Relativistic Structure of a Rotating Sphere in a Light Cylinder

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Abstract

This paper presents a relativistic model of a rotating dust sphere without gravity, in which the geometric deformation is caused solely by kinematics. The calculations are based on the method used by the author of the article to calculate a relativistic disk. Based on the analytical expression for the vertical deformation as a function of radius, angle and angular velocity, the effect of rotation on the geometry, perimeters and volume of the sphere is analyzed. Particular attention is paid to the asymmetry of expansion between the equatorial and polar regions.

1. **Introduction**

Hard disks have been studied quite well [1,2,3,4,5,6], which cannot be said about purely kinematic models of rotating disks and especially dust spheres - particles without internal pressure, cohesion and gravity. Rotating bodies in the special theory of relativity demonstrate complex deformations even in the absence of gravitational interactions and therefore require, in the author's opinion, separate consideration.
It should be noted that the entire picture of deformation in this article is considered from the point of view of a stationary observer located at the center of the sphere. In this model, the metric is studied as an observable geometric structure deformed without sources of the energy-momentum tensor - only due to kinematics.
Rotation breaks the symmetry of the metric in the direction of the Z axis, and the pseudo-interval will depend on the vertical nesting:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 - (\omega, r, \theta) dz^2$$

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2. Geometry of vertical deformation

In the work [7] it was possible to find the dependence of the geometry z = f (x, ω) = f (y, ω) of the dust disk on the angular velocity:

$$z=-\frac{\sqrt{3}c}{2\omega}\ln\left(1-\frac{x^2\omega^2}{c^2}\right) \qquad (1)$$

Using equation (1), we write an analytical expression for the vertical deviation z of the surface of rotation of the sphere, observed from the center along the axis of rotation:

$$z=r\cos(\theta)-\frac{\sqrt{3}c}{2\omega}\ln\left(1-\frac{r^2\sin^2(\theta)\omega^2}{c^2}\right) \qquad (2)$$

Where:
- r : radial coordinate
- θ: polar angle
- ω: angular velocity
- c: speed of light (further in the model it is assumed to be equal to 1)

Note that c/ω = R is the radius of the light cylinder, for R = 1 we obtain:

$$z=r\dot{c}os(\theta)-\frac{\sqrt{3}}{2}\ln(1-r^2\cdot\sin^2\theta) \qquad (3)$$

As can be seen from Fig. 1, when the sphere rotates, its surface “swells” in the direction of the axis of rotation. This deformation sharply depends on the radius of the sphere and is greater the closer the radius of the sphere is to the radius of the light cylinder, and therefore also depends on ω and θ.

3. Perimeters: equatorial and vertical

3.1 Horizontal (equatorial) perimeter:

The equatorial perimeter (in the XY plane) at $0 < z \leq 0.01$ is determined by formula (1):

$$PXY(\omega) = 2\pi \frac{1}{\omega}\sqrt{1-e^{\frac{-2z\omega}{\sqrt{3}}}} \qquad (4)$$

3.2 Vertical (meridian) perimeter:

In the second order of approximation:

$$PZX(\omega) \approx 2\pi\,(1 + 3\omega^2/8) \qquad (5)$$

4. Volume approximation

Assuming the symmetry of the deformation in the ZX and ZY planes, the volume of the rotating sphere in the second approximation in angular velocity ω can be expressed in general as:

$$V(\omega) \approx (\sqrt{3}\,\pi / 8)\,\omega\, r^4 + (\sqrt{3}\,\pi / 32)\,\omega^3 r^6 \qquad (6)$$

When r = 1, the expression is simplified:

$$V(\omega) \approx (\sqrt{3}\,\pi / 8)\,\omega + (\sqrt{3}\,\pi / 32)\,\omega^3 \qquad (7)$$

5. Comparison of the growth of geometric characteristics

Graph 2 compares the normalized ratios of volumes and perimeters:

- V/V0
- PXY/P0
- PZX/P0
- And also the ratios: V/PXY, V/PZX

- V0 and P0 - the volume and perimeter of the sphere along the equator before rotation.
- V=Vrel - increment of the sphere's volume.

It should be noted that the changes in the vertical perimeter and volume correlate well with each other at low angular velocities of the sphere's rotation, which indicates an acceptable approximation of these parameters. But this is not enough to reveal the simple proportionality of the $V \sim R^3$ form, since the shape deviates more and more from spherical as it approaches the boundary of the light cylinder. Its volume increases to about 20 percent.
The picture of the sphere shell ejection shown in Fig. 1 shows that during this ejection its velocity in the direction of the rotation axis Z should depend not only on the radius and polar angle, but also on the acceleration of the angular velocity. Indeed, the derivative ∂z/∂ω exists and shows the following dependence:

$$\partial z/\partial \omega = \frac{\sqrt{3}}{2\omega^2}\ln\left(1-r^2\omega^2\sin^2\theta\right)+\frac{\sqrt{3}r^2\omega\sin^2\theta}{1-r^2\omega^2\sin^2\theta} \qquad (8)$$

If we introduce the radius of the light cylinder and take it equal to R=1, then from equation (3) we obtain a simpler expression for the derivative:

$$\frac{dw}{dr}=\cos(\theta)+\frac{\sqrt{3}r\sin^2(\theta)}{1-r^2\sin^2\theta} \qquad (9)$$

Approaching the radius of the light cylinder R=c/ω, the derivative increases sharply, which may be associated with ejections of shells or instability of the metric. The highest rate of change of the metric (see graph 3) is observed, as expected, at the equator.

7. Discussion

Asymmetric swelling, strong dependence on angular acceleration in the absence of a gravitational source allow us to speak of purely kinematic deformations of the metric.
It should be expected that the vector of tangent (tangential) velocity receives additional distortion from the change in geometry along the perpendicular direction. Therefore, a possible movement of the metric itself is observed, and in the upward direction, in the direction of the rotation axis, the Z axis. The movement of any material object inside this moving geometry cannot but affect its trajectory. In such an anisotropic, dynamic metric, signs of nonequivalent times along different directions can be observed.

8. Principles of geometric redistribution as a hypothesis.

Based on the analytical expressions for the perimeters, an important pattern can be traced: with increasing angular velocity ω, the equatorial perimeter PXY decreases, while the vertical perimeter PZX increases.
This behavior indicates a redistribution of metric geometry: compression in one direction is partially or completely compensated by expansion in another. Here, the conservation law in the classical sense does not manifest itself, but in geometric terms it can resemble anisotropic expansion of the metric.
For a central observer (located in the center of the sphere), such a violation of the uniformity of the metric movement is equivalent to a localized expansion of the Universe in a certain direction.

9. Possibility of observing anisotropy in cosmology

As a hypothesis: a directed change in the metric in the presence of rotation leads to an uneven distribution of matter density in orthogonal directions.

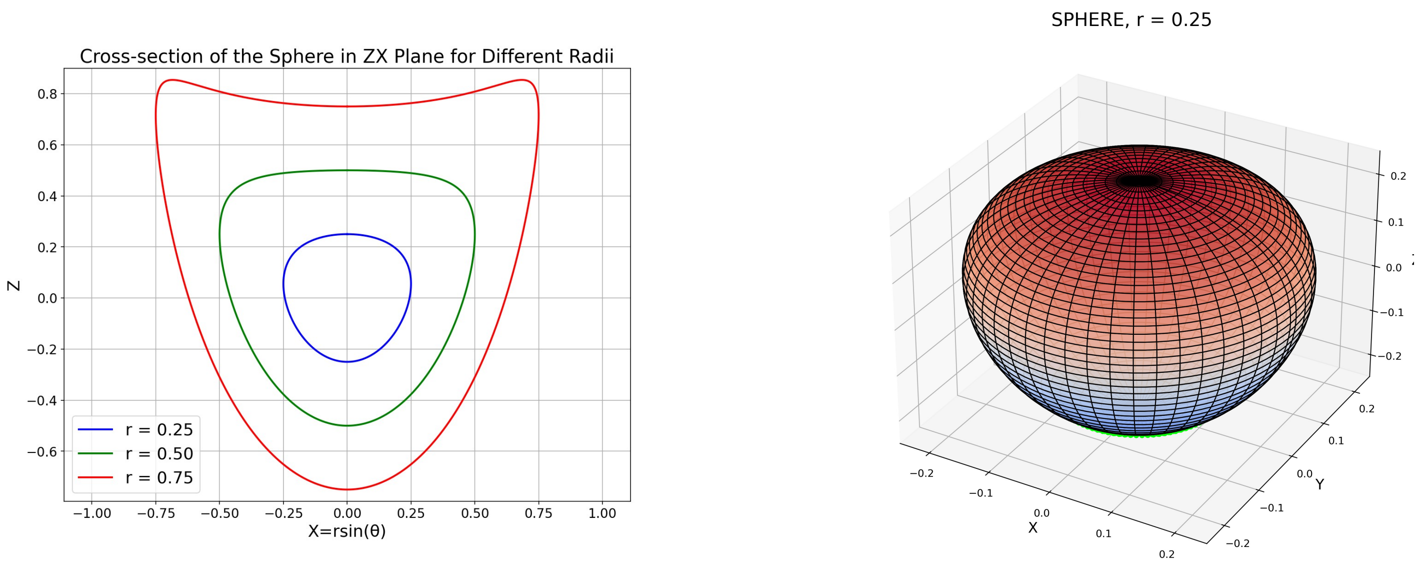
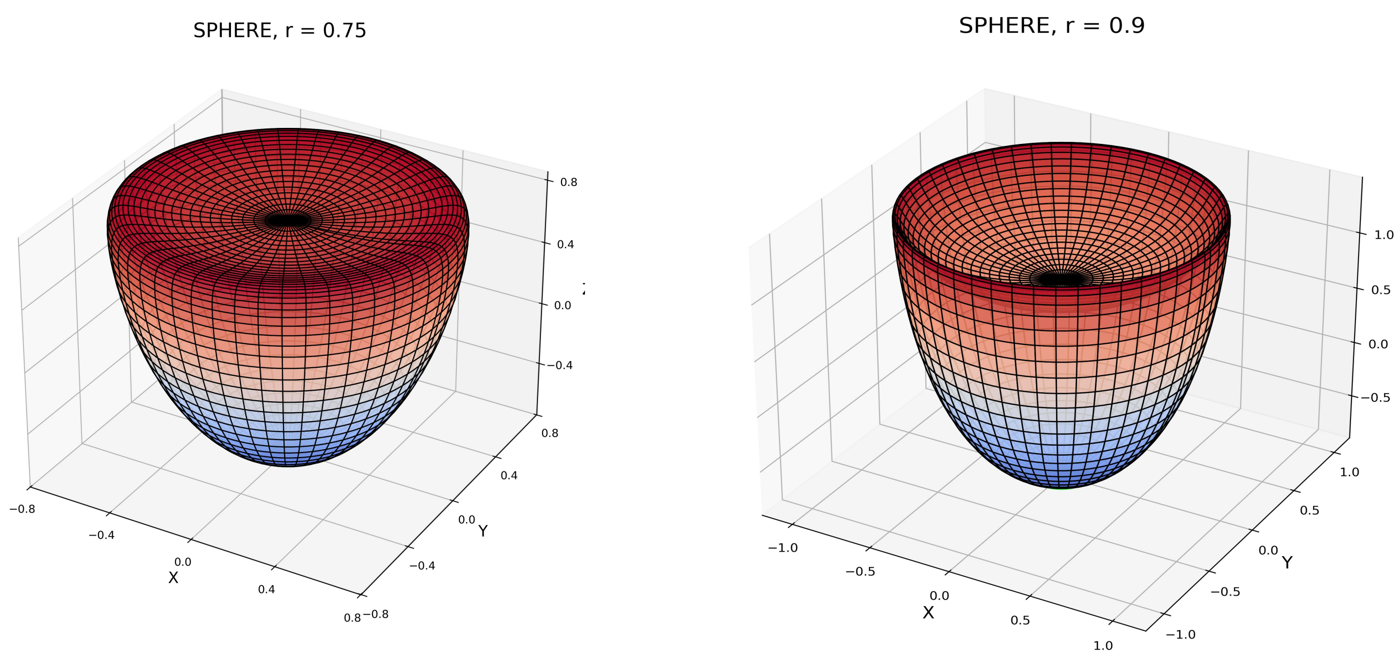

Cross-section of the Sphere in ZX Plane for Different Radii
Z
X=rsin(θ)
r = 0.25
r = 0.50
r = 0.75
SPHERE, r = 0.25
X
Y

Fig.1


SPHERE, r = 0.75
X
Y
SPHERE, r = 0.9
X
Y

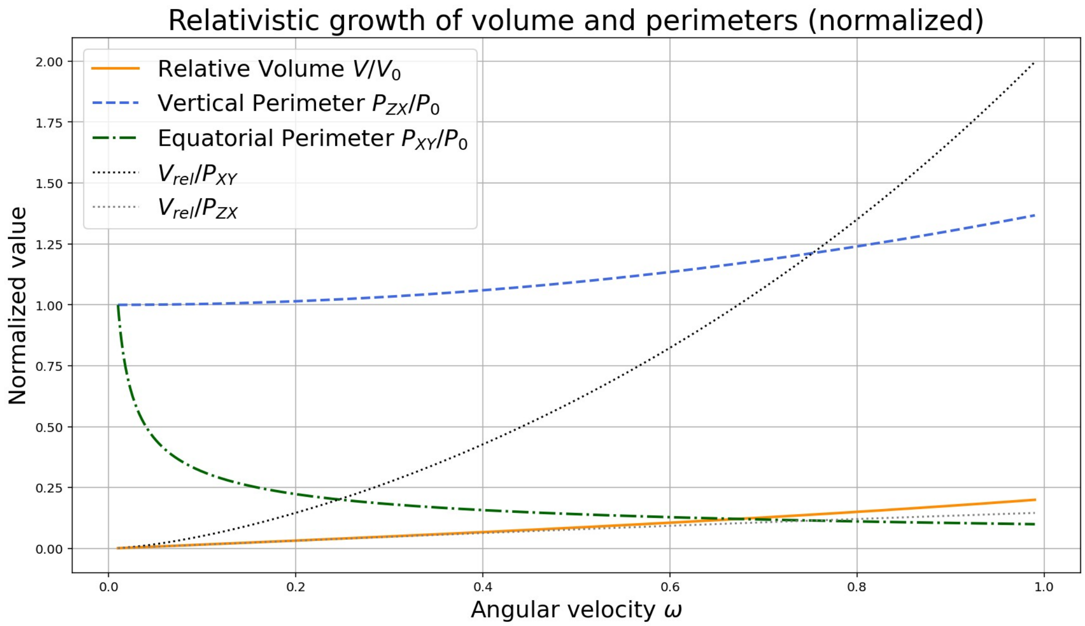

Relativistic growth of volume and perimeters (normalized)
Relative Volume V/V_0
Vertical Perimeter P_{ZX}/P_0
Equatorial Perimeter P_{XY}/P_0
V_{rel}/P_{XY}
V_{rel}/P_{ZX}
Normalized value
Angular velocity ω
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
0.0
0.2
0.4
0.6
0.8
1.0

Fig.2

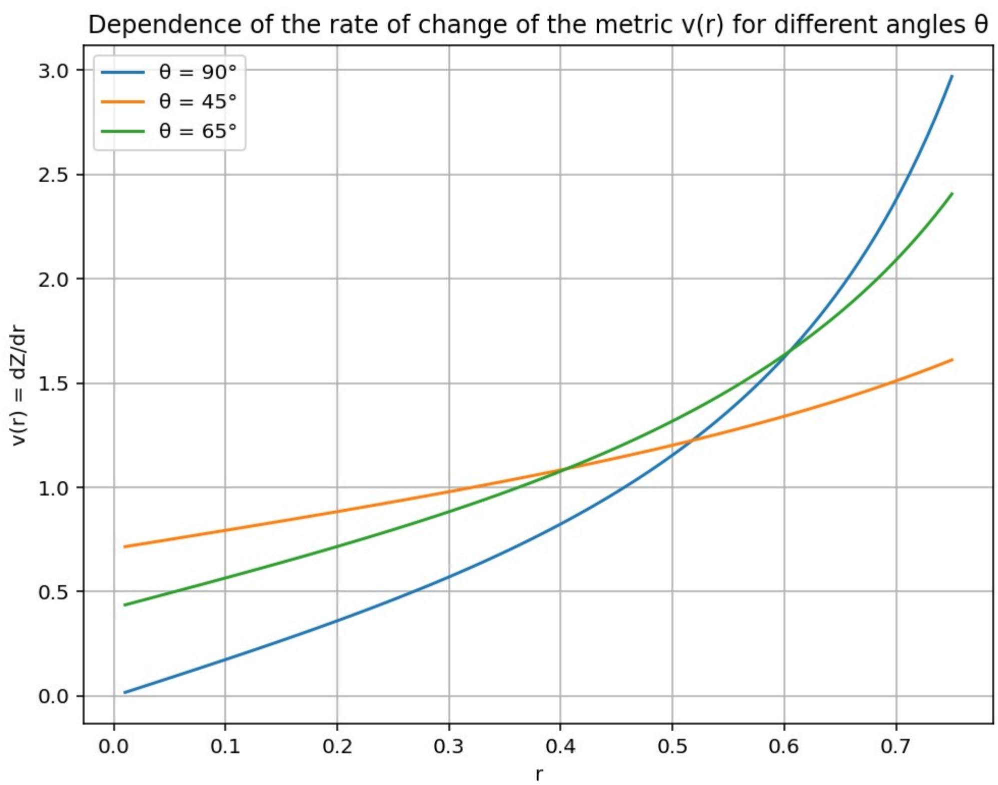

Dependence of the rate of change of the metric v(r) for different angles θ
θ = 90°
θ = 45°
θ = 65°
v(r) = dZ/dr
r
0.0
0.5
1.0
1.5
2.0
2.5
3.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Fig.3

Conclusion: Approximations of analytical expressions, plotting and visualization of relativistic deformation of spherical and disk models are performed using the C++ and Python 3.12 programming languages with the use of standard numerical analysis libraries and graphical modules.

Литература

1. Einstein, A. (1911). “Zum Ehrenfesten Paradoxon”. Phys. Zeitschrift 12:509

2.Ø. Grøn, Relativistic description of a rotating disk. Am. J. Phys. 43, 869-976 (1975)

3. M. Born, *Ann. Phys. (Leipzig)* 30, 1 (1909).

4. L. D. Landau and E. M. Lifschitz, *The Classical Theory of Fields,* 3rd. ed. (Pergamon Press, 1971), pp. 233, 234.

5. Jri Bicak and Tomas Ledvinka, Relativistic disks as sources of the Kerr metric

6 .Phys. Rev. Lett. 71, 1669 – Published 13 September, 1993

7. A. Rozenkevich , Rigidly Rotating Disk Dust in the Special Relativity: https://vixra.org/abs/1107.0010, 2011