Equation with sum of four sixth degree integers equal -

-to another four sixth degree integers

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Abstract

There are numerical solutions available on Wolfram world of mathematics website (ref. # 4) for the equation (p^6+q^6+r^6+s^6)=2(a^6+b^6). In this paper the author has arrived at numerical solution by algebra. It is common knowledge that arriving at numerical solutions by algebra is difficult for degree four & above. Also on the internet the author has not come across any method for the above mentioned equation.

Consider the below equation:

$$(p^6 + q^6 + r^6 + s^6) = 2(a^6 + b^6) \ --- (1)$$

We have the Identity:

$$u^6 + v^6 = (x^6 - 3uvx^2(2x^2 - 3ab) - 2(uv)^3) \ ---- (2)$$

Where, x=(u+v)

In equation (1), we take, (a+b)=(p+q)=n & 5ab=3pq

Hence we have:

$$(a^6 + a^6) = (n^6 - 3abn^2(2n^2 - 3ab) - 2(ab)^3) --- \ (3)$$

$$(p^6 + q^6) = (n^6 - 3pqn^2(2n^2 - 3pq) - 2(pq)^3) --- \ (4)$$

Since, 3pq=5ab , eqn (4) becomes:

$$(p^6 + q^6) = \ (n^6 - 5abn^2(2x^2 - 5ab) - 2(pq)^3) --- \ (5)$$

From eqn (1) we have:

$$(r^6+s^6)=2(a^6+b^6)-(\,p^6+q^6)\;-----\;(7)$$

Substituting in the (RHS) of (7) from eqn (3) & (5) we get after some algebra:

$27(r^6+s^6)=27n^6-54abn^4-189a^2b^2+142a^3b^3$ ----- (8)

$$=(3n^2-2ab)(9n^4-12abn^2-71a^2b^2)\;\;--(9)$$

We now take, $x=3r^2,\;\;y=3s^2$

Thus we have: x^3+y^3=$(3n^2-2ab)(9n^4-12abn^2-71a^2b^2)$

Hence we take,

$$(x+y)=(3n^2-2ab)-------------(10)$$

$$(x^2+y^2)=(9n^4-12abn^2-46a^2b^2)------(11)$$

Solving for (x,y) in in eqn (10) & (11) we notice that in-order to have integer

solution, the determinant "w" for [eqn (10) & (11)] is as below:

$$w^2=(9n^4-12ab-96a^2b^2)\;-----\;(11)$$

$$eqn\ (11)\ has\ numericalsolution, (a,b,n,w)=(9{,}2{,}11{,}273)$$

$$Hence,\qquad x=\frac{1}{2}(3n^2-2ab+w)$$

$$y=\frac{1}{2}(3n^2-2ab-w)$$

Substitutin for (a,b,n,w)=(9,2,11,273) in above we get: (x,y)=(300,27)

Since, $x=3r^2, y=3s^2\quad we\ get: (r,s)=(10{,}3)$

& since, 3pq=5ab & (a,b)=(9,2)

We get (pxq)=30,

And since, $(p+q)=n=11\ we\ get,\quad (p,q)=(6{,}5)$

$$Hence\ (a,b)=(9{,}2)\quad \&\quad (p,q,r,s)=(6{,}5{,}10{,}3)$$

Therefore: We have the below numerical solution:

$$(6^6+5^6+10^6+3^6)=2(9^6+2^6)$$

References

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