Divergences between the mathematical and physical interpretations of the tensor

On Einstein's attempt to simplify the gravitational field equation (1915)

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Abstract

In 1915, Einstein, simultaneously with the gravitational field equation, obtained a simplified version of this equation. Schwarzschild found an exact solution to this version. The article discusses the features of the simplified version of the Einstein equation and its asymptotic properties.

Keywords: Ricci tensor, gravitational field equation, correspondence principle, Newtonian limit.

1. Introduction.

Einstein introduced Riemannian space into physics as a physical object and laid out the fundamentals of the necessary mathematical techniques that allow one to operate with the parameters of Riemannian space. This made it possible to consistently describe the equations of motion and the equation of the gravitational field, which were convincingly confirmed by empirical data, at least in the area of not very large fields.

The principles of Einstein's general theory of relativity were established in 1913 [1]. In 1915, Einstein obtained the equation of the gravitational field [1]

$$R_{\mu\nu}=\frac{8\pi G}{c^4}\left(T_{\mu\nu}-\frac{1}{2}g_{\mu\nu}T\right), \tag{1}$$

where, $R_{\mu\nu}$ is the Ricci tensor.

In the same article [1], Einstein made an attempt to simplify equation (1). The simplified Einstein field equation was used for some time in studies of solutions to the Einstein equation, but then was practically forgotten. However, the procedure for transforming the Einstein field equation and the prospect of studying the asymptotic continuation of the gravitational field equation turned out to be extremely interesting.

2. Einstein's equation of the gravitational field with a limited coordinate transformation

In works preceding the derivation of the gravitational field equation, Einstein investigated the division of the Ricci tensor into two parts

$$R_{\mu\nu}=A_{\mu\nu}+B_{\mu\nu},$$
$$A_{\mu\nu}=\frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}}-\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}, \tag{2}$$
$$B_{\mu\nu}=\frac{-\partial \Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}}+\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}=\frac{-\partial \ln\sqrt{-g}}{\partial x^{\mu}\partial x^{\nu}}+\Gamma^{\alpha}_{\mu\nu}\frac{\partial \ln\sqrt{-g}}{\partial x^{\alpha}}.$$

Einstein considered the possibility [2] of considering the field equation in a coordinate system with the condition $g=-1$:

“But if —g is positive and finite, then the thought naturally arises that we should now choose the coordinates so that this value becomes equal to 1. Later we will see that by such a restriction of the choice of the coordinate system a significant simplification of the laws of nature can be achieved.”

Next, Einstein makes a remark:

“But it would be a mistake to think that this method means a partial rejection of the general principle of relativity. We do not ask: "What will be the laws of nature that are covariant with respect to all transformations with determinant 1?" But we ask: "What will be the generally covariant laws of nature?" Only after these laws have been established, do we simplify their expression by means of a special choice of coordinate system.”

It is obvious that by choosing the coordinates in such a way that the condition $\sqrt{-g}=1$ is satisfied, the second term $B_{\mu\nu}=0$ reduces Einstein's equation (1) to a system of equations in empty space [1]

$$\begin{cases} \dfrac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}}-\Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha}=0, \\ \sqrt{-g}=1. \end{cases} \tag{3}$$

1 In the following we consider the field equation in “empty space”, i.e. without “matter” and electromagnetic field.

It was precisely this form of the system of equations (3) that Schwarzschild solved [4]. Due to the complexity of this solution, this version of Einstein's equation has now practically disappeared from the literature.

Despite the fact that Schwarzschild obtained an exact solution to equation (3) for a point source and it coincided with subsequent similar solutions to equation (1), Einstein's choice of the condition $\sqrt{-g}=1$ does not seem sufficiently justified.

3. Tensorial nature of parts of the Ricci tensor

Einstein proved that parts of the Ricci tensor (2) are tensors. In doing so, he assumed that $\sqrt{-g}=1$. However, this statement is also true in the general case.

Theorem: The simplified Christoffel symbol $\Gamma^{\alpha}_{\mu\alpha}$ is a 4-vector, and $A_{\mu\nu}$ and $B_{\mu\nu}$ are tensors.

P r o o f. We use the formula for the coordinate transformation of Christoffel symbols (equation (85.15), [4])

$$\Gamma^{\iota}_{\mu\alpha}=\Gamma'^{\gamma}_{\nu\xi}\frac{\partial x^{\iota}}{\partial x'^{\gamma}}\frac{\partial x'^{\nu}}{\partial x^{\mu}}\frac{\partial x'^{\xi}}{\partial x^{\alpha}}+\frac{\partial^2 x'^{\gamma}}{\partial x^{\alpha}\partial x^{\mu}}\frac{\partial x^{\iota}}{\partial x'^{\gamma}}$$

Let's put $\iota=\alpha$ and $\gamma=\xi$ in this expression, then the formula will be simplified:

$$\Gamma^{\alpha}_{\mu\alpha}=\Gamma'^{\xi}_{\nu\xi}\frac{\partial x^{\alpha}}{\partial x'^{\xi}}\frac{\partial x'^{\nu}}{\partial x^{\mu}}\frac{\partial x'^{\xi}}{\partial x^{\alpha}}+\frac{\partial^2 x'^{\xi}}{\partial x^{\alpha}\partial x^{\mu}}\frac{\partial x^{\alpha}}{\partial x'^{\xi}}=\Gamma'^{\xi}_{\nu\xi}\frac{\partial x'^{\nu}}{\partial x^{\mu}}+\frac{\partial^2}{\partial x'^{\xi}\partial x^{\mu}}x'^{\xi}$$

The last term is obtained using the theorem of differentiation of a composite function. According to the theorem on the equality of mixed derivatives, the order of differentiation can be changed. Then it turns out that the last term is equal to zero, and

$$\Gamma^{\alpha}_{\mu\alpha}=\Gamma'^{\xi}_{\nu\xi}\frac{\partial x'^{\nu}}{\partial x^{\mu}}.$$

Thus,$\Gamma^{\alpha}_{\mu\alpha}=C_{\mu}$ transforms as a covariant 4-vector and is therefore a covariant 4-vector.

Covariant differentiation of $\Gamma^{\alpha}_{\mu\alpha}$ generates the tensor $-B_{\mu\nu}$:

$$\frac{D\Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}}=\frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}}-\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}=-B_{\mu\nu}.$$

It follows from the proven theorem that since $R_{\mu\nu}$ there is a tensor, then from $R_{\mu\nu}=A_{\mu\nu}+B_{\mu\nu}$ it follows that $A_{\mu\nu}$ is also a tensor quantity.

4. Asymptotic properties of the Einstein equation

Let us consider Einstein's equation (3) in a more convenient form

$$A_{\mu\nu}+B_{\mu\nu}=0 \quad (4)$$

Let us estimate the value of B^{μ}_{ν} in the Newtonian approximation, which is described by the simplest metric with an accelerated origin

$$ds^2=\left(1+\frac{2\varphi}{c^2}\right)dt^2-dx^2-dy^2-dz^2, \tag{5}$$

here $\varphi(x)$ is the Newtonian potential.

Taking into account that the spatial components of the metric tensor and its determinant differ slightly from unity, we obtain from the equation (5)

$$B^0_0=\Gamma^1_{00}\Gamma^0_{10}=\frac{-1}{c^2}\left(\frac{\partial\varphi}{\partial x}\right)\frac{1}{c^2+2\varphi}\left(\frac{\partial\varphi}{\partial x}\right)\approx-\frac{1}{c^4}\left(\frac{\partial\varphi}{\partial x}\right)^2.$$

The last quantity is proportional to the known field energy density in the Newtonian approximation ([8] § 106, problem 1):

$$f^0_0=\frac{-(\nabla\varphi)^2}{8\pi G}.$$

This contradicts Einstein's assumption $\sqrt{-g}=1$, which implies $B^{\mu}_{\nu}=0$. On the contrary, this is a good reason to accept the gravitational field energy-momentum tensor as a result of the comparison

$$\frac{\partial\Gamma^{\alpha}_{\mu\alpha}}{\partial x^{\nu}}-\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\alpha\beta}=\frac{8\pi G}{c^4}f^{\mu}_{\ \nu}. \tag{6}$$

Now we can recall that in the programmatic article of 1913 [5] Einstein saw the equation of the gravitational field (contravariant) as

$$\Delta_{\mu\nu}(\gamma)=\kappa\left(\Theta_{\mu\nu}+\vartheta_{\mu\nu}\right), \tag{7}$$

where on the left is a tensor that depends on the components of the fundamental tensor and their derivatives of order e higher than the second. Einstein formulated an important principle in his comments to equation (7) [5]:

> *"These equations satisfy a requirement, in our opinion, which is obligatory for a relativistic theory of gravity; namely, they show that the gravitational field tensor $\vartheta_{\mu\nu}$ is a source of the field on a par with the tensor of material systems $\Theta_{\mu\nu}$. The exceptional position of the energy of the gravitational field in comparison with all other types of energy would lead to unacceptable consequences (emphasis added)."*

where on the left is a tensor that depends on the components of the fundamental tensor and their derivatives of order no higher than the second. Einstein formulated an important principle in his comments to equation (7) [5] t^{λ}_{σ}:

$$\kappa t^{\lambda}_{\sigma}=\frac{1}{2}\delta^{\lambda}_{\sigma}\sum_{\mu\nu\alpha}g^{\mu\nu}\Gamma^{\alpha}_{\mu\alpha}-\sum_{\mu\nu\alpha}g^{\mu\nu}\Gamma^{\alpha}_{\mu\sigma}\Gamma^{\lambda}_{\nu\alpha}. \tag{8}$$

Note that this expression has a certain similarity with the tensor f^{μ}_{ν} (6). It seems that Einstein was close to the correct solution. However, the quantity t^{λ}_{σ} is not a tensor and cannot be included in the field equation. In addition, as it turned out later [6], [7]:

> *"There is no reason to understand t^4_4 as the density of the energy of the gravitational field, and $\left(t^1_4, t^2_4, t^3_4\right)$ as the components of the flux density of gravitational energy. However, one can assert the following: if the volume integral of t^1_4 is small compared to the volume integral of the density of the "material" energy T^4_4, then the right-hand side certainly represents the loss of energy by the material system. Only this was used in my present and previous works on gravitational waves." [6]*

This quote from Einstein correlates with the opinion of L.D. Landau and E.M. Lifshitz [4], who believed that the energy density of the gravitational field is not included in Einstein's equation due to the smallness of the energy density:

> *"Gravitational interaction plays a role only for bodies with a sufficiently large mass (due to the smallness of the gravitational constant). Therefore, when studying the gravitational field, we usually have to deal with macroscopic bodies. Accordingly, for T_{ik} we usually need to write expression (94.9)."*

5. Discussions with others researches

5.1 From Research 1

“Dear Research 2 dear Research 3:

since long time, Author is completely on the wrong track. He still thinks that the contracted Christoffel symbol is a tensor. But it has been shown several times to him that the contracted Christoffel symbol is not a tensor in general, see

Figure 1:

This is a proof that the contracted Christoffel symbol in general is not a tensor

$$ds'^2 = g'_{\mu\nu}\, dx'^{\mu}\, dx'^{\nu} = g_{\kappa\lambda}\, dx^{\kappa}\, dx^{\lambda} = ds^2$$

$$\Rightarrow \quad g'_{\mu\nu} = g_{\kappa\lambda} \frac{\partial x^{\kappa}(x')}{\partial x'^{\mu}} \frac{\partial x^{\lambda}(x')}{\partial x'^{\nu}} = g_{\kappa\lambda}\, \bar{\alpha}^{\kappa}_{\mu}(x')\, \bar{\alpha}^{\lambda}_{\nu}(x')$$

$$\Rightarrow \quad g' = g\, [\det \bar{\alpha}(x')]^2 \;, \quad g' = \det g'_{\mu\nu} \;, \quad g = \det g_{\kappa\lambda}$$

$$\Rightarrow \quad |g'| = |g|\, [\det \bar{\alpha}(x')]^2$$

$$\Rightarrow \quad \sqrt{|g'|} = \sqrt{|g|}\, |\det \bar{\alpha}(x')| \;, \quad \sqrt{|g'|} > 0 \;, \quad \sqrt{|g|} > 0 \;, \quad \Rightarrow |\det \bar{\alpha}| = \det \bar{\alpha} > 0$$

$$\Rightarrow \quad \sqrt{|g'|} = \sqrt{|g|}\, \det \bar{\alpha}(x')$$

$$\Gamma'^{k}_{kl} = \frac{\partial \ln \sqrt{|g'|}}{\partial x'^{l}} = \frac{\partial \ln\left[\sqrt{|g|}\, \det \bar{\alpha}(x')\right]}{\partial x'^{l}} = \frac{\partial \ln \sqrt{|g|}}{\partial x'^{l}} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}}$$

$$\Gamma'^{k}_{kl} = \frac{\partial x^{m}(x')}{\partial x'^{l}} \frac{\partial \ln \sqrt{|g|}}{\partial x^{m}} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}}$$

$$\Gamma'^{k}_{kl} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial \det \bar{\alpha}(x')} \frac{\partial \det \bar{\alpha}(x')}{\partial x'^{l}} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + \left[\det \bar{\alpha}(x')\right]^{-1} \underbrace{\frac{\partial \det \bar{\alpha}(x')}{\partial x'^{l}}}_{\neq 0 \text{ (in general)}}$$

For non-linear, which are general coordinate transformations, the Jacobian matrix $\bar{\alpha}$ depends on the coordinates. In case of linear coordinate transformations the Jacobian matrix $\bar{\alpha}$ does not depend on the coordinates. This is why the last term in the last line shown above vanishes only in case of linear but not for general coordinate transformations.

Source: [8]

Another argumentation against Author "theorem" is found on this website:

https://physics.stackexchange.com/questions/736958/is-the-contracted-christoffel-symbol-a-tensor [9]. However Author doesn't accept any correct disproof (as shown above) against his "theorem".

Another argumentation against Author "theorem" is found on this website: [9]

Figure 2:

The coordinate transformation law (from coordinates x to coordinates y) for the Christoffel symbol is:

$$\Gamma^i_{kl}(y) = \frac{\partial y^i}{\partial x^m}\frac{\partial x^n}{\partial y^k}\frac{\partial x^p}{\partial y^l}\Gamma^m_{np}(x) + \frac{\partial^2 x^m}{\partial y^k \partial y^l}\frac{\partial y^i}{\partial x^m}$$

Therefore the transformation law for the contracted Christoffel symbol would be:

$$\Gamma^i_{il}(y) = \frac{\partial y^i}{\partial x^m}\frac{\partial x^n}{\partial y^i}\frac{\partial x^p}{\partial y^l}\Gamma^m_{np}(x) + \frac{\partial^2 x^m}{\partial y^i \partial y^l}\frac{\partial y^i}{\partial x^m}$$

$$\Gamma^i_{il}(y) = \delta^n_m \frac{\partial x^p}{\partial y^l}\Gamma^m_{np}(x) + \frac{\partial}{\partial y^i}\left(\frac{\partial x^m}{\partial y^l}\right)\frac{\partial y^i}{\partial x^m}$$

$$\Gamma^i_{il}(y) = \frac{\partial x^p}{\partial y^l}\Gamma^n_{np}(x) + \frac{\partial}{\partial x^m}\left(\frac{\partial x^m}{\partial y^l}\right)$$

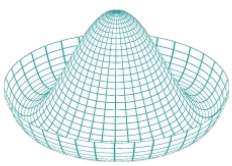
My question is, does the term $\frac{\partial}{\partial x^m}(\frac{\partial x^m}{\partial y^l})$ equal zero? My reasoning is that since partial derivatives commute, the order of differentiation can be reversed:

$$\frac{\partial}{\partial x^m}\left(\frac{\partial x^m}{\partial y^l}\right) = \frac{\partial}{\partial y^l}\left(\frac{\partial x^m}{\partial x^m}\right) = \frac{\partial}{\partial y^l}(\delta^m_m) = 0$$

That would mean that the contracted Christoffel symbol transforms like a tensor, specifically a vector, and therefore is a tensor. However, I am unsure if it's correct to commute partial derivatives of different coordinates.

Source: [9].

Figure 3:



Is the contracted Christoffel symbol a tensor?

The coordinate transformation law (from coordinates x to coordinates y) for the Christoffel symbol is: $$\Gamma^i_{kl}(y)=\frac{\partial y^i}{\partial x^m} \frac{\partial x^n}{\partial y^k} \frac{\partial x^p}{\partial y^l} \Gamma^m_{np}(x) +\frac{\partial ^2x^m}{\partial y^k \partial y^l} \frac{\partial y^i}{\partial x^m}$$ Therefore the transformation law for the contracted Christoffel ...

physics.stackexchange.com

Source: [9].

However Author doesn't accept any correct disproof (as shown above) against his "theorem".

5.2 From Author:

“Dear Research 2,

My friend Stefan is completely unadapted to the perception of mathematics. He has some strange ideas about linearity. I spent a lot of time selecting textbooks and citations from Wikipedia - all useless. Be careful with Stefan, I feel sorry for him. I suspect that he has a mental deviation and it is useless to discuss with him. He complained to me that no one wants to discuss his work. He clearly has an exaggerated opinion of his contribution to science. Of course, his work contains interesting points, but nothing more.

With respect, Author.”

5.3 From Author:

“Dear Research 2,

I noticed Stefan's link https://physics.stackexchange.com/questions/736958/is-the-contracted-christoffel-symbol-a-tensor [9]

They are discussing my theorem. They simply do not know the Clairaut-Schwartz theorem. This theorem has been known since the 18th century. And it is a mandatory part of the standard course of mathematical analysis. Some time ago I gave a link to this theorem to Research 1.
https://en.wikipedia.org/wiki/Symmetry_of_second_derivatives#Schwarz's_theorem [10]
I don't understand why modern physicists (including Russians) don't know the basics of mathematical analysis. This theorem was familiar to Einstein and is often found in the works of physicists of that time.
Best regards, Author”

Reply:

“Dear Author,

since long time, you is completely on the wrong track. He still thinks that the contracted Christoffel symbol is a tensor. But it has been shown several times that the contracted Christoffel symbol is not a tensor in general, see

Figure 1, section 5.1 ... Regards, Research 2.”

5.4 From Author

“Dear Research 1 and Research 2,
There is a proof. No one has found an error in it. Until an error is found, the theorem is true. Examples that, according to their authors, contradict the theorem. Contain unproven statements. For example, Research 1repeats an unproven "refutation" that contains an expression and Research 1 claims that it is not equal to zero. This is not proven.

Figure 3:

This is a proof that the contracted Christoffel symbol in general is not a tensor

$$ds'^2 = g'_{\mu\nu}\, dx'^{\mu}\, dx'^{\nu} = g_{\kappa\lambda}\, dx^{\kappa}\, dx^{\lambda} = ds^2$$

$$\Rightarrow\ g'_{\mu\nu} = g_{\kappa\lambda} \frac{\partial x^{\kappa}(x')}{\partial x'^{\mu}} \frac{\partial x^{\lambda}(x')}{\partial x'^{\nu}} = g_{\kappa\lambda}\, \bar{\alpha}^{\kappa}_{\mu}(x')\, \bar{\alpha}^{\lambda}_{\nu}(x')$$

$$\Rightarrow\ g' = g\,[\det \bar{\alpha}(x')]^2\ ,\ \ g' = \det g'_{\mu\nu}\ ,\ \ g = \det g_{\kappa\lambda}$$

$$\Rightarrow\ |g'| = |g|\,[\det \bar{\alpha}(x')]^2$$

$$\Rightarrow\ \sqrt{|g'|} = \sqrt{|g|}\,|\det \bar{\alpha}(x')|\ ,\ \ \sqrt{|g'|} > 0\ ,\ \ \sqrt{|g|} > 0\ ,\ \ \Rightarrow |\det \bar{\alpha}| = \det \bar{\alpha} > 0$$

$$\Rightarrow\ \sqrt{|g'|} = \sqrt{|g|}\, \det \bar{\alpha}(x')$$

$$\Gamma'^{k}_{kl} = \frac{\partial \ln \sqrt{|g'|}}{\partial x'^{l}} = \frac{\partial \ln\left[\sqrt{|g|}\, \det \bar{\alpha}(x')\right]}{\partial x'^{l}} = \frac{\partial \ln \sqrt{|g|}}{\partial x'^{l}} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}}$$

$$\Gamma'^{k}_{kl} = \frac{\partial x^{m}(x')}{\partial x'^{l}} \frac{\partial \ln \sqrt{|g|}}{\partial x^{m}} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial x'^{l}}$$

$$\Gamma'^{k}_{kl} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + \frac{\partial \ln \det \bar{\alpha}(x')}{\partial \det \bar{\alpha}(x')} \frac{\partial \det \bar{\alpha}(x')}{\partial x'^{l}} = \bar{\alpha}^{m}_{l}(x')\, \Gamma^{k}_{km} + [\det \bar{\alpha}(x')]^{-1} \underbrace{\frac{\partial \det \bar{\alpha}(x')}{\partial x'^{l}}}_{\neq 0\ \text{(in general)}}$$

For non-linear, which are general coordinate transformations, the Jacobian matrix $\bar{\alpha}$ depends on the coordinates. In case of linear coordinate transformations the Jacobian matrix $\bar{\alpha}$ does not depend on the coordinates. This is why the last term in the last line shown above vanishes only in case of linear but not for general coordinate transformations.

Source: Adapted Author.

The last paragraph demonstrates mathematical illiteracy. Open denial of mathematical definitions.
Denial of the definition of linearity of transformation. It was ignorance of this definition that led to the catastrophic error of Landau [4] and his followers.

At the same time, there is a proof of the theorem published in chapter The energy-momentum tensor of the gravitational field as a correction to the Einstein equation [11]

Figure 4:

Theorem 1. The magnitude of the simplified Christoffel symbol $\Gamma^{\alpha}_{\mu\alpha}$ is a 4-vector, $A_{\mu\nu}$ and $B_{\mu\nu}$ are tensors.

This is Proof.

We used the formula for the coordinate transformation of Christoffel symbols (Eq. (85.15), [4]).

$$\Gamma^{\iota}_{\mu\alpha} = \Gamma'^{\gamma}_{\nu\xi} \frac{\partial x^{\iota}}{\partial x'^{\gamma}} \frac{\partial x'^{\nu}}{\partial x^{\mu}} \frac{\partial x'^{\xi}}{\partial x^{\alpha}} + \frac{\partial^2 x'^{\gamma}}{\partial x^{\alpha} \partial x^{\mu}} \frac{\partial x^{\iota}}{\partial x'^{\gamma}}$$

If we are to add the following to this expression, $\iota = \alpha$ and $\gamma = \xi$, then the formula will be simplified as seen below:

$$\Gamma^{\alpha}_{\mu\alpha} = \Gamma'^{\xi}_{\nu\xi} \frac{\partial x^{\alpha}}{\partial x'^{\xi}} \frac{\partial x'^{\nu}}{\partial x^{\mu}} \frac{\partial x'^{\xi}}{\partial x^{\alpha}} + \frac{\partial^2 x'^{\xi}}{\partial x^{\alpha} \partial x^{\mu}} \frac{\partial x^{\alpha}}{\partial x'^{\xi}} = \Gamma'^{\xi}_{\nu\xi} \frac{\partial x'^{\nu}}{\partial x^{\mu}} + \frac{\partial^2}{\partial x^{\mu} \partial x'^{\xi}} x'^{\xi}$$

The last term is transformed according to the composite function differentiation theorem. According to the theorem on the equality of mixed derivatives, the order of differentiation can be changed. Then it turns out that the last term is equal to zero, and

$$\Gamma^{\alpha}_{\mu\alpha} = \Gamma'^{\xi}_{\nu\xi} \frac{\partial x'^{\nu}}{\partial x^{\kappa}}.$$

Source: [4].

It is not serious to deny the obvious. The tensor character of the abbreviated Christoffel symbol is already visible in the definition

$$\Gamma^{i}_{ki} = \frac{1}{2} g^{im} \frac{\partial g_{im}}{\partial x^{k}}$$

since the product of a contravariant tensor and a covariant tensor is a scalar.

There is no reason to refuse publication if there are no serious doubts about the correctness of the theorem. Only emotions.

Author”

6. Conclusions

If we do not neglect the field energy density, then, according to Einstein, we are obliged to include the energy-momentum tensor (6) in Einstein's equation (1)

$$R_{\mu\nu} = \frac{8\pi G}{c^4} \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T + f^{\mu}_{\nu} \right), \quad (9)$$

from where, taking into account equality (6), we obtain a consistent equation of the gravitational field [7]

$$\frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x^{\alpha}} - \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right). \quad (10)$$

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