From Kerr to Schwarzschild: A Thought Experiment on Black Hole Evolution

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Abstract

This paper presents a thought experiment on the conservation of angular momentum in Kerr black holes, focusing on the Penrose process as a mechanism for energy extraction. We derive the relationships between the black hole's mass, angular momentum, and extractable energy, highlighting the impact of negative-energy particle ejection on rotational dynamics. For a solar-mass Kerr black hole, we calculate the ring singularity radius and demonstrate that up to 29.29% of the mass-energy can be extracted while preserving angular momentum constraints. The study investigates the physical implications within the ergosphere and beyond the event horizon, addressing unresolved questions about angular momentum in extreme gravitational environments. Notably, we find that the energy required to reduce the ring singularity radius to near-zero (0^+) matches the Penrose process's maximum extractable energy, suggesting a novel evolutionary pathway: a Kerr black hole may transition into a Schwarzschild black hole upon complete rotational energy depletion. Connections to contemporary research, such as magnetic reconnection, contextualize these findings within modern astrophysics.

1 Introduction

The Kerr black hole, a rotating solution to Einstein's field equations, is distinguished by its angular momentum, which introduces complex dynamics absent in the static Schwarzschild case [1]. The ergosphere, a region where spacetime is dragged by the black hole's rotation, enables processes like the Penrose mechanism, where energy extraction is achieved by exploiting negative-energy states [2]. This process inherently couples the black hole's mass and angular momentum, raising fundamental questions about their conservation and the long-term evolution of rotating black holes.

This study investigates how angular momentum is conserved in a Kerr black hole during energy extraction via the Penrose process. We derive key quantities—such as the irreducible mass, extractable energy, and ring singularity radius—and analyze their implications for rotational dynamics. A significant finding is that the energy required to reduce the ring singularity radius to near-zero (0^+) matches the maximum energy extractable through the Penrose process, suggesting a potential evolutionary pathway: a Kerr black hole may transition into a Schwarzschild black hole upon exhausting its rotational energy. Grounded in general relativity, this work is enhanced by insights from recent studies, such as magnetic reconnection as an alternative mechanism [5], aiming to bridge classical theory with contemporary astrophysical applications and propose a novel perspective on black hole evolution.

2 Theoretical Framework

2.1 Kerr Black Hole Geometry

The Kerr metric in Boyer-Lindquist coordinates is:

$$ds^2 = -\left(1 - \frac{r_s r}{\Sigma}\right) dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \left(r^2 + \alpha^2 + \frac{r_s r \alpha^2}{\Sigma} \sin^2\theta\right) \sin^2\theta d\phi^2 - \frac{2 r_s r \alpha \sin^2\theta}{\Sigma} dt d\phi, \tag{1}$$

where $r_s = 2GM/c^2$, $\alpha = J/Mc$, $\Sigma = r^2 + \alpha^2 \cos^2\theta$, and $\Delta = r^2 - r_s r + \alpha^2$. The dimensionless spin parameter is $a = Jc/GM^2$, with $0 \leq a \leq 1$. The event horizon is at $r_+ = GM/c^2 + \sqrt{(GM/c^2)^2 - \alpha^2}$, and the ergosphere extends to $r_e = GM/c^2 + \sqrt{(GM/c^2)^2 - \alpha^2 \cos^2\theta}$.

The black hole's angular momentum J and mass M are related to its rotational energy, with the angular velocity at the horizon given by:

$$\Omega = \frac{\alpha c}{r_+^2 + \alpha^2}. \tag{2}$$

2.2 Penrose Process and Angular Momentum

In the Penrose process, a particle enters the ergosphere and decays into two fragments. The fragment with negative energy falls into the black hole, reducing its mass and angular momentum, while the other escapes with increased energy. The differential relation is:

$$dM = \Omega dJ, \tag{3}$$

ensuring conservation of energy and angular momentum. The negative-energy particle's contribution adjusts the black hole's parameters according to:

$$\delta M = E < 0, \quad \delta J = L < 0, \tag{4}$$

where E and L are the energy and angular momentum of the infalling particle, respectively.

3 Derivation and Analysis

3.1 Irreducible Mass and Extractable Energy

The irreducible mass M_{irr}, the mass remaining after all rotational energy is extracted, is:

$$M_{irr} = M\sqrt{\frac{1}{2}\left(1+\sqrt{1-a^2}\right)}, \tag{5}$$

where M is the initial mass and a is the spin parameter [3]. The extractable mass is:

$$M_{ext} = M - M_{irr} = M\left[1-\sqrt{\frac{1}{2}\left(1+\sqrt{1-a^2}\right)}\right]. \tag{6}$$

The corresponding energy is:

$$E_{ext} = Mc^2\left[1-\sqrt{\frac{1}{2}\left(1+\sqrt{1-a^2}\right)}\right]. \tag{7}$$

For $a = 1$ (maximal rotation):

$$E_{ext} = Mc^2\left(1-\frac{1}{\sqrt{2}}\right) \approx 0.2929Mc^2, \tag{8}$$

indicating that up to 29.29% of the mass-energy can be extracted, with the angular momentum decreasing proportionally.

3.2 Ring Singularity Radius

For a solar-mass Kerr black hole ($M = 1.9891 \times 10^{30}\,\text{kg}$), the ring singularity's radius is determined by $\alpha = J/Mc$. For $a = 1$, $J = GM^2/c$, so:

$$r = \frac{GM}{c^2} = \frac{6.67430 \times 10^{-11} \cdot 1.9891 \times 10^{30}}{(2.99792458 \times 10^8)^2} \approx 1477.98\,\text{m}. \tag{9}$$

This radius defines the equatorial plane of the ring singularity, distinct from the event horizon, and is critical for understanding the spatial extent of rotational effects.

3.3 Angular Momentum Conservation

The Penrose process conserves total angular momentum by transferring it between the black hole and the escaping particle. The change in the black hole's angular momentum is:

$$\Delta J = -L_{ext}, \tag{10}$$

where L_{ext} is the angular momentum of the escaping particle. For maximal extraction ($a \to 0$), J reduces to zero, consistent with M_{irr}.

3.4 Ring Singularity Collapse and Evolutionary Implications

In this section, we investigate the energy required to reduce the ring singularity radius, $r_{\text{sing}} = \frac{J}{Mc}$, to a near-zero value ($0^+$) and compare it with the maximum energy extractable via the Penrose process. This analysis leads to a novel hypothesis regarding the evolutionary path of Kerr black holes, proposing a transition to a Schwarzschild state upon complete extraction of rotational energy.

The ring singularity radius is directly tied to the black hole's angular momentum J:

$$r_{\text{sing}} = a \cdot \frac{GM}{c^2},$$

where $a = \frac{Jc}{GM^2}$ is the spin parameter. For $r_{\text{sing}} \to 0^+$, $J \to 0$ (i.e., $a \to 0$), necessitating the emission of all rotational energy. This rotational energy is quantified as:

$$E_{\text{rot}} = Mc^2 \left[1 - \sqrt{\frac{1}{2}\left(1 + \sqrt{1 - a^2}\right)}\right].$$

For a maximally rotating Kerr black hole ($a = 1$):

$$E_{\text{rot}} = Mc^2 \left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.2929Mc^2,$$

representing the total energy that must be emitted to reduce J to zero, collapsing the ring singularity into a point singularity ($r_{\text{sing}} = 0$).

This value is identical to the maximum energy extractable through the Penrose process, as derived in Section 3.1:

$$E_{\text{ext}} = Mc^2 \left[1 - \sqrt{\frac{1}{2}\left(1 + \sqrt{1 - a^2}\right)}\right] \approx 0.2929Mc^2 \text{ for } a = 1.$$

For a solar-mass black hole ($M = 1.9891 \times 10^{30}\,\text{kg}$):

$$E_{\text{rot}} = 0.2929 \cdot 1.9891 \times 10^{30} \cdot (2.9979 \times 10^8)^2 \approx 5.24 \times 10^{46}\,\text{J},$$

matching the energy required for $r_{\text{sing}} \to 0^+$. This equivalence suggests that the Penrose process can theoretically exhaust the black hole's rotational energy, driving a from 1 to 0.

This finding motivates a new evolutionary pathway for Kerr black holes. By fully extracting the rotational energy via repeated Penrose processes, a Kerr black hole ($a > 0$, ring singularity) transitions to a Schwarzschild black hole ($a = 0$, point singularity). The final irreducible mass becomes:

$$M_{\text{irr}} = M\sqrt{\frac{1}{2}(1 + 0)} = \frac{M}{\sqrt{2}},$$

with the event horizon radius expanding to $r_s = \frac{2GM_{\text{irr}}}{c^2}$, and the ergosphere disappearing. This "spin-down" process delineates a distinct evolutionary phase, differing from accretion or merger, which typically increase J and M.

Astrophysically, achieving this transition requires an environment where Penrose extraction outpaces accretion, such as an isolated Kerr black hole with minimal inflow. The extracted energy ($0.2929Mc^2$) could manifest as high-energy jets or radiation, providing potential observational signatures distinct from those of rotating black holes. This hypothesis connects the classical dynamics of energy extraction to the long-term evolution of black holes, potentially leading to their evaporation via Hawking radiation from a non-rotating state.

4 Discussion

The derived extractable energy of $0.2929Mc^2$ aligns with established theoretical limits [4], yet the fate of angular momentum beyond the event horizon remains speculative. The ergosphere facilitates negative-energy states, enabling energy extraction via the Penrose process; however, the interior dynamics—potentially influenced by quantum effects or singularities—remain beyond direct observation. Recent studies propose that magnetic reconnection in the accretion disk could enhance energy extraction, preserving angular momentum through electromagnetic coupling [5]. Our analysis further reveals that the energy required to reduce the ring singularity radius to near-zero (0^+) precisely matches this extractable energy, suggesting that the Penrose process could exhaust the black hole's rotational energy. This finding implies a potential transformation of a Kerr black hole into a Schwarzschild black hole, with the ring singularity collapsing into a point singularity as $J \to 0$.

5 Conclusion

This thought experiment demonstrates that the Penrose process in Kerr black holes conserves angular momentum while enabling significant energy extraction, with a maximum of 29.29% of the initial mass-energy. This extractable energy highlights the pivotal role of rotational energy in black hole thermodynamics. Moreover, the equivalence between this energy and that required to collapse the ring singularity radius to 0^+ unveils a novel evolutionary scenario: a Kerr black hole, upon complete depletion of its rotational energy via the Penrose process, may transition into a Schwarzschild black hole. This spin-down process marks a significant departure from traditional evolutionary paths dominated by accretion or mergers. Future research could explore quantum corrections near the singularity, observational signatures through gravitational waves, or astrophysical conditions facilitating this transition, building on this classical foundation to deepen our understanding of black hole evolution.

References

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