The physical meanings behind Wick rotation and the energy-momentum complex functions

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Abstract: Through Wick rotation, and the holomorphic and residue theory of energy-momentum complex functions, some formulas for wave theory, relativity, and quantum mechanics are derived. Also, the connections implied by various branches of physics such as classical mechanics, relativistic mechanics, wave theory, thermodynamics, and quantum mechanics are sought.

Keywords: Wick rotation, energy-momentum tensor, entropy, physical waves, Schrödinger's equation

1 Energy-momentum tensor

The world we live in can be expressed as a space-time point in three-dimensional space and one-dimensional time. For convenience, we only study one-dimensional space and one-dimensional time.

Define space-time points.

$$w = r + it = re_r + ite_t \qquad (1)$$

We use a space-time coordinate system where r is the position vector, t is the time coordinate, and t = cs, where c is the speed of light and s represents seconds

Define the time-space tensor.

$$dw = dr + idt = dre_r + idte_t \qquad (2)$$

Define the energy-momentum tensor.

$$T = mv + im = mve_p + ime_m \qquad (3)$$

Where m is mass and the momentum p is defined as p = mv.

we find that T is an energy-momentum complex function.

Based on this, we make a basic assumption: Any substance in the world can be represented by a complex function T.

Consider T as a function of m, r, and t, i.e.

$$T = mv(r,t) + im(r,t) \qquad (4)$$

According to the properties of complex functions, we can draw some conclusions.

$$(mv)^2 + m^2 = cons\tan t \qquad (5)$$

$$|dw|^2 = |dr|^2 + |dt|^2 = \text{constant} \qquad (6)$$

2 Wick rotation

Wick rotation

$$t' = \text{-i}t \qquad (7)$$

The quadratic Wick rotation

$t'' = -t$

Regarding

$$e^{i\theta} = \cos\theta + i\sin\theta$$

When θ=π/2.

Namely

$$e^{i\frac{\pi}{2}} dw = idw = idr - dt$$

We call this transformation the space-time inversion transformation.

Perform a quadratic space-time inversion transformation.

$$i \times idw = -dr - idt$$

In terms of time, it is the result of a second Wick rotation.

3 Special relativity

About

$$|dw|^2 = |dr|^2 + |dt'|^2 = \text{constant}$$

Perform a Wick rotation.

$t = -\mathrm{i}t'$

Get:

$$|dr|^2 - |dt|^2 = \text{constant}$$

For photons, we need to

$$|dr|^2 - |dt|^2 = 0 \tag{8}$$

Wick rotation is equivalent to the transformation between different inertial coordinate systems in special relativity.

4 Tensor transformation rules

From the conclusion of special relativity, it can be concluded that:

$$dt_0 = \sqrt{1 - v^2}\, dt$$

$$m = \frac{m_0}{\sqrt{1 - v^2}}$$

that is:

$$T = \frac{m_0}{dt\sqrt{1 - v^2}}(dr + idt) = \frac{m_0}{dt_0} dw$$

Therefore:

$$T = mv + im$$

And

$$dw = dr + idt$$

The tensor transformation rules are the same.

5 Class space

Regarding

$$dw = dr + idt$$

When | v |<1, it is called timelike.
When | v |>1, it is called spacelike,Then

$$T = \frac{m_0}{idt\sqrt{v^2 - 1}}(dr + idt) = -i\frac{m_0 v}{\sqrt{v^2 - 1}} + \frac{m_0}{\sqrt{v^2 - 1}}$$

Due to the same tensor transformation law between T and dw, space-time inversion is performed. Set

$$u = \frac{1}{v} < 1$$

Such that

$$T = -i\frac{m_0 \frac{1}{u}}{\sqrt{(\frac{1}{u}^2 - 1)}} + \frac{m_0}{\sqrt{(\frac{1}{u}^2 - 1)}}$$

In a spacelike region, a particle has a displacement described by the changes in time dt and displacement dr, and the particle has a mass m.

By

$$m = \frac{m_0}{\sqrt{1 - u^2}}$$

and its
p=mu
The physical quantities are obtained by observing the properties of spacelike in terms of timelike.

$$T_- = mu - im \quad (9)$$

$$dw_- = dr - idt \quad (10)$$

Therefore, regarding the so-called superluminal matter, it can be considered to be matter in negative time, resulting in a velocity lower than the speed of light.That is to say, space-time inversion is the transformation between timelike intervals and spacelike intervals.
The above results are consistent with the results of the second wick rotation of time.
We observe in positive time and have:

$$T_+ = -mu + im \quad (11)$$

$$dw_+ = -dr + idt \quad (12)$$

6 matter wave

In the natural unit system, $c = \hbar = 1$.
Phase velocity

$$v_p = \frac{\omega}{\kappa} \tag{13}$$

$$v_p = \nu\lambda = \frac{2\pi\nu}{2\pi/\lambda} = \frac{m}{p}$$

Here u_g should be the velocity of the material in the spacelike region. When observed in the positive time of the timelike, the space-time is reversed, and the energy-momentum is also reversed. Therefore, the so-called group velocity is:

$$u_g = \frac{p}{m} = \frac{d\omega}{d\kappa} \tag{14}$$

7 The speed after Wick rotation

The velocity is:

$$v = \frac{dr}{dt}$$

By performing a Wick rotation on time, we obtain:

$$v' = -i\frac{dr}{dt}$$

By performing a second Wick rotation on time, we obtain:

$$v'' = -\frac{dr}{dt}$$

The speed is:

$$|v| = \left|\frac{dr}{dt}\right|$$

There is no change to the Wick rotation.

8 ΔT and Δ(Δw)

T=mv+im and dw=dr+idt have the same tensor transformation rules. That is to say: Δ T and Δ (Δ w) also have the same tensor transformation rules.
The temperature is:

$$T = \frac{\mathrm{d}E}{dS} = \frac{dm}{dS} \tag{15}$$

In one-dimensional space and one-dimensional time, we obtain:

$$\frac{m_0}{\Delta t_0}\Delta\ (\Delta t) = \Delta m = TdS = bT \tag{16}$$

When the temperature T remains constant

$dS \propto dm$

Where b is a constant.

$$\frac{1}{2} \le \Delta m \Delta t = \frac{m_0}{\Delta t_0}\Delta\ (\Delta t)\ \Delta t = m\Delta\ (\Delta t)$$

So:

$$\Delta\ (\Delta t) \neq 0 \tag{17}$$

The absolute temperature scale cannot reach absolute zero degrees.

9 Wave particle duality

Order

$$L(r,v) == \frac{1}{2}\sum_{i,j} v_i m_{ij} v_j = \frac{1}{2}\sum_{ij} m_{ij} \times \frac{dr_i}{dt} \times \frac{dr_j}{dt}$$

The path integral is

$$K(r,t;r_0,t_0) = \lim_{n\to\infty}\left(\frac{m}{i2\pi\Delta t}\right)^{\frac{n}{2}} \int Dre^{i\int dtL}$$

Consider a very small time interval

$$t \to t_0 + \Delta t'$$

Get:

$$K'(r_0+\Delta r, t_0+\Delta t'; r_0, t_0) = <(r_0+\Delta r)|U(t_0+\Delta t', t_0)|r_0> = C\lim_{n\to\infty}\left(\frac{m_0}{i2\pi\Delta t'}\right)^{\frac{n}{2}} e^{i\Delta t' \times \frac{1}{2}m_0 \times \frac{\Delta r}{\Delta t'} \times \frac{\Delta r}{\Delta t''}} \tag{18}$$

To perform a Wick rotation

$\Delta t' = \text{-i}\Delta t$

we obtain

$$K(r_0+\Delta r, t_0+\Delta t; r_0, t_0) = C\left(\frac{m_0}{2\pi\Delta t}\right)^{\frac{n}{2}} \int dre^{-\int dtL} = C\left(\frac{m_0}{2\pi\Delta t}\right)^{\frac{n}{2}} e^{-\Delta t \times \frac{1}{2}m_0 \times \frac{\Delta r}{\Delta t} \times \frac{\Delta r}{\Delta t}} \tag{19}$$

Be aware

$$\frac{m_0}{\Delta t_0}\Delta\ (\Delta t) = bT$$

The T here is temperature.

Compare with Maxwell's velocity distribution function.

$$f(|v|) = 4\pi\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{m}{2kT}|v|^2} \bullet |v|^2 \tag{20}$$

We can consider that treating "particles" as "waves" is equivalent to the Wick rotation:
$\Delta t = \mathrm{i}\Delta t'$

10 Antimatter

If we observe a particle in a reference frame, according to equations (12) and (14), when v ≠ 1, matter has four types of energy-momentum tensor:

$$T_{++} = +mv + im \qquad T_{+-} = -mv + im$$

$$T_{-+} = +mv - im \qquad T_{--} = -mv - im$$

$$T_{+} = \pm mv + im = -T_{-} = \mp mv + im \tag{21}$$

The first indices in the T represent positive and negative substances, '+'represents positive matter,' -'represents antimatter;The second symbol represents rotation, '+'represents left rotation, and '-' represents right rotation.

We observe T_{-} as $-T_{-}$, and we observe the negative time dw_{-} as $-dw_{-}$, with the time direction becoming positive and the displacement direction being opposite.

These four types of energy-momentum tensor exhibit different spins in the forward and reverse directions of momentum. For electrons, there are four types of energy-momentum tensor with different spins and charges. For photons,due to the rest mass being zero and there being no space-time inversion, there are only two types of energy-momentum tensor. That is to say, photons do not have antimatter.

11 Gravitation

When there is no pole effect in the set D, the energy-momentum tensor T is a holomorphic function of the space-time point w.

That is

$$T(m_0, r, t) = p(r,t) + im(r,t)$$

where T is holomorphic in D.

There is an object with a mass of M that has a first-order pole characteristic, and a particle with a mass of m is moving around the object with mass M along a closed curve C.

If f (ξ) is a holomorphic function related to M and m in a closed region bounded by C, then in one-dimensional space and one-dimensional time, the energy change of a particle with mass m at a certain position is:

$$V(w) = \frac{1}{2\pi i}\int_{c} \frac{f(\zeta)}{\zeta - w} d\zeta \tag{22}$$

Order:

$$r = |w - \xi|$$

According to the residue theorem, we obtain:

$$V(w) = \frac{1}{2\pi i}\int_c \frac{f(\zeta)}{\zeta - w} d\zeta = -\frac{GMm}{r} \tag{23}$$

G is a constant. So:

$$F = -\frac{\partial V}{\partial r} = \frac{\partial \frac{GMm}{r}}{\partial r} = -\frac{GMm}{r^2} \tag{24}$$

Direction outward.

M and m can be mass or some form of energy, such as electric charge.

12 Increasing entropy and decreasing entropy

When a photon with energy $2\pi\nu$ is subjected to a gravitational force with a distance of r and a mass of M, according to equation (23) and first approximation, we obtain:

$$2\pi\nu(1 - \frac{Gm}{r}) = 2\pi\nu_1 \tag{25}$$

This is red shift.At this point, the photon energy decreases, which we call entropy reduction.

And when an object is subjected to a repulsive force generated by an object with mass -M:

$$2\pi\nu(1 + \frac{Gm}{r}) = 2\pi\nu_2 \tag{26}$$

It's blue shift.At this point, the photon energy increases, which we call entropy increasing effect.

The matter in space is subject to the increase or decrease of entropy caused by the source point of potential energy, namely the pole. In the region of the universe where we are located, it is currently expanding, where the effect of increasing entropy is greater than the effect of decreasing entropy.

13 Wave function

The energy-momentum tensor can be represented by a wave function:

$$\varphi(p) = \frac{1}{(2\pi)^{3/2}} \int \psi(r) e^{-ipr} d^3 r \tag{27}$$

$$\psi(r) = \frac{1}{(2\pi)^{3/2}} \int \varphi(r) e^{ipr} d^3 p \tag{28}$$

That means we can

T=mv+im

Expressed as coordinate representation, momentum representation.

or as:

$$\Phi = Ae^{i(pr - Et)} \tag{29}$$

It is a function of time, space, and rest mass.

14 Macro and Micro

A bit of information represents the amount of information when there are two equally likely possibilities and one of them is chosen. For example, one certain system has 2^r possible states,Then its amount of information is *r* bits.

As we know an electron has 2^2 possible existing states (it has two electric charges and two spinning states, which is measured by people now). If you want to specify what electron it is, you need 2 digits of binary. So a sole electric, information is 2 bits. As for the photon's spinning, it has two ways of motion which parallels the matter's moving direction or opposite of it. Its least possible existing states are 2^1. So a sole photon, information is 1 bit which is measured by people now. For a sole system which is formed by lots of photons, their left and right spins are equal and scattering evenly. The whole system information is 0 bit.

Obviously, the minimum capacity of information in elementary particles is 1 bit.

Considering a reversible system without external work, according to the principle of energy sharing.

$$\overline{E} = \frac{3}{2}kT$$

The T here is temperature.

In a specific system,we can obtain:

$$E \propto T$$

$$dS = \frac{dm}{T} = \frac{dE}{T}$$

$$\frac{dE}{E} = d(InE)$$

If $dS \geq 0$, then:

$$dm \geq 0 \tag{30}$$

From the theory of relativity, we get:

$$dv \geq 0$$

v stands for the speed of elementary particles of the sole special irreversible system.

It shows that the direction of the increase of entropy is to make the particle's ultimate elementary particles reach the speed of light. The particle will break up into the ultimate elementary particles whose rest mass is zero at last.According to the theory of relativity, a substance with non-zero rest mass cannot be accelerated to the speed of light. However, particles such as electron-positron pairs can annihilate and transform into a pair of photons. We know that these particles can be converted into photons under certain conditions.

Therefore, a photon, which travels at the speed of light and which is the limit of infinite subdivision of elementary particles, is a particle with zero rest mass and has an information amount of 1 bit.

Now,let's take a look at the following points and set $w = \infty$,we get:

$$\mathrm{f}(w) = \frac{1}{2\pi i}\int_c \frac{f(\xi)}{\xi - w} = 0 \tag{31}$$

thus

$$\sum_{n=1}^{\infty} \underset{w\to\infty}{f(w)} = 0$$

The total mass of the universe is 0.

Therefore, we can consider that the total mass, total momentum and other physical fundamental quantities of the universe are zero.As a whole, the universe is characterized by statistical (i.e. macroscopic) properties, and its microscopic information simply does not exist. The values of physical quantities such as time, space, mass, etc., only have non-zero values in local regions of the universe.

15 General relativity

A photon is subject to the gravitational force of a substance with a distance of r and a mass of M. Based on equations (5) and (23), and a first-order approximation, we obtain:

$$(1+\frac{GM}{r})^2 e_p^2 + (1-\frac{GM}{r})^2 e_m^2 \approx e_P^2 + e_M^2 \qquad (32)$$

$$T = e_P + ie_M = (1+\frac{GM}{r})e_p + (1-\frac{GM}{r})e_m \qquad (33)$$

Due to the same tensor transformation law between the active tensor T and dw, for photon motion, we obtain:

$$dw = e_R + ie_T = (1+\frac{GM}{r})e_r + i(1-\frac{GM}{r})e_t \qquad (34)$$

Among them, P, M, R, and T are Cartesian coordinates.

Roughly enough:

$$\begin{aligned} dr' &= (1-\frac{GM}{r})dr \\ dt' &= (1+\frac{GM}{r})dt \end{aligned} \qquad (35)$$

It is the result of general relativity.

16 First order differentiation of energy-momentum tensor

When a mass point is in a region without poles, equation (4) is the holomorphic function of w, which yields:

$$F = \frac{\partial (mv)}{\partial t} = -\frac{\partial m}{\partial r}$$

At dm/dt=0, dm=dE， If E is mechanical energy, then:

$$Fdr = -dE \qquad (36)$$

This is the Newton equation.

17 Schrodinger's equation

By

$$T = p(r,\tau) + im（r,\tau）$$

$$T'' = \frac{\partial^2 p}{\partial r^2} + i\frac{\partial^2 m}{\partial r^2} = \frac{\partial^2 T}{\partial r^2}$$

$$T'' = \frac{\partial^2 m}{\partial r \partial \tau} - i\frac{\partial^2 p}{\partial r \partial \tau} = -i\frac{\partial^2 T}{\partial r \partial \tau}$$

Resulting in:

$$-i\frac{\partial^2 T}{\partial r \partial \tau} = \frac{\partial^2 T}{\partial r^2}$$

$$-i\frac{\partial T}{\partial \tau} = dr\nabla^2 T$$

Replace T with Ψ to represent probability.
Considering one-dimensional time and three-dimensional space, for the right side of the equal sign, the value of a certain physical quantity is 1/4π in one direction.
At one wavelength, i.e. dr=λ:

$$-i\frac{\partial \Psi}{\partial \tau} = \frac{\lambda}{4\pi}\nabla^2 \Psi$$

By:

$$m = h\nu$$

$$c = \nu\lambda$$

$$\hbar = \frac{h}{2\pi}$$

$$t = c\tau$$

The Schrodinger equation for obtaining.

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \Psi \qquad (37)$$

18 Second order differentiation of energy-momentum tensor

For equation (4), we obtain:

$$\frac{d^2 p}{dr^2} + i\frac{d^2 m}{dr^2} = -\frac{d^2 p}{dt^2} - i\frac{d^2 m}{dt^2}$$

$$\frac{d^2 T}{dr^2} + i\frac{d^2 T}{dt^2} = 0$$

Assuming T=ϕ and performing a Wick rotation, we obtain:

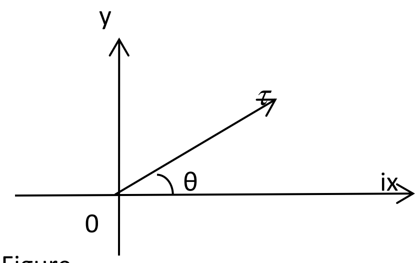
$$\frac{d^2 \varphi}{dr^2} - i\frac{d^2 \varphi}{dt^2} = 0 \qquad (38)$$

This is the wave equation.

19 Other issues

（i）Is there any imaginary time?

Let time be two-dimensional:



Figure

$$\tau = \mathrm{y} + \mathrm{ix}$$

$$|\tau| = \sqrt{\mathrm{x}^2 + y^2}$$

Where

$$\mathrm{x}, y \in R$$

y is real time,x is imaginary time.

Time is a plane,imaginary time lies in the multiplicity of history, that is, multiple paths.

（ii）Is there a physical meaning behind conformal transformation?

$$T = |T| e^{i\theta}$$

$$T' = \frac{1}{T} = \frac{1}{|T|} e^{-i\theta}$$

The modulus of a physical quantity is the reciprocal of its original value, does it mean that the smaller mass will become larger?

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