Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 22/7 and 355/113

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Abstract

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π-e formula and the natural end of the elements determined by us, i.e., the 112th element Cn[*]. In this paper, based on these formulas and the two approximate rates of π which are 77/2 and 335/113, we deduce new formulas of the fine-structure constant and the speed of light in atomic units. This is also to answer physicist Feynman's question whether the fine-structure constant is related to π. Except our previous answers and explanations, our new additional answer is that it is also amazingly related to the approximate rates of 77/2 and 335/113 which were deduced by Chinese ancient mathematician Chongzhi Zu (AD 429-500).

Keywords: the fine-structure constant, the speed of light in atomic units, formulas, π, approximate rates, 22/7, 355/113.

1. Definitions of the Fine-structure Constant and Physicist Feynman's Comment

The fine-structure constant (α) is a centennial mystery of physics. It was found and introduced by physicist Sommerfeld in 1916, it has three definitions as follows.

$$\alpha = \frac{\lambda_e}{2\pi a_0}, \quad \alpha = \frac{2\pi r_e}{\lambda_e}, \quad \alpha = \frac{v_e}{c} = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$$

$$\alpha = \frac{1}{137.035999\cdots} \approx \frac{1}{137.036} \approx \frac{1}{137}$$

The following was physicist Feynman's comment on the fine-structure constant.

It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: ***is***

it related to pi or perhaps to the base of natural logarithms? *Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!* [1]

2. Our Previous Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units

In our previous papers, we gave the formulas of the fine-structure constant and the speed of light in atomic units based on 2π-e formula and the natural end of the elements, i.e., the 112th element Cn* [2-14]. They are listed as follows.

$2\pi - e$ formula:

$$2\pi = \left(\frac{e}{e^{\gamma_c}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \frac{e^2}{(\frac{4}{3})^7} \cdots$$

$$(2\pi)_{Chen-k} = \left(\frac{e}{e^{\gamma_{c-k}}}\right)^2 = e^2 \frac{e^2}{(\frac{2}{1})^3} \frac{e^2}{(\frac{3}{2})^5} \cdots \frac{e^2}{(\frac{k+1}{k})^{2k+1}}$$

Formulas of the fine-structure constant:

$$\alpha_1 = \frac{\lambda_e}{2\pi a_0} = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112 + \frac{1}{75^2}} = 1/137.035999037415379188517229528 74$$

Relationships with nuclides:

$^{82,83,84}_{36}Kr_{46,47,48}$ $^{112}_{48}Cd_{64}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{185,187}_{75}\mathrm{Re}_{110,112}$ $^{188}_{76}Os_{112}$ $^{209}_{83}Bi^{*}_{126}$ $^{209}_{84}Po^{*}_{125}$

$^{285}_{112}Cn^{*}_{173}$ $^{312}_{125}Ch^{ie}_{187}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$

$$\alpha_2 = \frac{2\pi r_e}{\lambda_e} = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112 - \frac{1}{64\cdot 3\cdot 29}} = 1/137.035999111872962758119209477 93$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{63,65}_{29}Cu_{34,36}$ $^{100}_{44}Ru_{56}$ $^{112}_{48}Cd_{64}$ $^{140,142}_{58}Ce_{82,84}$ $^{136,137,138}_{56}Ba_{80,81,82}$ $^{157}_{64}Gd_{93}$ $^{169}_{69}Tm_{100}$

$^{188}_{76}Os_{112}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{257}_{100}Fm^{*}_{157}$ $^{278}_{109}Mt^{*}_{169}$ $^{285}_{112}Cn^{*}_{173}$ $^{2\cdot157}_{126}Ch^{ie}_{188}$ $^{2\cdot173}_{137}Fy^{ie}_{209}$ $^{426}_{169}Ch^{ie}_{257}$

Formulas of the speed of light in atomic units constructed in our previous papers:

$$c_{au} = \frac{c}{v_e} = \frac{4\pi\varepsilon_0\hbar c}{e^2} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$$

$$= \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot 141} - \frac{1}{14\cdot 112(2\cdot 173+1)+13+\frac{7}{72}})}$$

$$= 2\sqrt{56(83 + \frac{157}{188} - (\frac{1}{8\cdot 141} + \frac{1}{56^2(2\cdot 173+1)+26+\frac{7}{36}}))}$$

$$= 137.035999074644$$

$c_{au} = \frac{1}{\sqrt{\alpha_1\alpha_2}}$ is consitent with Maxwell formula of the speed of light $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Based on the above formula, we construct the following more precise formulas:

$$c_{au} = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot 141} - \frac{1}{14\cdot 112(2\cdot 173+1)+13+\frac{7}{72+1/50}})}$$

$$= 2\sqrt{56(83 + \frac{157}{188} - (\frac{1}{8\cdot 141} + \frac{1}{56^2(2\cdot 173+1)+26+\frac{7}{36+1/100}}))}$$

$$= 137.035999074644170968 2$$

Relationships with nuclides:

$^{56}_{26}Fe_{30}$ $^{83,84}_{36}Kr_{47,48}$ $^{100}_{44}Ru_{56}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$ $^{118,119,120}_{50}Sn_{68,69,70}$ $^{136,137,138}_{56}Ba_{80,81,82}$

$^{140,142}_{58}Ce_{82,84}$ $^{157}_{64}Gd_{93}$ $^{166,167,168}_{68}Er_{98,99,100}$ $^{169}_{69}Tm_{100}$ $^{173}_{70}Yb_{103}$ $^{188}_{76}Os_{112}$ $^{200}_{80}Hg_{120}$ $^{209}_{83}Bi^{*}_{126}$

$^{209}_{84}Po^{*}_{125}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{257}_{100}Fm^{*}_{157}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{2\cdot 137}_{107}Bh^{*}_{167}$ $^{276}_{108}Hs^{*}_{168}$ $^{285}_{112}Cn^{*}_{173}$ $^{300}_{120}Ch^{ie}_{180}$

$^{2\cdot 157}_{126}Ch^{ie}_{188}$ $^{344,2\cdot 173,348}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{355,357}_{141}Ch^{ie}_{214,216}$ $^{400}_{157}Ch^{ie}_{243}$ $^{420}_{168}Ch^{ie}_{252}$ $^{426}_{169}Ch^{ie}_{257}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

$$c_{au} = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot 141} - \frac{1}{14\cdot 112(2\cdot 173+1)} + \frac{1}{22603891794 + \frac{130}{137 - \frac{1}{2\cdot 7\cdot 73}}})}$$

$$= 137.035999074644170968261216427 08$$

Note: $22603891794 = 6\cdot 11\cdot 41(128\cdot 13 - 1)(32\cdot 157 - 1)$

Relationships with nuclides:

$^{73}_{32}Ge_{41}$ $^{93}_{41}Nb_{52}$ $^{125,130}_{52}Te_{73,78}$ $^{157}_{64}Gd_{93}$ $^{238}_{92}U^{*}_{146}$ $^{240}_{94}Pu^{*}_{146}$ $^{330}_{130}Ch^{ie}_{200}$ $^{370}_{146}Ch^{ie}_{224}$

$$c_{au} = \sqrt{112(168 - \frac{1}{3} + \frac{1}{4\cdot 141} - \frac{1}{14\cdot 112(2\cdot 173+1)+13} + \frac{1}{3045979902845 - 36/91})}$$

$$= 137.035999074644170968261216427 08$$

Note: $3045979902845 = 5(2\cdot 27\cdot 31\cdot 103 - 1)(2\cdot 3\cdot 17\cdot 47\cdot 67)$

Relationships with nuclides:

$^{27}_{13}Al_{14}$ $^{91}_{40}Zr_{51}$ $^{103}_{45}Rh_{58}$ $^{155}_{64}Gd_{91}$ $^{165}_{67}Ho_{98}$ $^{173}_{70}Yb_{103}$ $^{231,232}_{91}Pa^{*}_{140,141}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

3. Formulas of the Fine-structure Constant and the Speed of Light in Atomic Units Based on 22/7 and 355/113

Based on the above formulas of the fine-structure constant, the above formulas of the speed of light in atomic units and the approximate rates of π which are 77/2 and 335/113, we deduce new formulas of the fine-structure constant and the speed of light in atomic units as follows.

$$\alpha_1 = \frac{36}{7(2\pi)_{Chen-112}} \frac{1}{112+\frac{1}{75^2}} = 1/137.035999037415379188517229528 74$$

Use $2\pi \approx 44/7$ to replace $(2\pi)_{Chen-112}$:

$$\alpha_1 = \frac{36}{44} \frac{1}{112+\frac{16}{7\cdot 19}+\frac{1}{2\cdot 3\cdot 44\cdot 61-\frac{8}{3\cdot 19+\frac{1}{9\cdot 17+\frac{20}{3\cdot 19+\frac{5}{4\cdot 3\cdot 17}}}}}}$$

$$= 1/137.0359990374153791885172$$

$$\alpha_2 = \frac{13(2\pi)_{Chen-278}}{100} \frac{1}{112-\frac{1}{64\cdot 3\cdot 29}} = 1/137.0359991118729627581192094779 3$$

Use $2\pi \approx 710/113$ to replace $(2\pi)_{Chen-278}$:

$$\alpha_2 = \frac{13}{10}\frac{71}{113}\frac{1}{112-\frac{8}{\frac{44\cdot 19}{7}-\frac{1}{7}}+\frac{16}{\frac{3\cdot 44\cdot 71(2\cdot 7\cdot 83+1)}{7}+\frac{7}{113+\frac{1}{60+7/55}}})}$$

$$= 1/137.035999111872962758119$$

In the above formulas of α_1 and α_2, the rate of 44/7 and the factors of 44, 71 and 113 appear miraculously, so we suppose they should be reasonable and precise.

$$c_{au} = \frac{c}{v_e} = \frac{1}{\alpha_c} = \frac{1}{\sqrt{\alpha_1 \alpha_2}}$$

$$= \frac{10}{6}\sqrt{\frac{7}{13}\frac{44}{7}\frac{113}{710}}(112+\frac{1}{\frac{2\cdot 3\cdot 44}{7}-\frac{1}{7}}-\frac{8}{\frac{3\cdot 5\cdot 44(2\cdot 83+1)(4\cdot 83-1)}{7}+\frac{2}{41}})$$

$$= 137.035999074644170968$$

$$c_{au}=\frac{10}{6}\sqrt{\frac{7}{13}\frac{44}{7}\frac{113}{710}}(112+\frac{1}{\frac{2\cdot 3\cdot 44}{7}-\frac{1}{7}}-\frac{8}{\frac{3\cdot 5\cdot 44(2\cdot 83+1)(4\cdot 83-1)}{7}+\delta_c})$$

$$=137.03599907464417096826121642708$$

$$\delta_c=\frac{2}{41+\frac{1}{16\cdot 61-\frac{1}{4\cdot 9\cdot 29+1/4}}}$$

In the above formulas of c_{au}, the rate of 44/7 appears miraculously, so we suppose they should be reasonable and precise. It is worth noting that the factors 6, 10 and 14 (112=8×14) should correspond to the senary, decimal and forteenary number systems respectively [15], the factor 112 corresponds to the natural end of elements, i.e., the 112th element $_{112}Cn^*$, the factor 83 corresponds to the end of stable elements and the start of radioactive elements, i.e., the 83th element $_{83}Bi^*$, and the factor 137 corresponds to the end of hydrogen-like elements which was supposed by physicist Feynman, i.e., the 137th ideal extended element $_{137}Fy^{ie}$.

4. Discussion and Conclusion

The famous ancient Chinese mathematician Chongzhi Zu proposed the two eminent approximate rates of π which were 22/7 and 355/113 around in AD 460-480. The mathematicians in later generations (1944-2005) deduced the following formulas about them [16].

$$\pi=\frac{22}{7}-\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$$

$$\pi=\frac{355}{113}-\frac{1}{3164}\int_0^1\frac{x^8(1-x)^8(25+816x^2)}{1+x^2}dx$$

These above formulas could be expressed in 2π format as follows.

$$2\pi=\frac{44}{7}-2\int_0^1\frac{x^4(1-x)^4}{1+x^2}dx$$

$$2\pi=\frac{710}{113}-\frac{1}{2\cdot 7\cdot 113}\int_0^1\frac{x^8(1-x)^8(25+16\cdot 3\cdot 17x^2)}{1+x^2}dx$$

These precise formulas imply that 22/7 and 355/113 or 44/7 and 710/113 are not only the approximative rates of π or 2π, but also the scientific simulation of π or 2π. In other words, they should have scientific meanings. And after so many years from Chongzhi Zu's era, we have found what their real scientific meanings are and have applied them to construct reasonable formulas for the fine-structure constant and the speed of light in atomic units which are extremely important in physics.

Besides our previous formulas and explanations, this is another answer to physicist Feynman's question whether the fine-structure constant is related to π. Our answer is yes but in a very unexpected, subtle and reasonable way.

Reference

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