Reversibility and Determinism in an Expanding Universe

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Abstract

A graph-theoretical toy model is presented that allows us to study the degrees of freedom in globally hyperbolic discrete causal structures governed by a deterministic local rule. We observe that expanding structures compared to structures static in sizes behave markedly different. In expanding causal structures, the degrees of freedom can not all be localised in the past, and it is not meaningful to treat these degrees of freedom as initial conditions. As a result, the past no longer determines the future and determinism loses its meaning. This invalidates the argument of free will being incompatible with deterministic laws.

1 Introduction

In his Nobel lecture Wigner draws our attention to the duality between laws of nature on the one hand and initial conditions on the other.[1] As Wigner remarks, since Newton's time, the former have become increasingly precise beyond anything reasonable; while we know virtually nothing about the latter. Yet, it is the combination of both that determines the behaviour of our universe. Wigner further states that, provided the laws of nature are complete, the set of initial conditions should take the shape of degrees of freedom that can be chosen arbitrarily. Thus we conclude that the description of the universe consists of laws of nature complemented with degrees of freedom acting in the past and referred to as 'ínitial conditions'.

This, however, begs the question why the degrees of freedom associate with the laws of nature should act solely in the past and not, for instance, partly in the past and partly in the future? At first sight, this might seem a strange question to ask. However if one zooms in on the mathematical and computational well-posedness of the development described by the laws of physics, the answer to this question no longer is obvious. In fact, as we will show, the same laws applied to a universe static in size and also to an expanding universe, require markedly different sets of degrees of freedom.

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Starting point is a graph-theoretical toy model for the universe that is build on a simple local rule for information processing. We observe that this toy model allows for a global causal structure satisfying the requirement generally referred to [2] as 'global hyperbolicity'. This global hyperbolicity manifests itself as an internally consistent flow of information emenating from degrees of freedom referred to as a Cauchy subset. This in turn gives rise to an emergent chronology. Investigating the degrees of freedom along with the chronology allows us to demonstrate 1) how irreversibility manifests itself in the model's information processing, and 2) that the degrees of freedom emerging act as 'forks in the causal path'.

It should be clear from the above that the toy model presented here does *not* aim to provide us with a realistic description of our universe. Rather, the model is intended to hone our intuition as it forces us to focus on the key relevant concepts - such as global hyperbolicity and Cauchy degrees of freedom - that provide a foundation for the emergence of irreversibility and non-determinism.

2 Toy model

Our toy model is graph-theoretical in nature. Basing our model on a graph structure allows us to implement a discrete causal structure without commiting to a specific spacetime geometry. In addition, the following three definitions ensure that the key mathematical concepts 'causal structure', 'local field equation', 'Cauchy surface' and 'global hyperbolicity' all carry over into this discrete model:

1. A *spacetime grap*h is a finite connected graph that contains solely degree-3 vertices ('bulk' vertices) and degree-1 vertices ('boundary' vertices), and that is equiped with a discrete scalar field: each vertex carries a value from the set of integers $\{0, 1, .., k-1\}$.

2. The vertex value allocation algorithm is *greedy* in nature and defines a causal structure: each bulk vertex with known value that comes with exactly two neighbors with known values *causes* the third neighbor to be assigned a value (Fig. 1b). This value assignment is such that a local field equation is obeyed: the value W of the bulk vertex, together with the values X , Y and Z of its three neighbors (Fig.1a) satisfies $f(W) + X + Y + Z \equiv 0 \ (mod\, k)$. Here $f(..)$ denotes an arbitrary mapping from the set $\{0, 1, .., k-1\}$ into itself.

3. A spacetime graph that contains a *Cauchy subset* is referred to as *globally hyperbolic*. Here, a Cauchy subset is a subset of the vertices in a spacetime graph such that when all of these Cauchy vertices carry arbitrary assigned values, iteratively applying the greedy vertex value allocation algorithm leads to a value assignment process in which ultimately all vertices receive a value in accordance with the field equation.

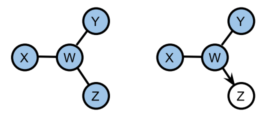

Fig 1a. Bulk (degree-3) vertex with its neighbors. Fig 1b. If a bulk vertex carries an assigned value, and so do precisely two of its neighbors, the value of the third neighbor (Z) can be computed greedily by invoking the field equation (Cf. definition 2). This value assignment process we will refer to as 'the local greedy rule'.

It should be clear that the specific choice of the function $f(..)$ in the local field equation is not in any way relevant for the causal structure that emerges. The only relevant aspect in this context is that a greedy algorithm operates that deterministically assigns values whenever a bulk vertex with known value comes with a single neighbor with undetermined value. To visualize the flow of information emerging from this greedy process (the 'direction of computation') in graph diagrams, we assign an arrow to the edge joining the central vertex used in the greedy rule to the single neighboring vertex that gets assigned a value, as in Fig.1b. By applying this arrow allocation throughout the greedy process, a causal pattern emerges that visualizes the direction of computation. In the following we will study the causal patterns emerging, and how these are related to the occurence of Cauchy subsets.

3 Cauchy subsets

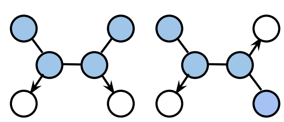

Fig 2. Spacetime graph with two bulk vertices. Using the field equation, with the values of four vertices being given, two boundary vertices (white nodes) can be assigned a value. Note that two distinct configurations of known values (blue nodes) both lead to consistent assignment of all six vertex values.

We get some key insights into the properties of Cauchy sets by considering minimalistic example spacetimes. Adding a second bulk vertex to the closed neighborhood shown in Fig.1, the spacetime graph shown in Fig. 2 results. We observe that two distinct configurations of four vertices with known values invariably lead to a consistent greedy assignment of values for all six vertices (as indicated by the causal arrows in Fig. 2).

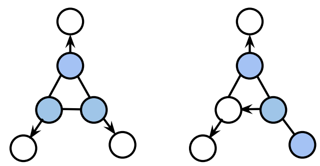

Fig 3. Spacetime graph with six vertices. With the values for three vertices being given (vertices with blue shading), the values for the other three vertices follows from iteratively applying the local update rule (represented by the arrows).

In Fig. 3 a spacetime graph with three bulk vertices and three boundary vertices is shown. Starting from specific configurations containing three vertices with assigned values (shaded in blue), by iteratively applying the greedy value allocation rule to the bulk vertices, the value of all six vertices can be determined. We conclude that the vertices shaded in blue in Figs. 2 and 3 we can refer to as *Cauchy subsets*, and therefore the spacetime gaphs in these two figure are *globally hyperbolic*. Also the spacetime graph with eight bulk vertices and four boumdary vertices shown in Fig. 4 is globally hyperbolic with multiple Cauchy subsets being present. We observe that without exception, in all these globally hyperbolic spacetime graphs the causal arrows trace out paths from the vertices in the Cauchy subsets to the boundary vertices.[1]

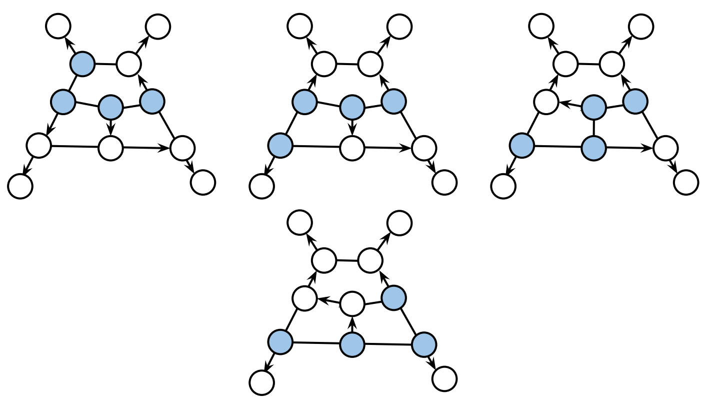

Fig 4. Cauchy subsets (shaded in blue) for a spacetime graph with twelve vertices.

From studies like these, we learn the following:

1. For a spacetime graph to contain a Cauchy subset, it must have at least three boundary vertices.

[1] Note that 'zero-length' paths do occur in case boundary vertices are present in the Cauchy subset.

2. If a spacetime graph admits a Cauchy subset, it must admit multiple of these.

3. A Cauchy subset of a spacetime graph with N boundary vertices contains exactly N vertices.

4. Not all vertices of a Cauchy subset can be located on boundary vertices.

5. For a given Cauchy subset, the causal arrow pattern generated is that of a set of directed causal paths from each Cauchy vertex to a distinct boundary vertex, such that no closed causal loops are created. Considering that the nodes neighboring a causal path all influence the values along that path, a closed causal loop is manifest whenever edges dressed with arrows in combination with single isolated edges between paths (or between distinct vertices of the same path) form a closed loop.

As should be clear from the above, a globally hyperbolic spacetime graph guarantees a deterministic process for assigning vertex values. More precisely, if all vertices in a Cauchy subset have a well-defined value, the vertices away from this Cauchy subset can all be assigned values based on a local greedy (computational complexity class P) algorithm. It might therefore be tempting to interpret each of the Cauchy subsets of a spacetime graph as a potential set of locations for the initial conditions to the development of the vertex values. However, such an interpretation would be misleading (or at least premature), as we have not identified a chronology in our model. Moreover, a specific Cauchy subset can not be seen as 'ultimate cause' for the development of vertex values across the full spacetime graph as multiple Cauchy subsets are always present, and we would need additional information to single out one of them as ultimate cause. An interpretation we can make however, is to interpret the vertex values in each Cauchy set as degrees of freedom associated with the deterministic dynamics. Note however, that the presence of multipole Cauchy susets prevents us from localizing these degrees of freedom.

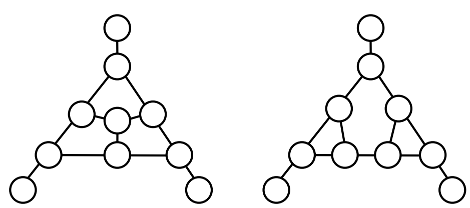

Fig. 5. Spacetime graphs that are not globally hyperbolic. No Cauchy subset can be identified: three or less nodes with assigned values don't lead to assignment of values for all nodes, four or more assigned values lead to conflicting value assignments.

Spacetime graphs that are not globally hyperbolic lack a deterministic process for assigning vertex values: no set of nodes can be identified such that

- when assigned arbitrary values - a unique developoment results in which all vertices get allocated a value. Either the value allocation process stalls before all vertices have been allocated a value, or conflicting value allocations happen. Examples of non-globally hyperbolic spacetime graphs are shown in Fig. 5.

4 Static vs expanding spacetimes

So far we have considered small spacetime graphs with few vertices. We now focus on spacetime graphs that are globally hyperbolic and that can systematically be extended to ever larger structures. Two distinct avenues for doing so open up. Shown in Fig. 6 are two globally hyperbolic spacetime graphs based on a honeycomb structure. Both graphs have six boundary vertices, and hence both come with Cauchy subsets consisting of six vertices. Moreover, both graphs allow for an embedding on a cylinder, i.e. for a representation on a cylinder that does not necessitate any edge crossings. As it happens, in both graphs the direction of computation obeyed by the local greedy update rule (indicated with arrowheads, as in Figs. 2 - 4) is upwards. The obvious distinction between the two graphs is that when we follow the direction of computation, the graph on the left-hand side does not change width, while the the graph on the right-hand side expands in the direction of computation. We will refer to spacetime graphs like the one on the left as *static* spacetime graphs, and those like the one on the right as *expanding* spacetime graphs.

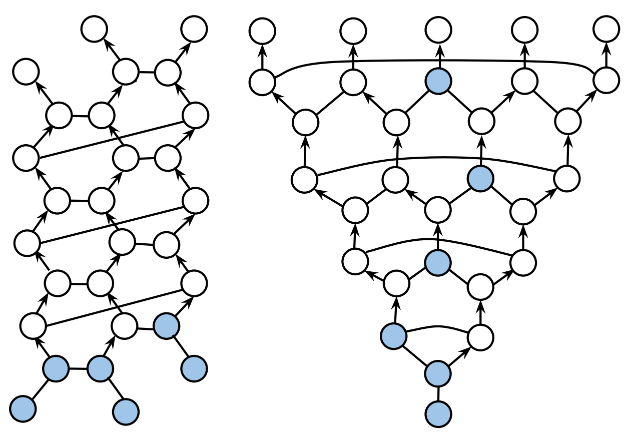

Fig 6. Globally hyperbolic spacetime graphs based on a honeycomb structure. Both graphs have six boundary vertices, and both have one out of the mulitiple Cauchy subsets highlighted in blue. The graph on the left is referred to as a static spacetime graph, the one on the right as an expanding spacetime graph.

An important difference between both types of graphs is that in static graphs the collection of Cauchy subsets define a foliation over the spacetime with the direction of computation typically perpendicular to this foliation. In expanding graphs a typical Cauchy subset consists of isolated bulk vertices, and such a foliation is not observed. Also, static graphs allow for Cauchy subsets that reverse the causal direction (the direction of the computation). This is not the case for expanding graphs: in an expanding spacetime graph the causal direction is always in the direction of expansion (see Fig. 7).

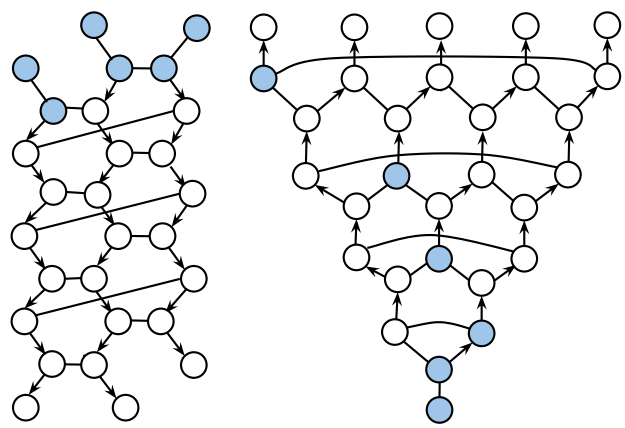

Fig 7. Alternative Cauchy subsets for the spacetime graphs shown in Fig. 6. In case of the static graph, we manage to reverse the causal direction. For the expanding graph this is not possible: we can select an alternative Cauchy subset, but this won't change the direction of computation.

Both static and expanding graphs can be extended in the causal direction. Doing so for a static graph doesn't change the number of boundary vertices, nor the size of the Cauchy subsets. Extending expanding graph in the causal direction, however, does increase the number of boundary vertices as well as the number of vertices in the Cauchy subsets. This is visualised in Fig. 8. A direct consequence is that for expanding spacetime graphs, knowing the past in all its details doesn't determine its full future. To predict the future, one also needs to know the values of Cauchy vertices located in that future. This issue is irrelevant for static spacetime graphs as these don't require Cauchy vertices located in the future.

We conclude that while we seem comfortable refering to physics laws as 'reversible' and 'deterministic', as if these concepts are independent of the characterristics of the causal geometry they operate on, here we have an instance where we are forced to reconsider this position. The present model confronts us with a local field rule that for static causal structures leads to reversible and deterministic dynamics, while for a expanding causal structures the resulting dynamics is not reversible nor deterministic.

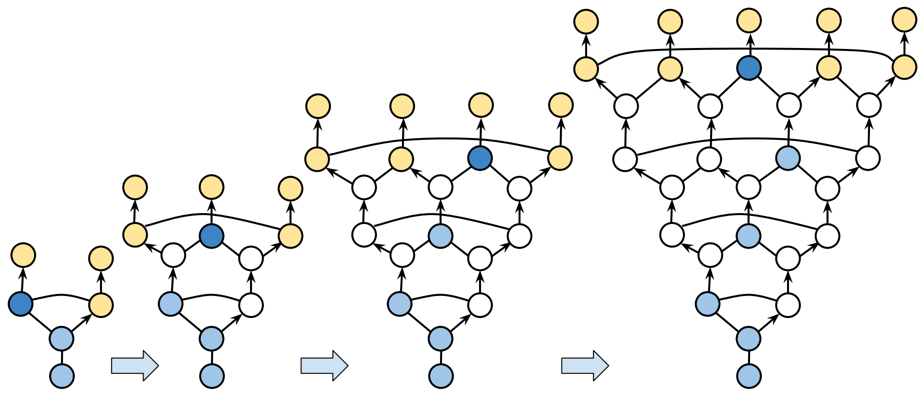

Fig 8. Development of an expanding spacetime graph. Each additional step in the development (vertices highlighted in orange) requires the value of an additional Cauchy vertex (highlighted in dark blue) to be fixed.

5 Discussion

In the above we studied a 1+1 dimensional toy model for static and expanding flat spacetimes. The key result obtained is that one-and-the-same local dynamics that leads to determinism and reversibility when operating on a spacetime static in size, fails to do so when operating on an expanding spacetime. We arrived at this conclusion by taking seriously the ordering in the computational proces, and interpreting this as the underlying causal ordering. This causal ordering manifests itself as bi-directional in a static spacetime, and as uni-directional in an expanding spacetime. The degrees of freedom in these causal orderings take the shape of a Cauchy subsets. These degrees of freedom can not be localised as a multiple Cauchy subset are always present. For expanding spacetimes none of the Cauchy subset can be localised solely in the past, and therefore the past fails to determine the future.

The toy model in which irreversibility and lack of determinism is obseved, is flat and expanding, just like our universe.[3, 4] Therefore these oservations can not be brushed away as irrelevant. Rather, it should focus our attention on the expansion of the universe as the very origin for the arrow-of-time and as a basis for libertarian free will. What emerges is a picture of a growing block universe (Cf. Fig. 8). A block universe taking the shape of 'a growing garden of forking paths'. This is a view markedly different from the various views building on Boltzmann's thinking,[5] and a view arguably orthogonal to the compatibalist view on free will.

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[5] For an overview, see: Price, H. (1996): *Time's Arrow and Archimedes' Point*, Oxford University Press.

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