An Adaptive Quantum Evidential Combination Rule for Open Set Recognition

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Abstract

By exploiting the computational potential of quantum computing beyond the computational power of classical computing, an adaptive quantum algorithm of generalized evidential combination rule (AQ-QECR) is proposed to reduce the computational complexity of QECR in the creditability and plausibility levels with no information loss.

Keywords: Quantum evidence combination rule, Quantum algorithm, Adaptive quantum evidential combination rule, Open set recognition

1. AQ-QECR: Adaptive Quantum Evidencial Combination Rule

The AQ-QECR algorithm consists of the following three steps.

Step 1: Initialization of the quantum states of GQBBA

In a quantum frame of discernment (QFOD) $|\Phi\rangle = \{|\phi_1\rangle, |\phi_2\rangle, \cdots, |\phi_n\rangle\}$, let $\mathbb{Q}_{\mathbb{M}}$ be the GQBBA. Then the corresponding quantum superposition state of $\mathbb{Q}_{\mathbb{M}h}$ could be generated by the following rule:

$$|\mathbb{Q}_{\mathbb{M}h}\rangle = \sum_{|\psi_j\rangle \in 2^{|\Phi\rangle}} \varphi_h(|\psi_j\rangle)\,|\psi_j\rangle, \tag{1}$$

in which

$$|\psi_j\rangle = \overset{n}{\underset{i=1}{\otimes}} |\delta_a^i\rangle = |\delta_j^n\rangle \cdots |\delta_j^2\rangle\,|\delta_j^1\rangle, \tag{2}$$

$$\delta_j^i = \begin{cases} 1, & |\phi_i\rangle \in |\psi_j\rangle, \\ 0, & |\phi_i\rangle \notin |\psi_j\rangle. \end{cases} \tag{3}$$

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Step 2: Deployment of the combination quantum circuit

Subsequent to the initialization stage, the GQBBAs will be combined by a series of specific quantum operators, designated as U_1^C and U_2^C. After setting up U_1^C, the density operator $\rho_{M_{12}}$ of the mixed state on the output qubits as follows:

$$\begin{aligned}\rho_{M_{12}} = &\sum_{|\psi_t\rangle \in 2^{|\Phi\rangle}} \left(\sum_{\substack{\cap|\psi_j\rangle=|\psi_t\rangle \\ \cup|\psi_j\rangle\neq|\emptyset\rangle}} \left(\prod_{1\le h\le 2} |\varphi_h(|\psi_j\rangle)|^2 \right) |0\rangle|\psi_t\rangle\langle\psi_t|\langle 0| \right) \\ &+ \sum_{\substack{\cap|\psi_j\rangle=|\emptyset\rangle \\ \cup|\psi_j\rangle=|\emptyset\rangle}} \left(\prod_{1\le h\le 2} |\varphi_h(|\psi_j\rangle)|^2 \right) |1\rangle|\emptyset\rangle\langle\emptyset|\langle 1| \,. \end{aligned} \tag{4}$$

The operator U_2^C uses these two inputs to obtain the desired quantum state. After setting up U_2^C, the density operator $\rho_{M_{1\cdots k}}$ of the mixed state on the output qubits as follows:

$$\rho_{M_{1\cdots k}} = \sum_{|\psi_t\rangle \in 2^{|\Phi\rangle}} \left(\sum_{\substack{\cap|\psi_j\rangle=|\psi_t\rangle \\ \cup|\psi_j\rangle\neq|\emptyset\rangle}} \left(\prod_{1\le h\le k} |\varphi_h(|\psi_j\rangle)|^2 \right) |0\rangle|\psi_t\rangle\langle\psi_t|\langle 0| \right) \tag{5}$$

$$+ \sum_{\substack{\cap|\psi_j\rangle=|\emptyset\rangle \\ \cup|\psi_j\rangle=|\emptyset\rangle}} \left(\prod_{1\le h\le k} |\varphi_h(|\psi_j\rangle)|^2 \right) |1\rangle|\emptyset\rangle\langle\emptyset|\langle 1| \,. \tag{6}$$

Step 3: Measurement of quantum superposition state

In addition, there are two different measurement functions in AQ-QECR to meet different levels of need, the creditability level and the plausibility level. In the event that the objective is to generate a full generalized basic belief amplitude assignment (GBBAA), the measurement operator U_C^M should be deployed. The U_C^M operator consists of the following measurement operators:

$$U_C^M = \{\mathcal{M}_{|0\rangle|\emptyset\rangle}, \mathcal{M}_{|0\rangle|\psi_1\rangle}, \cdots, \mathcal{M}_{|0\rangle|\psi_j\rangle}, \mathcal{M}_{|1\rangle|\emptyset\rangle}, \mathcal{M}_{|1\rangle|\psi_1\rangle}, \cdots, \mathcal{M}_{|1\rangle|\psi_j\rangle}\}, \tag{7}$$

$$\mathcal{M}_{|i\rangle|\psi_j\rangle} = |i\rangle|\psi_j\rangle\langle\psi_j|\langle i| \,. \tag{8}$$

Once the measurement operator U_C^M has been applied, the full combined GBBAA can be generated as follows:

$$K_G = \sum_{\substack{\cap|\psi_j\rangle=|\emptyset\rangle \\ \cup|\psi_j\rangle\neq|\emptyset\rangle}} \prod_{1\leq h\leq k} |\varphi_h(\psi_j)|^2 = \Pr(|0\rangle|\emptyset\rangle), \tag{9}$$

$$M(|\psi_t\rangle) = \frac{\sum\limits_{\substack{\cap|\psi_j\rangle=|\psi_t\rangle \\ \cup|\psi_p\rangle\neq|\emptyset\rangle}} \prod\limits_{1\leq h\leq k} |\varphi_h(|\psi_j\rangle)|^2}{1-K_G} = \frac{\Pr(|0\rangle|\psi_t\rangle)}{1-\Pr(|0\rangle|\emptyset\rangle)}, \tag{10}$$

$$M(|\emptyset\rangle) = \frac{\sum\limits_{\substack{\cap|\psi_j\rangle=|\emptyset\rangle \\ \cup|\psi_j\rangle=|\emptyset\rangle}} \prod\limits_{1\leq h\leq k} |\varphi_h(|\psi_j\rangle)|^2}{1-K_G} = \frac{\Pr(|1\rangle|\emptyset\rangle)}{1-\Pr(|0\rangle|\emptyset\rangle)}. \tag{11}$$

If the objective is to classify directly, another measurement function, designated U_{Pl}^{M}, is proposed. The U_{Pl}^{M} consists of the basic one qubit measurement operator as follow:

$$U_{Pl}^{M} = \{\mathcal{M}_{|0\rangle}, \mathcal{M}_{|1\rangle}\}, \tag{12}$$

$$\mathcal{M}_{|i\rangle} = |i\rangle\langle i|, \tag{13}$$

After the measurement operator U_{Pl}^{M} has been applied, the decision D could be generated as follow:

$$D = \phi_v, \quad v = \arg\max_{v}\{\Pr_v(|1\rangle)\}, \tag{14}$$

where ϕ_{n+1} represents $\emptyset$, i.e. elements outside the FOD, and $\Pr_v(|1\rangle)$ is the probability of getting $|1\rangle$ when measuring the v-th qubit of the output qubits.

2. Conclusion

Both levels of the proposed AQ-QECR could exponentially reduce the computational complexity of quantum evidence combination rule [1] with no information loss.

References

[1] F. Xiao, Quantum X-entropy in generalized quantum evidence theory, Information Sciences 643 (2023) 119177.