The method of dividing the 60° angle into three equal parts

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The problem of dividing a 60 ° angle into three equal parts in modern mathematics has not yet been solved. This involves the infinite extension of this trigonometric function in a generalized perspective. After research, it was found that the solution is located between 0r and one-thirds of r. The former represents a curve, while the latter represents a horizontal line. This article aims to utilize the relationships between various shapes to divide the 60 ° angle into three equal parts, which can then be extended to any angle less than 180 ° between r and two-thirds of r.

Keywords: 60° angle trisection; Bold attempt; Between 0r and one-thirds of r; trigonometric function

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Introduction

In the long history of geometry, the problem of dividing angles into three equal parts has always been a fascinating topic. Since ancient Greece, mathematicians have been exploring how to accurately divide an arbitrary angle into three equal parts using only a ruler and compass. Although the problem of trisecting angles has been proven to be unsolvable in general, this challenge has inspired countless mathematicians and propelled the development of mathematical theory.This article will use the strings of a circle and isosceles trapezoids to solve this problem, hoping that readers can patiently watch.

Literature review

Mathematician Wanzel once proved that it is impossible to directly make a trisecting angle. The reason is that cos20° is a root of the equation ($8x^3$-6x-1=0) (derived from the triple angle formula). This equation is a cubic equation, and its solution cannot be expressed by finite square root operations. Therefore, it is not possible to determine the length of the line segment at cos20°, but he ignored the chord of the circle, which also represents the circle(Pierre Wantzel,1837).

Lindemann proved the transcendence of π, indirectly supporting the unsolvable trisecting angle. His work shows that many problems related to ruler and gauge drawing, such as squaring circles, involve transcendental numbers and cannot be solved through ruler and gauge drawing.(lindemann, 1882).

Cos20° is a root of the equation ($8x^3$-6x-1=0) (derived from the triple angle formula).This equation is a cubic equation, and its solution cannot be expressed by finite square root operations.But my answer is not contradictory to the views of two famous scholars, because my starting point is the properties of circular chords and isosceles trapezoids.

Result

60° divide into three equal parts

Due to the slow processing of compass shapes on computers, paper images are used. This article will use the simplest method to draw to ensure that it is not too complicated.

(1) As shown in Figure 1, draw a circle with radius r and dot O, and then draw a 60° equilateral triangle inside the circle, which is Δ ABC. Divide the BC edge into two parts, with the dividing point being N, which is connected to vertex A and extended to M to form the dividing line NAM;

(2) Divide ∠ B and ∠ C into four parts, each at a 45° angle, with three parts forming one angle at 45° and the other angle also at 45°. The three-quarters of the two angles intersect at point A', connecting three points to form an isosceles right triangle (180-45-45=90). Connect A'A and O'B to obtain angle AA'B, then divide 90° into three equal parts to obtain an angle of 30°, and obtain points E and F of the three equal parts;

(3) Draw a circle with A'B and A'C as the centers, and the intersection point of the arc and the bisector NAM is A". Connect A" C and A"B to obtain the angle BA" C. Because the central angle is twice the

circumference angle of the circle, ∠ BA"C is 45°. Then divide the 45° angle into three equal parts to obtain a 15° angle, and obtain the points E' and F' of the three equal parts. The same applies to the 180 ° divide, where the three are in a straight line;

(4) The center of the 60° angle is between the center of the 45° angle and the center of the 90° angle;connect point E, point F, point E ', and point F' to form an isosceles trapezoid, which expands from the trisecting segment EF to the remaining trisecting segment E'F'. As the angle decreases, the chord length of its three equal angles also decreases. The chord lengths of the three equal angles are the EF lines of the isosceles trapezoid. The straight line of the middle string continuously shrinks as the angle decreases. You can imagine having a semicircle with a constant chord length. The angle of the semicircle decreases continuously and becomes a straight line, while the chord length of the three equal angle changes from r to two-thirds r.

(5) Draw a circle with A as the center and AB as the radius. The intersection points with the isosceles trapezoid are E"and F", which are also the 60° bisector points. Obtain the line segment E"F" that divides into three equal parts. It is an isosceles trapezoid that divides 45 ° and 90 ° equally, and this trapezoid naturally has equal division. You can draw a semicircle and divide it into three equal parts, with the 45 °, 90 °, and 180 ° points on a straight line.

(6)After obtaining an angle of 20°, we can obtain 18 equal circular parts, and then obtain 9, 36, and 72 equal circular parts;

(7)The same applies to Y equal parts. After dividing into 36 equal parts, each angle is 10 degrees. Find the middle angle or half of the angle between 140° degrees and 70° degrees, and then construct an isosceles trapezoid. Any angle can be divided into 7 equal parts; Similarly, 50° and 100° can also be found and divided into 5 equal parts. For example, 50°-100°, when divided into five equal parts, each angle is 20°. Observe the chord lengths of the third 10° and 20° and construct an isosceles trapezoid. After 72 equal divisions, with each angle being 5 degrees, find 85° and 170°;

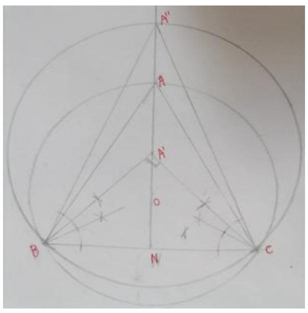

Figure 1

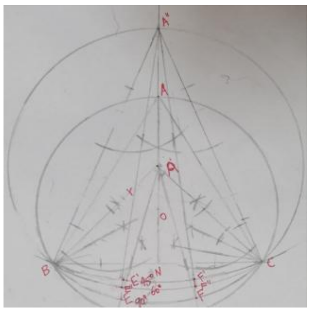

Figure 2

Any angle divided into three equal parts

To summarize, divide the points into three equal parts at 90 ° and 45 °, and connect the four points to construct a trapezoid. From Figure 3, it can be seen that there are two sides with an extension angle of ∠ X less than 180 °, resulting in an isosceles triangle A'OB '. Make a vertical bisector so that AB equals A'B '.

Then draw a circle with OA 'as the radius, where the arc intersects with the trapezoid. The connecting intersection point is the three equal parts of any angle. Then we need to start dividing it into any portion at any angle.

(1) If there are Y equal divisions (odd division and even division), you can first draw Y small angles similar to X degrees. Combine Y small angles into a large angle(large angle less than 180 °). Then draw the angle bisector at a large angle, where the intersection of the bisector and the arc is the center of the circle. Draw a circle with half the chord length at a small angle to obtain the angle at a small angle. Similarly, draw a double small and large angle, and then form an isosceles trapezoid. This trapezoid can achieve Y equal division of any angle. For example, figure 4.

(2) If the degree of X is greater than 180°, using 360° - x° for processing can achieve Y equal parts.

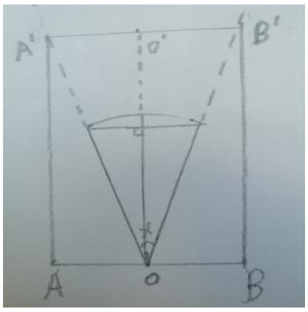

Figure3

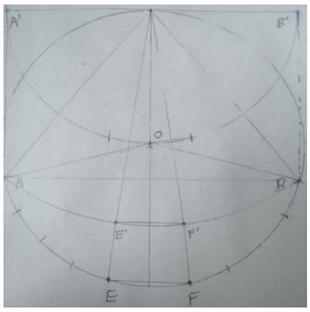

Figure4 Any angle divided into 10 equal parts.

The chord length of a three equal circle

The vertex angles of the three isosceles triangles are 45°, 60°, and 90°, respectively. The base is the same, both are $\sqrt{3}$. The top corner corresponds to the bottom edge. The ratio of the waist lengths of three isosceles triangles (expressed as trigonometric functions) is $(\sqrt{3}r\sin67.5°/\sin45)$: $\sqrt{3}r$: $(\sqrt{3}r\sin45°/\sin90°)$. Then draw a circle with the waist circumference of the three triangles as the radius, and calculate the chord lengths corresponding to 15°, 20°, and 30°: $[(\sqrt{3}r\sin67.5°/\sin45°) \times (\sin82.5°/\sin15°]$: $[\sqrt{3}r\times(\sin80°/\sin20°)]$: $[(\sqrt{3}r\sin45°/\sin90°) \times (\sin75°/\sin30°)$.

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