TWO METHODS OF CALCULATING GRAVITATIONAL FORCE WHICH LEAD TO A NEW LAW OF GRAVITATION

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Abstract. *For over 300 years, the force of gravitational interaction has been represented by only one physical law: Newton's law of gravity F = GMm/r^2. Here we show that Newton's law of gravity is not the only law of gravitational interaction. There is one more undiscovered law of two-body gravity in physics. The new law of gravity is: F = mR^3/T^2r^2. This is the second law of gravity. It does not exist in Newtonian dynamics. This law of gravity does not include the gravitational constant G. It does not include the central mass M. The new law of gravity includes the characteristics of the elliptical orbit in the form of the Kepler constant R^3/T^2 as parameters. Here we reveal a method for obtaining a new law of gravity. The same method allows one to obtain a new law of gravity for N bodies.*

Keywords: *Newton's law; N-body problem; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant Λ.*

1. Introduction

For over 300 years, the force of gravitational interaction has been represented by only one physical law: Newton's law of gravitation F = GMm/r^2. At the same time, Newton's law of gravitation does not provide a complete description of gravity. Newton's law cannot explain the shift in the perihelion of planets [1]. The gravitational anomalies revealed in the dynamics of stars show that at large distances Newton's law is not satisfied and has significant discrepancies with observations [2]. Newton's law of gravitation shows the force of gravitational interaction between two bodies. Newton's law describes gravity only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies in the Universe.

Numerous attempts have been made to modify Newton's law without changing its form [3-5]. But all attempts at "cosmetic" modification of Newton's law of gravitation have not yielded results. Gravity theories have not led to the discovery of an alternative law of gravitation free from the shortcomings of Newton's law. The possibility of the existence of another law of gravity, not similar to Newton's law, has not been studied.

Here we show that Newton's law of gravity is not the only law of gravitational interaction. Two new laws of gravity are presented.

2. Two methods for obtaining the law of gravitational force

In physics, much attention has been paid to attempts to reveal the mechanism of gravity and the cause of gravity. To do this, all the attention of scientists switched to the development of theories of gravity. But there is still no complete theory of gravity.

Newton was not interested in the causes of gravity. Newton discovered the fundamental law of Nature without having a theory of gravity. He devoted all his attention to the method of obtaining the law of gravitational force. The heuristic potential of Newton's method turned out to be more powerful than the heuristic potential of theories of gravity. After Newton, the topic of developing methods for obtaining the law of gravity was practically not developed. The development of this topic can lead to the discovery of a law of gravity more perfect than Newton's law, without relying on future theories of gravity.

Here are two methods for obtaining the law of gravity of two bodies. The first method is based on the use of the parameters of the central body and the inverse square law. The second method is based on the use of the parameters of the orbit and the inverse square law.

Newton introduced the mass of the central body and the inverse square law into his law of gravitation. In this method, there is no need to take into account the orbital parameters. The inverse square law and the central mass are sufficient to obtain the gravitational force. Newton, in his letter to Hooke dated December 13, 1679, stated regarding elliptical orbits: «*Your acute Letter having put me upon considering thus far ye species of this curve, I might add something about its description by points quam proximè. But the thing being of no great moment…*» [6]. For the method that Newton used, it is sufficient to know the mass of the central body. There is no need to take into account the parameters of the elliptical orbit.

The second method is a new method. The inverse square law and the orbital parameters are sufficient to obtain the gravitational force. The second method does not need to use the parameters of the central mass.

Two different methods lead to different laws of gravitation. One of them is Newton's law, the other is the new law of gravitation.

2.1. Central body method.

The first method for obtaining the law of gravitational force is based on the use of the mass of the central body. The acceleration under the action of the central body of mass **M** is determined by the expression: **a = GM/r^2**. In this case, we obtain Newton's law of gravity **F = GMm/r^2**. This method does not use the orbital parameters.

2.2. Orbit method.

The second method for obtaining the law of gravitational force is based on the use of orbital parameters instead of the central mass. The semi-major axis **R** of the ellipse and the period of revolution of the body **T** are used as parameters of the elliptical orbit. Acceleration is determined by the expression: $\mathbf{a = R^3/T^2r^2}$. The orbit method leads to a new law of gravity $\mathbf{F = mR^3/T^2r^2}$. This is the second law of gravity for the gravitational interaction of two bodies. It is not in Newtonian dynamics. This law of gravity does not include the gravitational constant **G**. It does not include the central mass **M**. The new law of gravity includes the characteristics of the elliptical orbit in the form of the Kepler constant $\mathbf{R^3/T^2}$ as parameters. As we can see, Newton's law of gravity is not the only law of gravitational interaction. Unlike the Bertrand problem [7 - 9], the Orbit method uses integral trajectory parameters. This simplifies the calculation of acceleration, which is also an integral parameter. The integral trajectory parameters directly lead to a new law of two-body gravity.

3. Applying the central body method to the N-body system yields a new law of gravity $F = mc2\sqrt{\Lambda}$.

If we apply the central body method to the gravity of N bodies, it is easy to obtain a new law of gravity that takes into account the gravitation of all N bodies in the Universe. To do this, we move from the N-body gravity problem to the two-body problem, where the central body is the N-body system. The N-body system is considered as a single object. To derive the law of gravity of N bodies, we need to know the integral parameters of the N-body system. The integral parameters of the N-body system directly derive the acceleration, which is also an integral parameter. This path leads to a new law of gravity of N bodies.

If the N-body system is the Universe, then the problem has several solutions depending on the choice of the integral characteristic of the Universe. With respect to the Universe, the integral parameters are defined. These are the mass of the Universe Mu, the radius of the Universe Ru, the cosmological constant Λ, and the time of the Universe Tu. Of all the parameters of the Universe, the cosmological constant Λ is measurable. The acceleration caused by the N-body system is determined by the expression: $a = c2\sqrt{\Lambda}$. When applied to the N-body system, the central body method yields the following law of gravity: $F = (mc^2)\sqrt{\Lambda}$. This is a new law of gravity - the law of cosmological force. It does not exist in Newtonian dynamics. Instead of the gravitational constant G, the new law of gravity includes the cosmological constant Λ. The cosmological force does not obey the inverse square law and has a linear dependence on the mass of the body. The same formula for the law of gravity: N bodies ($F = (mc^2)\sqrt{\Lambda}$) was obtained as a solution to the inverse N-body problem [10].

4. Conclusion.

Newton's law of gravity is not the only law of gravitational interaction. In gravity, two fundamental physical laws remained undiscovered. Two methods for obtaining the law of gravitational force are described, which lead to two new laws of gravity **$F = (mc^2)\sqrt{\Lambda}$, $F = mR^3/T^2r^2$.** These laws of gravity are not present in Newtonian dynamics. These laws of gravity do not include the gravitational constant G. They complement Newtonian dynamics and allow us to overcome the shortcomings and limitations of Newtonian dynamics. In particular, they explain the rotation curves of galaxies without involving the dark matter hypothesis. The law of gravity $F = mR^3/T^2r^2$ describes the gravitational interaction of two bodies. This is how it is similar to Newton's law $F = GMm/r^2$. The advantage of the law of gravitation $F = mR^3/T^2r^2$ is that the parameters R and T are known from observations with greater accuracy than the central mass M and the gravitational constant G. This law of gravitation was hinted at by Robert Hooke in his correspondence with Newton in 1679 [11, 12]. This was before the publication of Philosophiæ Naturalis Principia Mathematica.

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