Irrational Numbers Do Not Exist

by

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The Rational Number *ρ*

Let *r* and *n* be integers whose absolute values approach *infinity*, then

$$\lim_{|r|,|n| \to \infty} \frac{r}{n} = \rho \qquad (1)$$

Any rational number that is very near to *r*/*n* will converge to *ρ*

$$\lim_{|r|,|n| \to \infty,\ \to \infty} \left(\frac{r}{n} + \frac{k}{n}\right) = \lim_{|r|,|n| \to \infty} \left(\frac{r}{n} + 0\right) = \rho$$

for any small integer *k*.

Because we are dealing with large integers, it is not necessary to assume that they are relatively prime. What is relevant is the limit of their ratio.

Proof that √2 is rational

Let $\sqrt{2} = \lim_{r,n \to \infty} \frac{r}{n}$, where *r* and *n* are very large integers.

Squaring both sides

$$\lim_{r,n \to \infty} \frac{r^2}{n^2} = 2$$

Since

$$\lim_{r \to \infty} r^2 = \lim_{n \to \infty} 2n^2$$

$$\infty = \infty$$

we can not know whether the large integers *r* and *n* are both even.

Adding 1 to *r* and getting the limits below

$$\lim_{r,n \to \infty} \left(\frac{r+1}{n}\right) = \lim_{r,n \to \infty} \left(\frac{r}{n} + \frac{1}{n}\right) = \lim_{r,n \to \infty} \left(\frac{r}{n} + 0\right) = \sqrt{2}$$

$$\lim_{r,n \to \infty} \frac{(r+1)^2}{n^2} = \lim_{r,n \to \infty} \frac{(r^2+2r+1)}{n^2} = \lim_{r,n \to \infty} \left(\frac{r^2}{n^2} + \frac{2\sqrt{2}}{n} + \frac{1}{n^2}\right)$$

$$= \lim_{r,n \to \infty} \left(\frac{r^2}{n^2} + 0 + 0\right) = \lim_{r,n \to \infty} \left(\frac{r^2}{n^2}\right) = 2$$

Therefore, √2 is a rational number.

$$\sqrt{2} = \frac{14142135623.\ldots\ldots d}{10000000000.\ldots\ldots 0} = 1.4142135623\ldots\ldots d$$

where *d* is the last digit of $\sqrt{2}$.

Remarks: There are *no* irrational numbers. The numbers *e*, π,√3, √5, … are *all* rational numbers defined by (1).