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The Existence of Chance

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Abstract

This paper aims to study a particular case to provide a rational basis for recognizing the existence of chance by finding its real trace, that is, specifying its practical meaning, which consists of the following statement known as the Statistical Law of Large Numbers:

If an event E has a constant probability p of occurrence on any one trial, and has occurred m times in n trials, then, if the relative frequency of E, m/n, approaches the value of a limit point l and the accuracy of the approximation increases as the number of trials increases, we have l = p.

The argument we propose is based on the concepts of "event" and "trial", formulated by the author himself, and their direct implications.

Keywords: Chance; Gambling System; Random Event; Statistical Law of Large Numbers; Trial.[1]

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1. Introduction

Is chance real, true? Everyday life and some formal analyses seem to lead us to a negative answer, suggesting the idea that chance, or randomness, are relative and not absolute notions; indeed, it seems that the same phenomenon can be considered random based on some evidence and no longer be so in the presence of a stronger, more refined evidence[2].

It is therefore necessary to decide what exactly is to be understood by "chance", that is, whether it denotes an inescapable attribute of reality, as indeterminism suggests, or whether, as determinism and necessitarianism hold, it denotes our non-exhaustive knowledge of the exact causes underlying and figuring out various phenomena.

If, instead, our knowledge/evidence is such as to allow us to prove by itself the impossibility of predicting a certain phenomenon, we are in the presence of a random event. Examples of events of this last type are easily found in dice games, meteorology, etc., that is, in all those situations in which minimal or imperceptible variations of the starting conditions – so small as to be impossible for us to record or to exclude them – produce macroscopic differences crucial for the event to occur or not[3]. An event is therefore random if it, because of its instability, its complexity, or any other relevant property, refers in a total and exclusive way to a cognitive state, sufficient to affirm its absolute imponderability.

The objectivity that characterizes this notion of randomness, however, is such in relation to the living being, to the world in which it lives and therefore to what in this world is a cognitive patrimony common to all. Note, then, that those who support this position would seem to have to refrain, perforce, from taking a decision for or against indeterminism necessarily. In fact, they could neither deny determinism nor exclude that, also in the context of determinism, unpredictable events can be found, which are such because of the set of conditions/knowledge to which they refer.

But is it possible to dispel this doubt? In other words, is it possible that the concept of randomness is marked by truthfulness, that is, is characterized by objectivity in the strict sense, of an ontological type, and not only by what is subjectively universal – i.e., valid for all men –?

Yes, so it is, if we take some interpretations within post-quantum sciences, in which "real" is only what can be measured and where a distinction

[2] By the term *evidence* we refer here to all the knowledge we possess when we are preparing to argue about a given phenomenon. When the very broad meaning just outlined is attributed to evidence, it is impossible to speak of randomness in the absence of evidence. The randomness of a phenomenon will therefore always be related to evidence.

[3] See M.C. GALAVOTTI, *Probabilità*, La Nuova Italia, Milano, 2000, pp. 70-79.

between gnoseological unpredictability and ontological indeterminacy cannot exist: in other terms, pre-assumption of a certain ontological reality – beyond those possible experimental measures – would signify the fall into metaphysics.

Yes, it is. Provided that we reject the traditional distinction between a truthful reality and a subjective one, we reject the firm belief that the world is divided into a truthful development in space-time and a consciousness that merely sees and thinks about this evolution. And it is precisely this last point of view that we wish to propose here.

Reality is no longer something independent of us and of our knowledge, something that can go ahead autonomously without our contribution. It is not the set of objects, material substances, of what is outside of us, to which to oppose the internal sphere of subjectivity. Rather, the real thing[4] becomes the objective result of the necessary and constant interaction between something that is seen and what is known about this something, the experience of their fusion, which will take on different forms and degrees in relation to the different contexts that are treated.

Chance can therefore still be considered a name for our ignorance, specifying however that by "our ignorance" we are not referring to our lack of information on reality, but to a precise state of reality itself, in which we are inevitably involved and which changes because of the variation of our knowledge on what we see about it. Changing our knowledge about reality is in fact equivalent to changing it.

At this point it becomes necessary to give to this position a coherent logical structure and an operational sense. And that is exactly what we will try to do in the next two paragraphs.

2. Event and Trial

In close connection with what explained in the introduction, we argue in the following our denunciation of a truthful randomness, focusing on three key concepts which we will be discussing at once and that are closely related: *event and trial*.

The notion of event that we adopt here can be summarized in the following two basic points:

ε1) By *event*, we mean a thing that refers totally and exclusively to a well-defined set (or class) of conditions, such that, when this set of conditions is realized – i.e., when all the conditions that constitute it are realized –, and

[4] For *real thing* we mean here simply "what is (appears)".

only in this case, the thing gives rise to an ascertainable fact by which we can always determine whether or not the event has occurred(presented) [5].

ε2) When we affirm that some event A is *random,* we mean that the set of conditions C, or more briefly set C or only C, to which event A refers, holds the whole class of reasons necessary and sufficient for there to be no immanent way of predicting whether A will or will not occur when C. The event A is instead *certain* (*impossible*) with respect to C, if the set C includes the necessary and sufficient conditions for A to occur (not to occur) when C[6] (See [8] pp. 21-24). It follows that every event of C is necessarily random, certain, or impossible[7].

The single realization of the set of conditions C, called *trial of C*, can understandably take place, at least mentally, more than once – in this case we

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[5] The event, realized the class conditions to which it relates, gives rise to a verifiable fact by which it can always be inferred whether the event occurred. By *fact* we mean, therefore, a happened that can be described by a boolean proposition of the type «the thing or entity x has the property y» that makes sense, that is, for which a criterion is given by which it is technically possible to attribute it the truth value: *true* (See [1] pp.13-14). Since, as is obvious, the meaning and truth value of a proposition depend on some aspects of the context in which it is inserted – e.g., the phrase «there is a pen on that table» makes sense and is a proposition because it is placed, even implicitly, in environments where one knows perfectly well what «table» and «pen» are and how one recognizes their presence; in some of these areas, it could thus be (judged) true, in others false etc. –, one can speak of a fact (as of an event) only in relation to a set of conditions.

[6] When we talk about randomness, certainty, or impossibility of any event, we always mean randomness, certainty, or impossibility with respect to the set of conditions to which the event refers.

By use of the three known symbols of logical implication, introduced as follow: "$\Leftrightarrow$" as short for "co-implies", or "is logically equivalent to", "$\nRightarrow$" as short for "not imply" and "$\Rightarrow$"as short for "implies", we can summarize the concepts of randomness and certainty (impossibility) of an event, respectively, by the following relations:

$$\{C \Leftrightarrow [«C \nRightarrow A» \text{ and } «C \nRightarrow \text{not } A»]\}\ ,\ \ C \Leftrightarrow\ «C \Rightarrow A \text{ (not A)}»$$

where, for the sake of brevity, "C" stands for "C is realized," "A" for "A occurs," and "not A" for "A does not occur."

[7] Let us first show that $S_1 = (C\Rightarrow A) \Leftrightarrow S_2 = [C\Rightarrow(C\Rightarrow A)]$, where A is an event of C.

Recall in this regard that, from a logical point of view, the two relations $C\Rightarrow A$ e $C\nRightarrow A$ are necessary and mutually exclusive, and that each of these two options implies C; in fact neither $C\Rightarrow A$ nor $C\nRightarrow A$ would make sense without the realization of C, since, by ε1, A presupposes C.

Now, $S_2 \Rightarrow S_1$ is a self-evident truth. Conversely, if the realization of C is a sufficient condition for the occurrence of A, i.e., if $C\Rightarrow A$, then, if S_2 were false (i.e., if $C\nRightarrow(C\Rightarrow A)$, we would have at least one classification, one example in which C is realized and A does not occur. But this would contradict the assumption that $C\Rightarrow A$. Consequently, it must be $C\Rightarrow (C\Rightarrow A)$. Finally, we prove that

$$S_3 = [C \nRightarrow (C \Rightarrow A)] \Leftrightarrow S_4 = [C\Rightarrow (C\nRightarrow A)].$$

If S_3 were false, that is, if $C\Rightarrow(C \Rightarrow A)$, then, evidently, $C\Rightarrow(C\nRightarrow A)$ could not be true; therefore, $S_4 \Rightarrow S_3$.

Vice versa, suppose that $S_3 = C\nRightarrow(C\Rightarrow A)$ is true. Since $S_1 \Leftrightarrow S_2$, we get $C\nRightarrow A$, necessarily – it cannot occur $(C\Rightarrow A)$, if C is true; otherwise, we would have $C\Rightarrow(C\Rightarrow A)$ against the hypothesis: S_3 true –. It must therefore be $C\Rightarrow(C\nRightarrow A)$. Hence, $S_3\Rightarrow S_4$. It follows that $S_3 \Leftrightarrow S_4$, and so that $(C\nRightarrow A)\Leftrightarrow[C\Rightarrow(C\nRightarrow A)]$.

We thus proved that [$C\Rightarrow$ «$C\nRightarrow$ ($\Rightarrow$) "A" "not A"»] $\Leftrightarrow$ «$C\nRightarrow$($\Rightarrow$) "A" "not A"». The statement that every speak of *repeatable trial of C* – or only once, when it coincides with a totality of repeatable trials. The various trials, being exactly individual realizations of the same set of conditions, can only be identical and mutually independent, in the sense that the outcome of each of them does not influence and is not influenced in any way by that of another or the others. The set of all physical or conceptual trials of C – i.e., of all thinkable ones – will be written down with H_C. H_C is hence a singleton[8] or an infinite countable set.

Regarding each single trial, certainty, randomness, and impossibility are qualities that are not restricted to events, but also extend to facts; so, they are properties applicable not only to propositions concerning future events, but also to propositions that describe past or present events. This can be easily proved as follows:

Without loss of generality, suppose we do not know which of the random events A and -A[9], of the set of conditions C, is the one that, by happening, caused the occurrence of a certain circumstance B. Since B inevitably occurred together with one and only one of the two events A and -A, we have (with respect to B) the following and conflicting hypotheses:

H_1: «Event A has occurred» H_2: « Event -A has occurred».

Now, due to ε1, it must be possible to carry out at least one T-test – even if it were only a direct verification – that allows one to find whether A has occurred, and therefore to check which of the two hypotheses H_1 and H_2 is proven true, and which is refuted. Two complementary events are so defined:

$\hat{H}$: "the hypothesis H_1 is confirmed" -$\hat{H}$: "the hypothesis H_2 is confirmed",

both inherent to the set of conditions $\hat{C}$, which consists of the condition "C has been fulfilled" and all the other conditions that, together, define the properties of the T-test and the technical modalities of its execution.

Also note that the event $\hat{H}$ (-$\hat{H}$) occurs in any trial of $\hat{C}$ only if the event A (-A) has already occurred in the corresponding trial of C. Hence, if the premises $\hat{C}$ are true, the conclusion $\hat{H}$ or -$\hat{H}$ does not necessarily follow,

event of C must be certain, random, or impossible is thus, with good reason, a theorem.

[8] Recall that by singleton we mean a set that has exactly a single element.

[9] Let -A be the complementary event of A. We remember that two events are said to be complementary if they are incompatible and necessary, that is when, in the same realization of the set of conditions to which they refer, the occurrence of the one excludes the occurrence of the second – incompatible – but one of the two must necessarily occur – necessary –. For example, consider the single roll of a dice. As it is easy to guess, the following two cases may occur: either an even number comes out or an odd number comes out. The two events are complementary since one of the two will necessarily occur and one excludes the other.

because A(-A) is a random event of C and its occurrence, since it predates the T-test building, is independent of all T characteristics and specificities.

So, both $\hat{H}$ and $-\hat{H}$ are random events with respect to $\hat{C}$ [10].

On the other hand, as can easily be seen, the element "C has been realized", which constitutes the singleton $\hat{C}_H$, is a necessary and sufficient condition not only for the hypotheses H_1 and H_2 to make sense, for them to be, but also for the impossibility of foreseeing their correctness or falsity when C has been realized, or, what it is the same, whenever $\hat{C}_H$ is realized [11]. Both the hypotheses H_1 and H_2 satisfies thus the two requirements ε1 and ε2 characterizing the notion of event; therefore, H_1 and H_2 are two random events with respect to the set of conditions $\hat{C}_H$.

Hence, the property of being a random event can apply as well to that which has happened and which we can discover to be a fact. But no fact can be a random event; a fact is something whose truth it is impossible to doubt. It would seem to be facing a contradiction. It seems, as argued by Pierre-Simon de Laplace, that chance should be thought of negatively, not as a concrete reality, but simply as an epistemic fact, that is, as a deprivation of knowledge. It would then seem that chance/contingency cannot be truthfully characterized, that is, marked by ontological objectivity but must depend solely on the fact that we have "partial" knowledge of the data necessary for certain prediction. It seems to be, but it is not.

We explain ourselves better by proving the following important result (*Mister Tanatò's Paradox*):

R_1: Let A be an event with respect to a well-defined and realizable set of conditions C. If A is a certain or random event of C, then A occurs in at least one trial of C and vice versa.

Proof: The property is clear if A is a certain or impossible event of C. Suppose then that Mister Tanatò, a man sentenced to death, could avoid the death penalty in a way completely free from his will. Specifically, a coin M will be flipped (condition set C): if HEAD gets out (random event A), Mister Tanatò will have won his life challenge and will continue his existence; if

[10] If it were $\hat{C} \Rightarrow \hat{H}(-\hat{H})$, given that $\hat{H}(-\hat{H}) \Rightarrow A(-A)$, we would have $\hat{C} \Rightarrow A(-A)$ and thus $C \Rightarrow A(-A)$, being the occurrence of A(-A) completely independent of the T-test. But this would be absurd since A and -A are two random events of C and therefore $C \nRightarrow A(-A)$.
Finally, it is obvious that $\hat{C} \Leftrightarrow$ [«$\hat{C} \nRightarrow \hat{H}$» and «$\hat{C} \nRightarrow -\hat{H}$»], since $[C \nRightarrow A(-A)] \Leftrightarrow \{C \Rightarrow [C \nRightarrow A(-A)]\}$ (See footnote 7), $[C \nRightarrow A(-A)] \Rightarrow [\hat{C} \nRightarrow \hat{H}(-\hat{H})]$ and $\hat{C} \Rightarrow C$.

[11] If the conditions that form the set $\hat{C}$ yield that it is impossible to predict whether the event $\hat{H}$ occurs in any trial of $\hat{C}$, the conditions of $\hat{C}_H$, a fortiori, are necessary and sufficient to make it impossible to decide whether the hypothesis H_1 is true or false. In fact, $\hat{C}_H$ is a subset of $\hat{C}$, the realization of $\hat{C}_H$ is compatible with that of $\hat{C}$ and the occurrence of $\hat{H}$ $(-\hat{H})$ co-implies the truth of H_1 (H_2).

instead TAILS (random event -A) comes out, he will be promptly administered intravenously a lethal substance that will kill him instantly.

Since A is a random event with respect to the realizable set C, we cannot exclude that, in the immediately following instant of time τ to the toss of the coin M, the sentenced Tanatò has won the said bet and, hence, he has managed to survive, escaping his executioner.

At the time τ, Mister Tanatò might then find himself as a member of a special circle of the appearing (of reality), thus, still forced to keep going towards to one or more possibilities that will be ineluctably offered or imposed on him, including that of his own death. So, simultaneously, there must be a set of realized conditions, let us call this Č, respect to which the existence of Mister Tanatò is a random event.

On the other hand, at the same time τ, to affirm the impossibility of the existential fact of Mister Tanatò, of finding him still alive, it would be equivalent to supplying notification of his death – which therefore happened at least one moment before τ – to public. In other words, at the instant τ, this would mean that Mister Tanatò never existed, or as they say that he has already passed away in all possible worlds. But if so, one would have to admit that the set of conditions $Č_1$: {"The event -A occurred in every possible (thinkable) toss of the coin M"} has realized.

Hence, to avoid facing an antinomy[12], given the arbitrariness in the choice of Mister Tanatò and the experiment (launch of M) in which he is an interested spectator, we must recognize the veracity of the $\mathbf{R_1}$-statement, which is thus a theorem.

Remark 2.1: Let C be any realizable set of conditions. Let A be any certain or random event of C.

Because of the definition of an event, the occurrence of A in any trial of C is necessarily predictable – if A is certain – or always unpredictable – if A (and therefore -A) is random –. It follows, for $\mathbf{R_1}$ result, that the randomness of A co-implies the truth of the proposition $\mathbf{r_1}$: « "A occurs in a trial of C" is sometimes true». For these considerations, it seems that the thesis of indeterminism, which admits the ontological reality of chance/contingency, can be argued, since experience shows that $\mathbf{r_1}$ is true in at least one instance – consider, for example, the outcomes of repeatedly tossing of a fair coin –.

The validity of remark 2.1 assumes that randomness is a truthful characteristic of reality – of the type of the mass of the universe – which

[12] That is, of coming across the concomitant presence of two sets of conditions Č $\supseteq${"C has been realized"} and $Č_1$, both realized at time τ, the union of which is such as make the nonexistence of Mr. Tanató (in τ) <u>a fact</u> – it is unequivocally a fact if $Č_1$ is realized –whose truth turns out to be foreseeable and unpredictable at the same time.

involves the knowledge/evidence of the one who uses it, since it may otherwise also involve the happened, the latent fact – what has happened may not yet have happened –. Evidence thus sensitively affects what is real. Reality, therefore, can no longer be thought of as something independent of the knowledge of the one who thematizes it – who would then have only "degrees of confidence" about the manifestation of the phenomena by which it is affected – but what appears to us is what is, because to experience what directly appears to us is exactly to experience reality. So that conditions which for some observers are indeterminate, i.e., which are known, which appear to them (and thus are real) in the form of unknown variables, may then, paradoxically, appear to others (and thus belong to reality) as constants, i.e., may, for the latter, be precisely specified. Variables and constants therefore have the same dignity as real things and can be objects of experience and knowledge.

What are the boundaries within which the case acts? What quantitative limits does it have to respect? The next section will deal with this last issue.

3. Probability and relative frequency of an event

The percentage of repeatable trials, in which a given random event occurs, cannot experimentally be found, because we never have all the possible achievements of the class of conditions that define an event. If we think about the "last night it snowed" event, it is technically impossible to repeat independently «last night» an arbitrary number of times. We could do several numerical simulations on the computer (compatible with what is known about the weather situation of yesterday evening) and record the percentage of cases in which the simulation gives snow. In doing so, however, we would only get a correct approximation of our situation. The repeatability of the complex of «last night» conditions is indeed only theoretically possible if the circumstances of «last night» could be reconfigured in the same way as they were configured. To deal with such idealizations, it is necessary to involve probabilities. Probabilities replace uncertainty with something more usable and consistent.

Probability theory uses statements of the type «$P(E) = \lambda$», where E is any event and λ a numeral.

The expression «$P(E) = \lambda$» should read as follows: «the probability of the event E is λ». We now introduce another event F. This allows us two new combinations of symbols: $E \vee F$ and $E \& F$ each of which is another event. The event $E \vee F$ corresponds to the occurrence of three distinct cases: E, or F, or E and F, while the event E&F corresponds to the occurrence of the only case: E and F. We complete our presentation of probabilities with four basic rules:

1) $P(E) \geq 0$.

2) $P(-E) = 1-P(E)$.

3) If E and F are incompatible events, then $P(E \vee F) = P(E) + P(F)$.

4) $P(E\&F) = P(F\&E) = P(F)*P(E|F)$, where $P(E|F)$ is «the probability of event E, supposedly verified the event F».

All the other rules of the calculation of probabilities can be deduced from these through an exclusively logical-mathematical reasoning[13].

There are different points of view on what the statements of probability mean and how their veracity can be found. Limited to the needs of this discussion, we will consider probability as the numerical measure of the possibility of an event taking place and therefore as something truthful, independent of any human judgment and constant from person to person.

Our interpretation of the probability of an event is based on the following result:

R_2: Let A be an event related to a well-defined set of conditions C. Suppose that A has a constant probability p of occurring in each trial of C and that it occurs μ times in ν trials of C, with $\nu > 1$. Let H_C is the set of all (physical or conceptual) trials of C.

Then, the relative frequency with which A occurs, μ/ν, converges to the probability p of A, i.e., it stabilizes around the p-value, and the approximation improves as the number ν of trials of C increases.

Proof: A theorem in probability theory, called the «Strong law of large numbers»[14], implies that, if the probability of event A occurring in each trial of C is constant and equal to p, then, with a probability of 1, the relative frequency of A, μ/ν, in the set H_C of all independent trials of C converges to p. The set H_C is thus, by definition, an infinite countable set, having assumed that $\nu > 1$. Worth noting is also that a probability of 1 does not necessarily mean that the event will happen, although it is clearly possible.

Now, if it were admissible that the relative frequency of A (in H_C) does not converge to p, then even "the relative frequency of A does not converge to p" would be a possible outcome of class H_C – understood as a single trial – and

[13] See M. D. RESNIK, *Choices*, University of Minnesota Press, Minneapolis, 1987, trad it. *Scelte*, franco muzzio & c. editore spa, 2003, pp. 76-89.

[14] See M. SPIEGEL – A. SRINIVASAN – J. SCHILLER, *Schaum's Outline of Theory and Problems of PROBABILITY AND STATISTICS*, The Mc Graw-Hill Companies Inc., New York 2000, pp. 80-88.

thus a random event with respect to that set of conditions, let us call it C', whose single realization consists in performing all the trials of C[15].

If it did, by $\mathbf{R_1}$, the relative frequency of the event A of C would simultaneously have to converge and not converge to p (in H_C), since a unique set, H_C, consisting of all trials of C is conceivable. A contradiction has been reached. The $\mathbf{R_2}$ claim is then true.

In summary: in a «long» trial sequence of C, in which the probability p of the event A of C always be the same, the relative frequency of A is about the probability p, the approximation improving as the number of trials (of C) increases. In other terms, if A and -A are two complementary events, then both A and -A are present in n_A and $n_{\bar{A}}$ "meta-real worlds", respectively, such that the ratio $n_A/(n_A + n_{\bar{A}})$ $[n_{\bar{A}} /(n_A + n_{\bar{A}})]$ is equal to the probability p [1-p] of the event A [-A]. In addition, the greater the number of trials of C that we record as having concretely occurred, which we subtract from the sum $n_A + n_{\bar{A}}$, the smaller ordinarily will be the absolute deviation between the relative frequency of A in these trials and its probability p of occurrence.

The $\mathbf{R_2}$ result, known as the statistical law of large numbers, summarizes our conception of probability - i.e., it stands for the meaning we attach to probability assessments - and thus allows us to apply probability calculus to practical cases. An interesting example of this can be found in the following:

Problem (of the certain winner). Let C be the set of conditions that define the following experiment: «Two players X and Y alternately roll a pair of unrigged dice. The first player to throw a sum of 7 wins and ends the game. We assume that the game is a priori unlimited». Show that:

i) A: "the game is not won by either player" is an impossible event with respect to C.

ii) The methodical choice a priori of infinite subsequences of repeatable trials of C does not change the probability of a specific event of C (*impossibility of a gambling system*).

Solution: Let $\hat{C}$ be the class of conditions defining the following experiment:

"X or Y throws the pair of unrigged dice from experiment C". The probability P(B) of the event B: "X or Y gets a seven in a double throw with unrigged dice" of $\hat{C}$, as it is easy to calculate, is equal to 1/6. If player X is the

[15] Note that the expression «the relative frequency of A does not converge to p», by construction, makes sense and it is likely to have a truth value if and only if it refers to the set of conditions C'. This expression, therefore, describes here an event that can legitimately be assumed not to be impossible, even though, by the "strong law of large numbers", it must be assigned a probability value of zero – zero probability, however, does not necessarily mean impossibility –.

first to throw, then X will win in one of the mutually exclusive cases listed below with their respective odds:

- X wins on the first toss. Probability = 1/6.
- X loses on the first roll, then Y loses, then X wins. Probability = =(5/6)*(5/6)*(1/6) = 25/216.
- X loses on the first toss, then Y loses, then X loses, then Y loses, then X wins.
Probability = (5/6)*(5/6)*(5/6)*(5/6)*(1/6) = 625/7776.
- ………………………………………………………………… ;
- ………………………………………………………………… .

The probability that X wins the game is then:

(1/6) +(5/6)*(5/6)*(1/6) +(5/6)*(5/6)*(5/6)*(5/6)*(1/6) +…= (1/6)*[1+ +$(5/6)^2$ +$(5/6)^4$ +…+$(5/6)^{2n}$ + +…] = (1/6)/{1-$(5/6)^2$} = 6/11, being the expression in square brackets, as known from the Mathematical Analysis, the development of a geometric series of common ratio $q = (5/6)^2$.

In like manner, assuming that Y is the second to throw, the probability that Y wins the game is equal to:

(5/6)*(1/6) +[$(5/6)^2$]*(5/6)*(1/6) +... = (5/6)*(1/6)*[1 +$(5/6)^2$ +$(5/6)^4$ +… + +$(5/6)^{2n}$ +…] = (5/6)*(6/11) = 5/11.

It is worth noting that the probability P(A) of a tie is equal to 1-[(6/11) + + (5/11)] = 0, but this data is not sufficient to say that event A (draw) is impossible, because events with a probability of zero are not necessarily impossible. Probability theory does not seem to be able to solve the problem of the certain winner placed above. It does not seem, but it is.

In this regard, we note that each trial of C, by construction, consists of a sequence of equiprobable trials of Ĉ[16]. Specifically, if X or Y were the winner of the game, there would be at least one realization of C consisting of a finite sequence of trials Ĉ – of at least one term if X (the first to throw) were the winner; or two if he had won Y – in which the event B has occurred once.

Moreover, if it were possible that no one between X and Y wins the game, then of course B might not occur for an entire infinite sequence of trials of Ĉ, that is, for a sequence whose terms form an equipotent set[17] to the set of all the trials of Ĉ. In this latter hypothesis, one could then not rule out the possibility that the event -B occurs in every single trial of Ĉ. From all this,

[16] That is to say that there is no reason, either factual or in principle, to believe that any of these trials (of Ĉ) is more likely than any other to be a constituent of a fixed trial of C. It follows that the randomness or certainty of event B is a sufficient condition for it to be possible for the game to end with a winner.

[17] Remember that two sets are said to be equipotent when their elements can be put in one-to-one correspondence, that is, if each element of the first can be associated with one and only one element of the second and vice versa.

because of $\mathbf{R_1}$, it easily follows that the game always has a winner, always a limit, if and only if B has occurred in at least one trial of $\hat{C}$.

So, by $\mathbf{R_1}$, if A were a certain or random event of C, then none of the possible (thinkable) repeatable trials of $\hat{C}$ could have resulted in a 7, i.e., in none of these trials could event B have occurred.

On the other hand, since $P(B) = 1/6 > 0$, there must be, by $\mathbf{R_2}$, at least ν trials of $\hat{C}$ in which event B occurred μ times, with $\mu/\nu = (1/6)$. Hence, if A were a random or certain event of C, there would be a clear contradiction.

A is thus an impossible event of C (thesis ***i***).

Similarly, if it were possible that the relative frequency of B in a whole infinite subsequence of trials of $\hat{C}$, selected regardless of their outcomes, gives a value greater or less than 1/6, then it would be possible that $F(B) \neq P(B) = 1/6$, where F(B) is the relative frequency of the event B in each sequence of all the trials of $\hat{C}$. But this would contradict the $\mathbf{R_2}$ result, according to which it must be $F(B) = P(B) = 1/6$.

So, it is impossible to find any pre-established function or rule that can select a priori, among all the repeatable trials of $\hat{C}$, an infinite subsequence in which, as regards the total number of its elements, the frequency of B is not equal to its probability $P(B) = 1/6$ (thesis ***ii***).[18]

4 Conclusions

We have achieved our goal, which was to show how randomness can be truthfully characterized, that is, marked by an ontological objectivity. By recognizing chance in the reality, we have also proved the rationality of the statistical law of large numbers, thereby giving an example of how determinism can appear from underlying indeterminism.

Our ignorance of things, from which the chance arises, therefore consists not so much in the unknowability of such things in their entirety as in the delineation of a precise state of reality constituted by the conjunction between everything that is seen and the wealth of knowledge that one owns on what is seen. It follows that there cannot be any kind of partial knowledge, that there is no reality without knowledge, that the complete annihilation of our knowledge entails (our) true non-existence. If, on the other hand, we were to gradually eliminate all phenomena that we study or could study – cease to be occupied with that matter, cease to hear that sound…–, everything that is or could be the object of our knowledge, with the exeption of itself, we would at some point arrive at a theoretical state of appearing, at that which is not but

[18] See R. D'AMICO, *Chance and The Statistical Law of Large Numbers,* in Journal of Mathematical Economics and Finance, [S.l.], v. 7, n. 2, p. 41-53, dec. 2021. ISSN 2458-0813. https://doi.org/10.14505/jmef.v7.2(13).03.

legitimately aims to be, at the mediator between the non-real and the real: the possible, the certain or random event.

Reality therefore depends on our knowledge. The fact that mathematical knowledge is the certain and rigorous form of knowledge par excellence would thus seem to explain why mathematics is effective in physics, why its precise language is the most suitable for describing reality. Artificial intelligence (AI), by collecting and analysing data and identifying patterns, can help us improve our knowledge, but it cannot replace us.

Hence, the randomness of phenomena is a feature of reality and probability is an intrinsic aspect to reality itself and it is no longer an instrument at the service of an imperfect knowledge. Possibility and probability thus become two notions both endowed with that empirical as well as truthful character that is claimed by the natural sciences.

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