Two new methods based on 6x ± 1 equations to break all types of evens in sum of two primes

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Abstract.
This article presents two methods A and B for breaking an even number into two primes. Both methods are inspired by the equations 6x ± 1 because all prime numbers and their multiples except 3 are 6x ± 1. Both methods can be very useful for even conversion in sums of two primes or to study the Goldbach's strong conjecture. Both methods can have applications in computer science.

Keywords. Method. Prime. Composite. Goldbach's strong conjecture. Congruence. Modulo. Sum of two primes.

Abbreviations. *GSC : Goldbach's strong cnjecture. P : prime. C : composite.*

INTRODUCTION

I have recently reported in several papers that Goldbach's strong conjecture (GSC) depends on the presence of two equidistant primes p and q such that $p < S/2$ and $q > S/2$ and such that $S/2 – p = q – S/2$ therefore $S = p + q$ [1 - 6]. In addition I have shown that GSC depends closely on the gaps between primes especially gaps = 6 or 4 [1, 5]. I have also shown that GSC might hold true to infinity [4].

In the present paper, I give a detailed description of two methods (A and B) to convert an even number into two primes according to GSC. Both of these methods are based on the equations $6x \pm 1$. Actually, there are three types of even numbers 6X; 6X - 2 and 6X + 2 whose conversion into sum of two prime numbers depends on the equations $6x \pm 1$ given that $6X = (6x – 1) + (6x' + 1)$; $6X – 2 = (6x – 1) + (6x' – 1)$ and $6X + 2 = (6x + 1) + (6x' + 1)$.

RESULTS

Method A

I. Even S = 6X

1. Be an even S ≥ 8 (or ≥ 4 for any integer n = S/2) . Determine whether 6X ; 6X + 2 or 6X – 2.
2. S = 6X → S = Odd1 + 0dd2 = (6x + 1) + (6x' – 1) or S = (6x - 1) + (6x' + 1)
3. Calculate all Odds 6x – 1 < S/2 (denoted **O6--**) and 6x – 1 > S/2 and < S (**O6-+**). Caculate all 6x + 1 < S/2 (**O6+-**) and all 6x + 1 > S/2 and < S (**O6++**). Exclude any 3n or 5n or the prime factors of S. Either S 6X = (O6--) + (O6++) or S 6X = (O6+-) + (O6-+).
4. In function of the Unit digit of S → select (O6--) and (O6++) with right partition unit digits. Do the same with (O6-+) and (O6+-)
5. Determine p < S/2 or π (S/2).
6. S = 6X → Calculate S – (6x – 1) = 6X + 1 ; and S – (6x + 1) = 6X – 1. Both 6X + 1 or 6X – 1 can either be prime or composite (C) but ≠ 3n.
7. Calculate congruence of S and (O6++) or (O6+-) modulo (p). If S ≡ (O6-+) mod (p) or S ≡ (O6++) mod (p) → Calculate **S = xp + r** ; **O6-+ = x'p + r** and **O6++ = x''p + r**.
8. If x – x' = 1 or x – x'' = 1 → S = p + q (p and q primes). If x – x' > 1 or x – x'' > 1 then S = p + c or S = p + c' such that c = np or c' = n'p.

Example 180.

180 = 6 x 30 therefore 180 is 6X. S/2 = 90.
Tables 1 show 6x – 1 and 6x + 1 numbers < 90 and > 90 but < 180.

Tables 1 *: Selection by the 6x ± 1 equation of odd numbers < S = 180 and either < S/2 = 90 or > S/2.*

O6--	O6++
11	97
17	103
23	109
29	121
41	127
47	133
53	139
59	151
71	157
77	163
83	169
89	

O6-+	O6+-
7	101
13	107
19	113
31	119
37	131
43	137
49	143
61	149
67	161
73	167
79	173

Tables 2 show the next step which is selection of 6x – 1 and 6x + 1 numbers by their Unit digits. Only primes with right unit digits are involved in sum S. Prime numbers p < S/2 and q > S/2 such that S = p + q are selected (Tables 3). Results are confirmed by the calculation of congruence modulo (p) to explain why S = p + q in some cases and why S = p + c (c composite) in the other cases. Table 3 shows the final conversion of S = 180 in sums of two primes.

Table 2 *: Selection by Unit digits of odd numbers < S = 180 and either < S/2 = 90 or > S/2. Let us note odd numbers < 180 such XN with N their unit digit. Then 180 = XN1 + XN2 such that XN1 < S/2 and XN2 > S/2. We have 180 = X**1** + X**9** or 180 = X**9** + X**1** ; and 180 = X**3** +X**7** or 180 = X**7** + X**3**. Indeed 0 (unit digit) = 1 + 9 or 0 (unit digit) = 3 + 7.*

O6-- (1)	O6++	O6--	O6++
11	109	17	103
41	139	47	133
71	169	77	163

29	121	23	97
59	151	53	127
		83	157
		3	177

O6-+	O6+-	O6-+	O6+-
31	109	7	113
61	119	37	143
71	149	67	173

19	101	13	107
49	131	43	137
79	161	73	167

Table 3 *: S = p + q such that p and q are primes with p< S/2 and q > S/2 (S = 180 ad S/2 = 90).The method gives all possible sums.*

p	q
3	177
7	173
13	167
17	163
19	161
23	157
29	151
31	149
41	139
43	137
71	269
53	127
67	113
71	109
73	107
79	101
83	97

Calculation of congruence

- 180 = c + q = 131 + 49 → 180 ≡ 131 mod (7) → 180 – 131 = 7n and → 180 – 131 = 7 x 7 = 7^2. 180 – 131 = X → 180 = (7 x **25**) + 5 and 131 = (7 x **18**) + 5 → **25 – 18 = 7 > 1** → X is composite = 7n = 7^2 (composite).
- 180 = c + q = 77 + 103. 180 ≡ 103 mod (7) and 180 ≡ 103 mod (11) → 180 – 103 = (7 x 11)n → 180 – 131 = 7 x 11 = 77 (composite).
- 180 = p + q = 83 + 97 → 180 ≡ 97 mod (83) → 180 = (83 x 2) + 14 and 97 = (83 x 1) + 14. 180 – 97 = X and so X = (**2** x 83) + 14 – (**1** x 83) + 14 → **2 – 1 = 1** → X = p = 83 (prime). Given that 180≇97 mod (p ≠ 83), **83 is prime** (*p ≠ 83 means any p different from 83*).
- 180 = p + q = 19 + 161 . 180 ≡ 161 mod (19) → 180 = (**9** x 19) + 9 and 161 = (**8** x 19) + 9. 180 – 161 = X and so X = (**9** x 19) + 9 - (**8** x 19) + 9. → 9 **– 9 = 1** → X = p = 19 (prime). Given that 180 ≇ 161 mod (p ≠ 19), 19 **is prime** (*p ≠ 19 means any p different from 19*).

II. Even S = 6X – 2.

1. Be S = 6X – 2.

(6X – 2) – (6x + 1) = 6X – 3 = 3N. (6X – 2) – (6x – 1) = 6X – 1.

Determine π (S/2) with p < S/2. Calculate S – p. Either S – p = 3N or S – p = 6X – 1. Then, remove all **3N**. Note S – p = 6X – 1 leads to either primes or composites ≠3N*). S – p allows us to search for prime numbers p and q such that p + q = S.*

<u>**Example S = 154 (Table 4)**</u>. Note 154 is 6X + 4 and 6X + 4 = 6X – 2.
Either 154 = 150(6X) + 4 = (6 x 25) + 4 ; or 154 = 156 (6X) – 2 = (6 x 26) – 2.
π (154/2) = π (77) = {3 ; 5 ; 7 ; 11 ; 13 ; 17 ; 19 ; 23 ; 29 ; 31 ; 37 ; 41 ; 43 ; 47 ; 53 ; 59 ; 61 ; 67 ; 71 ; 73}.

- *<u>**Table 4**</u> . Search for primes by S – p (p < S) (S = 154 ; S/2 = 77). X in the right column means P or C.*

π (154/2)	**154 – p = Xn**	**Xn**
3	151	X
5	149	X
7	147	3N
11	143	X
13	141	3N
17	137	X
19	135	3N
23	131	X
29	125	3N
31	123	3N
37	117	3N
41	113	X
43	111	3N
47	107	X
53	101	X
59	95	5N
61	93	3N
67	87	3N
71	83	X
73	81	3N

S = p + q	
P < S/2	**> S/2**
3	151
5	149
<u>11</u>	<u>143</u>
17	137
23	131
41	113
47	107
53	101
71	83

<u>**Calculation of congruence**</u>

- 154 = 3 + 151 ; .154≡151 mod (3) → 154 = (3 x 51) + 1 and 151 = (3 x 50) + 11 → 51 – 50 = 1 → Given that 154≆151 mod (p > 3) 151 is prime.
- 154 = 11 + 143. Give that 154 and 143 share two common factors 11 and 13, 143 is not prime.
- 154 = 17 + 137. 154≡137 mod (17) → 154 = (9 x 17) + 1 and 137 = (8 x 17) + 1 → 9 – 8 = 1 → Given that 154≆151 mod (p ≠ 17) 137 is prime.
- And so on.

Finally, S = 154 is broken 8 times in sum of two primes (Table 5). *Note that this method gives all possible sums of p + q for any even S ≥ 8 with q > p or S ≥ 4 if q ≥ p.*

- ***Table 5*** *: Final conversion S = p + q for S = 154.*

S = p + q	
P <S/2	**q > S/2**
3	151
5	149
17	137
23	131
41	113
47	107
53	101
71	83

- Note Units digits are ignored with S = 6X – 2 numbers because we calculate S – p = X such that p and X have the right unit digits.

III. Even S = 6X + 2.

2. Be S = 6X+ 2.

(6X + 2) – (6x – 1) = 6X + 3 = **3N**. (6X + 2) – (6x + 1) = 6X + 1

Determine π (S/2) with p < S/2. Calculate S – p. Either S – p = 3N or S – p = 6X + 1. Then, remove all **3N (Table 6)**. S – p = 6X + 1 with 6X + 1 prime or composite (c).

- ***Table 6***. *Example 98. Calculation of 98 – p with p < 98/2 = 49. Numbers 3N or 5N are shown. The numbers 5N are recognized by their unit digit = 5.*

Π (98/2)	98 – p	Xn
3	95	5N
5	93	3N
7	91	X
11	87	3N
13	85	5N
17	81	3N
19	79	X
23	75	3N
29	69	3N
31	67	X
37	61	X
41	57	3N
43	55	5N
47	51	3N

Calculation of congruence

- 98 = 7 + 91 to exclude because 91 and 98 sharing one common factor.
- Note prime factors of S can be excluded since the begining as aforementionned.
- Conversion S = p + q for S = 98 are therefore 98 = 19 + 79 ; 98 = 31 + 67 and 98 = 37 + 61. The method gives all possible sums.

Method B

IV. **Algorithm based on a postulate analogous to Goldbach's strong conjecture (GSC)**

1. ***Proposition A***: « *All primes p and q are equidistant and symmetrical at an integer n = p +q/2 except 2 and 3* ». Therefore p + q = 2n. ***This is true*** because any sum of two odds is even. If you add together the primes p and q, you always get an even number S ≥ 4 and the two primes are equidistant at S/2.

2. ***Proposition B***: « *Any even number can be broken down into the sum of two primes* ». Even if any even number can be the sum of any two odd numbers (*Proposition A*), ***we can't deduce from this that any even number ≥ 4 is the sum of two primes***. Even if the addition of two known primes always produces an even number, *we can't deduce from this that an even number can be broken into the sum of two primes*. Hence *breaking any even number ≥ 4 into the sum of two primes cannot be true until it has been demonstrated*. Till now, this propisition known as GSC is not mathematically solved.

3. *Proposition C*: « *A proposition is true until proven false by a counterexample or mathematical demonstration* ». ***We can then assume that for any natural number n ≥ 4 there exists a value t such that t < n and such that n - t and n + t are both primes***.

- Let p = n - t and q = n + t and thus 2n = p + q. This proposition is analogous to the GSC. It can be applied to break an even number into the sum of two primes until a counterexample is found. Note here q ≥ p. GSC means an even S can be broke down to two primes p and q such that S = p + q and such that p < S/2 and q > S/2.

IVa. Even S = 6X.

To break an even 6X in two primes we calculate n = 6X/2 and then n – t and n + t. Because 6X – (6x – 1) = 6X + 1 ; and 6X – (6X + 1)= 6X – 1 ; **t** = 6x – 1 or **t** = 6x + 1. Let us pose n = S/2 and then we calculate S/2 – (6x – 1) and S/2 + (6x – 1) ; or S/2 – (6x + 1) and S/2 + (6x + 1).

Example S = 204 and S/2 = 102. We first calculate 102 – p such that p < 102 (we add 1 although not prime). Then we calculate 102 – C with C a composite odd number ≠ 3n (Table 7). Both p and C are 6x ± 1.

- ***Table 7*** *: Search for primes by S/2 – p (p primes < S/2) and S/2 – C (composites). Example S = 204 and S/2 = 102. Primes p and q are highlighted.*

102 – p	p (except 1)	102 + p
101	**1**	103
99	3	105
97	**5**	107
95	7	109
91	**11**	113
89	13	115
85	17	119
83	19	121
79	23	125
73	**29**	131
71	31	133
65	37	139
61	41	143
59	43	145
55	47	149
49	53	155
43	59	161
41	**61**	163
35	67	169
31	**71**	173
29	73	175
23	**79**	181
19	83	185
13	**89**	191
5	**97**	199
1	101	203

102 – C	C ≠ 3n	102 + C
77	25	127
67	**35**	137
57	45	147
53	**49**	151
47	**55**	157
37	**65**	167
27	75	177
25	77	179
17	85	187
11	91	193
7	95	197

Table 8 shows how 204 can be broken 14 times in two primes. This method gives all possible sums p + q.

Tables 8. *S = p + q. S = 204*

204 = p + q
5+199
7+197
11+193
13+191
23+181
31+173
37+167
41+163
47+157
53+151
67+137
73+131
97+107

IVb. Even S = 6X + 2 and S/2 = 6X - 2

(6X – 2) – (6x + 1) = 6X – 3 = **3N**. And (6X – 2) + (6x + 1) = **6X – 1**.
(6X – 2) – (**6x – 1**) = **6X – 1**. And (6X – 2) + (**6x – 1**) = 6X **–** 3 = **3N**.
For **n = S/2 = 6x – 2**, we can't use the (n – t) and (n + t) methods such as (t = 6x - 1) or (t = 6x + 1) because we'll always have a 3N and therefore never S = p + q.
As shown below (Tables 9 + 10 ; n = S/2 = 124) using numbers t = 6x – 1 or t = 6x + 1 we always get 3N numbers and therefore cannot break S = 248 in two primes. Numbers 6x – 1 can be obtained by 5 + 6n equation while the 6x + 1 ones by 7 + 6n ($n \geq 0$).

Example S = 248. 248 = (6 x 41) + 2 ; and 124 = (6 x 20) + 4 → 248 is 6x + 2 and 248/2 = 124 is 6x + 4 or 6x – 2. We then calculate n – t and n + t such that n = S/2 = 124 and t = 6x – 1 numbers (Table 9).

- ***Table 9*. S = 248**. *We calculate n – t and n + t such that n = S/2 = 124 and t = 6x – 1. The 6x – 1 numbers are obtained by 5 + 6n equation. We have no case S = p + q. Note 3N numbers present on each line.*

124 – t	t = 6x – 1	124 + t
119	5	129
113	11	135
107	17	141
101	23	147
95	29	153
89	35	159
83	41	165
77	47	171
71	53	177
65	59	183
59	65	189
53	71	195
47	77	201
41	83	207
35	89	213
29	95	219
23	101	225
17	107	231
11	113	237
5	119	243

We then calculate n – t and n + t such that n = S/2 = 124 and t = 6x + 1 numbers. We get the same results as with t = 6x – 1 due to constant presence of the 3Ns, which prevents us from breaking the number into two primes (see below). Note 6x + 1 odd numbers are obtained with 7 + 6n equation (Table 10).

- ***Table 10*** : Example S = 248. *We calculate n – t and n + t such that n = S/2 = 124 and t = 6x + 1 numbers obtained by 7 + 6n equation. We have no case S = p + q. Note 3N numbers present on each line.*

124 – t	t = 6x + 1	124 + t
117	7	131
111	13	137
105	19	143
99	25	149
93	31	155
87	37	161
81	43	167
75	49	173
69	55	179
63	61	185
57	67	191
51	73	197
45	79	203
39	85	209
33	91	215
27	97	221
21	103	227
15	109	233
9	115	239
3	121	241

While by contrast, we can use the odd-numbers which are multiples of 3 noted here as **O3N**, which are in order 3; 9; 15; 21; 27; 33....O3N. We then set t= O3N and calculate **n – O3N** and **n + O3N** for S = 248 and S/2 = 124. The O3N allows us to break 248 in two primes as shown below (Table 11).

- ***Table 11*** *: Breaking an even number S = 248 ; S/ 2 = 124 which is 6X – 2 in two primes by calculating S/2 – t and S/2 + t such that t = O3N (note n = S/2). Primes p and q are highlighted.*

124 - O3N	O3N	124 + O3N
121	3	127
115	9	133
109	**15**	139
103	21	145
97	**27**	151
91	33	157
85	39	163
79	45	169
73	51	175
67	**57**	181
61	63	187
55	69	193
49	75	199
43	81	205
37	**87**	211
31	93	217
25	99	223
19	**105**	229
13	111	235
7	117	241

The number is therefore broken 6 times in two primes as below (Table 12). The method gives all possible sums.

- ***Table 12*** *: S = p + q with S = 248.*

S = p + q
7+241
19+229
37+211
67+181
97+151
109+139

IVc. Even S = 6X - 2 and S/2 = 6X + 2
The even S = 196 and S/2 = 98.
196 = (6 x 32) + 4 is 6X - 2 whereas 98 = (6 x 16) + 2 therefore 6X + 2.
(6X + 2) – (6x – 1) = 6X + 3 = **3N**. And (6X + 2) + (6x – 1) = **6X + 1**
(6X + 2) – (**6x + 1**) = **6X + 1**. And (6X + 2) + (**6x + 1**) = 6X **+** 3 = **3N**. Therefore we cannot use n – t ad n + t such that t = 6x – 1 neither t = 6x + 1 because as seen with the previous case ; there will always be 3N numbers that prevents us from breaking the even S in sum of two primes. We only use O3N this time. We then break the numbers in two primes (Table 13).

- ***<u>Table13</u>*** *: Search for primes by n - t and n + t with n = S/2 and t = O3N. Note* ***95*** *is not 03N (the last line of the table 95 = 5 x 19) but it is required to check if the number S = 3 + q. Note that 6X + 2 and 6X – 2 even numbers might have 3 as an additive prime on the contrary to even 6x.*

98 – t	t = O3N	98 + t
95	3	101
89	**9**	97
83	**15**	113
77	21	119
71	27	125
65	33	131
59	**39**	137
53	45	143
47	**51**	149
41	57	155
35	63	161
29	**69**	167
23	**75**	173
17	**81**	179
11	**87**	185
5	93	191
3	**95**	193

- ***<u>Table 14</u>****: The number 196 (S/2 = 98) can therefore be broken 9 times in two primes.*

S = p + q
3+193
5+191
17+179
23+173
29+167
47+149
59+137
83+113
89+107

Table 14 shows all possible conversions in sums of two primes of S = 196.

CONCLUSION

This article presents two methods for breaking an even number into two primes. Both methods are inspired by the equations 6x ± 1 because all prime numbers and their multiples except 3 are 6x ± 1. Method A finds prime numbers p and q with the correct unit digits to form the even number S = p + q. Congruence rules confirm or explain why S - p = q or S - p = c (composite).

Method B is based on proposition C which, explained differently, suggests that prime numbers are always formed in pairs from an integer n ≥ 4. A prime number never appears alone but together with another one and both of them are symmetrical and equidistant to the integer n from which they come. In an interval |[0▬ n ▬ 2n] where we would then have at least one value t such that t < n and such that p = n - t and q = n + t are primes and 2n = p + q. The value t differs depending on whether the even number S is 6X; 6X - 2; 6X + 2. Method B shows that for 6X ; t = 6x - 1 or t = 6x + 2. Whereas it is necessary that t = 03N (multiple of 3 that are odds) to obtain pairs of prime numbers (p; q) such that S = p + q frm evens which are 6X – 2, or 6X + 2.

Both methods are not only useful to break evens in two primes but can help investigating the Goldbach's strong conjecture (GSC). There is a very common confusion to avoid, which is that even if prime numbers added together always give an even number; this does not mean at all that any even number can be broken into two prime numbers. To break an even number into two prime numbers, a method is required that can be simple or very complex. To prove the GSC, it is necessary to demonstrate by a theorem that an integer n ≥4 always produces two prime numbers p and q such that p = n - t and q = n + t (t < n). Such a theorem still does not exist. For the time being; it can only be achieved using simple or complex methods or algorithms. Both methods here are simple and accessible and could generate new algorithms of even conversion in sums of two primes.

Finally, this article defends the idea that prime numbers are formed in pairs from the same integer n ≥ 4. For the moment, only methods and congruence rules can help us achieve this, but is this fact demonstrable? Can it obey a theorem that also solves the GSC? Time will tell.

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