

Proof that the Legendre conjecture is unprovable

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Abstract

In this paper, we attempt to prove the unprovability of the Legendre Conjecture, a known open problem. The method is similar to the author's "Guessing that the Riemann Hypothesis is unprovable using nonstandard analysis."

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Assume that the following proposition is unprovable.

proposition 1.

$$\pi(x) = li(x) + O(x^{\frac{1}{2}+\epsilon})$$

For more details, please refer to the author's paper [1]. This unprovability is mainly due to the existence of the following "distorted function". We will proceed with the discussion using nonstandard analysis by imitating the following method. For a certain prime number P , set $\pi(P)$ to $M \gg 1$ instead of 1. Since the use of infinity is allowed in nonstandard analysis, we will set P to infinity and create a consistent negation model.

If

$$\pi(x) < li(x) + Kx^{\frac{1}{2}+\epsilon}$$

is true in the normal axiom system, and

$$\pi(x) > li(x) + Kx^{\frac{1}{2}+\epsilon}$$

is true by taking the "distorted function" in nonstandard analysis, then this proposition is unprovable. (It can be disproved.)

Theorem 1. (*Independence of the Legendre Conjecture*) *It is unprovable (independent proposition) that there is at least one prime number between n^2 and $(n + 1)^2$.*

Consider the Legendre conjecture. Consider a prime number P and the next prime number Q . Then, in nonstandard analysis, let P and Q be infinite, as nonstandard analysis allows. Then, consider an arbitrarily wide interval between P and Q by counting the same number many times. This leads to a consistent model, providing a counterexample to the Legendre conjecture in nonstandard analysis.

If nothing is done. In other words, in normal mathematics, the Legendre Conjecture is "probably" correct. However, in a model based on nonstandard analysis, the Legendre Conjecture is false. In other words, the Legendre Conjecture is an unprovable proposition, that is, an independent proposition.

If the model in nonstandard analysis includes a positive proof of the Legendre Conjecture, which has common ground with normal mathematics, there is no way that a negative model can be created in nonstandard analysis. The negative proposition of the Legendre Conjecture is provable, but the positive proposition is unprovable since the negation can be created by one of the extensions.

References

- [1] "'Guessing that the Riemann Hypothesis is unprovable using Non-Standard Analysis" T. Nakashima, Vixra. org. Number theory