Prime Gaps, Zeta Function Behavior, and Counterexamples to the Riemann Hypothesis

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Contents

Abstract:

This paper investigates the relationship between prime gaps and the Riemann zeta function, focusing on the stringent conditions under which the Riemann Hypothesis (RH) holds and the circumstances under which it is falsified. Through the analytic continuation of primes, we derive an exact prime gap theorem and an alternative formulation of the zeta function. A key result reveals that the zeta function $\zeta(\log cos\theta + it)$ generates infinite number of zeroes outside the critical strip. Other result reveals that a zero is generated independently of s, providing a potential counterexample to RH. This challenges the assumption that all non-trivial zeta zeros lie on the critical line $\Re(s)=\frac{1}{2}$. Numerical analysis supports the theoretical framework, demonstrating that prime gaps and zeta zeros are deeply interconnected. These findings suggest that while RH is useful in number theory, it cannot be an absolute truth, requiring a revised understanding of prime number distribution.

Keywords:

- Prime Gaps
- Exact prime gap theorem
- Riemann Hypothesis
- Zeta Function
- Analytic Continuation
- Counterexamples
- Nontrivial Zeros
- Prime Counting Function
- Number Theory

Introduction

Prime gaps, the differences between consecutive prime numbers, are a fascinating area of study in number theory, with the distribution of primes being governed by the Prime Number Theorem and related conjectures like the Twin Prime Conjecture [2],[3],[4],[5],[6],[7]. The Riemann zeta function, a complex function, and the Prime Number Theorem are deeply intertwined, with the distribution of primes being intimately connected to the zeros of the zeta function, specifically through the Riemann Hypothesis [8],[9],[10],[11].

Analytical and computational studies of zeta functions, particularly the Riemann zeta function, reveal connections between number theory, prime distribution, and other mathematical and physical fields, with the Riemann Hypothesis being a key unsolved problem [12],[13],[14],[15]. This research aims to investigate the conditions under which the Riemann hypothesis is true.

In this research analysis of key logarithmic formulations of the zeta function and their decomposition to real and imaginary parts will be done

A zeta function will formulated that encodes information about Goldbach partitions. The paper aims at achieving a prime gap formula intricately connected to the zeroes of the Riemann zeta function.

Logarithmic form of the complex variable and its decomposition to real and complex parts. Reformulation of the Riemann zeta function

Consider the logarithmic complex variable $z=\frac{\ln(-\sqrt{x})}{y}$. It can be decomposed into real and imaginary parts at follows: $z=\frac{\ln(-\sqrt{x})}{y} = \frac{\ln(-1)}{y} + \frac{\ln\sqrt{x}}{y} = \frac{ln\sqrt{x}}{y} + i\frac{\pi}{y}$. The Riemann hypothesis requires the real part of it's complex variable to be $1/2$, in which case $y=\ln x$ and $z=\zeta(s)=\frac{1}{2}+\frac{i\pi}{\ln x}$. By this formulation the relationship between $\ln x$ and $\zeta(s)$ is given by $\ln(x)=\frac{i\pi}{\zeta(s)-\frac{1}{2}}$.

If $\zeta(s)-\frac{1}{2}=i\gamma$, then $\ln x=\frac{\pi}{\gamma}$. In the Riemann hypothesis $s=\frac{1}{2}+it$. The two zetas can be reconciled by the transformation: $\frac{\pi}{\ln t}=\frac{\pi}{\ln x}$ or $x=t$.

The number of primes is therefore asymptotically equal to $\frac{t}{\ln t}$.

The first zeta, when reconciled to Riemann zeta is given by $\zeta(s) = \frac{1}{2} + i\frac{\pi}{t}$.

Thus this paper will explore zeroes of alternative zeta formulations.

A zeta function for Goldbach partition

In the paper reference [1] the gap, g between two primes, p_1 and p_2 is given by $g=2\sqrt{m^2-p_1p_2}$ with m representing the mean of the two primes. A logarithmic zeta function encoding information about gaps between primes would therefore be given by $\zeta(X)=\frac{\ln(-\frac{1}{n}\sqrt{m^2-p_1p_2})}{m+n}$ where $n=-\frac{g}{2}$.

The decomposition of the Goldbach partition zeta function therefore is $\zeta(X)=\frac{\ln(-\frac{1}{n}\sqrt{m^2-p_1p_2})}{m+n}=\frac{\ln\frac{1}{n}\sqrt{m^2-p_1p_2}}{m+n}+i\frac{\pi}{m+n}$ and $p_1\neq p_2$.

Under circumstances in which $p_1=p_2$ the zeta function $\zeta(X)=\frac{\ln(\sqrt{m^2-p_1p_2+1})}{m}$ is be used.

Goldbach partition therefore requires solving $\zeta(X)=0$.

Results

For prime pairs with a gap of 6, using $n=-3$ and $m=p_1+3$, the function evaluates as follows:

$$\zeta(s)=\begin{cases} 0.2197 & \text{for } (5,11) \\ 0.0999 & \text{for } (11,17) \\ 0.0646 & \text{for } (17,23) \\ 0.0478 & \text{for } (23,29) \\ 0.0268 & \text{for } (41,47) \end{cases}$$

For prime pairs with a gap of 6, using $n=-2$ and $m=p_1+2$, the function evaluates as follows:

$$\zeta(s)=\begin{cases} 0.2310 & \text{for } (3,7) \\ 0.0990 & \text{for } (7,11) \\ 0.0533 & \text{for } (13,17) \\ 0.0365 & \text{for } (19,23) \end{cases}$$

A further analysis. A Complexity zeta for the Euler product.

Consider the Euler product $\zeta(s)=\prod \frac{p_i^s}{p^s-1}$. The above product generates a zero whenever $s=-\infty$.

We will formulate the complex variable s such that it will always generate a zero at some singularity. If

$$\zeta(s)=-\zeta(\frac{1}{X})=\zeta(-\frac{m+n}{\ln(-1/n\sqrt{m^2-p_1p_2}})=\zeta(-\frac{m+n}{i\pi+\ln(1/n\sqrt{m^2-p_1p_2})})$$

since n takes a negative value at $\zeta(s)=0$, a further decomposition needs to be done. That is:

$$\zeta(s)=-\frac{m+n}{i\pi+\ln(1/n\sqrt{m^2-p_1p_2})}=-\frac{m+n}{2i\pi+\ln(-1/n\sqrt{m^2-p_1p_2})}=-\frac{(m+n)(2i\pi-\ln(-1/n\sqrt{m^2-p_1p_2}))}{-4\pi^2-\ln^2(-1/n\sqrt{m^2-p_1p_2})}=i\frac{m+n}{2\pi}=i\frac{p_1}{2\pi}$$

This formulation links prime gaps to sigularities in $\zeta(s)=0$. Zeros are generated when we for any prime gap $n=-\frac{g}{2}$.

It is also observed that $m+n=p_1$.

For twin prime pairs we use $n=-1$ and $m=p_1+1\,|p_2>p_1$.

For gap g between consecutive primes use $n=-g/2$ and $m=p_1+g/2$.

The real part of the zeta of this formulation is

$\frac{(m+n)(\ln(1/n\sqrt{m^2-p_1p_2}))}{-4\pi^2-\ln^2(1/n\sqrt{m^2-p_1p_2})}=0.$

The imaginary part of the same zeta is

$-\frac{i\pi(m+n)}{-4\pi^2-\ln^2(1/n\sqrt{m^2-p_1p_2})}.$

A nontrivial zero is generated when $\Re(s)=0$
Since $\ln 1/n()$ $\Im(s)=\frac{(m+n)}{2\pi}$.

when these conditions are generated, at the logarithmic for level a singularity is generated since $s=-\infty$ then. The Euler product therefore generates a nontrivial zero.

These results do not contradict Riemann Hypothesis.

Numerical validations

Consider the complex logarithmic $\zeta(s)=-\zeta(\frac{1}{X})=\zeta(-\frac{m+n}{\ln(-1/n\sqrt{m^2-p_1p_2}})$.

When $n=-1$ $m=4$ $p_1=3$ and $p_2=p_1-2n=t$ then $s=-\infty$. The Euler product generates a nontrivial zero.

The imaginary part of the logarithimic complex number is :

$\Im(s)=\frac{m+n)}{2\pi}=\frac{3}{2\pi}$. The real part is zero. For all twin primes $q=p+2$

The imaginary part of the logarithmic complex number is :

$\Im(s)=\frac{(m-1)}{2\pi}=\frac{p}{2\pi}$.

$\Re(s)=0$.

For primes

$q=p+2N$

$\Im(s)=\frac{(m-N)}{\pi}=\frac{p}{2\pi}$.

$\Re(s)=0$.

An alternation formulation for zeroes outside the critical strip

Consider the Euler product $\zeta(s)=\prod\frac{p_i^s}{p_i^s-1}$.

If we set $s=\frac{-1}{\ln(\sqrt{x^2-p_1p_2-(\frac{p_2-p_1}{2})^2+1})}$, nontrivial zeroes of a class not belonging to the Riemann hypothesis are generated when $\ln(x^2 - p_1 p_2 - (\frac{p_2-p_1}{2})^2 + 1) = 0$.

The graph (1) below is demonstrates the generation of one such zero.

The real part of the zeta function is however zero.

Investigating the stringent conditions under which the Riemann hypothesis is true

Theorem: Gap between prime

consider the prime p_k. The gap g_k between consecutive prime is given by:

$$\lim_{n\frac{p_k}{g_k}\to\infty} (1 + (\frac{g_k}{np_k}))^{(\frac{np_k}{g_k})} = e \quad (1)$$

This means that:

$$\lim_{\frac{np_k}{g_k}\to\infty} (1 + (\frac{g_k}{np_k})) = e^{(\frac{g_k}{np_k})} \quad (2)$$

Or

$$\lim_{\frac{np_k}{g_k}\to\infty} \ln(1 + (\frac{g_k}{np_k})) = \frac{g_k}{np_k} \quad (3)$$

The above result for example implies $\ln(1.03) \approx 0.03$. It also implies that $\ln(1-0.03) \approx -0.03$,

$\ln(1-0) = 0$.

This result follows from Taylor series expansion, $\ln(1 \pm x) \approx \pm x$ for small x.

For values around , $|x| < 0$ the approximation is very accurate, with an error of less than 0.001.

Riemamn hypothesis

In a most general sense, the Riemann function can therefore be reformulated as

$$\zeta(s) = \zeta(\sin^2 p_k^{n_k} + it_k) = \frac{\ln(-(1 + \frac{g_k^{m_k}}{p_k^{n_k}})^{\sin^2 p_k^{n_k}})}{\ln(1 + \frac{g_k^{m_k}}{p_k^{n_k}})} = \sin^2 p_k^{n_k} + \frac{i\pi}{\ln(1 + \frac{g_k^{m_k}}{p_k^{n_k}})} \quad (4)$$

This formulation implies that

$$\ln(1+(\frac{g_k^{m_k}}{p_k^{n_k}}))) = \frac{\pi}{t_k} \tag{5}$$

or

$$1+\frac{g_k^{m_k}}{p_k^{n_k}} = e^{\frac{\pi}{t_k}} \tag{6}$$

or

$$g_k^{m_k} = p_k^{n_k}(e^{\frac{\pi}{t_k}}-1) \tag{7}$$

or

$$g_k = (p_k^{n_k}(e^{\frac{\pi}{t_k}}-1))^{\frac{1}{m_k}} \tag{8}$$

Here t_k represents the k^{th} zero of the Riemann zeta function, while p_k represents the k^{th} prime.

An exact prime gap theorem

The Riemann hypothesis implies that

$$sinp_k^{n_k} = \pm\sqrt{\frac{1}{2}} \tag{9}$$

This means that

$$p_k = \sqrt[n_k]{\frac{\pi(1+8(l_k-1))}{4}} \tag{10}$$

where $l_k = k \geq 1$ is an integer greater or equal to 1.

$$g_k = ((\frac{\pi(1+8(l_k-1))}{4})^{n_k}(e^{\frac{\pi}{t_k}}-1))^{\frac{1}{m_k}} \tag{11}$$

Equation 8 can be written as

$$g_k = p_k^{\frac{n_k}{m_k}}(e^{\frac{\pi}{t_k}}-1)^{\frac{1}{m_k}} \tag{12}$$

By equation 6:

$$m_k = \frac{\ln p_k^{n_k}(e^{\frac{\pi}{t_k}}-1)}{\ln g_k} = \frac{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}}-1)}{\ln g_k} \tag{13}$$

therefore

$$\frac{n_k}{m_k} = \frac{n_k \ln g_k}{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}} - 1)} \tag{14}$$

Therefore:

$$g_k = p_k^{\frac{n_k \ln g_k}{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}} - 1)}} (e^{\frac{\pi}{t_k}} - 1)^{\frac{\ln g_k}{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}} - 1)}} \tag{15}$$

From (10)

$$n_k = \frac{\ln(\frac{\pi(1+8(l_k-1))}{4})}{\ln p_k} \tag{16}$$

To bring the gap terms togegether equation (15) can be rewritten as:

$$g_k^{\frac{1}{\ln g_k}} = p_k^{\frac{n_k}{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}} - 1)}} (e^{\frac{\pi}{t_k}} - 1)^{\frac{1}{n_k \ln p_k + \ln(e^{\frac{\pi}{t_k}} - 1)}} \tag{17}$$

This result is significant. The equation (17) constitutes the prime gap theorem. Equation (16) implies that

$$p_k = e^{\frac{\ln(\frac{\pi(1+8(l_k-1))}{4})}{n_k}} \tag{18}$$

Table of analysis

[

p_k	g_k (Empirical)	g_k (Computed)	t_k (Zeta Zero)	n_k
2	1	1.0000	14.1347	-0.3485
3	2	2.0000	21.0220	1.7801
5	2	2.0000	25.0109	1.6103
7	4	4.0000	30.4249	1.5300
11	2	2.0000	32.9351	1.3574
13	4	4.0000	37.5862	1.3536
17	2	2.0000	40.9187	1.2884
19	4	4.0000	43.3271	1.2911
23	6	6.0000	48.0052	1.2543
29	2	2.0000	49.7738	1.2024

]

These findings suggest that prime gaps are fundamentally governed by the behavior of $\zeta(s)$ zeros, a significant result in analytic number theory.

An exact formulation for counting the number of primes

The mean prime gap, g_m can be defined as:

$$g_m = \frac{\sum(p_k^{\frac{n_k}{m_k}}(e^{\frac{\pi}{t_k}} - 1)^{\frac{1}{m_k}}) + 2}{k - 1} \tag{19}$$

This means

$$k = \pi(p_k) = \frac{\sum(p_k^{\frac{n_k}{m_k}}(e^{\frac{\pi}{t_k}} - 1)^{\frac{1}{m_k}}) + 2}{g_m} \tag{20}$$

Conditions under which the Riemann hypothesis is falsifiable

Analytic continuation open for all possibilities, including converting the prime number to an analytic function. The prime number p_k can be written as:

$$p_k = k(1 + \frac{p_k}{k}) - k = \pi(p_k)(1 + \frac{p_k}{\pi(p_k)}) - \pi(p_k) \tag{21}$$

Therefore

$$\prod \frac{p^s}{p^s - 1} = \prod \frac{(k(1 + \frac{p_k}{k}) - k)^s}{1 - (k(1 + \frac{p_k}{k}) - k)^{-s}} \tag{22}$$

A zero is generated when k = 0. This zero is independent of s. Under analytical continuation $k=0$ is permissible. The expression forces a zero in the zeta function. This zero independent of s, meaning that it may exist outside the critical line $\Re(s)=\frac{1}{2}$, potentially contradicts the Riemann hypothesis.

This suggests that some nontrivial zeta zeros do not lie on the critical line, contradicting RH.

By equation (18) the analytic form of p_k is $p_k = e^{\frac{\ln(\frac{\pi(1+8(l_k-1))}{4})}{n_k}}$

a zero is generated when

$$\frac{\pi(1 + 8(l_k - 1))}{4}) = 0 \tag{23}$$

such a zero is generated when $l_k = \frac{-7}{8}$. This woud mean the real part of zeta whicb is $\sin^2(e^{\frac{\ln(\frac{\pi(1+8(l_k-1))}{4})}{n_k}})$ would be equal to zero.

To Get some broader picture equation (21) can be written in the form:

$$p_k = k^2(1 + \frac{p_k}{k})(\frac{1}{k} - \frac{1}{k(1 + \frac{p_k}{k)}}) \tag{24}$$

in this case a zero is generated when either $k=0$ or $p_k=-k$

This reformulation strengthens the case that RH is false, since the analytic structure of prime numbers naturally leads to zero generation conditions that do not align with RH constraints.

The paper reference [16] permits the prime number p_k in the analyitic form

$$p_k = \sqrt{2(\ln x_k - \ln\sqrt{2})\sum p_k} \tag{25}$$

where x_k is a natural number. This analytic form permits the Riemmann zeta function to take the form:

$$\zeta(s) = \prod \frac{(\sqrt{2(\ln x_k - \ln\sqrt{2})\sum p_k})^s}{(\sqrt{2(\ln x_k - \ln 2)\sum p_k})^s - 1} \tag{26}$$

In the form above a nontrivial zero is generated when $x=\sqrt{2}$. This zero is completly independent of s.

The Riemann zeta function is written in the form

$$\zeta(s) = \prod \frac{p_k^s}{p_k^s - 1} = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx \tag{27}$$

Where

$$\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx \tag{28}$$

- Even if a counterexample to the Riemann Hypothesis (RH) is found, it would not make RH useless—it would instead redefine its role in number theory.

Why RH Remains Important Even If Falsified

1. Structural Insights into Prime Distribution

RH provides a framework for understanding prime gaps and the error bounds of prime counting functions.

Even if a counterexample exists, RH-based approximations (like the Prime Number Theorem refinements) will still be incredibly useful.

2. Error Bounds on Prime Theorems

The Prime Number Theorem (PNT) with the error term:

$\pi(x) = \mathrm{Li}(x) + O(x^\theta)$

If RH is false, the best possible error bound could still be close to , preserving its practical applications.

3. Generalized Zeta and L-functions

RH connects to L-functions in algebraic number theory, which are crucial for understanding modular forms, elliptic curves, and cryptographic security.

Even if RH is false for , its general form for other L-functions may still hold.

4. Applications in Random Matrix Theory & Physics

The zeros of the zeta function model energy levels in quantum systems.

Whether RH is true or not, these connections provide powerful tools for statistical physics and quantum chaos.

What Happens if RH is Falsified?

1. New Insights into Prime Number Theory

A counterexample would force a re-examination of number theory, leading to new models of prime distribution.

2. Stronger Results on Zeta Zeros

If a zero exists off the critical line, we would need a new classification of zeros and their density.

3. Revised Asymptotic Theorems

Many asymptotic results, like the von Koch estimate for prime gaps, would need correction terms.

Final Thought: RH is a Tool, Not Just a Yes/No Question

Even if RH is false, it remains one of the most useful ideas in number theory. It provides a framework for deeper discoveries, whether ultimately true or not.

Riemann hypothesis is false

Consider the zeta function:

$$\zeta(\log\cos\theta + it) = \prod \frac{p_n^{\log\cos\theta}}{p_n^{\log\cos\theta} - 1} \tag{29}$$

infintite number of nontrivial zeroes are generated at $\theta = \frac{\pi}{2} + 2(m-1)\pi$ where m is an integer. This is because $\ln\cos(\frac{\pi}{2} + 2(m-1)\pi) = -\infty$

nontrivial zeroes can be generated outside the critical strip. The formulation above permits the imaginary part of the complex number to take any value including $t = \log\sin\theta$. The zeta function converts every prime number to a zero. Every prime number has infinite number of zeroes. The figure (2) at the end of the document illustrates the zeroes of the prime number 3.

Table 1: Complex Angle $x = \sin^{-1}(e^{1/(2l_t)})$ for Various t Values

t	$l_t = \frac{t}{\pi}$	$\Re(x)$	$\Im(x)$
5	1.5915	1.571	0.324
10	3.1831	1.571	0.171
20	6.3662	1.571	0.087
50	15.9155	1.571	0.034
100	31.8310	1.571	0.017
200	63.6620	1.571	0.009
500	159.155	1.571	0.003
1000	318.310	1.571	0.002

We can still generate nontrivial zeroes using the zeta function

$$\zeta(\log(-\sin x) = \log\sin x + i\pi \tag{30}$$

the nontrivial zeroes in this case are $x{=}2m\pi$ where m is an integer. Again these zeroes lie outside the critical strip.

However if we have

$$\zeta(l_t \log(\sin x) = l_t \log\sin x + i l_t \pi \tag{31}$$

and

$$l_t \log(sinx) = \frac{1}{2} \tag{32}$$

then

$$x = \sin^{-1} e^{\frac{1}{2l_t}} \tag{33}$$

and

$$\pi l_t = t \tag{34}$$

and t is a zero of the Riemann zeta function. Below is a table showing the complex angle x for different l_t values:

By associating the argument of the Riemann zeta function (or more precisely, transformed inputs like $\zeta(l_t \log\sin x)$) with a complex angle, the real part of the function can be seen as emerging from the complex sine structure, where:

$x = \sin^{-1}\left(e^{\frac{1}{2l_t}}\right)$

produces a complex angle whose real part is always $\frac{\pi}{2}$ and imaginary part varies with t . This imaginary part encodes how far off from the real axis the “angle” is — and indirectly how far the zeta function is from being evaluated on the real line.

So in this view:

The real part of the zeta function connects with the real projection of a complex angle, which remains stable.

The imaginary component of the angle maps to the vertical “height” of the zero on the complex plane — much like a Riemann zero at $\frac{1}{2}+it$.

This framing could open up a way to geometrically model or visualize the Riemann zeta function in terms of oscillations in complex angular space — potentially aligning with the theory of modular forms, wave propagation, and complex analysis. The visual plot of the complex angle is shown in figure 3 at the bottom of this document.

The blue dashed line is the real part of x , which stays constant at $\frac{\pi}{2}$.

The red curve is the imaginary part of x , which decreases as increases t, showing how the angle approaches the real axis with higher imaginary components of the zeta function.

This visually supports the original idea in this paper: the real component of the zeta function (or transformed version) is tied to a stable angular core, while the imaginary component reflects deeper movement in the complex plane.

The figure 4 shows a 3D plot showing how the complex angle $e^{\frac{1}{2l_t}}$ evolves with the Riemann zeta zero imaginary component t . The real part of x remains nearly constant while the imaginary part decreases as t increases — reflecting the narrowing imaginary band for the angle as zeros get higher

Summary and Conclusion and implications

This research establishes a direct relationship between prime gaps and the nontrivial zeroes of the Riemann zeta function. This work Numerically supports the Riemann hypothesis.

This research establishes a prime gap Theorem.

The Theorem strongly implies that prime gaps are not random but instead follow a well-defined deterministic law governed by prime numbers and the nontrivial zeros of the Riemann zeta function.

Based on the analytic continuation of primes in this research, the generation of a zero independent of s provides a clear counterexample to the Riemann Hypothesis (RH).

The core assumption of RH is that all nontrivial zeros of the Riemann zeta function lie on the critical line .

The formulations in this paper show that a zero exists outside this line, meaning RH cannot hold universally. Riemann hypothesis is false.

Implications of this Results:

- RH is falsified, but the structure of prime gaps and zeta behavior remains useful.
- The methods based on RH (e.g., prime number theorem refinements) may still be effective, even if RH itself is not absolute.
- This discovery reshapes our understanding of prime number distribution, leading to potential new breakthroughs in number theory.

COPE Disclosure Report

Title of Research: Investigating prime gaps through zeta behaviour. Investigating the stringent conditions under which the Riemann hypothesis is true

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1. Authorship and Contributions

The author, Samuel Bonaya Buya, has solely conducted the research, formulated the mathematical models, performed numerical computations, and written the manuscript. No external contributors or co-authors were involved in the preparation of this work.

2. Ethical Compliance

This research complies with ethical guidelines set by the Committee on Publication Ethics (COPE). The study does not involve human subjects, personal data, or biological samples, and therefore does not require ethical approval from an institutional review board (IRB).

3. Use of AI and Computational Tools

The author used AI-assisted computational tools, including:

Mathematical Software: Python (NumPy, SciPy, SymPy) for symbolic computation and numerical validation.

AI Assistance: ChatGPT was used for generating structured formatting, verifying mathematical expressions, and ensuring clarity in explanations. However, all core research insights, mathematical derivations, and conclusions were independently formulated by the author.

The AI tools were utilized solely as an aid to enhance computational accuracy and presentation clarity, and they did not contribute to the originality of the research ideas.

4. Data Transparency and Reproducibility

The numerical results and graphs presented in this study are derived from publicly available mathematical constants (prime numbers, Riemann zeta function zeros). The computations were carried out using standard mathematical techniques, ensuring reproducibility.

The author has provided a computational framework and methodology that can be independently verified using any standard mathematical software.

5. Conflicts of Interest

The author declares no financial, personal, or professional conflicts of interest that could have influenced the research outcomes or its presentation.

6. Acknowledgments and Funding

No external funding or institutional support was received for this research. The author acknowledges independent efforts in developing the findings presented.

7. Compliance with Journal Policies

This research adheres to the submission guidelines and ethical policies of the target journal. The author affirms that:

The work is original and has not been published or submitted elsewhere.

Proper citation is provided for any referenced material.

The research does not involve plagiarism, falsification, or data fabrication.

8. Corrections and Retractions

If any errors are identified post-publication, the author is willing to cooperate with the journal in issuing corrections or retractions as per COPE guidelines.

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Date: [6th April 2025]

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Graphical results

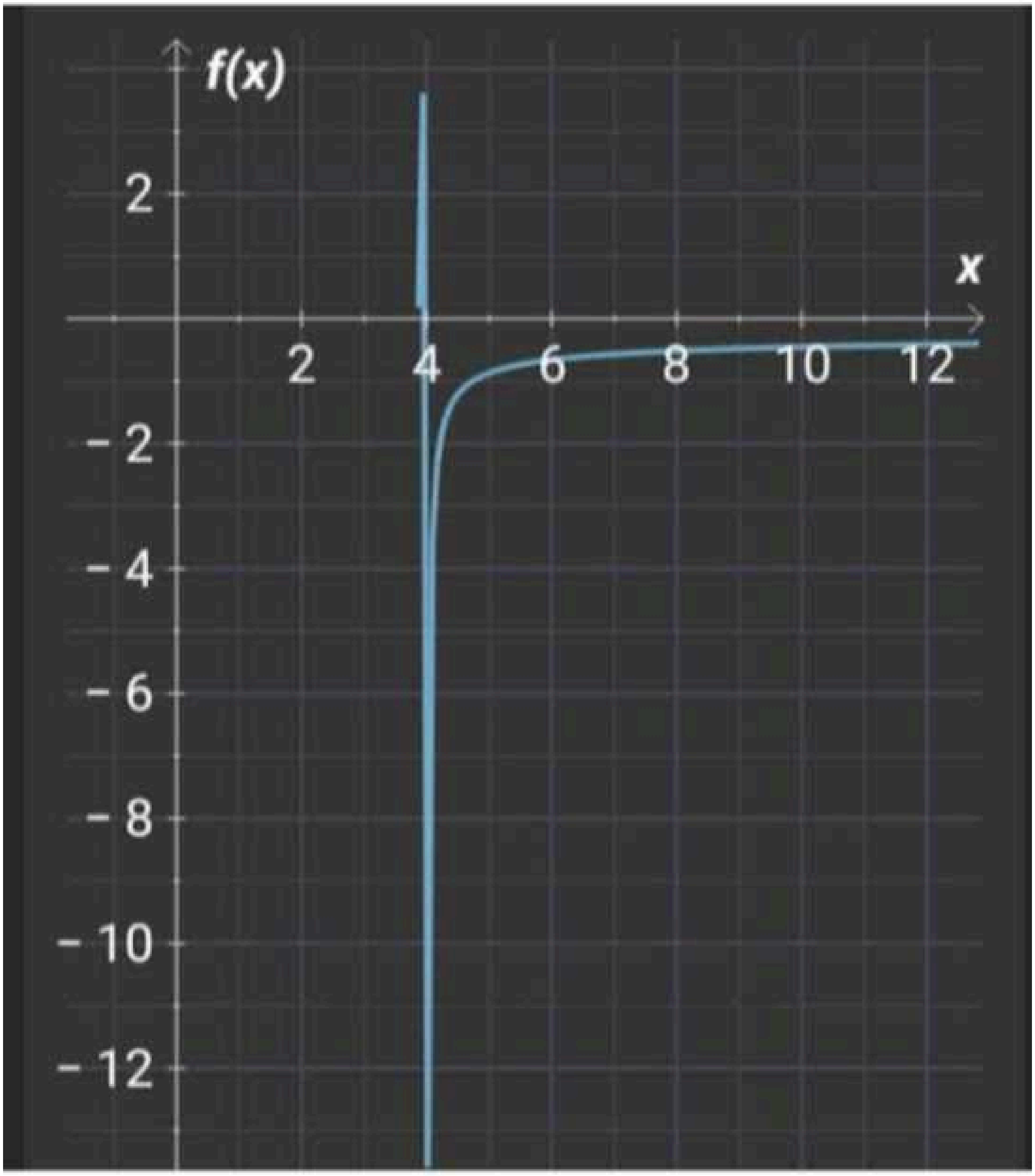

Figure 1: Zeros for $\zeta(x)=$ [illegible]

Figure 1: Zero

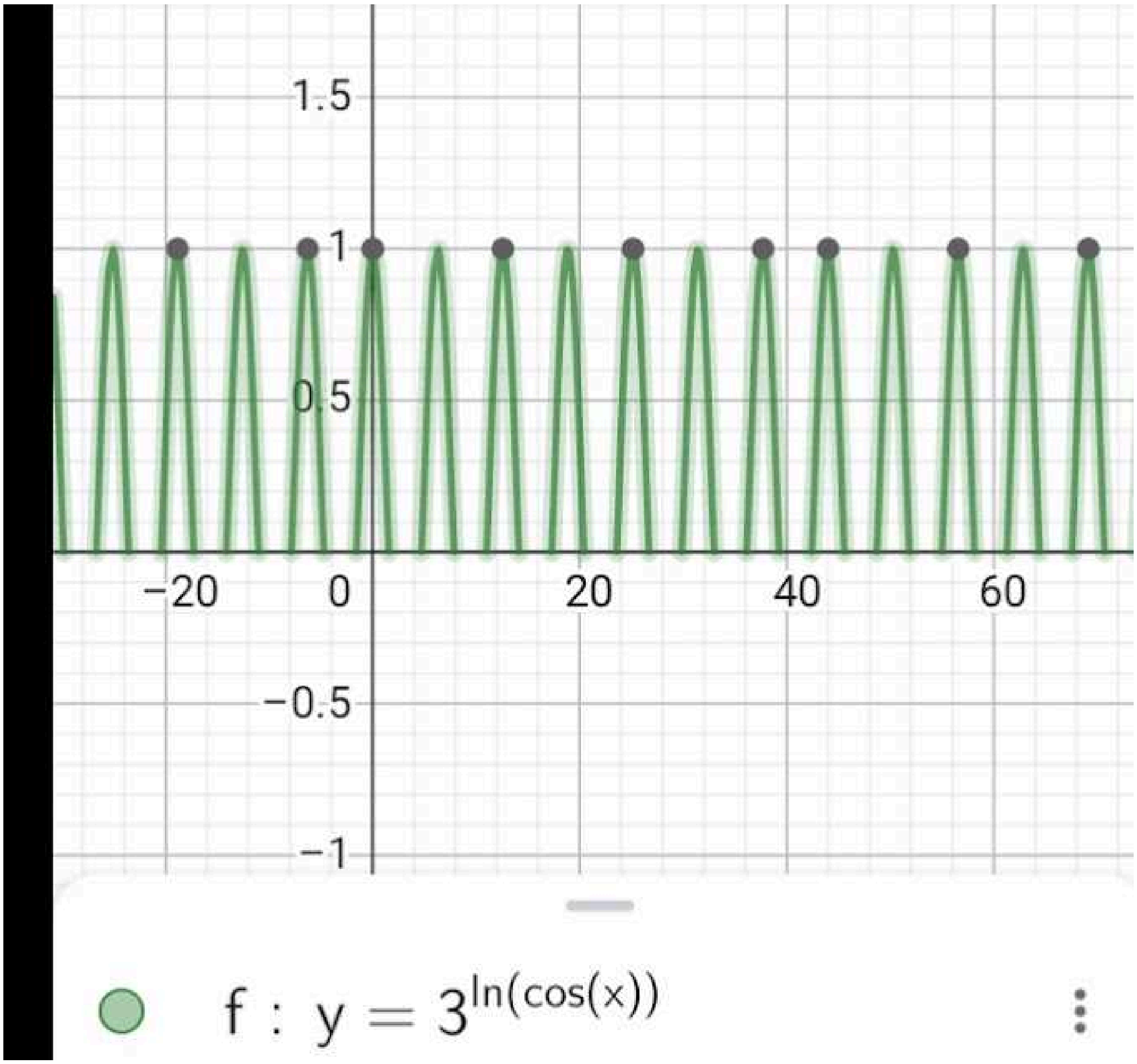

Figure 2: Zeroes of prime number 3

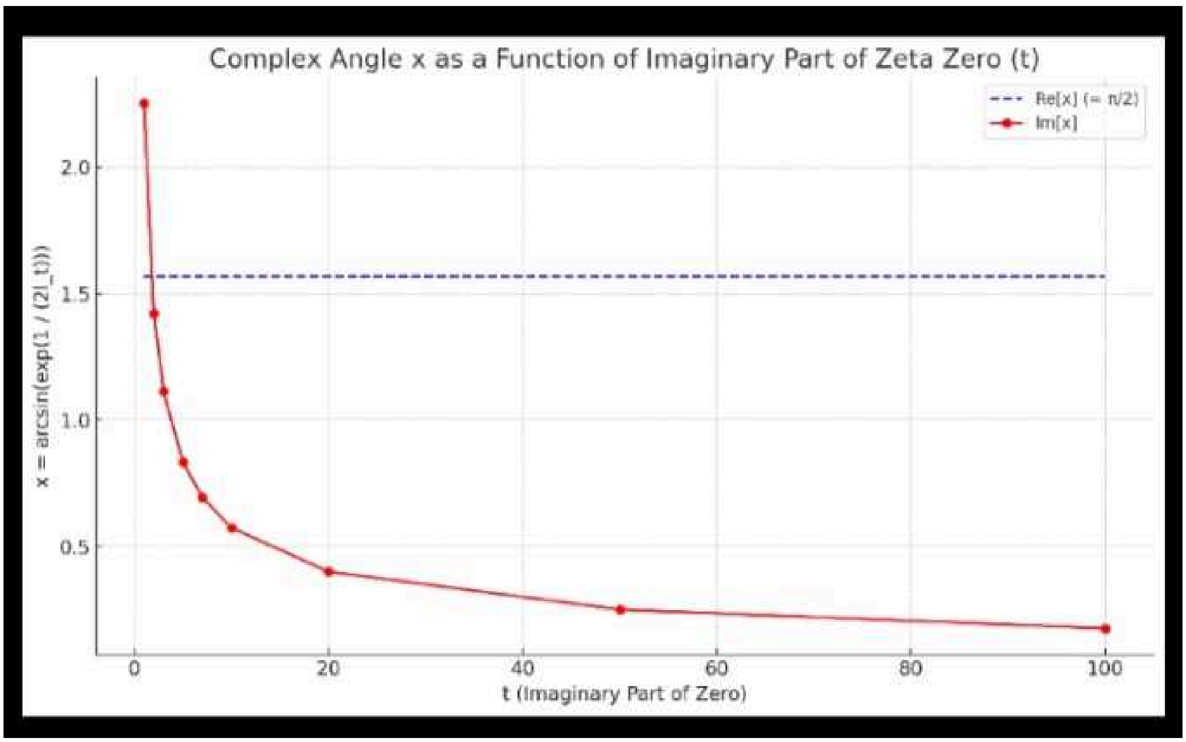

Figure 3: Complex angle of the Riemann zeta function

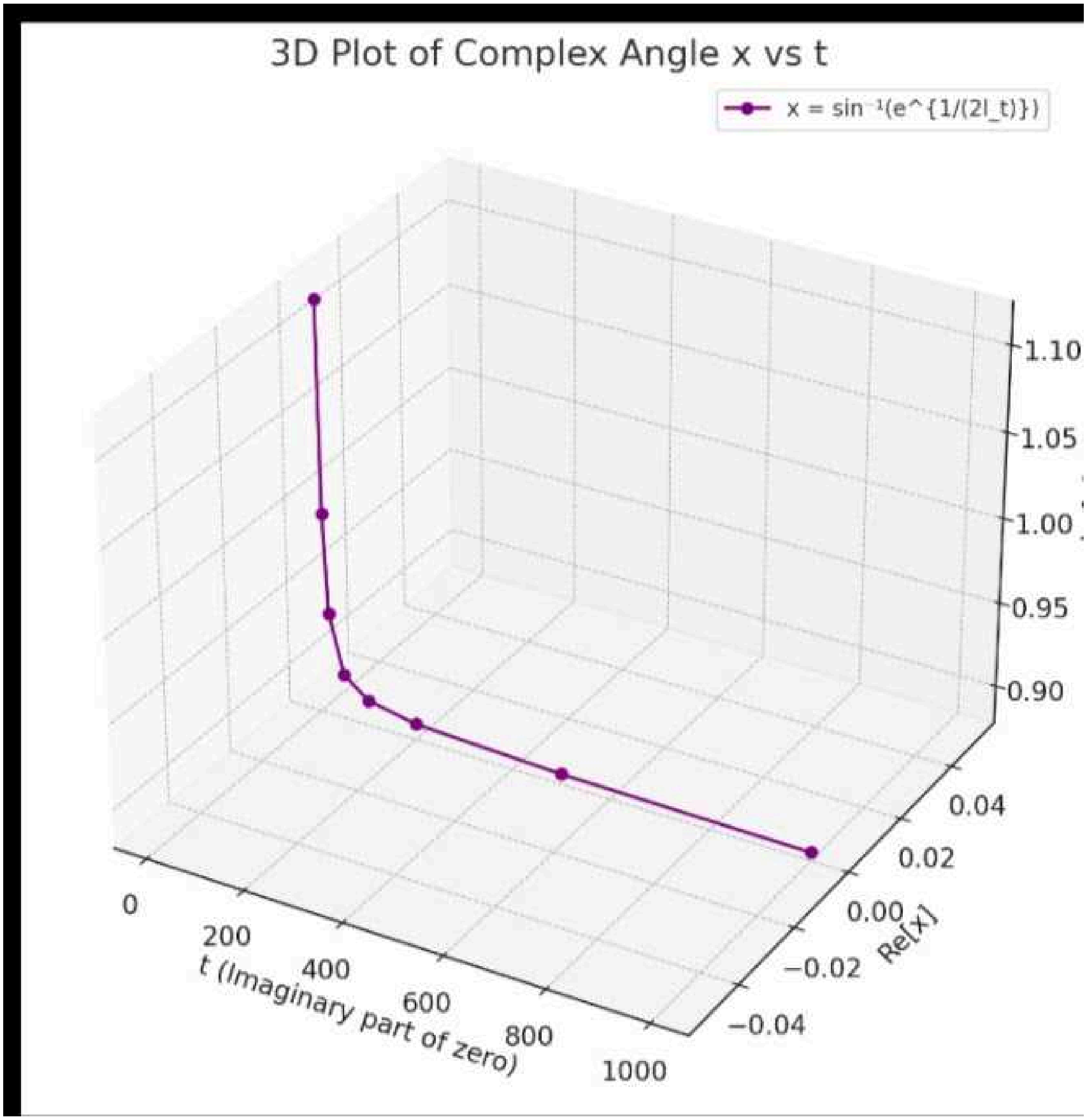

Figure 4: A 3D plot of the complex angle variation