mathbf{u}}{c} \times \frac{\mathbf{r}}{r^3}, \qquad (3.8)$$

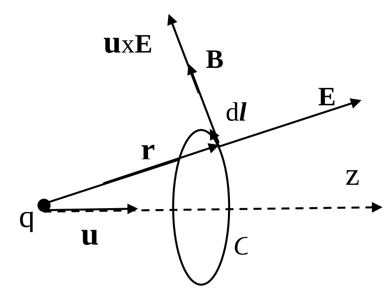

Figure (3.2). Fixed circular curve *C*, coincident with the lines of the fields **B** and **u** x **E**, element d***l*** at position **r**, charge q with velocity **u** along the z - axis direction.

which corresponds to the Biot-Savart law [3,4,5]. Such a problem cannot be addressed by Ampère's law [3,4,5,7,8], which provides a second evidence for the generality of relation (3.1), the generalized Ampère's law.

iii) **Finitely current-carrying wire**.

In this case, the resulting electric field $\mathbf{E}_r$ due to the current-carrying wire is also zero. However, at any position **r** (see the figures (3.3) and (3.4)), there are moving electric field lines, those associated with the electric field **E** of the charge carriers moving at velocity **u**, which generate the current I; let λ be the linear charge density of these carriers. By integrating Coulomb's law [4,5,7,8] over this finite linear charge distribution, we obtain for the electric field, with moving lines, the expression

$$\mathbf{E} = \frac{\lambda}{r}\left[\left(\sin(\theta_2)\text{-}\sin(\theta_1)\right)\hat{\mathbf{r}} - \left(\cos(\theta_2)\text{-}\cos(\theta_1)\right)\hat{\mathbf{z}}\right] \qquad (3.9)$$

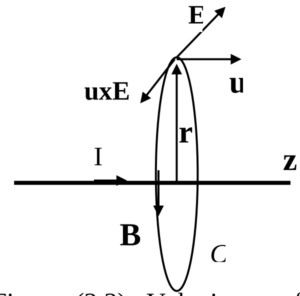

Figure (3.3). Velocity **u** of the E -field lines at **r**, and the lines of the **u** x **E** and **B** fields coinciding with curve *C*.

Thus, using the generalized Ampère's law (3.1), the result (3.9), and the symmetry shown in Fig. (3.3), we obtain the magnetic field,

$$\mathbf{B} = \frac{1}{c}\,\mathbf{u}\times\mathbf{E} = \frac{\lambda}{cr}\left(\sin(\theta_2)\text{-}\sin(\theta_1)\right)\mathbf{u}\times\hat{\mathbf{r}}. \qquad (3.10)$$

Since $\mathbf{u} = u\hat{\mathbf{z}}$ and λu = I, the expression results as

$$\mathbf{B} = \frac{I}{cr}[\sin(\theta_2)\text{-}\sin(\theta_1)]\hat{\mathbf{z}}\times\hat{\mathbf{r}}, \qquad (3.11)$$

the expected result according to the literature [3,4,5,7,8], where the Biot-Savart law (3.8) integration is used. In this problem, the usual Ampère's law does not apply either.

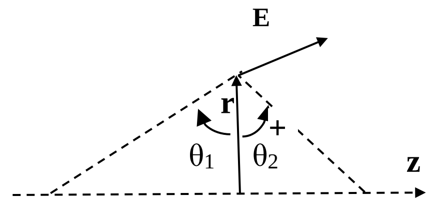

Figure (3.4). Geometry for the field **E** presented in equation (3.9).

iv) **Infinite current plane.**

Let σ be the surface charge density of the carriers generating the linear current density i in the plane. Thus, according to Fig. (3.5), the electric field whose lines move with velocity **u** is

$$\mathbf{E} = 2\pi\sigma\,\hat{\mathbf{r}}, \qquad (3.12)$$

which can be obtained using Gauss's law [7,8], where $\hat{\mathbf{r}}$ is the unit normal vector to the plane. Then, using relation (3.1) and Fig. (3.5), we write the expressions

$$\mathbf{B} = \frac{1}{c}\,\mathbf{u}\times\mathbf{E} = \frac{1}{c}2\pi\sigma(\mathbf{u}\times\hat{\mathbf{r}}) = \frac{1}{c}2\pi\sigma u\hat{\mathbf{z}}\,, \qquad (3.13)$$

with $\hat{\mathbf{z}}$ defined in the relation $\mathbf{u}\times\hat{\mathbf{r}} = u\hat{\mathbf{z}}$, and since σu=i, the linear current density, we write for **B** the well-known formula [3,7,8].

$$\mathbf{B} = \frac{2\pi i}{c}\hat{\mathbf{z}}. \qquad (3.14)$$

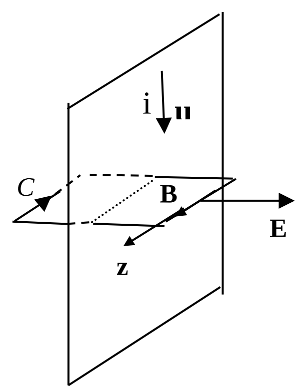

Figure (3.5). **B** field line, curve C, **u**, and the direction of the planar current distribution i.

v) **A long solenoid with current**.

Let us consider, as a final example in this section, a long cylindrical charge distribution, with linear charge density λ, rotating around its z-axis with angular velocity **w**, as shown in Fig. (3.6). Here, the moving electric field lines are those of the field

$$\mathbf{E} = \frac{2\lambda}{r}\,\hat{\mathbf{r}} \qquad (3.15)$$

and it follows, using the symmetry results of this problem found in the literature [4,5,7,8] and the modified Ampère's law (3.1), for the magnetic field the expression

$$\mathbf{B} = -\frac{2\lambda}{cr}\mathbf{u}\times\hat{\mathbf{r}} = \frac{2\lambda}{c}w\hat{\mathbf{z}}\,. \tag{3.16}$$

Since $\lambda w = 2\pi i$, where i is the linear current density, the solution is given by

$$\mathbf{B} = \frac{4\pi\, i}{c}\,\hat{\mathbf{z}}\,. \tag{3.17}$$

And for the case of a long solenoid with concatenated turns, where we identify that $\lambda w = 2\pi nI$ with n being the turn density and I the current in the turns, the expression follows as

$$\mathbf{B} = \frac{\mathbf{4\pi\, nI}}{\mathbf{c}}\,\hat{\mathbf{z}}\,, \tag{3.18}$$

well-known in literature [3,4,5,7,8].

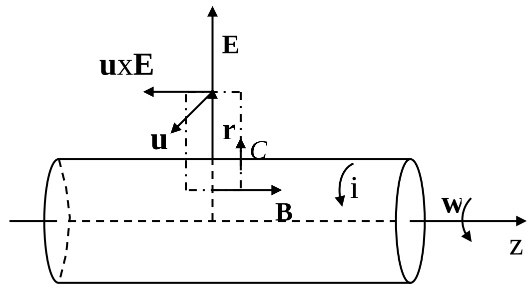

Figure (3.6). The fields **u** x **E** and **B**, the curve *C*, and the linear current density i for the rotating charged cylindrical shell with angular velocity **w**.

4. Reformulation of Magnetic Force and the Final Forms of Maxwell's Equations.

In this section, we present a reformulation of the magnetic force based on experimental results from two problems. The first is the dependence of cyclotron frequency [4,5,7,8] on the velocity of a charged particle in a uniform magnetic field. The second problem involves the magnetic field due to the nucleus in the spin-orbit interaction within an atom [3,5], analyzed from the perspective of an observer at rest with respect to the atom's nucleus, a problem for which no solution exists within usual electromagnetic theory. With these considerations in mind, we propose the expression for the magnetic force **F** on a particle with charge q_i and velocity $\mathbf{v}_i$ in a magnetic field **B** as follows:

$$\mathbf{F} = \alpha_i \frac{q_i}{c}\,\mathbf{v}_i \mathrm{x} \mathbf{B}\,. \tag{4.1}$$

Where, in this new formula, $\mathbf{v}_i$ is the velocity of the particle relative to the magnetic field lines, and $\alpha_i = \left(1 - \frac{v_i^2}{c^2}\right)^{\frac{1}{2}}$. Thus, observers in different inertial frames will measure the same magnetic force if **B** does not depend on the frame of reference, in accordance with classical mechanics. We remember that the usual magnetic force law depends on the observer, as in this law, $\mathbf{v}_i$ is the velocity of the particle relative to the observer, and $\alpha_i = 1$.

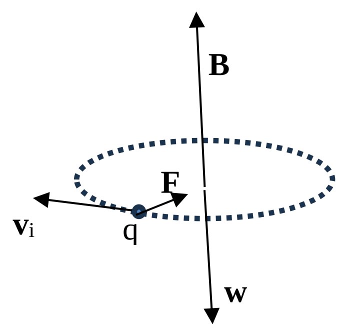

Figure (4.1). Path of a particle with charge q_i in a uniform magnetic field **B**.

Now, to address the first problem mentioned above, let us consider a particle with mass m_i, charge q_i, and velocity $\mathbf{v}_i$ in a uniform magnetic field **B**, as shown in Fig. (4.1). Under the present reformulation of the magnetic force, the particle follows a circular motion that satisfies the following relationships:

$$\mathbf{F} = m_i\mathbf{w}_i \times \mathbf{v}_i = \alpha_i \frac{q_i}{c}\,\mathbf{v}_i \mathrm{x} \mathbf{B} = -\,\alpha_i \frac{q_i}{c}\,\mathbf{B}\mathrm{x}\mathbf{v}_i, \tag{4.2}$$

from which, for $\mathbf{w}_i$, the cyclotron frequency of the particle, the following relation results:

$$\mathbf{w}_i = -\alpha_i \frac{q_i}{m_i c}\,\mathbf{B}, \tag{4.3}$$

which shows a dependence on the particle's velocity, consistent with results found in the literature [3,4,5,7,8]. In this new treatment presented here, the mass m_i remains constant, independent of velocity, in accordance with classical mechanics.

The problem of the magnetic force between two long current-carrying wires is essential here, as it highlights the need for a complementary modification to Faraday's law (2.24) and consequently to the Maxwell-Ampère law (2.25), for reasons of symmetry. Let us then consider two long, parallel wires carrying currents I_1 and I_2, as shown in Fig. (4.2), and use the result from item i) in Section 3 to write the expression for the magnetic field of current I_2 at the position of wire 1 as follows:

$$\mathbf{B_2} = \frac{1}{c}\mathbf{u}_2 \times \mathbf{E_2} = \frac{2\lambda_2}{cr}\mathbf{u_2} \times \hat{\mathbf{r}} = \frac{2I_2}{cr}\hat{\mathbf{z}} \times \hat{\mathbf{r}} \tag{4.4}$$

where $\mathbf{u}_2$ is the velocity of the charge carriers in wire 2, or the velocity of the electric field lines $\mathbf{E}_2$, due to these carriers, at position $\mathbf{r}$. Using the force law (4.1), assuming a low-velocity regime where $\alpha_1 = \left(1 - \frac{v_1{}^2}{c^2}\right)^{\frac{1}{2}} \cong 1$, and that $dq_1\mathbf{u_1}$ is equivalent to $I_1 d\boldsymbol{l}_1$ (with dq_1 being the charge of the carriers in $d\boldsymbol{l}_1$ moving with velocity $\mathbf{u_1}$), we can write the expressions for the magnetic force $\mathbf{F}_{12}$, which is the force on segment $d\boldsymbol{l}_1$ of wire 1 due to wire 2, as follows:

$$d\mathbf{F}_{12} = \frac{dq_1\mathbf{u}_1}{c} \times \left(\frac{1}{c}\mathbf{u}_2 \times \mathbf{E_2}\right) = \frac{I_1 d\boldsymbol{l}_1}{c} \times \left(\frac{2I_2}{cr}\hat{\mathbf{z}} \times \hat{\mathbf{r}}\right). \tag{4.5}$$

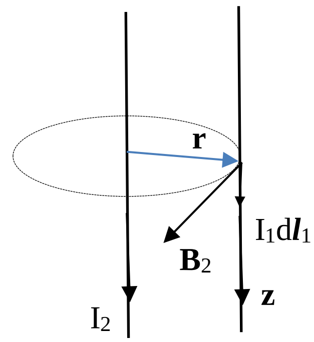

Figure (4.2). Some quantities used in calculating the magnetic force between two long current-carrying wires.

Let us remember that, according to the new law of magnetic force (4.1), $\mathbf{u}_1$ is the velocity of charge dq_1 relative to the lines of magnetic field $\mathbf{B}_2$. It is important to note that the configuration of the $\mathbf{B}_2$ field lines is stationary relative to wire 2, which applies to any closed current circuit, such as the magnetic field of a current loop.

By integrating relation (4.5) over $d\boldsymbol{l}_1$ along a length l_1 of wire 1 and solving the cross products, we obtain the result for the force per unit length between the wires:

$$\boldsymbol{f}_{12} = \mathbf{F}_{12}/l_1 = -\frac{2I_1I_2}{c}\frac{1}{cr}\hat{\mathbf{r}} = -\frac{\mu_0 I_1 I_2}{2\pi}\frac{1}{r}\hat{\mathbf{r}}, \tag{4.6}$$

which agrees with the literature [3,4,5,7,8], and where it is verified that $\boldsymbol{f}_{12} = -\boldsymbol{f}_{21}$. If we had kept the factor $\propto_i \equiv \propto_1 \neq 1$, which appears in the reformulated law of force (4.1), the result (4.6) would be

$$\boldsymbol{f}_{12} = -\propto_1 \frac{\mu_0 I_1 I_2}{2\pi}\frac{1}{r}\hat{\mathbf{r}}, \tag{4.7}$$

and then we would have $\boldsymbol{f}_{12} \neq -\boldsymbol{f}_{21}$, that is, a symmetry breaking, which can be restored if we introduce the factor $\propto_2$ in expression (4.4) for the magnetic field $\mathbf{B}_2$, redefining it as

$$\mathbf{B_2} = \frac{1}{c}\propto_\mathbf{2} (\mathbf{u}_2 \times \mathbf{E_2}), \tag{4.8}$$

resulting, then, the new expression for $\boldsymbol{f}_{12}$.

$$\boldsymbol{f}_{12} = -\propto_1\propto_\mathbf{2} \frac{\mu_0 I_1 I_2}{2\pi}\frac{1}{r}\hat{\mathbf{r}}, \tag{4.9}$$

and then, we again have that $\boldsymbol{f}_{12} = -\boldsymbol{f}_{21}$. As a consequence of the reformulation of the magnetic force in (4.1) and the result we proposed for the magnetic field of a long wire in (4.8), Faraday's law (2.24) and the Maxwell-Ampère law (2.25), due to symmetry, must be rewritten as follows:

Lei de Faraday: $$\oint_C \mathbf{E}\cdot d\boldsymbol{l} = -\frac{1}{c}\oint_C \propto (\mathbf{v}\times\mathbf{B})\cdot d\,\boldsymbol{l} - \frac{1}{c}\int_S \frac{\partial}{\partial t}\mathbf{B}\cdot d\mathbf{a}, \tag{4.10}$$

Lei de Maxwell - Ampère: $$\oint_C \mathbf{B}\cdot d\boldsymbol{l} = \frac{1}{c}\oint_C \propto (\mathbf{v}\times\mathbf{E})\cdot d\,\boldsymbol{l} + \frac{1}{c}\int_S \frac{\partial}{\partial t}\mathbf{E}\cdot d\mathbf{a}, \tag{4.11}$$

where here $\propto = \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$. We recall that **v** is the velocity of the electric and magnetic field lines relative to the position **r** of the element d***l***. Here we reconsider the interpretation that the first term on the right side of Faraday's law (4.10) is the circulation of magnetic force per unit charge, if curve *C* is a real circuit, as interpreted just above equation (1.9). We should also note that the usual Maxwell's laws (1.3) and (1.4) result from expressions (4.10) and (4.11) in the regime of low field line velocities, where $v \ll c$ so that $\propto \cong 1$, as we showed in section 2.

Next, using the new expression for the magnetic force and the reformulated Maxwell-Ampère law (4.14), we will address a problem that persists in the literature without a coherent explanation. This problem is the presence of a magnetic field at the position of an electron in an atomic orbit relative to the rest frame of the atomic nucleus. Here, we are referring to the problem of spin-orbit interaction, which results in the splitting of the atom's energy spectrum, known as fine structure [3,5].

The reformulated Maxwell-Ampère law (4.11) makes evident the existence of a magnetic field at a point **r** moving relative to the lines of an electrostatic field **E**. For example, a point charge q at rest, as outlined in Fig. (4.3), produces a magnetic field **B** at position **r**, which moves with velocity **v´** along with curve *C*, according to law (4.11), and we have sufficient symmetry to write

$$\mathbf{B} = -\frac{1}{c}\mathbf{v}' \times \mathbf{E}, \qquad (4.12)$$

where we consider $v' \ll c$, and $\mathbf{E} = \frac{q\mathbf{r}}{r^3}$, it is the field of the charge at rest [3,4]. And here, keeping in mind the force law (4.1) and, as is known in the literature [3,4,5,7,8], no magnetic force is observed on a charge q' moving in an electrostatic field, we conclude that in this situation the velocity of the charge q' relative to the magnetic field line **B**, which coincides with the curve *C*, is zero. However, if the charge q' has some motion beyond translation, such as an intrinsic magnetic moment, interactions will occur. In the case of an orbital electron, we have its spin magnetic moment $\boldsymbol{\mu}_s$, and the interaction can be quantified by the torque,

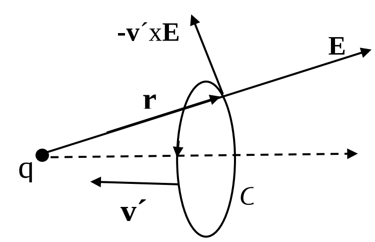

Figure (4.3). Curve *C* with velocity **v´**, coinciding with the field lines -**v**' x **E** and **B**, and the charge at rest.

$$\mathbf{T} = \boldsymbol{\mu}_s \times \mathbf{B}, \qquad (4.13)$$

or by the energy

$$U = -\boldsymbol{\mu}_s \cdot \mathbf{B}. \qquad (4.14)$$

Thus, the experimental results on spin-orbit interaction [3,5] corroborate the generalized Maxwell-Ampère law (4.11), because this, along with the force law (4.1), provides an answer for the magnetic field **B** in relation to the rest frame of the nucleus.

In the next section, we present a concrete and real medium to correspond to the luminiferous ether, an idea that continues to resonate in scientific literature to the present day [9].

5. "The Luminiferous Ether".

To conclude our proposal, let us present the medium through which the electromagnetic wave propagates, "the luminiferous ether", which we assert to be the electromagnetic field itself, that is, not a material medium but an immaterial field. To understand this concept, let us present an initial sketch considering a particle with charge q_2 moving uniformly at velocity $\mathbf{v}_2$, using the notation ($\mathbf{E}_2$, $\mathbf{B}_2$, $\mathbf{v}_2$) to designate the electromagnetic field of this particle. If this particle experiences a brief impulse of duration δt, for instance, as it passes through a narrow potential ramp, acquiring a new velocity $\mathbf{v}_2'$, a wave pulse is generated [8] and propagates radially from the charge at the speed of light c relative to the medium, which we here reaffirm as the electromagnetic field of the particle ($\mathbf{E}_2$, $\mathbf{B}_2$, $\mathbf{v}_2$). The field lines have the same velocity as the source, the charged particle, just before the pulse emission, so that the center of the radial lines of the electric field $\mathbf{E}_2$ retains the velocity $\mathbf{v}_2$ of the particle before the emission of the pulse. As the pulse propagates, the electromagnetic field ($\mathbf{E}_2$, $\mathbf{B}_2$, $\mathbf{v}_2$) transitions into the ($\mathbf{E}_2'$, $\mathbf{B}_2'$, $\mathbf{v}_2'$), with the charge, now moving at velocity $\mathbf{v}_2'$, at the center of the radial lines of the electric field $\mathbf{E}_2'$.

Taking into account the time t elapsed since t = 0, when the pulse was emitted, the following electromagnetic field distribution results [8]: the field ($\mathbf{E}_2'$, $\mathbf{B}_2'$, $\mathbf{v}_2'$) is established in $R_1 \equiv \{ r < ct \}$, where ct is the radial distance from the center of the $\mathbf{E}_2$ field lines to the internal surface of the emitted pulse; the field ($\mathbf{E}_2$, $\mathbf{B}_2$, $\mathbf{v}_2$) remains in region $R_3 \equiv \{ r > c(t + \delta t)\}$, where c(t+δt) is the radial distance from the center of the electric field $\mathbf{E}_2$ lines, that is where the charge would be if it were still moving at $\mathbf{v}_2$, to the external surface of the pulse. The pulse, located in region $R_2 \equiv \{ct > r < c(t + \delta t)\}$, advances radially at velocity c in the medium ($\mathbf{E}_2$, $\mathbf{B}_2$, $\mathbf{v}_2$) toward region $R_3 \equiv \{ r > c(t + \delta t)\}$. Figures to visualize this outline are found in reference [8]. In the region where the pulse is located, the electromagnetic field exhibits transverse oscillations to the radial direction, satisfying a wave equation compatible with Maxwell's equations [3-5] and the reformulated Maxwell equations (4.13) and (4.14).

This concept of an immaterial medium through which the electromagnetic wave propagates brings, as a first consequence, a new relation for the Doppler effect of light, where now we must consider that the source of the electromagnetic wave moves with the medium through which the wave propagates, resulting in the classical formula [3,5]

$$\frac{f'}{f} = \left(1 - \frac{\mathbf{v}_0 \cdot \hat{\mathbf{n}}}{c} \right), \tag{5.1}$$

where the restriction that the source and the medium are in relative rest must be considered, $\mathbf{v}_0$ is the detector's speed, of the frequency f´, in relation to the medium or source, $\hat{\mathbf{n}}$ indicates the direction of wave propagation, and f is the source emission frequency.

We conveniently leave, as will be noted, for this paragraph a brief discussion on the electromagnetic interaction between two charged particles. First, let us observe that a particle with charge q_1, released with velocity $\mathbf{v}_1$ to interact with charge q_2, which is in the situation presented in the first paragraph of this section, experiences a force whose characteristics depend on the region where particle q_1 was released. Here, as confirmed by the literature [3,5,8], the concept of force between two charged particles lacks meaning, and the idea of interaction at a distance does not hold. What occurs is an interaction between particle q_1 and an electromagnetic field ($\mathbf{E}$, $\mathbf{B}$, $\mathbf{v}$) at the position of q_1, which we express by the new force law:

$$\mathbf{F} = q_1 \mathbf{E} + \propto \frac{\mathbf{u}}{c} \times \mathbf{B}, \tag{5.2}$$

where $\mathbf{u} = \mathbf{v}_1 - \mathbf{v}$, $\propto = \left(1 - \frac{u^2}{c^2}\right)^{\frac{1}{2}}$ and the fields $\mathbf{E}$, $\mathbf{B}$, and the velocity $\mathbf{v}$ have magnitudes inherent to the region where the charge q_1 is located. In region 1, for example, we would have the set of magnitudes ($\mathbf{E}'_2$, $\mathbf{B}'_2$, $\mathbf{v}'_2$). This means that the particle q_1 experiences forces expressed differently in different regions. Here, we also note that the concept of force loses meaning unless we present an expression for the reaction force of charge q_1 on the field ($\mathbf{E}$, $\mathbf{B}$, $\mathbf{v}$). However, this situation does not prevent the treatment of such interactions, as these can be addressed using the concepts of conservation of energy, linear momentum, and angular momentum, as already established in the literature [3,4,5,8]. This is because the particles q_1 and q_2, when propelled by each other's fields, emit electromagnetic pulses as a reaction, carrying energy, linear momentum, and angular momentum. Such difficulty in the concept of force was not observed in the interaction between two long current-carrying wires, discussed in section 4, which led to equation (4.6). Similarly, this issue does not arise in the interaction between any two closed current loops [3,4,5], as in these cases, the magnetic fields due to closed circuits are stationary, and the conduction charges are confined and maintained with their conduction velocity throughout the closed circuit [3,4,5,8], not propelled during the interaction.

So far, we have not presented an expression for the electric field **E** of a moving charged particle. To do this, let us consider the modified electromagnetic force law (5.2) and the situation in which two charges q_1 and q_2 are at rest in reference frame S, and they are kept in this state by fixed instruments in S that measure a purely electric force between them. Then, according to law (5.2), the force $\mathbf{F}_1$ on q_1 as measured by an observer in S must be

$$\mathbf{F}_1 = q_1\mathbf{E}_2 \;\text{ e }\; \mathbf{E}_2 = \frac{q_2\mathbf{r}}{r^3}, \tag{5.3}$$

where $\mathbf{E}_2$ is the Coulomb field of the particle q_2, at rest, at the position **r** of q_1. Now let us consider system S' that moves relative to S with velocity -**v**; then the charges q_1 and q_2 are observed by an observer in S' to have velocity **v**. According to the force law (5.2), the force between these charges is still purely electric, because the relative velocity between charge q_1 and the lines of magnetic field $\mathbf{B}_2$, due charge q_2, is zero, and we write

$$\mathbf{F}_1' = q_1\mathbf{E}_2' \quad \text{e} \quad \mathbf{E}_2' = \frac{q_2\mathbf{r}}{r^3}, \tag{5.4}$$

where we state that the observer in S' will measure the same force as the observer in S, according to classical mechanics, **F** = **F**' for these particles confined to uniform rectilinear motion. Thus, we conclude that the electric field of a charged particle is independent of the velocity of the inertial reference frame in the theory we are proposing here.

6. Conclusion.

The plan proposed in the introduction of this work has been fully accomplished. Maxwell's equations (1.3) and (1.4), respectively Faraday's law and Maxwell-Ampère's law, have been reformulated, resulting in forms that present greater generality and symmetry. The generalized Ampère's law, which results from the reformulated Maxwell-Ampère law, has been successfully applied, solving various problems already known in the electromagnetism literature, where they are addressed using the Biot-Savart law and/or Ampère's law, among which we mention the infinite straight current wire and the finite straight current segment.

The magnetic field observed in the spin-orbit interaction in an atom, which results in the splitting of its energy levels known as fine structure, is obtained using the modified Maxwell-Ampère law from an inertial reference frame at the nucleus of the atom, which cannot be done with the usual Maxwell-Ampère law.

The expression for the magnetic force was also modified to explain: (i) why the magnetic field responsible for the spin-orbit interaction does not exert a force on the electron in translation; and (ii) the dependence of the cyclotron frequency of a charged particle in a uniform magnetic field on the particle's velocity, maintaining the particle's mass constant, as per classical mechanics.

Finally, we presented the electromagnetic field itself as the special medium corresponding to the luminiferous ether, in which electromagnetic waves propagate, leading to a reinterpretation of the Doppler effect and a new understanding of the interaction force between two particles.

Thus, in this paper, we have established fundamental elements, including a special medium through which electromagnetic radiation propagates, to guide us in the development of a possible classical electromagnetic theory. Our hope is that academics of electromagnetic theory discuss, complement, and refine these ideas toward a classical electromagnetic theory.

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