What Does It Mean that Quantum Mechanics and Classical Mechanics Can Be Used Together?

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Abstract

Nowadays, the notion that 'quantum mechanics and classical mechanics are incompatible' is firmly held in people's minds. One of the paths of the physics revolution was to break this notion and achieve the goal of combining classical mechanics with quantum mechanics. The Schrödinger equation for gravitational potential energy was derived by replacing the potential energy function. This equation can describe classical mechanical systems, and is a mathematical foundation that can be combined with classical mechanics and quantum mechanics.. Mathematically speaking, the application of Hamiltonian operator and Schrödinger equation is not limited by whether the system is microscopic or macroscopic. The method of using the Schrödinger equation to solve problems is called the wave dynamics method (quantum mechanics method). The classical mechanical system can use the Schrödinger equation. This indicates that classical mechanics and quantum mechanics can be combined for the same system. As long as there is no superstition about the absolute dominance of existing quantum mechanical explanations such as uncertainty, superposition, and coherence in the microscopic world (Plus establishing a ring electronic structure model), the combination of classical mechanics and quantum mechanics can be used in practice. Multiple successful examples of using quantum mechanics without combining classical forces have been provided with the help of references.

Keywords: Schrödinger equation for gravitational potential, Quantum mechanics, Classical mechanics, Compatibility, The noumenon of wave function, Electron spin magnetic moment.

Important statement
The research results of this article will cause a significant revolution in physics: the quantum mechanics revolution it will cause is that "classical mechanics and quantum mechanics can be compatible, giving birth to new quantum chemistry calculation methods"; The revolution in the theory of gravitational material structures led to the birth of the theory of non-point material structures.

1 Introdunction

Currently, there is no unified understanding of certain interpretations of quantum mechanics. Many people are dissatisfied with the existing explanations of quantum mechanics. The specific form of electron spin and the source of electron spin magnetic moment are still unsolved mysteries. This indicates that there are still several large dark clouds floating in the current physics sky. In the fundamental field of physics, there have been no major disruptive innovations for over a century (only limited progress within the current theoretical framework). To dispel that dark cloud and break through the existing framework of physics, there must be significant disruptive innovations. It is not difficult to make two important discoveries by analyzing the existing Schrödinger equation. Firstly, the Schrödinger equation utilizes the classical electrodynamic potential energy function. This indicates that the Schrödinger equation itself suggests that we can simultaneously use quantum mechanics and classical electrodynamics. We cannot mathematically prove that the Schrödinger equation or Hamiltonian operator cannot be used in classical electrodynamics. Secondly, we cannot prove mathematically that 'the mass m in the Schrödinger equation has a finite upper bound' (usually judged based on experience or old concepts). Some people may say that this upper limit can be determined based on experiments. However, these judgment methods lack mathematical basis. If there is no such upper limit, it cannot be guaranteed that the Schrödinger equation cannot be used to describe macroscopic systems. Once the Schrödinger equation can be used to describe macroscopic systems, quantum mechanics can be combined with classical mechanics to use. It can be seen that we cannot directly deny the combination of classical mechanics and quantum mechanics using mathematical methods. The third section of this article and references [1,2] demonstrate through application examples that we can separately or simultaneously describe the same object using quantum mechanics and classical mechanics. After deriving the Schrödinger equation of gravitational potential energy, this point is more intuitively explained. If someone believes that using the Schrödinger equation does not equate to using quantum mechanics (wave dynamics), and that using classical electrodynamics formulas and gravitational potential energy formulas does not equate to using classical mechanics methods, then they can only be accused of sentimentality or sophistry.

There are now three original innovations emerging: establishing the Schrödinger equation for gravitational potential energy, which can be used to describe classical planetary systems; Create wave element (non-point) material structure theory; Combining classical mechanics and quantum mechanics to solve practical problems. They are closely related and form a chain of evidence (*i.e.* formed a chain of evidence). Their theoretical value lies in their ability to change the old notion that "classical mechanics and quantum forces are incompatible" and promote the birth of new theories. In addition, the birth of the gravitational potential Schrödinger equation [3-7] revealed that the original Schrödinger equation can also be used to describe macroscopic systems.. Their practical value lies in the birth of new methods in quantum mechanics and the simplification of quantum chemical calculations. Provided multiple successful calculation examples such as electron spin magnetic moment, "Earth's orbital motion", and hydrogen molecules.

In fact, in order to achieve the combination of quantum theory and classical mechanics, in addition to establishing the Schrödinger equation for gravitational potential energy, the author also modified the existing quantum mechanics interpretation system (attempting to establish localized realism quantum mechanics [8]) and used a ring electronic structure model [9-13]. This model determines the conditional applicability of the planetary structure model in atoms and molecules, and is the structural basis for the applicability of classical mechanics theory in microscopic systems [1,2,14]. This electronic structure model is also a strong wind that disperses dark clouds in the physics sky.

In the third section, the author compared the calculation results of the Schrödinger equation of gravitational potential energy and Newton's laws of mechanics for the ideal orbital system of the Earth (the two calculation results are consistent). The energy of hydrogen atoms in the background of planetary models was calculated using the Schrödinger equation. The calculation results are consistent with the Bohr hydrogen atom model method (without denying the quantum mechanics method under the background of uncertainty principle).This is also a successful example of combining classical mechanics and quantum mechanics for use. Even assuming that classical mechanics and quantum mechanics can be combined, it can still predict the characteristics of this wave of physics revolution storm. The situation is, the author has truly achieved the combination of quantum mechanics and classical mechanics through examples [1,2,8,10,15].

If you want to deny the research results of this article, you must deny that the Schrödinger equation for gravitational potential energy is logically untenable. Or deny that it is unusable and meaningless. Or deny that the author has implemented all examples of combining quantum mechanics with classical mechanics.

The classical mechanics referred to in this article are Newtonian mechanics and classical electrodynamics, not relativity (relativity is not classical mechanics but the theory of spacetime in modern physics that can represent classical mechanics). Therefore, the contradiction (or incompatibility) between relativity and quantum mechanics does not mean that Newtonian mechanics and classical electrodynamics cannot be combined with quantum mechanics.

2. The Difference in Value (or Meaning) between the Schrödinger Equation of Earth's Revolution and the Original Schrödinger Equation

The differences referred to in the title of this section include differences in form, usage methods (and/or scope), and meanings. Schrödinger initially established the Schrödinger equation for hydrogen atoms. Among them, the time-dependent Schrödinger equation (one-dimensional) is

$$i\hbar\frac{\partial}{\partial t}\,\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze^2}{r}\psi\,. \qquad (1)$$

References [1-5] and appendix A provide the Schrödinger equation for gravitational potential energy. Taking the ideal orbital motion system of the Earth as an example, its form is

$$-\frac{i\hbar}{2}\frac{\partial}{\partial t}\,\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{GMm}{R}\psi = E\psi\,. \qquad (2)$$

There are significant differences between the first and third terms of Eqs. (1) and (2). Equation (2) can be used to describe the ideal orbital motion of planets such as Earth. Equation (1) cannot be used to describe the motion of planets in the solar system.

The three laws of Newtonian mechanics include the law of inertia, Newton's second law of motion, and the laws of action and reaction. The existing quantum mechanics denies that microscopic particles can be in a static state and a definite motion state, and considers them to be in an uncertain state. Therefore, Newton's laws of motion (*i.e.* classical mechanics) do not apply in quantum mechanics. The initial Schrödinger equation indeed cannot describe the inertial motion state, acceleration motion state, and the relationship between force and reaction force of an object. People have used the concept that microscopic particles have no definite state, and therefore believe that microscopic particles cannot remain stationary regardless of whether they are under force or not, which does not comply with Newton's three laws. In wave mechanics, It can only use the initial electromagnetic potential energy Schrödinger equation.

After using $V=-\frac{GMm}{R}$ for potential energy, it is impossible to deny the applicability of the classical mechanical formula F= V/r=$-\frac{GMm}{r^2}$ for planetary models. After using the de Broglie relationship $\lambda_d=h/mv$, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=\frac{1}{2}mv^2\psi$. Among them, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ is the kinetic energy operator (Note! λ_d is the wavelength of the Broglie wave of a moving object, mv is the momentum of the moving object). The Hamiltonian operator describes the overall motion of objects in a system, and its relationship between wavelength and momentum can only be expressed using the Broglie relationship. If describing the internal motion of electrons, use the relationship $\lambda_e=h/2m_ec$ (which will be used in the next section). The validity of the Viry theorem in macroscopic confinement systems indicates that Newton's second law of $F=ma=mv^2/r$ apply {Newton's second law can be used in conjunction with Eq. (2)}. As long as Eq. (2) does not violate Newton's second law, Eq. (2) does not violate Newton's first law. This indicates that Eq. (2) is applicable to classical mechanics. The use of Schrödinger equation and wave function is the foundation of quantum mechanics methods. It can be seen that Eq. (2) indicates that quantum forces have a compatible theoretical basis with classical mechanics. The combination of quantum mechanics and classical mechanics has a theoretical basis. This indicates that classical mechanics and quantum mechanics can be combined for use. The section on "Discussion, Debate, and significance" in this article also explains that if there is a microscopic gravitational confinement system, Eq. (2) can also describe it. Similarly, if the bound state of two Lamp grass ball with opposite signal charges forms a uniform circular motion system (classical mechanics system), it can also be described using the original Schrödinger Eq. (1). This analytical conclusion can greatly reinforce the above conclusion. This is a completely new understanding that emerged after exporting Eq. (2).

The simplest understanding or recognition is that "as long as the distance r in the potential energy function is selected as a certain value when solving the equation, Eqs. (1) and (2) can be used to describe macroscopic systems applicable to classical mechanics. As long as the range of the distance r in the potential energy function is set between 0 and ∞, Eqs. (1) and (2) can be used to describe microscopic systems. The Schrödinger equation itself cannot logically limit its application in macroscopic or classical mechanics.

Equation (1) itself cannot deny or limit their applicability to deterministic objects. The choice of a constant value or a range of 0- ∞ for r is determined by factors outside of this formula (mainly subjectively determined by people's lack of confidence in the understanding of the state of microscopic particles). If a certain value of *r* is chosen, Eqs. (1) and (2) are both applicable to classical mechanics.

The de Broglie relationship is a direct connection between macro and micro, rather than just 'compatibility'. The compatibility and combination of classical mechanics and quantum mechanics are not only theoretical, but also achievable in practice References [1, 5, 8-10] list many such application examples. This greatly enhances the persuasiveness of the conclusions presented in this section. The ring electronic structure model in references [10-12] supports the combination and compatibility between local realism and determinism, as well as classical mechanics and quantum mechanics. In the process of scientific research, anything that can change human thinking and concepts is a major event.

3. The fundamentals of mathematics-physics and application examples of the combination of quantum mechanics and classical mechanics for use

We have four reasons to use the method of combining quantum mechanics with classical mechanics. Firstly, the derived gravitational potential energy equation can be used to describe planetary model systems. Secondly, when the mass *m* of the described object increases to a certain extent, it becomes a macroscopic object, and pure mathematical methods cannot determine the finite upper limit of the mass *m* in the Schrödinger equation (From this, it can be seen that we cannot use pure mathematical methods to deny that the Schrödinger method can be used to describe macroscopic objects. That is, we cannot deny the use of classical mechanics methods while using quantum mechanics methods). Thirdly, the use of the Schrödinger equation generally requires the expression of potential energy in classical electrodynamics (Since the classical potential energy expression has been used, it cannot be denied that classical mechanics calculation methods have been employed). Fourthly, multiple successful computational examples of the joint use of quantum mechanics, classical mechanics, and classical mechanics can be provided. Below is a more detailed explanation.

We will ompare the results of classical mechanics calculations and quantum mechanics calculations for the same system. This operation includes an application example of the Schrödinger equation for gravitational potential energy.

The watershed between the micro world and the macro world (and also the watershed between the applicability of quantum mechanics and classical mechanics) is determined by the quality of the system. As long as the quality of the system reaches a certain value, the quantum properties of the system will disappear, and classical mechanics can be applied. It is not difficult to see from the Schrödinger equation [such as equation (1)] that we cannot mathematically find the upper limit of the mass m in the equation. That is to say, we cannot mathematically limit the applicability of the Schrödinger equation to the microscopic world (or the uncertainty background). If the Schrödinger equation containing wave functions is used, it is equivalent to using quantum mechanics methods. The above situation is that it is difficult to accurately find the watershed between the applicability of quantum mechanics and classical mechanics by using mathematical methods (which can only be found through experience or experimental methods). This is why it is difficult for us to find theoretical barriers for the combination of quantum mechanics and classical mechanics, but we can find successful examples of both theory and application.

3.1. Comparison of quantum force calculation results and Newtonian mechanics calculation results for the Earth's orbital system

The combination of classical mechanics and quantum mechanics for use refers to that can describe a system using both classical mechanics and quantum mechanics simultaneously. We can provide multiple such examples and compare the solution of Eq. (2) in a deterministic context with the computational results of Newtonian mechanics. Listed the solutions of the Schrödinger equation for hydrogen atoms under deterministic and uncertain backgrounds. For Eqs. (1) and (2), the solution under deterministic background is applicable to the planetary model (*i.e.*, classical mechanical system), and r is taken as a constant value; The solution under uncertain background is not suitable for planetary models, where r is an uncertain value in the range of 0-∞.

According to Newton's laws of motion, the ideal state for the Earth to move around the Sun is a uniform circular motion. In this case, the magnitude of the centripetal force of the Earth is $F = m\frac{v^2}{R}$, and the magnitude of the Earth's attraction to the Sun is $F = \frac{GMm}{R^2}$. The magnitude of these two forces is equal. In this way, the potential energy of the Earth's revolution under the background of Newtonian mechanics is $V = -\frac{GMm}{R} = -mv^2$. The kinetic energy E_k of the Earth's orbital motion is $\frac{1}{2}mv^2$. $E = \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2$. The total energy of the Earth's orbital motion is $E = \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2$. Next, we will calculate the total energy E based on Eq. (2). Using the de Broglie relationship and $\upsilon = \lambda\nu$, the first term of Eq. (2) is $-\frac{i\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{1}{2}mv^2\psi$. According to $\lambda = h/mv$ or the Hamiltonian operator, the second term of equation (2) is $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{1}{2}mv^2\psi$ (where the energy is the kinetic energy of the Earth). On the premise of acknowledging that the Earth has a definite motion orbit, the third term of equation (2) is $-\frac{GMm}{R}\psi = -mv^2\psi$ (where the energy is the Earth's potential energy). Calculate the algebraic sum of the energies in these three terms, and obtain the energy E of the fourth term in equation (2) as $-\frac{1}{2}mv^2$. This result is consistent with the conclusion calculated using Newtonian mechanics. This indicates that equation (2) as the Schrödinger equation for the Earth's revolution is completely correct and practical. Please note that in $\upsilon = \lambda\nu$, ν is the frequency of the de Broglie wave. Its relationship with the energy of de Broglie waves is E=hν. In reference [18], it was demonstrated that the phase velocity and group velocity of de Broglie waves are consistent, thus avoiding the difficulty of their phase velocity being greater than the speed of light.

3.2. Quantum mechanics calculation methods for hydrogen atoms in the context of planetary models

This is consistent with the calculation method using the Schr ö dinger equation in the context of planetary models (or deterministic backgrounds). It is actually a method of simultaneously using (or mixing for use) quantum mechanics and classical mechanics. The specific approach is to establish a one-dimensional partial differential Schrödinger equation, and take a finite definite value for the distance *r*. This approach satisfies the structure of the planetary model. This method is different from the mathematical calculations in Bohr's planetary model method in pure classical mechanics. For the energy of the ground state hydrogen atom, Bohr calculated it using pure classical methods, while this article calculated it using the Schrödinger equation. The similarity between the two is that they both use the planetary model or the determinacy of classical mechanics.

Eq. (2) is a combination of the derivative of the wave function with respect to time and the Hamiltonian operator acting on the wave function, which is clearly also the Schrödinger equation. Eq. (2) can be successfully applied to classical mechanical systems. This proves that the Schrödinger equation can describe classical mechanical systems (including planetary model systems). We restore the gravitational potential energy function in Eq. (2) to the electromagnetic interaction potential energy function, thus obtaining $-\frac{i\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze}{r}\psi = E\psi$. Compared with Eq. (2), it can be seen that this equation can be used to describe the hydrogen atom in the planetary model. In the context of planetary models, the calculation results of the first three terms of this equation are: $-\frac{i\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{1}{2}mv^2\psi$, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi = \frac{1}{2}mv^2\psi$, $-\frac{Ze}{r}\psi = -mv^2\psi$ (For hydrogen atoms, the mass m in the equation is the electron mass me). If the radius of the electron orbit is a_0, then $-\frac{Ze}{r} = -m_e v^2 = -\frac{e^2}{a_0} = -2624$ kJ/mol. The E in the fourth term $E\psi$ of the equation is equal to -1312 kJ/mol. This method is obviously not Bohr's old quantum theory method, but one of the quantum mechanics methods (using deterministic concepts in quantum mechanics). It is this method that determines that we can simultaneously use quantum mechanics and classical mechanics to describe a constrained system (*i.e.*, we can combine quantum mechanics and classical mechanics to use). We do not deny the solution of " $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze}{r}\psi = E\psi$" in the context of uncertainty (this solution is quantized with a value of $E = -1312/n^2$ kJ/mol). Bohr used old quantum theory to calculate the hydrogen atom in planetary models, mainly using classical electrodynamics methods. The calculation results are the same as those obtained using the Schrödinger equation under deterministic background in this article.

It is this method that determines that we can simultaneously use quantum mechanics and classical mechanics to describe a constrained system (*i.e.*, we can combine quantitative mechanics and classical mechanics to use). The Schr ö dinger equation (especially the steady-state Schrödinger equation) consists of two parts: the potential energy function part and the partial differential part of the wave function. The partial differentiation of wave functions clearly belongs to the category of wave mechanics methods (quantum mechanics methods). It is difficult for us to exclude the expression and calculation of potential energy functions from the scope of classical mechanics methods. It is not difficult to see that as long as one is not absolutely bound by the concept of uncertainty (*i.e. r* takes a certain value), using the Schrödinger equation is equivalent to using both classical mechanics and quantum mechanics simultaneously. It can be said that the rejection of classical mechanical methods (*i.e.*, the belief that the two are incompatible) by quantum mechanical methods stems from a conceptual idea rather than mathematical logic. The derivation and application of the Schrödinger equation for gravitational potential energy more intuitively reflect this point.

Taking hydrogen molecules as an example, references [6,8,14,15] used a new method that combines classical mechanics and quantum mechanics: Using classical electrodynamics to establish the mechanical equilibrium skeleton structure of hydrogen molecules, then establishing the Schrödinger equation for this structure, and using the Schrödinger equation and classical mechanics to calculate the bond length and dissociation energy of hydrogen molecules. There are multiple examples of calculations for diatomic molecules like this [14]. There are over a hundred examples of quantum mechanics calculations under deterministic backgrounds [15].

4. A Ring Electronic Structure Model that Can Qualitatively and Quantitatively Explain the Source of Electron Spin Magnetic Moments

For a hundred years, the specific form of electron spin explaining the source of electron spin magnetic moment has not been well done. The mystery of electron spin magnetic moment is a big dark cloud looming over physics. References [16-17] support the "solid ring electronic structure model". References [8,9] established a wave ring electronic structure model. After using $m=E/c^2$, there is a logical connection between these two electronic structure models. The solid-state ring particle structure ring model is just a small step forward from the point particle model. The wave ring, or wave element particle structure model, is a revolutionary step forward.

Based on the experimental fact that high-energy photons decay into electrons and anti electrons, I assume that electrons are composed of wave rings. Specifically, electrons are composed of circularly polarized photons with clockwise rotation. And anti electrons are composed of circularly polarized photons with anti rotation. A basic plane polarized photon is decomposed into a clockwise circularly polarized photon and an counterclockwise circularly oscillating photon. The momentum and energy of waves are also divided into two parts simultaneously. In this case, the energy and momentum of basic circularly polarized light are only half of those of plane polarized light, but the wavelength and frequency are the same. This determines the following relationships: $p_e = \frac{1}{2}p_{plane}$; $h\nu_{\text{circle}} = \frac{1}{2}h\nu_{plane}$. The way waves form electrons is by transforming circularly polarized photons from linear propagation to propagation along a small circle, thus forming a particle with a stationary center of mass (in this case, the electron radius $r_e = \lambda/2\pi$). The linear momentum of a fundamental circularly polarized photon in a free electron is $p_e = m_e c$, and the energy is $E_e = m_e c^2 = \frac{1}{2}h\nu_{plane}$. Nearby, p_e is the intrinsic momentum of the electron's motion, and m_e is the electron's rest mass. In this way, the spin angular momentum of the electron is $\vec{p}_e \times \vec{r}_e$. The size of the electron radius $\vec{r}_e$ is equal to $\lambda/2\pi$. The intrinsic motion momentum of electrons is a plane wave with a $p_e = \frac{1}{2}p_{\text{plane}}$. So, the spin magnetic moment of the electron is $\vec{L}_e = \vec{p}_e \times \vec{r}_e$.

If the magnetic moment of an electron rotating counterclockwise on a horizontal plane is considered positive, then flipping such an electron loop 180 degrees results in a negative spin magnetic moment for the electron. If only these two values are taken, then $\vec{L}_e = \vec{p}_e \times \vec{r}_e = \pm\frac{1}{2}\hbar$. If we only consider the magnitude of the electron spin magnetic moment, then there are

$$L_e = p_e \cdot r_e = \frac{1}{2}\hbar. \quad (3)$$

Please note that! The direction of the electron spin magnetic moment obtained from the experiment is not exactly two opposite directions, but multiple directions. For planar electromagnetic waves, p_{plane}=2$m_e c$. For plane polarized light and de Broglie waves, the momentum Hermitian operator is

$$\hat{p} =- i\hbar\frac{\partial}{\partial x}. \quad (4)$$

For basic circularly polarized light, p_{circular}=$m_e c$. This determines that the momentum operator of the intrinsic motion of electrons has an additional coefficient of 1/2 compared to the ordinary momentum Hermitian operator.

$$\hat{p}_{\mathrm{e}} = \hat{p}_{circle} =-\frac{i\hbar}{2}\frac{\partial}{\partial x}. \quad (5)$$

The correlation functions of basic circularly polarized light and plane polarized light can both take the form of Eq. (6).

$$\psi\ (x,t) = Ae^{-i2\pi(\nu t - x/\lambda)}. \quad (6)$$

However, compared to the planar polarized light before decomposition, the amplitude A, energy value h ν, and momentum value p of the decomposed circularly polarized light differ by two times. When calculating its partial derivatives, there is no difference in form between the results obtained.

Now let's derive the electron spin operator. For wave functions, we are accustomed to using Eq. (6). However, we assume in reference [7] that electrons are composed of fundamentally circularly polarized light. Therefore, when deriving the electron spin magnetic moment operator, the intrinsic motion operator of the electron must use Eq. (5). Considering the relationship between basic circularly polarized light and electrons, which is represented by the operator $E_{\text{circle}} = \frac{1}{2}E_{plane} = \frac{1}{2}h\nu$, $p_e = m_e c$, $\lambda_e = h/(2m_e c)$. We replace p_e in $L_e = p_e \cdot r_e = p_e\frac{\lambda}{2\pi}$ with the operator $\hat{p}_{\text{circle}}$ in equation (5), and can obtain

$$\hat{L}_e =-\frac{\hbar^2}{2mc}\frac{\partial}{\partial x}. \quad (7)$$

We can verify Eq. (7) by finding $\frac{\partial}{\partial x}\psi =- i\frac{2\pi}{\lambda}\psi$. According to the relationship between the angular momentum of charge and its spin magnetic moment in classical electrodynamics, the magnitude of the electron spin magnetic moment is $\mu_e = \frac{e}{2mc}L_e$. Therefore,

$$\mu_e = \frac{e\hbar}{4mc}. \quad (8)$$

It is not difficult to see that we have quantitatively determined the relationship between the self magnetic moment of an electron and its composition, structure, and internal movement mode. That is, the rigorous logic reveals that the spin magnetic moment of electrons originates from their intrinsic motion. The method is to break through the constraints of the standard model material structure theory and the point particle material structure theory to establish the wave element material structure theory. By replacing the operator L_e in Eq. (7) with the operator $\mu_e = \frac{e}{2mc}L_e$, we can obtain

$$\hat{\mu}_e =-\frac{e\hbar^2}{4m^2c^2}\frac{\partial}{\partial x}. \quad (9)$$

In fact, the orbital magnetic moment of the extranuclear electrons of hydrogen atoms during orbital motion is 274 times that of the spin magnetic moment of free electrons [8]. This is a basic assumption. With it, it is convenient to use planetary models in atomic and molecular systems (that is, to combine classical mechanics with quantum mechanics). Based on this assumption, it can be predicted that conducting Stern Gerlach experiments using alpha particle beam, electron beam, and hydrogen atom beam will gradually increase the experimental effect. The quantitative ratio of the splitting effect is "proton (alpha particle): electron: hydrogen atom"=1:1836:1836 × 274.

The ring electron model (wave element electronic structure model) discussed in this section is the foundation of the planetary hydrogen atom model. It is also one of the theoretical foundations for the combination of classical mechanics and quantum mechanics methods for hydrogen atoms.

5. Chemical Calculation Examples Combining Quantum Mechanics and Classical Mechanics

We have theoretically demonstrated above (especially after deriving the Schrödinger equation for the Earth's revolution) that classical mechanics can be combined with quantum mechanics. Because the wave function and Schrödinger equation in the two elements of Eq. (2) belong to the thinking method of quantum mechanics, and the results calculated from $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi$ and the gravitational potential energy function are both classical mechanical quantities. Additionally, the Eq. (2) is the energy relationship equation of a planetary system. The method of describing macroscopic mechanical systems using the Schrödinger equation is the idea-method of combining classical mechanics and quantum mechanics. We can establish a deterministic mechanical equilibrium system for microsystems. Eq. (2) can be used to describe a uniform circular motion using the concept of force.

Reference [2,8,15] lists several successful computational examples that combine classical mechanics with quantum mechanics. Reference [8-10] provides a clearer explanation of the method of combining classical mechanics with quantum mechanics. The possibility of combining classical mechanics with quantum mechanics has been predicted in theory and realized in practice. This is destined to be a major event in the history of physics development.

6. Conclusion

The research work introduced in this article has formed a complete chain of evidence: firstly, theoretically, the Schrödinger equation for gravitational potential energy applicable to the combination of classical mechanics and quantum mechanics can be derived (Established the Schrödinger equation that can describe macroscopic mechanical systems, intuitively expressing that classical mechanics and quantum mechanics can be combined for use); Secondly, it can provide a convenient wave element electronic structure model for the combination of classical mechanics and quantum mechanics, and this model can successfully explain the source of the burst spin magnetic moment; Thirdly, many successful examples of the combination of classical mechanics and quantum mechanics have been found. The first point is the mathematical evidence of quantum mechanics. The second point is the theoretical evidence of material structure, which is also a theoretical advantage (solving the problem of the source of electron spin magnetic moment that cannot be solved by other theories). The third point is practical evidence (or theoretical application evidence). These contents are referred to as positive evidence chains.

By exporting and using the Schrödinger equation of gravitational potential energy, it intuitively reflects that the Schrödinger equation itself does not exclude the joint use of quantum mechanics and classical mechanics. This article supports the validity of planetary models, which still have a certain range of applicability in quantum mechanics. Taking hydrogen atoms as an example, a planetary model that only wants to obtain the deterministic ground state energy of the system is still applicable. If one wants to obtain quantized results, one can reject the planetary model in classical mechanical equilibrium systems.

This article does not deny the existing mathematical formal system of quantum mechanics, but suggests the conditional use of concepts such as uncertainty, state superposition, and coherence in existing interpretations of quantum mechanics. From another perspective, the research results of this article indicate that using methods compatible with classical mechanics and quantum mechanics can obtain accurate computational results.

The ability to combine quantum mechanics and classical mechanics for use means that the scientific revolution of this century is about to break out.

7. Discussion

Readers who have accepted existing physics theories generally choose negative answers to the following questions. The author has provided a detailed analysis of these issues. Please ask the reader to choose a new answer after reading these analyses.

7.1. Is replacing the electromagnetic potential energy in the original Schr ö dinger equation with gravitational potential energy still the Schrödinger equation? Is there a logical problem with this operation?

To answer this question, the first thing to consider is the composition, structure, and important position of the Schrödinger equation in quantum mechanics. The original Schrödinger equation was a partial differential equation in which the Hamiltonian operator acted on the wave function. To be more specific, the steady-state Schrödinger equation is a differential equation that contains a potential energy function and a derivative of the wave function. The bound state systems in the real world are not only bound by electromagnetic interactions, but also by gravitational interactions. The composition and structure of equation (2) are also the same. Therefore, equation (2) is also the Schrödinger equation. But it is not the original Schrödinger equation (we will discuss the difference between the two in **Section** 7.9). However, equation (2) can describe the Schrödinger equation for macroscopic bound systems. At the beginning of the birth of quantum mechanics, quantum mechanics was also called wave mechanics due to the use of wave functions and the treatment of the described system as completely (purely) waves (this is the process of "wave functionalization"). “Wave functionalization for Using Schrödinger equation” is an important step in the application of quantum mechanics. Therefore, it can be said that using the Schrödinger equation is equivalent to using quantum mechanical methods (at least involving quantum mechanical methods). Even if the Schrödinger equation is not the axiomatic starting point of quantum mechanics, but an expression for the unitary evolution of states in Hilbert space, it cannot be denied that "using the Schrödinger equation is using quantum mechanics methods" (matrix mechanics has also been proven to be equivalent to wave dynamics). As long as the potential energy function in the Hamiltonian operator can be a gravitational potential energy function, the potential energy function in the Schrödinger equation can be replaced with a gravitational potential energy function.

7.2. Is the Schrödinger equation of gravitational potential energy meaningless?

The Schrödinger equation for gravitational potential energy is a differential equation containing wave functions and Hamiltonian operators that can describe macroscopic systems. This indicates that the Schrödinger equation can be used to describe planetary model systems (classical mechanics and quantum mechanics can be combined to use, meaning they are compatible). Since gravitational potential energy can be used to describe classical mechanical systems (such as planetary model systems, etc.), the Schrödinger equation for electromagnetic interaction potential energy can also be used to describe classical electrodynamic systems. For example, the equation $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\boldsymbol{\psi}-\frac{Ze^2}{r}\boldsymbol{\psi}=\boldsymbol{E\psi}$ can be used to describe both hydrogen atom systems in the context of uncertain electronic motion states and Bohr hydrogen atom model systems (hydrogen atom planet model systems). It can also be used to describe a classic electrodynamic constraint system in which a small lamp grass ball rotates around a large lamp grass ball [This is equivalent to the mass m in equation (1) being within the macroscopic range]. As long as there are enough examples of such calculations, it can change the inherent concept of humanity (*i.e*. the idea that "classical mechanics and quantum mechanics are incompatible") and give birth to new methods of quantum chemistry and quantum mechanics. When using the Schrödinger equation to describe macroscopic systems, the concept of "uncertainty" can be avoided and a definite value of r can be taken. This conclusion is precisely one of the meanings of the Schrödinger equation for gravitational potential energy (classical mechanics and quantum mechanics can be compatible, and both classical mechanics and quantum mechanics methods can be used simultaneously in the same system). The classical mechanical system cannot completely exclude the Schrödinger equation. This is a brand new concept that supports corresponding new theories and methods. We have discussed the significance of the research work introduced in this article multiple times in different places.

7.3. Has the author achieved the goal of combining classical mechanics methods with quantum mechanics for use? Can the number of successful application instances be large enough to exclude randomness?

Section 3 of this article introduces two computational examples that combine classical mechanics and quantum mechanics to use. Reference [1,2] introduces a detailed method for this joint use (taking a hydrogen molecule as an example). Reference [2] introduces several computational examples of small diatomic molecules (all of which are joint examples of classical mechanics and quantum mechanics). Reference [15] introduces over 100 calculation examples of atoms (or ions), all of which are under deterministic background. If there are more than three calculation examples of the same type, coupling factors can be excluded. Moreover, we have provided theoretical support for this method. For example, the Schrödinger equation, which can be used to describe classical mechanical systems, was derived, and a ring electronic structure model supporting atomic molecular planetary models was established. We do not completely deny the existing mathematical formal system of quantum mechanics, but suggest modifying the existing explanatory system of quantum mechanics. Therefore, existing examples of quantum mechanics applications are not counterexamples to the conclusions of this article. In this way, the old theory of quantum mechanics cannot be used as a standard to deny the successful application of the theory presented in this article.

The successful application of the combination of classical mechanics and quantum mechanics cannot be explained by coincidence. If you want to choose the negative answer to question 7.3, you must logically prove that the calculation method provided by the author has logical errors. It is not a fair approach to deny the conclusions of this article by using the phrase 'not in line with the existing theoretical framework of physics'.

7.4. What is the significance of comparing the results of classical mechanics calculations and quantum mechanics calculations for the same system?

In **Section 3**, we made the above comparison. If the calculation results of classical mechanics are consistent with those of quantum mechanics, it can indicate that the Schrödinger equation for gravitational potential energy is correct and effective.

It can also indicate that there is still a market for planetary models and determinism in quantum mechanics. Combining quantum mechanics with classical mechanics for use is clearly a new approach to quantum mechanics.

7.5. How did the author combine classical mechanics with quantum mechanics to use?

The most crucial thing is to break free from the shackles of old ideas. If old ideas are not changed, it is difficult to create new things. Even if I come up with something new, I won't believe it myself (just like the situation where "Planck came up with the idea of energy discontinuity, but later he didn't believe it himself"). If one does not break free from the constraints of old ideas, the difficulty of accepting new things created by others becomes even greater. This is a phenomenon repeatedly proven by human history. The new concept we have established is that the Schrödinger equation of gravitational potential energy can be used to describe classical macroscopic systems. This leads to compatibility between classical mechanics and quantum mechanics. In other words, a bridge has been established between classical mechanics and quantum mechanics. There are unified axioms in complex physical or objective reality (there are no absolute boundaries or conflicts between them). We use a ring electronic structure model to revive the planetary model (not completely reject it). We have established a classical electrodynamic equilibrium structure for the described binding system. It has been demonstrated through theoretical and practical examples that the Schrödinger equation for hydrogen atoms can also be used to describe planetary models of hydrogen atoms. As long as we change the concept that "uncertainty is absolute in microscopic systems" (i.e., r can take a constant value), the equation $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze^2}{r}\psi = E\psi$ can be used to describe the Bohr hydrogen atom (planetary model hydrogen atom). Specifically, even for microsystems, the choice of uncertainty is conditional (under certain conditions, certainty can be chosen, that is, certainty and uncertainty can be chosen based on purpose or conditions). The electronic structure model we created is necessary for establishing a planetary model microsystem. It can provide convenience for the combination use mentioned in the title of this article (see **Section** (4) of this article and references [8,14] for details).

Fair and honest readers can easily repeat this calculation method themselves.

7.6. Are there significant errors in existing physics theories? Or, are there still some unsolved mysteries?

As long as the development of scientific theories has not reached its end (no ultimate theory has emerged), the answer to the above question is affirmative. In this way, we must be cautious when denying new perspectives, theories, and ideas. Here, we only care about whether there are dark clouds in the current physics sky. Apart from that, the problem of the source of electron spin magnetic moment (the specific form of electron spin) has not been solved. This is a dark cloud in the sky in physics. New perspectives and theories that can dispel this dark cloud should be given due attention.

7.7. What are the similarities between the conclusion that classical mechanics and quantum mechanics can be combined to use and the conclusion that energy changes are discontinuous (energy quantization) in the past?

It is necessary to compare whether classical mechanics and quantum mechanics can be combined with the high-frequency formula and low-frequency formula of blackbody radiation (by interpolating the two). Wien and Rayleigh-Jeans proposed blackbody radiation formulas for different frequency bands. Before Planck, no one thought that the two could be combined (*i.e.*, no one thought of interpolating between the two). Before the author of this article, no one had thought of combining quantum mechanics with classical mechanics for use. The "union" discussed in this article is very similar in form to Planck's union (interpolation between the two). The result of Planck interpolation and its explanation (energy quantization) is a new perspective with significant importance. The author of this article also provided an explanation for the result to combine use of classical mechanics and quantum mechanics, stating that quantum mechanics and classical mechanics can be compatible, and there is still a certain market for determinacy in the microscopic world. This explanation is also a completely new perspective. That is to say, in terms of the novelty of the viewpoint, the author's behavior in this article is similar to Planck's behavior back then.

In the field of science, the initial reactions (not widely accepted in a timely manner) caused by the two are also very similar. At the beginning of Planck's proposal of energy quantization, the physics community basically denied it. Both Planck himself strongly denied his proposal of energy quantization. At present, the number of people who reject the new viewpoint that "classical mechanics and quantum mechanics can be used together" far exceeds the number of people who affirm it.

7.8. Can we preserve the mathematical formal system of quantum mechanics and only modify the explanatory system of quantum mechanics?

No one can deny that in the scientific community, there are far more people dissatisfied with the explanatory system of quantum mechanics than with the mathematical formal system of quantum mechanics. This phenomenon provides the soil for "transforming the explanatory system of quantum mechanics while preserving the mathematical formal system of quantum mechanics".

Admitting that classical mechanics and quantum mechanics can be combined to use means acknowledging that the methods and mathematical systems of quantum mechanics are still usable. This can only leave room for 'changing the old quantum mechanics interpretation system'. The first method: use the three-dimensional Schrödinger equation, where the radius r of the sphere takes an uncertain value from zero to infinity. Then find the solution to the equation. This is a method in the mathematical system of quantum mechanics under uncertainty. The second method is to use the one-dimensional Schrödinger equation, where the interaction distance r takes a determined single value; then, solve the Schrödinger equation. This is still a method in the mathematical formal system of quantum mechanics, but it is already a quantum mechanics method under deterministic background, which can use planetary models and classical mechanics. The transition from the first method to the second method only changed the old interpretation of quantum mechanics. It can be seen that the answer to the question in the title of this section is affirmative. From a mathematical perspective, the first and second methods are two purely mathematical alternatives. We can choose according to our needs (we cannot exclude one from a purely mathematical perspective and choose the other). From the perspective of physics (i.e., from the perspective of reality), the difference between classical mechanics and quantum mechanics does not mean that they are incompatible (with irreconcilable contradictions). If you want to deny that classical mechanics is incompatible with quantum mechanics, you must prove that the classical electrodynamic potential energy function and gravitational potential energy function cannot be used in the Schrödinger equation. Or prove that the mass m in the Schrödinger equation must be less than a gauge limit.

7.9. Which is more correct, equation (1) or equation (2)?

Equation (1) is the well-known one-dimensional Schrödinger equation. Its first term is different from the first term of Eq. (2). One of these two first items should be incorrect. Next, we will take hydrogen atoms as an example to further analyze based on the third section. Let the right side of the equal sign on equation (1) be equal to $E\psi$, and we can obtain $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze^2}{r}\psi = E\psi$. This is also the Schrödinger equation for hydrogen atoms. The solution under its planetary model background is -1312 KJ/mol (which is also the ground state energy of hydrogen atoms). Let the left side of equation (1) is also equal to $E\psi$, we can obtain $i\hbar\frac{\partial}{\partial t}\psi = E\psi$. Only when these two E are equal can Eq. (1) be guaranteed to hold. Otherwise, Eq. (1) is incorrect. By calculating the partial differential in $i\hbar\frac{\partial}{\partial t}\psi = E\psi$, $E=h\nu$ can be obtained. For electrons undergoing constrained motion, $\upsilon = \lambda\nu$, $\lambda = h/m_e\upsilon$. In this way, $E= h\nu = m_e\upsilon^2 =$ 2624 KJ/mol (according to Bohr's original method of calculating the hydrogen atom, the same result can also be obtained). The left side of equation (1) is not equal to the right side (and both the "symbol" and "size" are not the same). It can be seen that equation (1) is incorrect, while the calculation results in the third section indicate that equation (2) is usable.

7.10. Why are the "Schrödinger equation of gravitational potential energy and its significance" and "methods that can combine classical mechanics with quantum mechanics" difficult to be accepted by the academic community?

Both humans and human society have a cognitive inertia. This inertia leads to a relatively difficult process for new things to be accepted. For humans, they unconsciously maintain what they have already accepted. Deep recognition can also generate faith. Once faith is formed, it is inevitable to be emotional. Due to the widely accepted interpretation system of quantum mechanics and the notion that "quantum mechanics is incompatible with classical mechanics", it is normal for "new ideas that do not conform to these widely accepted concepts to encounter temporary trust crises".

There have been numerous instances in history where new theories or viewpoints were initially rejected and suppressed. The history of Copernicus' geocentric theory is a history of blood and tears. At the beginning of the birth of Riemannian geometry, the research results were also severely suppressed, and famous journals refused to publish them. Only a mathematical journal called Kleer withstood tremendous pressure and continued to publish articles introducing the research results of Riemannian geometry. The concept of energy quantization proposed by Planck did not receive much attention in the first five years. In the following 10 years, Planck regretted his initial statement and tried to conceal the assumption of energy discontinuity on many occasions. It has been working hard for nearly 7 years, hoping to eliminate this assumption. The attitude of Planck towards his innovative achievements can predict the views of orthodox theoretical physicists at that time on this quantization hypothesis. The research results introduced in this article are aimed at challenging the existing theoretical framework of physics. It is possible to predict the treatment it will receive in the 5 years after its proposal.

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Appendix A. Derivation of the Schrödinger Equation for Gravitational Potential Energy

There are several ways to express the momentum, kinetic energy, and potential energy of a planet in an ideal planetary motion system. As long as the values of the same physical quantities in different expressions are ensured to be the same, it is OK. Below, we have listed these relationships in a planetary system in a table.

Table S1. Several expressions of momentum of energy in planets and planetary systems

Content	**Planetary Velocity, υ**	**p Momentum,p**	**E_k Kinetic Energy, E_k**	**V Potential Energy,V**	**Total Energy of System, E_{total}**
Classical mechanical representation	υ	$m\upsilon$	$\frac{1}{2}m\upsilon^2$	$-\frac{GMm}{r}$ =− $m\upsilon^2$	$-\frac{1}{2}m\upsilon^2$
De Broglie wave representation	υ=$\lambda\nu_d = \frac{E_k}{p_d}$, υ=$E_k/m\upsilon$	$\frac{h}{\lambda_d}$	$\frac{1}{2}h\nu_d$	$-h\nu_d$	$-\frac{1}{2}m\upsilon^2$ = $-\frac{1}{2}h\nu_d$

From Table S1, it can be seen that the following relationship exists:

$$m\upsilon^2 = h\nu. \qquad \text{(S1)}$$

Due to the minimal impact of removing subscripts on understanding the following content, we will omit the subscripts of physical quantity symbols below.

In quantum chemistry, the Schrödinger equation for a hydrogen atom is $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{Ze^2}{r}\psi = E\psi$. This is the fundamental equation of quantum mechanics chosen by Schrödinger based on intuition. By replacing the potential energy function in this equation with the gravitational potential energy function in the bound system, the macroscopic system Schrödinger equation describing the planetary revolution can be obtained.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{GMm}{r}\psi = E\psi. \qquad \text{(S2)}$$

The E in the equation is the energy of the bound planetary system (excluding m_0c^2. At this point, it is the same as the Schrödinger equation for hydrogen atoms — the Schrödinger equation for microscopic systems). According to classical mechanics, Viry's theorem, and the values in Table S1, equation (S2) can be calculated to obtain the total energy of the ideal bound state system in the planetary model as E=$-\frac{1}{2}m\upsilon^2$. As long as the state of such a binding system remains unchanged, its total energy will not change (still is $-\frac{1}{2}m\upsilon^2$). In order to establish the steady-state Schrödinger equation of "time changes while the state of the system remains unchanged" (which must contain the partial derivative $\frac{\partial}{\partial t}\psi$ of the wave function with respect to time), we assume that

$$f(x,t)\frac{\partial}{\partial t}\psi = E\psi. \qquad \text{(S3)}$$

Find the specific form of the function term $f(x,t)$ and combine equations (S2) and (S3) to achieve the goal. Substituting $E = -\frac{1}{2}mv^2$ into equation (S3) and considering equation (S1) and $\frac{\partial}{\partial t}\psi = -i2\pi v\psi = -i\frac{hv}{\hbar}\psi$, , we can obtain: $f(x,t) = -\frac{1}{2}hv\psi \div \left(-i\frac{hv}{\hbar}\right)\psi$ and $f(x,t) = -\frac{i\hbar}{2}$. So, the steady-state Schrödinger equation for an ideal planetary system is

$$-\frac{i\hbar}{2}\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi - \frac{GMm}{R}\psi . \qquad \text{(S4)}$$

or

$$-\frac{i\hbar}{2}\frac{\partial}{\partial t}|\psi\rangle = \hat{H}|\psi\rangle. \qquad \text{(S5)}$$

In equation (S5), $V = -\frac{GMm}{r}$. This is different from $V = -\frac{Ze^2}{r}$ in the same form of the Schrödinger equation. The $-\frac{i\hbar}{2}$ in equation (S4) is also different from $\hbar$ in the original Schrödinger equation.