Generalized Dirac delta impulse and determinism obtained from the multivariate Gaussian

Ait-taleb Nabil *

Abstract

In this paper, we will propose to generalize the Dirac delta impulse to several dimensions. This generalization will be done by taking into account the one-dimensional version of the Dirac delta impulse. From a projection of the variance-covariance matrix, located inside the cone of positive semi-definite matrices, onto the boundary of the cone of positive semi-definite matrices, we will make the transition from Gaussian probability theory to determinism.

*Corresponding author: nabiltravail1982@gmail.com

1 Introduction

In this paper, we will recall the notion of Dirac delta impulse in the one-dimensional case. This classical definition is made from a limit computed from a one-dimensional Gaussian. We will then generalize this concept to several dimensions from a projection of a variance covariance matrix, initially located inside the cone of semi-definite positive matrices, onto the boundary of the cone of semi-definite positive matrices. We will also explain the reason why we have generalized the notion of Dirac delta impulse to several dimensions. From the result obtained in paper [1] page 4, this projection will show the transition between the domain of Gaussian randomness and the determinism.

2 Classical Dirac delta impulse obtained from a Gaussian

Before introducing the generalized Dirac delta impulse, we need to recall the approach to the Dirac delta impulse made by the one-dimensional Gaussian in order to make the analogy later. Recall that the one-dimensional Dirac delta impulse is made by the following limit of the Gaussian:

$$\delta(x) = \lim_{\sigma \longrightarrow 0} [\frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{x^2}{2\sigma^2}\}]$$

In what follows we will generalize this concept to several dimensions.

3 Generalized Dirac delta impulse obtained from the multivariate Gaussian

We define the generalized Dirac delta impulse $\delta(\vec{X}-\vec{\mu}_X)$ from the multivariate Gaussian $\mathcal{N}(\mu_X, \Sigma_{X^2})$ as follows:

$$\boxed{\delta(\vec{X}-\vec{\mu}_X) = \mathcal{P}_{\Sigma_{X^2}\longrightarrow\partial S_0^+}[\mathcal{N}(\mu_X, \Sigma_{X^2})] = \mathcal{P}_{\Sigma_{X^2}\longrightarrow\partial S_0^+}[(2.\pi)^{-\frac{n}{2}}|\Sigma_{X^2}|^{-\frac{1}{2}} \exp - \frac{(\vec{X}-\vec{\mu}_X)^t\Sigma_{X^2}^{-1}(\vec{X}-\vec{\mu}_X)}{2}]}$$

To understand the generalized Dirac delta impulse, we must initially consider a random Gaussian vector following the probability law $\mathcal{N}(\mu_X, \Sigma_{X^2})$.

$\mathcal{P}_{\Sigma_{X^2}\longrightarrow\partial S_0^+}$ then corresponds to the projection of the variance covariance matrix, located inside the cone of positive semi-definite matrices, onto the boundary ∂S_0^+ of the cone of positive semi-definite matrices. **This projection of the matrix is done by performing the spectral decomposition of the matrix Σ_{X^2}, by canceling the last eigenvalue and by returning to the starting basis**.

Onto the boundary of the cone ∂S_0^+, the determinant of the matrix is zero ($|\Sigma_{X^2}| = 0$), the matrix Σ_{X^2} is therefore singular and not invertible. **It is for these reasons that we can say that we have indeed generalized the one-dimensional Dirac delta impulse to multidimensional.(see the classic version in the previous section)**

As we demonstrated in the paper ([1] page 4) the boundary of the cone of positive semi-definite matrices ∂S contains **the predictability** and **the determinism**. The projection $\mathcal{P}_{\Sigma_{X^2}\longrightarrow\partial S_0^+}$ therefore expresses the transition from Gaussian probability theory to determinism.

The multivariate Gaussian $\mathcal{N}(\mu_X, \Sigma_{X^2})$ therefore infers a **randomness** vector $\vec{X}$ while the generalized Dirac delta impulse $\delta(\vec{X}-\vec{\mu}_X)$ infers a **deterministic** vector $\vec{X}$.

For the multivariate Gaussian, the transition from **randomness** to **determinism** is made with projection $\mathcal{P}_{\Sigma_{X^2}\longrightarrow\partial S_0^+}$.

4 Conclusion

In this paper, the generalization of the Dirac delta impulse was made by taking into account the classical one-dimensional version and the limit of the one-dimensional Gaussian. The limit then became, in a multidimensional case, a projection of a strictly positive-definite variance-covariance matrix onto the boundary of the cone of positive semi-definite matrices. This projection, giving the generalized Dirac delta impulse, made the transition between Gaussian randomness and the determinism. The multivariate Gaussian in fact infers randomness, while the generalized Dirac delta impulse obtained by projection infers determinism.

[1] Understanding when the correlations imply the predictability for the multiple Gaussian. Author: Ait-Taleb Nabil. Published on vixra in 2025

[2]Optimal stastical decisions. Author: Morris H.DeGroot. Copyright 1970-2004 John Wiley and sons.

[3]Computing the nearest correlation Matrix-A problem from finance. Author: Nicholas Higham.copyright 2002, The university of Manchester