Geometrical quantum theory and applications.

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Contents

Some words upfront.

This book is written for the scientific mind and requires a good knowledge of mathematics used in theoretical physics. The reader wishing for a more thorough philosophical as well as mathematical introduction to the subject is referred to two previous books of mine on the matter. They are much more elementary whereas this book is directly aimed at the trained scholar who wishes to see the beef right away instead of muddling through long philosophical considerations. Moreover, substantial extensions in content are made here some of which require quaternionic geometry as a kind of specialization of Riemann surfaces. This book is filled with spicy insights regarding the proper formulation of quantum mechanics and contains several highly nontrivial conjectures regarding the mental world based upon a physical-mental correspondence which is a consequence of the mathematics employed. As far as I know, I am the first author to do that and I would encourage people to experimentally verify these findings given that they display a remarkable connection between our psyche and the physical laws of the universe. Albeit we discuss the mental world at length here, we do not address for example epistemological adventures such as an explanation for consciousness: that is metaphysics and science shall never have anything to say about that. What we can do however is putting in consciousness parameters in our theory which, if surpassing a certain value, tells whether we are aware of a certain thought process or not. This is something which could even be detected in principle by means of chatting to a person and telling him or her to speak out every thought which occurs to them and, at the same time, measuring brain activity by means of an MRI scan. Indeed, it is already possible, as far as I know, to decode the thoughts of a person by suitably interpreting the electrical currents in the brain. You might want to ask a computer for the same thing and maybe it will provide you with the full data flow in some of its electrical components, all bit streams. A computer, in principle, could in this sense have an immaculate consciousness. It is important to use the very best possible words here regarding what we really want to say because human consciousness is often related to things such as coherence of speech, level of understanding, intelligence and so on. What people such as Roger Penrose want to point out is that humans seem to possess a level of understanding which goes beyond computable processes, something which he believes to be the limitations of a computer. I am not a specialist in this field but it seems rather unlikely to me that a computer will ever be able to answer questions regarding mathematical truths involving first order logic by wich I mean that it will not be able to provide for counterexamples if they were not programmed upfront. What you can do is feed it with the formal rules of first order logic such that it can perform formal manipulations with it or verify wether some reasoning is correct or not. You don't even have to provide it with all intermediate steps as long as there is a strict sequence, which can be decided upon by the Turing Machine at hand in a reasonable finite time, of basic steps from which the next line follows. Furthermore, if you feed it with enough proofs, then it might even learn some strategies of how to construct new theorems but there exist plenty of problems which would not be decidable. To give a silly example, if a computer were not told that $x \to |x|$ is not differentiable at $x = 0$, it would never ever be able to find out the truth of wether it is or is not everywhere differentiable (even if you would programme it to first look at a counterexample and check $x = 0$ first, then I will feed it with $|x - a|$ where a is a random number). It does not comprehend what the quantors forall and there exists mean since no physical logical gates exist which can decide upon those symbols, in contrast to conjuctions which are the foundations of zero order (or proposition) logic: I do not know in full detail how a computer verifies statements in proposition logic when abstract words are used for statements, but I presume it just gives a random binary code to these things (it does not matter what code, as long as all codes are different and as long as you do not use words which are pre-programmed as commands) and then executes this particular sentence using logical gates. We humans, on the other hand, understand intuitively what those quantifiers mean (and we even invented them!), something which allows us to search for counterexamples in a meaningful way or recognize at first glance that the absolute value is not differentiable at zero, because we understand the graph of a function (I must mention here that there is nothing exact about the Peano axioms given that you cannot define the quantifiers, free variables and so on, it is an intuitive understanding we have). A computer can create its own concepts and even deduce logical statements from those concepts and you could even programme it to create new statements using those concepts and let them wonder wether they are true or not. But it will never be able to decide upon its truth unless, by mere luck and pre-programmed strategy, it finds out a proof of it or its negation. We humans can also say when something is false even when no formal proof of its negation is decidable by an algorithm.

It appears to me that Roger seems to associate intelligence with human consciousness, something which is a matter of bad wording unless I misunderstood him. To me, it is much more mysterious as to where all those living, intelligent organisms come from and how they appear to construct their own world! Surely not by random selection out of a thoughtless process! We shall not adress this issue of origination either, which is clearly beyond computable scientific reach, but I shall discuss relationships between our mental processes and the physical world; in particular the constraints the latter puts upon the former. I do think that this is a worthwhile question to ask, which can be answered by science; it would not allow us to understand all details of evolution, but at least it could tell us what kind of evolution is merely possible or what kind of limitations are imposed upon it. It would also offer a better comprehension of who we are and why we are the way we are and perhaps, in the long run, allow us to manipulate humanity. This is the best science can do as far as I am concerned. This work should ultimately connect to biophysics and I am glad that giants in mathematics, such as Misha Gromov, have shown active interest in the field. Very much like Gromov, I think we are currently not ready yet to adress the problem of quantum gravity and hope that my work might provide for a serious contribution in that regard. At least, it suggests a very novel viewpoint which makes sense and this is briefly discussed in part 4 of this book. I am of the opinion that physics and mathematics are the two parents of all sciences and that ultimately every field should be held responible regarding insights in both. Historically, this has been so for chemistry, then biology and medicine, next computer sciences and now I believe it is time to interfere with the "sciences" of the mind! This ultimately should couple back to biology as I clearly argue in part 1 that there is much more to the mind than classial and quantum physics tell and a mindless mechanism shall never ever explain biological evolution. It is indeed our business to interfere with those sciences and it is my intention to give a kick-off start here. The psychologist and psychiatrist might react in a polite way by means of "what the fuck". But I am not interested in those fields as they currently stand; the idea which I want to put forwards is to engage in a higher level of abstraction which might open the door to a deeper understanding at least peeling off some of the mystery. Indeed, as the reader will notice in parts $1, 3$, the abstraction of concepts leads to specific predictions and insights which I have nowhere encountered before: precision of language shapes your insights into reality!

Regarding quantum field theory, the message of this book is loud and clear: the notion of a particle therein is completely wrong! What researchers do is to ascribe particle notions to the Killing fields of Minkowski, this is completely unphysical and bogus. They wrongly attach the notion of spin and momentum to changes of coordinate systems and the whole edifice of quantum field theory is based upon this. The notions of spin and momentum are far more general than this, they pertain to the tangent bundle and not spacetime! They are inherently present in any spacetime and not just Minkowski! This is a major insight and I discuss in detail as to why the old viewpoint in Minkowski results from ours by breaking an infinite dimensional symmetry group to a 10 dimensional global one. This is not just a minor point since people have always been interested in asymptotically Minkowkian (or de Sitter or anti de Sitter) spacetimes given that the maximal symmetries allowed them to define particles with respect to observers at infinity. Even string theorists do that all the time: there is I am afraid no physics in that. To be really bad, in my opinion, there is nothing of value to learn from the AdS-CFT correspondance, even if it holds. The result is that the notion of a field is no longer adequate, but you need bi-fields (or an entirely geometrical description which is relational) as I explain in part 1 in full detail. Another upshot is that you ask no longer for coordinate or position operators which are completely unphysical and redundant (pure gauge); the theory is entirely geometrical. Maybe, the reader who did not go through the entire process of learning quantum field theory might find these remarks obvious and they are also as such for me. But the relativity community likewise commits these sins in trying to interpret solutions to the Einstein equations where spacetime is seen from the perspective of an observer at infinity. So, in a way, this is a major upset which changes the entire game. Our theory is valid in any parallelizable spacetime, meaning one with a globally well defined tetrad, which is mandatory to define particles with non-zero spin; this condition implies that spacetime is time orientable as well as space orientable leading to the well known applications of Stokes theorem. It is by far the weakest condition any physical spacetime should satisfy. The first edition of this book was written in a lovely village near the coast in Andalusia during the winter time in 2019. This revised version eliminates many typo's, annoying mathematical errors and inaccuracies in the formulation albeit every crucial idea in the previous edition remained intact: it has cost me a serious effort, especially regarding the theory of the mental world, to make everything crystal clear. All remaining errors and inaccuracies are my responsability alone.

Part I

Introduction.

Chapter 1

General philosophical considerations.

We propose a general scheme in which to comprehend the world from a unified point of view. It is an open dabate on what form such theory should take; as is well known, the current accepted theory for microscopic physics is relativistic quantum theory or quantum field theory as it is usually called. There are two key observations here we start with, a conventional one and another which has not been discussed yet in the literature but which I believe is of utmost importance. The standard view is that quantum theory is incomplete; in the early interpretations, foundinding fathers insisted upon a double view on the world in the sense that the very formulation of quantum mechanics requires a classical world with macroscopic apparati having the usual properties. The act of measurement then on the quantum system was thought of as due to some sort of interaction between the classical and quantum world for which no dynamical prescription exists. Instead, people early on, insisted upon a rule of thumb, called the collapse of the wavefunction in order to bring waves to classical particles. This view has changed upon the time and the current state of affairs is that the entire world is quantum mechanical and “consciousness”, which deals with the Platonic world, projects the wavefunction onto more classical separated components whereas the entire quantum world is totally entangled and in superposition. There are very few researchers willing to deal with this subject, but in this book we shall not shy away from it and address the matter at least from a principled point of view. The second, and for most physicists probably the most important and controversial stance, is that the mathematical formulation of quantum theory is poor and inadequate and hinges too much upon properties which only hold in flat spacetime. By this, I want to say that the basic language of quantum theory is not a geometrical or intrinsic one in general. It is only intrinsic on flat spacetime but leads to a wide variety of unphysical ambiguities in curved spacetime. This is certainly so for the old quantum theory, which we call first quantization. The situation is somewhat better in the second quantization but in general issues regarding the proper representation of the theory as well as vacuum state (definition of particle) remain widely open. Recently, I was involved in a more novel approach which tries to eliminate the very last non-intrinsic aspects of quantum field theory and that concerned work done in the so called Sorkin-Johnston formulation of a free scalar field on a causal set. Here, one does not suffer from all problems in the continuum where products of field operators are not well defined and all calculations are formal and leading to infinities. The field operators on a causal set are well defined unbounded (and not distributional) operators so that products do have a well defined meaning if one is delicate about domain issues. However, even if this theory has potential to meaningfully deal with interactions, I remained dissatisfied for two reasons. One is inherent to problems with causal sets in particular, and that is that there is no obvious replacement, on a coarse grained scale, for "infinitesimal" Lorentz transformations so that the entire notion of spin gets lost. Also, as far as the theory stands, they would have to explain why their Pauli Jordan function is

$$i(G_R(x,y) - G_A(x,y)) = \Delta(x,y)$$

where $G_R(x,y) = \overline{G_R(x,y)}$ is the retarded Greens function and $G_A(x,y) = G_R(y,x)$. Indeed, the prescription

$$\tilde{\Delta}(x,y) = G_R(x,y) + G_A(x,y)$$

would be commensurable with the anti-commutator for real fields, so in a way, you need to add the information that the Pauli Jordan function treats the past and future asymmetrically. A second, and much more substantial objection was that the *definition* of a particle and vacuum state was entirely dependent upon the whole of spacetime and in particular growth towards the future would recalibrate those. Now, this kind of unphysicality is virtually not criticized in the literature and even embraced by means of the Hawking and Unruh effect. I do not consider those as deep indications of how to reconsile quantum theory with general relativity but merely expressions of the very failure of quantum theory in the first place. Indeed, science requires that we can locally define particles and experience shows that our Minkowskian definition works pretty fine on lab scales and that we do not even have to bother about the gravitational field. Indeed, how would it be possible to make any predictions regarding physics if we could not even grasp the objects we are talking about from a local perspective; we would be entirely lost. This view upon particles stems from the usual implementation of the Fourier transform, a notion which does not make any sense at all in the way people usually think about it in a general curved spacetime. The operational language attached to it by means of the Heisenberg commutation relations is a very bad way of speaking about wave-particle duality; one which only works in Minkowski and nowhere else. We shall propose an exclusively geometrical language for dealing with the quantum world where all those problems do not occur and which keeps a clear definition of spin and momentum even if those do not exist in the standard approach towards the subject. This subject, which we call level zero of reality, or the so called physical manifestation of elementary particles, is discussed in about seventy percent of this book.

The remaining part deals with mystery, things which we do not understand properly and which we never shall because the mathematical language falls rather short here. It is important to make a distinction between what one may call investigative minds and mechanical minds, the former being able to question every question whereas the latter only have a finite number of what one may call pointer questions. For example, I don't think a measurement apparatus asks itself the question why the wavefunction of an incoming particle collapses; it just deals with particles scattered and absorbed by it. Humans on the other hand can question why they see what they see, feel what they feel or think what they just thought. Investigative minds can never be described as there is an infinite number of questions one can ask and we shall deal only with spirits which are up to a truncation mechanical. Another issue regards your impact in the physical world of the questions you ask, here one can define a hierarchy or level by means of questions an entity of a certain hierarchy can ask related to questions of a mind in a higher hierachy without reduction of the wavefunction or physical impact as to speak. This is usually not considered in quantum theory as there any question asked has in principle an answer which reflects into reality. So, the higher spirits have an exclusively active influence on the world of the lower beings regarding their material observations; their questions and answers regarding the physical world having no impact whatsoever on the outside world. At a given point, spirits may interact democratically amongst one and another without there being a higher spirit which fully influences your perception. To understand why I introduce this concept of hierarchy, let us examine a few examples in the literature. For example, take an atom; ideally, the dynamics thereof is desribed by the standard model with the strong interactions dominating the nucleus and the electromagnetic interactions describing the interactions between electrons and the nucleus. Now, in no way is an atom state an eigenstate of this Hamiltonian, but the way the atom communicates with the outside world is approximately by means of a semiclassical Hamiltonian $H_{\text{eff}}(t)$ which is incredibly difficult to define; indeed, one might define

$$P(t)\text{Tr}_{\text{nucleons}}\left(|\Psi\rangle\langle\Psi|\right) = \text{Tr}_{\text{nucleons}}\left(e^{-iHt}|\Psi\rangle\langle\Psi|e^{iHt}\right).$$

From the property that the right hand side maps density matrices to density matrices in a linear fashion, we conclude that $P(t)$ must do the same thing; but in general $P(t)$ will not preserve the rank of the density matrix, so there is no associated canonical unitary mapping. Now, nature tells us, that the effective dynamics for a stationary atom reasonably separated from other atoms or molecules has an "effective" $P(t)$ which does preserve the rank and is associated to a unitary (time independent) operator in Minkowski. This means that the outside world should entangle democratically with our atom in the sense that all those entanglements which build up almost factorize; this is reasonable as on sufficiently large distance scales, the variation in the interactions is extremely small on the "effective" support of the wave functions in another atom so that the interaction between states in different atoms might be treated as constant and therefore state independent. Another complication shows up here which is that in quantum field theory, particles are indistinguishable and wavepackets of different atoms will have a nontrivial projection on one and another - in contrast to the standard Euclidean multiparticle approach where the atoms would be distinghuished and live in "distinct" Hilbert spaces. Therefore, taking the partial trace will inevitably cause for some interference effects between distinct atoms but those are in general expected to be small. There is something further to say, that is that $H_{\text{eff}}(t)$ treats the nucleus classically, if one merely investigates the electromagnetic properties of the atom, and that the only questions the atom gheist seems to be asking regarding the electrons is about their individual energy levels; if not so, we would not observe a black body radiation spectrum since that result hinges upon semiclassical considerations regarding transition probabilities between those energy levels where a supplementary equilibrium *assumption* regarding the interactions with the external electromagnetic field is made. Clearly, the elctron and nucleons are dumb and cannot ask questions about themselves nor their relationship to the outside world, but the atom can do better; but somehow it does not ask for positions of electrons. Now, when atoms join to form molecules, it are no longer the energy levels of the electrons in the individual atoms that count, but rather their energy with regard to the full semiclasical molecule Hamiltonian is of importance; for example, the so called valence electrons do feel both worlds whereas the lowest energy states are mostly confined to the one atom world - in that sense, you could argue that the atom still has free will regarding its lowest energy states and agrees with the molecule spirit on those. So, in a way, the atom spirits have become slaves of a communal spirit who asks the questions and the effective one atom density matrices of the real molecule world pure state are always a mixture of individual levels. The atom is never certain about itself (meaning the state of the electrons) and maybe, one can uphold the interpretation that it would consciously choose a particular projection on a definite one atom-electron state $|\Phi\rangle$ with probability

$$\langle\Phi|\text{Tr}_{\text{nucleons and electrons belonging to distinct atoms}}\left(e^{-iHt}|\Psi\rangle\langle\Psi|e^{iHt}\right)|\Phi\rangle$$

and say that it percieves its electron structure like that but no collapse of the wave function is associated with this. So this is an example of a situation where the molecule spirit is higher as the atom spirits are; the former determines the real spectral lines whereas the latter give their own interpretation to these findings. So this is really how our world operates I believe, we humans are related regarding our questions/observations concerning the outside world whereas the questions regarding our "internal" state can be partially free since the interactions going on at a microlevel are much stronger than those with the surroundings, so that corrections to the internal state due to the outside world are small and the resulting operator effectively commutes with the projection operator on the "exterior" states. In our example, our atom spirits may be free regarding questions about the nuclei that don't affect the electromagnetic interaction with the electrons given that the molecule spirits maybe only care about the electrons given that they are responsible for the binding. They won't influence the obervations of the molecule spirit since the latter just doesn't bother asking those questions. One can ask for the sense of such a worldview since, on the conscious level, questions and answers thereon are hardly reproducible; we seldomly think about the same thing twice in a short period of time and in any case, the anwers to the same question posed again will depend upon previous reflections. This is the same issue in a time dependent cosmology where it is in principle impossible to get the same state and dynamics out at different times. Another issue regards free will; if all probabilities of pointer questions have been fixed, then there is no such thing as free will which does not want to imply that there is no such thing as "purpose". It seems to me that nature has a tendency to ask more and more complex questions over time leading to evolution of species; the goal herein is to form more and more complex physical structures. So, in this way, nature is intelligent; it probes new questions and learns frpm previous trials. It is very well possible that this can be described up to some height by means of laws relating different questions and the global interactions theirof. As is highlighted in this book, the compatibility of questions we ask about nature requires a global notion of parallelism of how to interpret the state of the universe; a global conscious "now" say. So, in a way, fairly isolated organisms appear relatively free in asking internal questions without influencing others exclusively and/or consciously in a statistical sense. Now, given the broadness of such conception which is not to the end falsifiable, we might simply give up and say that the whole world is in hands of Gods as religious people often do. We however have the tools to capture a glimpse of the world and control part of it. On the other hand, some things are out of reach; to find and demarcate this boundary is the aim of science. Since it is unknown to us where this boundary resides, the good scientist is optimist and must wisely and slowely expand his grasp upon the world. The limitations are that we can only describe the world in our language implying that the language of less and more complex beings is unknown; physicists often interpret this complexity in terms of length scales.

That is, we assume the language of the tiniest beings which we cannot further subdivide as the basic one. This irreducibility, meaning inadequacy of destruction, may be entirely due to our own limitations and therefore our basic world view is always mathematically and linguistically irreducible meaning there is no sub object or sub language in an appropriate sense. Now, things are even more complex, this language of irreducibles is our way of speaking about them and not necessarily the way the irreducibles or elementary particles do. Their language may even be much more limited and irreducibility gets a different meaning there; so the linguistic part and to some extend, the mathematical construction, of irreducibility is ours. There is no fundamental objectivity in science; going over to more complexity and larger length scales, we may extrapolate and think that there is a human reduction meaning that the language of giants is reducible to the one of atoms. This stance is called *reductionism* in science and it is clear to me that it is false; we shall discuss the principle of weak reductionism and emergent variables at a later stage.

Our definition of hierarchy so far pertained to the influence our observations on the objective physical world and in particular whether spirits exclusively influenced the observations of lower order spirits where those of lower order did not affect some part of the physical world as observed by those spirits of a higher hierarchy, whereas the latter are insensitive to those changes the former can cause. There is a small detail here which is the best exemplified by the process of nuclear decay; in such case the atom spirit redines itself which has an impact on the very existence of the molecule spirit; so the reader must understand here that our hierachy pertains only to stable spirits which do not undergo a transition. Psychologists usually develop a second idea, which is one of the of so called mental leadership where consciously communicated thoughts or observations exclusively affect the thoughts of others regarding some class of "issues" C. However, it appears to me that mental leadership is an internal urge of the personal spirite nwho takes the physical action of speaking out its words and thereby influencing the brain of others and as a backreaction their spirit. Maybe, a predestination for mental leadership has a material grounding in the brain, but such effective variables would be extremely hard to describe from a point of view grounded in the cell, molecular or even atomic structure; nevertheless, the spirit must be able to recognize such patterns as pertaining to such issue and it is *that* what we cannot describe to the end. Slowely, we will learn there but our view on theories will expand once we better understand the proper questions (which does not mean of course that the fundamental material dynamics of our theory changes, but the way it manifests itself to us changes for sure, so our knowledge about the spiritual domain expands here). So, ultimately the issue of mental leadership is still a mental one even if some material traces are present which explain why you are more likely as such as others are. For example, one may look for a genetic basis correlated to a high IQ, but such a thing will never explain to the end why you will appear to be intelligent. It is this form of hierarchy that we shall discuss in part 3 of this book and we shall try to unravel principles of interactions between thoughts or other mental constructs which go reflect themselves in the physical structure of the brain; in doing so, we shall make a shortcut here, instead of going from thoughts of person A to physical communication thereof to person B who stores this information in his physical brain and then adapt its mental world to that information, we shall directly describe a causal theory between thoughts of distinct observers opening the door for thelepathy and subconscious communication. This may very well be a controversial stance but it is effectively much easier to find out some principles regarding mental comunication than passing through level zero of physical manifestation. Moroever, it is a fact that unconsciously, we are all connected in asking the proper questions about reality so that this extrapolation is maybe not that controversial after all.

The somewhat naive quote of physicist Feynman goes that "mathematics is the language in which nature speaks"; as we just discussed, maybe nature does not speak any language, but its living inhabitants do. The language we shall use is a quantitative one, it employs numbers! As far as I know, numbers allow for the most predictive statements to be made since there are a lot of internal manipulations you can perform upon them. Any other way of expressing relationships would be far more elaborate and require many more rules than caculus does; it is just not desirable at this point to walk that route but it might be a higher level of abstraction which is mandatory for the future. Now that we have discussed the general philosophy behind this book, let us now come to how the content is actually treated in a technical way in the several parts of this manuscript. Part 1, as the reader knows by now, constitues an introduction to the most basic mathematical ideas behind the realization of these concepts. Here, we shall make a couple of assumptions: that is (a) our irreducible constructs (mental as well as physical) correspond to unique mathemaical points and (b) there is a reality to mathematical points, which is that they are the setting for what we call the universe (c) all those points glue nicely together in a n dimensional manifold which provides us with continuous experience of things. Furthermore, we assume that if we see an elementary particle in a pointlike manifestation to us, that it is really pointlike. This naive correspondence between our perception and an underlying reality are obviously the first thing to try out; there is no point in creating an imaginary world on cannot access to "explain" something as long as one does not need it. Given these assumptions, Part 2 develops a theory of interactions amongst elementary particles and the resulting observations made by measurement apparati. Here, the description of an apparatus is rather elementary as we effectively do not take it as constituting of a bunch of particles, obeying the same laws, but "frozen" in constitution due to rapid projections of the constituting spirits. This is standard practise in quantum theory as calculations of an exact nature would become extremely complex and nobody has even dared to do so.

The reader will notice that Part 2 is highly mathematical in nature and that mathematical consistency helps one to sharpen ones image of the world; that is, it provides ou with new ideas which were inconceivable before. This point is often not understood by layman, that mere consistency of language provides one with new ideas regarding the topic one wishes to speak about. The spirit in this work is not described, it does what it is supposed to do and no furter questions are asked. A spirit distinguishes itself from what we call matter in the sense that it can observe itself. This implies, in contrast to what I suggested in a previous vrsion of this book, that there is no a priori need for an infinite number of gheists supervising one and another. In that sense is the spirit classical whereas the issues or questions it deals with are quantum mechanical. In this sense is the ultimate goal that the spirit just asks one question, which is "what do I think?" All the rest should be prescribed probabilistically by the dynamics; such dynamics being discussed in Part 3. What does happen, and which is the result of awareness or measurement, is that spirits redefine themselves all the time; an example is given by an elementary particle which is absorbed by the measurement apparatus; in that case the latter includes the former in its constitution. Part 3 tries to develop a principled approach regarding the attraction or repulsion between different people taking a stance at questions or issues; it also suggests for an interaction between ideas. Part four is a first exercise is the extension of our work on elementary particles to so called extended objects; that is those which have different manifestations to us than mere pointlike reduction such as strings and the universe itself. Here, we shall generalize our geometrical methods in a way which is very different from the standard procedure but which is far more intrinsic and unique. One could take the stance here that such ideas are gibberish and that ultimately every shape we see ultimately breaks up into pointlike obervations of its atoms, even the universe might consist out of a countable number og atoms, which is the point of view of cusal set theory. Whatever may be the truth, it is certainly worthwhile to make the exercise as the dynamics of suh object might account for the usual dynamics of point particles plus the effect of projection on the classical state. Indeed, a car remains a car and its shape is almost always identical to us; therefore it makes sense to describe a car as such and forget about the immense complexity of its fundamental being. We get pretty well away with coarse grained classical descriptions and may forget about the deeper underlying reality. So, in this vein, is the description of extended objects is one of higher spirits (higher than level zero for sure). Finally we remark that the universal "constants" c and $\hbar$ with the estimated known values in the literature in terms of standard units are to be regarded as applicable to physics in the vacuum. In particular, when psychic interactions are turned on, $\hbar$ may become considerably larger and certainly so up to an order of 10^2 in terms of standard units. Also c may become considerably smaller and even of the order 10^2 metres per second instead of 10^8 giving rise to a huge bandwith in speed of communication. We must insist upon the fact that our geometrical language below is the only one capabale of dealing with those extensions given that we start from Local Lorentz invariance and not global.

Chapter 2

Physics bottom up.

In this chapter, I shall revise basic classical and quantum physics from a geometrical point of view, in sharp contrast to what is usually done in the literature where one bestowes coordinates with an (intermediate) physical meaning. Before we start of, let me mention some modifications quantum theory should undergo which I have suggested in the past. The first one deals with the theoretical unlimited extend of particle wave packets versus whaat is really observed in nature. Regarding Young double slit experiments done with electrons by Merli eighty years ago, one saw that the ultimate spread of a wave packet was around the order of one millimeter. On dimensional grounds, one has a natural wave length which is the Compton length and is given by

$$l_c = \frac{\hbar}{mc}$$

which is aound an Angstrom or 10^{-11} meters for an electron. I argued then that in order to get the correct spread of such wave packet, one had had to consider the dimensionless gravitational potential on the surface of the earth

$$\frac{GM}{rc^2} = (\alpha_g)^{-1}]$$

which is around 10^{-9} and consider that

$$\Delta X = l_c \alpha_g \sim 10^{-2}$$

so that gravity adds information to the usual uncertainty principle. I also suggested that the electromagnetic field might do the same thing for electrons as here one disposes of a natural dimensionless quantity

$$\alpha_e = \frac{\hbar c}{\mu e^2} \sim 10^3$$

where μ is the permettivity of the vacuum. These are just suggestions but something like that is needed is one is going to give a theory to the collapse of the wavefunction.

One hundred years ago, Einstein revolutionized science by insisting upon the four dimensional character of spacetime; the consequences thereof still have to be properly understood today. Most physicists still do not reson intrinsically about nature which reflects itself in the use of symplectic geometry whose definition requires the choice of coordinate systems and so on. In a way, it puts the classical position and momentum variables on an equal ground which appears to be a big error. First, we would not like to speak about position variables since those are nongeometrical and second it seems to me that momenta live in tangent space and not in spacetime, something which has an entirely different geometrical meaning. Later on in this chapter, we shall define a much more intrinsic picture of paricle physics where the Poisson bracket is replaced by the usual commutator and position operators as well as momentum operators are all geometrical and not dependent upon some coordinate system. Clearly, the splitting of spacetime into space + time is a matter of the mind and not so much one of dynamics; certainly our predictions should not depend upon this. After we have dealt with the intrinsic geometrical formulation of classical physics, we shall do the same for the procedure of *second* quantization. Indeed, first quantization does not exist in my mind and probably does not make sense when one takes relativity seriously. Second quantization on the other hand can be regarded from two different points of view both of which we shall discuss in some detail in this chapter.

Let us first discuss a general principle from which both classical physics as free quantum field theory can be derived; rhe relevant quantity is work $\phi(\gamma, p(\gamma) \in \mathbf{B}$ where $\gamma : [a, b] \to \mathcal{M}$ is a curve joining an event x to an event y in spacetime (or space) $\mathcal{M}$ in arbitrary parametrization and p is a vector field on that line associated with the physical quantity of "momentum". We do not really know yet what momentum is but it represents a kind of weight or importance given to that motion. p must, a priori, not be proportional to $\dot{\gamma}$ as weight might sometimes be disfavourable to the current motion. Given that we all love calculus, $\mathbf{B}$ is a division algebra over the real numbers with standard operations $+, .$, that is $\mathbb{R}, \mathbb{C}$ disregarding the non-associative octonions. A frictionless theory is a dreamworld as no waste is produced; mathematically, this translates as follows, there exists an involution $\dagger$ and operation $\star$ such that

$$\phi(\gamma(b-s), p(\gamma(b-s))) \star \phi(\gamma(s), p(\gamma(s)) = 1_{\star}$$

and

$$\phi(\gamma(b-s), p(\gamma(b-s))) = \phi(\gamma(s), p(\gamma(s)))^{\dagger}.$$

It is worthwhile to comment upon those; the first one says that that reversing the process is arithmetically equivalent to taking the inverse, an expression that nothing gets lost, whereas the second says that the inverse has an arithmetical significance. This last stance is useful as inverting two processes must preserve the "distance" between them. No discussion about this viewpoint is allowed for.

As a consequence, the constant curve $\gamma_e(s) = x$ satisfies

$$\phi(\gamma_e(s), p(\gamma_e(s)))^2 = 1_\star$$

which for $\star = +$, $x^\dagger = -x$ and $\mathbf{B} = \mathbb{R}$ gives $\phi(\gamma(b-s), p(\gamma(b-s)) = -\phi(\gamma(s), p(\gamma(s))$ and $\phi(\gamma_e(s), p(\gamma_e(s))) = 0$. These simple observations give rise to the notion of work *and* classical physics. On the other hand, taking $\mathbf{B} = \mathbb{C}$, $\star = .$, $x^\dagger = \overline{x}$, we have that

$$\phi(\gamma(b-s), p(\gamma(b-s)) = \overline{\phi(\gamma(s), p(\gamma(s))}$$

and $|\phi(\gamma(s), p(\gamma(s))|^2 = 1$ what leads to the $U(1)$ Fourier waves in quantum theory. We shall first argue how classical physics arises.

2.1 The classical theory.

The idea is to write down a first order differential equation for the quantity of labour performed along a path up to some parameter value. Reparametrization invariance forces

$$\frac{d}{ds}\phi(\gamma(s), p(\gamma(s))$$

where the latter is, with a slight abuse of notation, the same as $\phi(\tilde{\gamma}_s, p(\tilde{\gamma}_s)$ for $\tilde{\gamma}_s$ the restriction of γ to the interval $[a, s]$. We demand that it is proportional to $\frac{d}{ds}\gamma(s)$; hence, the reversion property implies that

$$\frac{d}{ds}\phi(\gamma(s), p(\gamma(s)) = h(\frac{d}{ds}\gamma(s), \mathbf{F}(\gamma(s), p(\gamma(s))))$$

where h is the spatial metric, which is the old Newtonian expression with $\mathbf{F}$ having the meaning of force. Indeed, $\mathbf{F}(\gamma(s), p(\gamma(s)))$ cannot depend upon the history between a and s as otherwise the reversion condition does not hold in general; that is, it needs to be an ultralocal quantity. To complete the dynamics, Newton supposed that $p(\gamma(s))$ must maximally stimulate the direction in which the particle is moving implying that

$$p(\gamma(s)) = m\dot{\gamma}(s)$$

where $m > 0$ expresses the weight attached to persistance of the motion, called the physical mass. Another observation was of an Einsteinian nature, namely that the change of work should be equal to the change in an inherent physical property of the particle. Such invariant is to the lowest order given by the momentum squared

$$h(p(\gamma(s)), p(\gamma(s)))$$

which leads to

$$\frac{d}{ds}\left(\frac{m}{2}\left(\frac{d}{ds}\gamma(s)\right)^2\right) = \frac{d}{ds}\phi(\gamma(s), m\frac{d}{ds}\gamma(s))$$

and bestowes $\phi(\gamma(s), m\frac{d}{ds}\gamma(s))$ with the dimension of $\frac{\mathrm{kg.m}^2}{\mathrm{s}^2}$ which it should be, given that the notion of force must be associated to something intrinsic which is, in this case, the change of momentum

$$\mathbf{F}(\gamma(s), p(\gamma(s))) := \frac{d}{ds}p(\gamma(s)).$$

This is the simplest idea possible, given that the kinetic term is the lowest order invariant and m can be thought of as some material based constant. This leads to

$$\frac{m}{2}\left(\frac{d}{ds}\gamma(b)\right)^2 - \frac{m}{2}\left(\frac{d}{ds}\gamma(a)\right)^2 = \phi(\gamma(b), p(\gamma(b))) - \phi(\gamma(a), p(\gamma(a)))$$

and in a way generalizes a conserved quantity given that ϕ depends upon the entire path and not just the endpoints in general.

One could make higher derivative theories also in this way and allow for Newtonian laws with third order derivatives. These naturally appear in the context of backreactions in electromagnetism for example and allow for "unphysical" solutions with causality going backwards in time. For example, an electron would accelerate prior to turning on a lightbulb. Note also that the interpretation of γ as the physical path of the particle natually emerges given that Newtons law fixes it entirely given two "initial data".

2.2 Quantum theory.

Now, we derive quantum theory in the same vein. One notices that the obvious, but not only, candidate for an equation of motion is given by

$$\hbar\frac{d}{ds}\phi(\gamma(s), p(\gamma(s))) = -ig(p(\gamma(s)), \dot{\gamma}(s))\phi(\gamma(s), p(\gamma(s)))$$

where p is the so called energy momentum vector, g the Lorentzian spacetime metric and $\dot{\gamma}(s)$ the dimensionless velocity in units where the velocity of light is one. Notice that $\hbar$ is needed for dimensional reasons to get a nontrivial theory given that ϕ must in this case, contrary to the previous one, be a dimensionless number as any physical quantity is a real and not complex unitary number. On flat spacetime $\phi(\gamma(s), p(\gamma(s)))$ is, assuming the *law* that energy-momentum is conserved along γ meaning $\frac{D}{ds}p(\gamma(s)) = 0$, topological as it just depends upon the homotopy class or winding number of the curve. For Minkowski, such winding number is zero and the solution is given by

$$\phi(\gamma(b), p(\gamma(b))) = e^{-ip.(y-x)}$$

where $p = p(x)$ and $x = \gamma(a)$, $y = \gamma(b)$ which is the standard Fourier wave in y with base point x. Given that $e^{-ip.(y-x)}$ provides for a trivial unitary mapping between $e^{-ip.(z-y)}$ and $e^{-ip.(z-x)}$, the waves are identical up to a momentum dependent constant multiplicative $U(1)$ factor. In traditional quantumfield theory, this is precisely the impact of the translation symmetry in Minkowski. Now, unlike the previous case, there is no constraint on p in terms of γ and therefore, in order to come to a meaningful, Lorentz invariant, theory where everything is determined by the curve γ, we should integrate over a minimal Lorentz invariant shell in momentum space. The latter is given by $p^2 = \pm m^2$, where we have made the convention that the signature of the spacetime metric is given by $+ - --$. Since we further impose energy to be positive, only $p^2 = m^2$ remains and we arrive at the quantity

$$W(x, y) = \alpha \int_{\mathbb{R}^4} d^4p\theta(p^0)\delta(p^2 - m^2)\phi(\gamma, p)$$

which is, up to a constant, precisely the expression for the standard QFT propagator for scalar particles with "mass" $m > 0$. So, a consistent view upon a frictionless theory with our second choice of algebra, leads to free quantum field theory for scalar particles where the integration over the on shell momenta utters nothing but the Heisenberg uncertainty principle that if the positions x, y are known sharply, then the momentum is totally uncertain apart from the fact that it needs to be forwards pointing in time and on shell. Indeed, the Wightman function is all there is to free QFT on Minkowski; going over to interactions, it is desirable to define the Feynman propagator which expresses the idea that you must travel from the past to the future and there is no ambiguity regarding spacelike separated events since there $W(x, y) = W(y, x)$ at least in Minkowski. We have of course that $W(x, y) = \overline{W(y, x)}$ since that was the very requirement of a frictionless theory. Usually, $W(x, y)$ is interpreted as an amplitude for a particle to be born, or created, in x and annihilated in y. Since for spacelik separate events, the creation and annihilation processes at x and y can be swapped without altering the "propagator", we arrive at an expression for Bose-Einstein statistics, a desirable property in the general theory. In a general curved spacetime, ϕ depends upon the curve and not just the homotopy class due to the existence of local gravitational degrees of freedom. In light of Bose statistics, only *geodesics* give rise tp $W(x, y) = W(y, x)$ for x, y spatially separated, given that the scalar product is preserved under evolution. In part 2 we shall consider regularized Fourier waves satisfying satisfying some Schrodinger equation and the reader has to postpone his or her curiosity up till then. Note that this picture of quantization of the free particle is beautiful; free relativistic particles travel over timelike geodesics with an energy momentum squared equal to m^2c^4 where m is the rest mass. Here, we consider propagation of a particle as still being determined by geodesics dragging the on shell momenta over it and integrating them out, which is in a way the Heisenberg uncertainty principle and provides for a clear classical quaantum correspondence. In ordinary quantum theory, which is not a covariant theory and does not even provide one with the laplacian in curved spacetime as the "Hamiltonian constraint"[1]. Moreover, the

[1]Such a thing would require the momenta to be equal to the *covariant* derivatives which would imply that the momenta do not commute with one and another due to the nonvanishing of the Riemann tensor in sharp contrast to the usual standard lore.

first quantization procedure does not deliver any normalizable physical wave packages so that it is not sensible to speak about the classical limit[2] or the geodesic equation gets totally lost. One would expect in the standard second quantization for the geodesic equation to reappear in the propagator in order to provide for the correct classical limit; alas, this is only so in flat spacetime. In curved spacetime, the propagator has nothing to do with the geodesic equation which means that the classical limit is most likely wrong. I have not seen this very simple point adressed anywhere in the literature; in string theory, they escape this conclusion by not properly implementing all constraints which quantum theory says is not even possible! We shall adress this issue in Part 4 of this book.

2.3 Haute Weinbergian cuisine.

In this section, we shall further specialize ourselves into the geometrical description of as well classical and quantum theory from an operational point of view. Indeed, the reader may enjoy that we find the same propagator back by means of an entirely different procedure. Let us start with the relativistic theory of classical point particles with no internal structure. Consider a particle moving in a complex bundle $\mathcal{E}$ over spacetime $\mathcal{M}$, which locally trivializes as as $\mathbb{C}^n \times \mathcal{V}$ where $\mathcal{V}$ is an open set in $\mathbb{R}$, and consider the vector valued sections $(v^i(x))_{i:1\ldots n}$ which take values in the flat fiber which is endowed with a sesquilinear Cartan metric and $\mathbb{C}^n$ carries an irreducible representation of a compact gauge group $\mathcal{G}$ with generators τ_a. The vector sections are prone to local gauge transformations which require the introduction of a gauge connection $A^a_\mu(x)$. We shall do two things in this work: (a) we choose our kinematical variabls as such that everyghing is poored into manifestly quantum mechanical form with the standard Heisenberg commutation relations apart from a factor of $\hbar$ (b) the equations of motion are defined in a manifestly covariant fashion without recourse to any coordinate system whatsoever. Everything is expressed in terms of evolution of the worldline with as physical momentum, the four momentum of the particle itself. The "Hamitonian" is of first order in the momenta and of rather trivial nature. Technically, we shall not need a single worldline but a slight "thickening" thereof meaning that we consider worldvolumes $\gamma : (t, \vec{s}) \to \mathcal{M}$ where, say, $\vec{s} \in (-\epsilon, +\epsilon)^3$ and of course we are only interested in the equation at $s = 0$. Nothing depends upon that thickening but it is mandatory to make the math well defined. We shall be interested here with the time evolution of bundle vector sections. Before we proceed, let us make the following basic observations: as mentioned already, you should regard the wordline as an immersion $\gamma : \mathbb{R} \times (-\epsilon, +\epsilon)^3 \to \mathcal{M}$ and the momentum as the push forward of ∂_t which we denote by $(\partial_t)_\star$. Given that we shall work with functions $f, g : \mathcal{M} \to \mathbb{C}$ we can

[2] In no way can you make sense of the expectation value of the position and momentum operator

define the linear operator γ_f by

$$[(\gamma_f)(g)](t, \vec{s}) := f(\gamma(t, \vec{s}))g(\gamma(t, \vec{s}))$$

and

$$[p_\gamma g](t, \vec{s}) := i\frac{d}{dt}g(\gamma(t, \vec{s})).$$

We have moreover,

$$i[(\partial_t)(\gamma_f g)](t, \vec{s}) := i[(\partial_t)_\star f](t, \vec{s})g(\gamma(t, \vec{s})) + if(\gamma(t, \vec{s}))[(\partial_t)_\star(g)](t, \vec{s}).$$

This suggests to extend the definition of the momentum in this way to functions $\mathbb{R} \times (-\epsilon, +\epsilon)^3 \to \mathbb{C}$. The same comment holds for γ_f. In this vein,

$$[\gamma_g \gamma_f h](t, \vec{s}) = g(\gamma(t, \vec{s}))f(\gamma(t, \vec{s}))h(\gamma(t, \vec{s}))$$

$$[p_\gamma \gamma_f h](t, \vec{s}) := i\partial_t(f(\gamma(t, \vec{s}))h(\gamma(t, \vec{s})))$$

as well as

$$[\gamma_f p_\gamma h](t, \vec{s}) := if(\gamma(t, \vec{s}))\partial_t h(\gamma(t, \vec{s})).$$

Finally,

$$[p_\gamma p_\gamma h](t, \vec{s}) = -(\partial_t)^2 h(\gamma(t, \vec{s}))$$

which induces a complex algebra generated by

$$\gamma_g, p_\gamma$$

where γ varies over all immersions. This algebra is represented by means of linear operators on the function algebra

$$\mathcal{B} := C^\infty(\mathbb{R} \times (-\epsilon, +\epsilon)^3) \otimes C^\infty(\mathcal{M})$$

which may be given the structure of an Hilbert algebra in the usual L^2 sense. Concretely

$$[\gamma_f, \gamma_h](g) = 0 = [p_\gamma, p_\gamma](g), [p_\gamma, \gamma_f](g) = p_\gamma(f)\gamma_\star(g) = \gamma_{p_\gamma(f)}(g)$$

where $\gamma_\star$ is the pull back defined by the immersion γ. Here, the commutation relations employ the full $\mathcal{B}$ action but are understood to apply on $f, g, h \in C^\infty(\mathcal{M})$ and result in an element of $C^\infty(\mathbb{R} \times (-\epsilon, +\epsilon)^3)$. So, here we have the standard Heisenberg commutation relations without $\hbar$ of course. Covariant dynamics requires dynamics without "potential energy" terms; therefore, any force has to be implemented in the momentum what explains the bundle $\mathcal{E}$. Moreover, according to Einstein, the gravitational field can be gauged away in some point so that physically every particle is a free one meaning that, in absence of other force fields, the correct equation is the geodesic equation. Therefore, in that case, the covariant "Hamiltonian" becomes trivially the momentum p_γ; indeed

$$[\frac{D}{dt}\gamma_f](g) := [\frac{D}{dt}\triangle\gamma_f](g) = [p_\gamma, \gamma_f](g) = \gamma_{p_\gamma(f)}(g)$$

and

$$[\frac{D}{dt}(\partial_t)_\star](g) := [\frac{D}{dt}\triangle p_\gamma](g) = [p_\gamma, p_\gamma](g) = 0$$

where

$$[\frac{D}{dt}\triangle\zeta](g) = [\frac{D}{dt}, \zeta](g).$$

So, in this view the geodesic equation $\frac{D}{dt}(\partial_t)_\star = 0$ is implemented and the correct Hamiltonian is p_γ. There is nothing more to say really apart from the constraint $g(p_\gamma, p_\gamma) = m^2$ which is the mass energy relation. In case you consider gauge fields, the picture becomes slightly more complicated. Here, we are interested in the action of the Lie algebra on vector sections $v(x)$. We propose as Hamiltonian

$$H = i(\partial_t)_\star - q\gamma_{\tau_a A^a_\mu(\gamma(t,\vec{s}))\dot{\gamma}^\mu(t,\vec{s})}$$

which is nothing but the gauge covariant derivative along the worldline and we have extend our notion of γ to the action of matrices on vector sections given by

$$\gamma_{\tau_a A^a_\mu(\gamma(t,\vec{s}))\dot{\gamma}^\mu(t,\vec{s})} v := \tau_a A^a_\mu(\gamma(t,\vec{s}))\dot{\gamma}^\mu(t,\vec{s}) v(\gamma(t,\vec{s})).$$

Now, the momenta we shall be interested in are gauge covariant derivatives in commuting "space" directions $V(\gamma(t,\vec{s}))$ such that $[V, (\partial_t)_\star] = 0$. Hence, we define

$$p_{\gamma,V} := iV - q\gamma_{\tau_a A^a_\mu(\gamma(t,\vec{s}))V^\mu(t,\vec{s})}.$$

Hence, we propose as equations of motion

$$mg\left(\frac{D}{dt}p_\gamma, V\right) = w(\gamma(t,\vec{s}))^\dagger \left[H, p_{\gamma,V}\right] w(\gamma(t,\vec{s}))$$

and

$$Hw(\gamma(t,\vec{s})) = 0$$

where w is again a vector bundle section over $\mathcal{M}$. A small and elementary computation yields

$$mg\left(\frac{D}{dt}p_\gamma, V\right) = w(\gamma(t,\vec{s}))^\dagger \left(-q\tau_a(\nabla A)^a_{[\nu\alpha]}(\gamma(t,\vec{s}))\dot{\gamma}^\nu(t,\vec{s})\right) V^\alpha(\gamma(t,\vec{s}))w(\gamma(t,\vec{s}))$$

$$+w(\gamma(t,\vec{s}))^\dagger \left(q^2 f^{abc}\tau_c A_{a,\nu}(\gamma(t,\vec{s}))A_{b,\alpha}(\gamma(t,\vec{s}))\dot{\gamma}^\nu(t,\vec{s})\right) V^\alpha(\gamma(t,\vec{s}))w(\gamma(t,\vec{s}))$$

which is equivalent to the standard classical non abelian Yang Mills equations given that one can safely drop V^α from all considerations. Here, I mention that

$$[\tau_a, \tau_b] = if_{abc}\tau_c$$

and the Cartan metric is just δ_{ab}. Finally, we must insist upon

$$g((\partial_t)_\star, (\partial_t)_\star) = 1$$

where g is the Lorentzian metric of signature $+---$. This is all what is allowed in classical physics of point particles really. Notice that the dynamical content is completely implied by the commutator algebra which is precisely the same as in quantum mechanics.

We now look at relativistic quantum theory from the operational point of view in a way which is fully equivalent to the one in the previous section. Here, we use without too much restriction the results of Weinberg [1] and I refer the reader to that book if anything looks mysterious to him or her. I feel it is not my duty to rewrite an analysis wich takes around eighty pages to motivate what I do. Sometimes, I shall explicitely state all assumptions I am making as I deem appropriate and mandatory for the discussion. This story goes way back in time as the ideas expresed in this paper were already present some 15 years ago. In 2011, I wrote a book [3] about an operational approach to quantum theory with local vacua delineating a Fock-Hilbert bundle $\otimes_{x \in \mathcal{M}} \mathcal{H}_x$ over the spacetime manifold $\mathcal{M}$. However, the approach was troublesome and muddled with two "fundamental errors" of mine, not due to a lack of mathematical precision, but being the consequence of a poor understanding of what curved spacetime really means, something most authors don't really understand. This error found a natural solution in [4] written on generally covariant quantum theory from the point of view of the Dyson-Feynman "perturbative" expansion. Concretely, we assumed $\mathcal{H}_x$ to be constructed by means of a cyclic quasi-free vacuum state $|0\rangle_x$ and multiparticle states showing Bose or Fermi statistics constructed in the Fock way. The dynamical object was a unitary bi-field $U(x, y)$ mapping $\mathcal{H}_y \to \mathcal{H}_x$ and obeying a Schroedinger like differential equation

$$\frac{d}{dt}U(t, s) = iHU(t, s)$$

but then with the times t, s replaced by x, y. The two errors in the book originated from the mathematical implementation of this idea I conceived; first of all $U(t, s) = U(t)U^{\dagger}(s)$ and moreover the only covariant first order differential operator homogeneous in the spacetime coordinates is given by the covariant Dirac operator D. The first condition is equivalent to a "cohomology" condition

$$U(x, y)U(y, z)U(z, x) = 1$$

which turns out to hold in Minkowski or any maximally symmetric spacetime only and reflects the absence of local gravitational degrees of freedom. Consequently, the only solution I was able to find of my field equations was free quantum field theory on Minkowski. The Dirac operator gives all sorts of trouble meaning we have to replace the complex numbers by an appropriate Clifford algebra of signature $(1, 3)$ or $(3, 1)$. This gives rise to negative probabilities and huge problems with the spectral theorem even for finite dimensional Clifford bi-modules. The approach was clearly dead as it stood which I realised later on. The crucial realization was that you just cannot relate particle notions like that, such relation is path dependent and as we explained before, the natural paths are the geodesics. So the idea of a Hilbert bundle is adequate, but the correct differential equation for $U(y, x)$ needs to run over geodesics connecting

x with y in a fully reparametrization invariant way. Before we proceed, we make the convention that at any spacetime point any observer sees the same particles with identical masses and so on. Furthermore all unitary representations of the Poincaré algebra are the same; a Lorentz transformation merely relating one local vielbein to another. I will assume the point of view that the translations are only infinitesimally represented as our particles live on tangent space and it makes little sense to walk a finite distance away from the origin, albeit you can perfectly imagine this to be the case. Hence, if we have a vierbein $e_a(x)$ which we passively boost meaning $e_a(x) \to (\Lambda(x)^{-1})^b_a e_b(x)$ and $k^a \to \Lambda(x)^a_b k^b$ then the effect of this change on the states of the theory is given by $\Psi \to U(\Lambda(x))\Psi$. We denote the generators of the translations by P^a and J^{ab} for the boosts. The Lorentz algebra yields [1] that

$$U(\Lambda)P^a U(\Lambda)^\dagger = (\Lambda^{-1})^a_b P^b$$

and this is the only property we really need. The obvious candidate for a dragging law is being given by

$$\frac{d}{ds}U(w^a, x, e_a(x), s) = -iw_a P^a U(w^a, x, e_a(x), s)$$

where $w^a e_a(x)$ determines a unique geodesic connecting x with y and P_a equals the free four momentum generator, given by the expression

$$P_a = \sum_{\text{particles j, internal degrees}\,\sigma_j} \int \frac{d^3k}{\sqrt{k_0}} k_a a^\dagger_{k;j,\sigma_j} a_{k,j,\sigma_j}$$

and the final result has to be taken with respect to the *dragged* vierbein $e_a(x)$ in $y = \exp_x(w)$. This invites one to define quantities

$$U(w^a, x, e_b(y), e_b(x)) = U(\Lambda(y))U(w^a, x, e_a(x))$$

where $\Lambda(y)$ is the unique Lorentz transformation relating $(\exp_x(w)_\star)e_a(x)$ to $e_b(y)$. Therefore, if you change from reference frame at y you simply have to perform $U(\Lambda(y))U(w^a, x, e_b(y), e_b(x))$. Likewise, suppose you change of reference frame at x, meaning $e_a(x) \to (\Lambda^{-1})^b{}_a e_b(x)$ then $w_a \to (\Lambda^{-1})^b_a w_b$ and therefore our differential equation

$$\frac{d}{ds}U(\Lambda^a_b w^b, x, (\Lambda^{-1})^b{}_a e_b(x), s) = -i(\Lambda^{-1})^b_a w_b P^a U(\Lambda^a_b w^b, x, (\Lambda^{-1})^b{}_a e_b(x), s)$$

$$= -iU(\Lambda)U(w_a P^a U(\Lambda^a_b w^b, x, (\Lambda^{-1})^b{}_a e_b(x), s)U(\Lambda)^\dagger$$

which is up to an equivalence precisely the same equation as for $U(w^a, x, e_b(y), e_b(x))$ which shows that

$$U(\Lambda^a_b w^b, x, (\Lambda^{-1})^b{}_a e_b(x), (\exp_x(w))_\star((\Lambda^{-1})^b{}_a e_b(x))) =$$

$$U(\Lambda)U(w^a, x, e_b(x), (\exp_x(w))_\star(e_b(x)))U(\Lambda)^\dagger.$$

Therefore, in order to go from $(\exp_x(w))_\star((\Lambda^{-1})^b_{\ a} e_b(x))$ to $(\exp_x(w))_\star(e_b(x))$ we have to perform an inverse Lorentz transformation $U(\Lambda^{-1}) = U(\Lambda)^\dagger$ from the left resulting in

$$U(\Lambda^a_b w^b, x, (\Lambda^{-1})^b_{\ a} e_b(x), (\exp_x(w))_\star(e_b(x))) =$$

$$U(w^a, x, e_b(x), (\exp_x(w))_\star(e_b(x)))U(\Lambda)^\dagger$$

which gives our covariance properties of $U(w^a, x, e_b(y), e_b(x))$. The coincidence limit is of course fixed by $U(0, x, e_a(x)) = 1$. So, as before, we have a canonical transport equation relation particles with different spin and momenta to one and another. In order to get an equivalent viewpoint on the propagator, we have to introduce the notion of a bi-field. I shall only comment here for real scalar fields associated to bosonic particles of spin zero; consider the observer $x, e_a(x)$ then his field at the origin of tangent space is given by

$$\Phi(x, x, e_b(x)) = \int \frac{d^3k}{(2\pi)^3 2\sqrt{k^2 + m^2}} \left(a_k + a_k^\dagger \right)$$

which is a locally Lorentz invariant expression so I could drop reference towards $e_b(x)$ in its definition. Now, we propagate the field by means of

$$\Phi(w, x, e_b(x), e_c(\exp_x(w))) =$$

$$U(w^a, x, e_b(x), e_c(\exp_x(w)))\Phi(x, x)U(w^a, x, e_b(x), (\exp_x(v))_\star e_c(\exp_x(w)))^\dagger.$$

which transforms covariantly under local Lorentz transformations at y and is invariant regarding local Lorentz transformations at x. This leads one to define

$$\Phi(y, x, e_b(x), e_c(y)) = \sum_{w \in T^\star \mathcal{M}_x : \exp_x(w) = y} \Phi(w, x, e_b(x), e_c(y))$$

Now, the reader should understand that the Wigthman function of the previous section in this case equals

$$W(x, y) = \langle 0 | \Phi(y, y, e_b(y)) \Phi(y, x, e_b(y)) | 0 \rangle$$

which is independent of the local Lorentz frame at y given dat the vacuum is Lorentz invariant and we have dropped any reference towards $e_b(x)$ since it does not matter at all. Again, on Minkowski, one simply chooses y to be the origin 0 and one defines a field

$$\Phi(x) = \Phi(0, x, e_b(0))$$

whose transformation law has become a "global" one since it only depends upon the Lorentz frame at the origin. One can, in this framework always define commuting observables in case x, y are spatially separated, but those all need to be relational; they cannot depend freely upon the reference frames at x and y. It is not difficult, by using our definition, to see that for x and y spatially separated, there exist open neighborhoods $O(x), O(y)$ such that the smeared bi-field operators

$$\int_{O(y)} d^z y \sqrt{g(z)} h(z) \Phi(y, z, e_b(y))$$

and

$$\int_{O(x)} d^4 z \sqrt{g(z)} s(z) \Phi(y, z, e_b(y))$$

commute with one and another for arbitrary smearing functions h, s. This raises profound questions about the "psychic" interconnectedness of observers, something which was already implicitly present in the standard formulation but becomes here, due to the rich local quantum symmetry group, very explicit. Regarding interactions, it is clear that that one cannot write down expressions of the kind

$$\lambda \int_{\mathcal{M}} \sqrt{g(y)} \Phi(y, x) \Phi(y, z) \Phi(y, p) \Phi(y, q)$$

since it is impossible to make such vertex locally Lorentz invariant; the only thing one can do is to take propagators, which are locally Lorentz invariant and *define* more general operators using these special matrix elements. There is no way to write down directly a product of bi-field operators, that is simply meaningless.

Chapter 3

Fourier transform and generalized Heisenberg operators.

In this chapter, we deepen our understanding of the Fourier transformation and therefore propose for an entirely new propagator on curved spacetime whereas on Minkowki, our viewpoint coincides with the old one, as explained previously. Hence, consider a generic, time-orientable spacetime $(\mathcal{M}, g)$ and select a base point x, k^a a Lorentz vector at x defined with respect to $e_a(x)$ and y any other point in $\mathcal{M}$. Let $\gamma(s)$ be a curve from x to y and denote by $k^\mu(s)$ the parallel transport of $k^\mu(x) = k^a e^\mu_a(x)$ along γ, then we can define a potential $\phi_\gamma(x, k^a, y)$ by means of the differential equation

$$\frac{d}{ds}\phi_\gamma(x, k^a, \gamma(s)) = -i\dot{\gamma}^\mu(s)k_\mu(s)\phi_\gamma(x, k^a, \gamma(s))$$

with boundary condition $\phi_\gamma(x, k^a, x) = 1$. Then, one easily calculates that in Minkowski spacetime, the potential is independent from the choice of γ and is given by the following group representation

$$\phi(x, k^a, y) = e^{-ik_a(y^a - x^a)}$$

where the formula is with respect to global inertial coordinates defined by the vierbein $e_a(x)$. Minkowski is special in many ways: (a) every two events are connected by a unique geodesic (b) the ϕ_γ are path independent and define a group representation. Neither (a) nor (b) are true in a general curved spacetime which means we have to select for a preferred class of paths: the natural choice being that the *information* about the birth of a particle at x travels freely, meaning on geodesics which implies that we should sum over *all* distinct geodesics between x and y. This inspires one to consider the following mapping

$$\tilde{\phi} : T^\star\mathcal{M} \times T^\star\mathcal{M} \to U(1) : (x, k^a, w^a) \to \tilde{\phi}(x, k^a, w^a)$$

where $\tilde{\phi}(x, k^a, w^a)$ is defined as before by means of integrating the potential over the unique geodesic emanating from x with tangent vector w^a and affine parameter length one. One has then that

$$\phi(x, k^a, y) = \sum_{w:\exp_x(w)=y} \tilde{\phi}(x, k^a, w^a)$$

and although $\tilde{\phi}$ is more fundamental, we will sometimes switch between $\tilde{\phi}$ and ϕ by assuming that they are the same meaning that every two points in spacetime can be connected by a unique geodesic: this last assumption will be abbreviated to GS standing for "geodesic simplicity". In a general spacetime,

$$\tilde{\phi}(x, k^a, w^b) = e^{-ik^a w_a} = e^{ik^a e_a^\mu(x)\sigma_{,\mu}(x, \exp_x(w))}$$

where we assume in the last equality GS to hold and

$$\sigma(x, y) = \frac{1}{2}\epsilon L^2(x, y)$$

is Synge's function where $\epsilon = -1$ if x and y are connected by a spacelike geodesic and 1 if they are connected by a timelike geodesic and $L(x, y)$ denotes the geodesic length. Covariant derivatives of $\sigma(x, y)$ with respect to x will be denoted by unprimed indices μ, ν whereas their counterparts with respect to y are denoted with primed indices. It is clear that as usual the standard Fourier identities hold between the two tangent spaces at x, that is

$$\int_{T^\star \mathcal{M}_x} \frac{dk^a}{(2\pi)^4} \overline{e^{-ik^a w_a}} e^{-ik^a v_a} = \delta^4(w^a - v^a)$$

and

$$\int_{T^\star \mathcal{M}_x} \frac{dw^a}{(2\pi)^4} \overline{e^{-ik_a w^a}} e^{-il_a w^a} = \delta^4(k^a - l^a)$$

being the inverse Fourier transform. Under the hypothesis of GS, the first integral reduces to

$$\int_{T^\star \mathcal{M}_x} \frac{dk^a}{(2\pi)^4} \overline{e^{ik^a e_a^\mu(x)\sigma_{,\mu}(x,y)}} e^{ik^a e_a^\mu(x)\sigma_{,\mu}(x,z)} = \frac{\delta^4(y, z)}{\sqrt{-g(y)}\Delta(x, y)}$$

and the second one under the additional assumption of geodesic completeness (GC) becomes

$$\int_{\mathcal{M}} \frac{d^4y}{(2\pi)^4} \sqrt{-g(y)}\Delta(x, y) \overline{e^{ik^a e_a^\mu(x)\sigma_{,\mu}(x,y)}} e^{il^a e_a^\mu(x)\sigma_{,\mu}(x,y)} = \delta^4(k^a - l^a).$$

Here,

$$\Delta(x, y) = \frac{|\det(\sigma_{,\mu\nu'}(x, y))|}{\sqrt{-g(x)}\sqrt{-g(y)}}$$

is the absolute value of the Van Vleck-Morette determinant. Still working under the GS assumption, one recognizes the presence of a global coordinate system given by $\sigma_{,\mu}(x,y)$ which transforms as a co-vector under coordinate transformations at x; contracting with $e^{a\mu}(x)$, one obtains local Lorentz coordinates $\sigma^a(x,y)$ and momentum operators $i\frac{\partial}{\partial \sigma^b(x,y)}$ which transform as a local Lorentz covector such that

$$-i\frac{\partial}{\partial \sigma^b(x,y)}\phi(x,k^a,y) = k_b\phi(x,k^a,y)$$

meaning our generalized exponentials are eigenfunctions of the relative momentum operators. Also,

$$-\eta^{ab}\frac{\partial}{\partial \sigma^a(x,y)}\frac{\partial}{\partial \sigma^b(x,y)}\phi(x,k^a,y) = k^2\phi(x,k^a,y)$$

meaning that the above operator is to be preferred over the generalized d'Alembertian. In Minkowski spacetime, something special happens as

$$\sigma^b(x,y) = x^b - y^b$$

and one can substitute $-i\frac{\partial}{\partial \sigma^b(x,y)}$ by $-i\frac{\partial}{\partial x^b}$ or $i\frac{\partial}{\partial y^b}$. In other words, the x,y coordinates factorize and one can identify all pictures in this way and obtain *one* Heisenberg pair only. Indeed, I have stressed previously that the philosophy of Minkowski is misleading due to its translational invariance and the reader should appreciate that the latter just falls out from our formalism. Also, it is now clear that a generalized Heisenberg picture demands the condition of geodesic simplicity whereas there is no good physical reason why this should be the case: our geometric framework is far more interesting than that.

3.1 Elementary particles and a universal probability interpretation.

In this section, we shall work towards a theory for a single free spinless particle in a general curved spacetime, the extension towards multiple particles of higher spin being worked out in Part 2. Our first, preliminary, postulate of relativistic quantum theory consists in the statement that an *idealized* free, spin-0 particle of mass m and future pointing momentum k^a, created at x, is given by the Fourier wave $\phi(x,k^a,y)$ where $k^2 = \eta_{ab}k^ak^b = m^2$ is Einstein's energy-momentum relationship or the mass-shell condition. We have deduced that in a geodesically simple universe

$$(\eta^{ab}\frac{\partial}{\partial \sigma^a(x,y)}\frac{\partial}{\partial \sigma^b(x,y)} + m^2)\phi(x,k^a,y) = 0$$

which is the correct generalization to a GS spacetime of the Klein-Gordon equation in flat spacetime. In the literature however, one proposes the equation

$$(g^{\mu\nu}\nabla_\mu\nabla_\nu + m^2)\psi(x) = 0$$

which leads to a conserved current

$$j^{\mu}(\overline{\psi},\phi)(x) = i(\overline{\psi}(x)\nabla^{\mu}\phi(x) - \phi(x)\nabla^{\mu}\overline{\psi}(x))$$

on the space of solutions ψ, ϕ to the Klein-Gordon equation. Standard arguments in the old fashioned quantum theory then suggest that the correct probability interpretation is given by the charge of this current which determines the bilinear form

$$\langle\psi|\phi\rangle = i\int_{\Sigma} d^3x\sqrt{h(x)}n_{\mu}(\overline{\psi}(x)\nabla^{\mu}\phi(x) - \phi(x)\nabla^{\mu}\overline{\psi}(x))$$

where $h(x)$ is the determinant of the induced metric on the Cauchy hypersurface Σ and n_{μ} is the normal vector to it. So, this reasoning only holds in globally hyperbolic spacetime. There are two problems with this scalar product: (a) it is of indefinite signature meaning there are as many positive as negative norm states in a nondegenerate basis and (b) the probability density is not positive restricted to some positive norm solution meaning it cannot serve as the probability density associated to a generalized position operator. (a) is well known and reflects that the theory is not unique or covariant given that distinct splits of the total vector space of solutions in a positive and negative norm subspace determine different theories. (b) on the other hand is not well known and even true in Minkowski spacetime; indeed, the density for a superposition of two plane waves

$$\alpha e^{-ik_a x^a} + \beta e^{-il_a x^a}$$

on an inertial hypersurface Σ reads

$$\overline{(\alpha e^{-ik_a x^a} + \beta e^{-il_a x^a})}(k^0\alpha e^{-ik_a x^a} + l^0\beta e^{-il_a x^a}) + cc = 2k^0\,|\alpha|^2 + 2l^0\,|\beta|^2 + 2\mathrm{Re}(\alpha\overline{\beta}e^{-i(k_a - l_a)x^a})(k^0 + l^0)$$

where Re denotes the real part. By adjusting the phase of α we get at some value of x the expression

$$2k^0\,|\alpha|^2 + 2l^0\,|\beta|^2 - 2\,|\alpha|\,|\beta|\,(k^0 + l^0)$$

which can easily be made smaller than zero. Now, in Minkowski, unlike in any other spacetime, it is still possible to save the day as one can look for a canonical Heisenberg conjugate of the dynamical momentum operators and interpret those as position operators. It must be clear to the reader that the only possible position density is given by

$$|(T\phi)(x)|^2$$

where T constitutes a linear transformation of ϕ. To find this operator in Minkowski spacetime and generalize it to our setting later on, note that the correct scalar product between plane waves is given by

$$\langle e^{ik_a x^a}|e^{il_b x^b}\rangle_x = (2\pi)^3 k^0\delta(\vec{k} - \vec{l})$$

where the right hand side is Lorentz invariant given that the left hand side must be. Therefore, we obtain with

$$\psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \widehat{\psi}(k) e^{-ik_a x^a}$$

and

$$\phi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \widehat{\phi}(k) e^{-ik_a x^a}$$

that

$$\langle \psi | \phi \rangle_x = \int d^3k k^0 \overline{\widehat{\psi}(k)} \widehat{\phi}(k) = \int_{\mathbb{R}^3} d^3x \overline{(T\psi)}(t,x)(T\phi)(t,x)$$

where

$$(T\psi)(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{k^0} \widehat{\psi}(k) e^{-ik_a x^a}.$$

There is a canonical Lorentz invariant wave function associated to a particle being born at x which is given by a dimensionless multiple of

$$\delta_x(y) = \frac{1}{(2\pi)^3 m} \int \frac{d^3k}{k^0} e^{-ik_a(y^a - x^a)}$$

which is, as the notation suggests, a relativistic replacement of the $\delta^3(\vec{y} - \vec{x})$ function of dimension mass instead of mass^3. Indeed, as the reader may verify later on, we have that

$$\langle W(z,y) | \delta_x(y) \rangle_y = \delta_x(z) = \int d^3y \, T_y \delta_x(y) \overline{T_y W(z,y)}$$

where

$$W(x,y) = \int \frac{d^3k}{(2\pi)^3 k^0} e^{-ik_a(y^a - x^a)} = m\delta_x(y)$$

is the propagator and T_y means the T operator with respect to the y variable. These facts indicate the correct probability interpretation in a general curved spacetime given that the dimension of $(T\psi)$ is given by mass$^{\frac{3}{2}}$ which means that the dimension of ψ is given by mass. In our setting, particle notions depend upon the place where they are born and therefore also depend upon two points x, y which means one can consider two operators T_x and T_y applied to it, but here T_x has a slightly distinct meaning than before. Indeed, T_x means that the T operation is applied with respect to the Fourier waves $\phi(x, k^a, y)$ and the reader should keep this in mind. In a general curved spacetime, we do not dispose of analogues of $\langle | \rangle_x$ given that the derivative of Synge's function does not factorize; we can, however, generalize the T mappings and spatial scalar products in a canonical way. More specifically, regarding any particle state

$$\psi_x(y) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \, \widehat{\psi}(k) \phi(x, k^a, y)$$

we define

$$(T_{x,e_0}\psi_x)(y) = \sum_{w:\exp_x(w)=y} \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \sqrt{k^{0'}_{\star w}}\widehat{\psi}(k)\tilde{\phi}(x,k^a,w)$$

where $k^{0'}_{\star w}$ is the component of $k_{\star w}$ with respect to e_0 at y and the weight for a particle to cross a spatial, but not necessarily achronal, cross section Σ is given by

$$w_\Sigma(\psi_x) = \int_\Sigma d^3y\sqrt{h(y)}\,|(T_{x,e_0\perp\Sigma}\psi_x)(y)|^2$$

whereas the propagation, seen as a process of annihilation and recreation, is given by

$$P_\Sigma(\psi_x)(z) = \int_\Sigma d^3y\sqrt{h(y)}T_{x,e_0\perp\Sigma}\psi_x(y)\,\overline{T_{z,e_0\perp\Sigma}W(z,y)}.$$

There is no analogue of the propagation process in standard quantum mechanics in Minkowski where unitarity and the unique choice of an inertial Cauchy hypersurface Σ garantuee that

$$P'_\Sigma(\psi)(x) = \psi(x)$$

where P' is defined by means of the Klein-Gordon product explained above. In fact, if you think about this, the latter equation is not very natural given that propagation through a hypersuface is associated to a process where knowledge about the state of the particle has been gained and therefore, there is no reason why this should come without a cost. The latter translates itself into the loss of "unitarity" in our framework. Likewise do we define the weight of detection on a world tube W_Σ of spatial hypersurfaces Σ_t which correspond to the "time evolution" (towards the future) of a spatial surface Σ, given that the particle is annihilated at some event y such that $y \in W_\Sigma$. In our philosophy of strong measurements, we insist that a measurement corresponds to a process of renewal which is always associated to an annihilation: basically, the particle leaves one quantum world and enters another one. Obviously, the point of annihilation is born after Σ and the size of Σ is always very small, certainly below the micron scale. Hence, we define

$$d_{W_\Sigma}(\psi,y;\delta) = \int_{\Sigma_{t-\delta}} d^3z\sqrt{h(z)}\left|T_{y,e_0\perp\Sigma}\widetilde{\psi}_y(z)\right|^2$$

where $\widetilde{\psi}_{y|_{\Sigma'}} = \chi_{\Sigma_{t-\delta}}\psi_{|\Sigma'}$, $\Sigma_{t-\delta} \subset \Sigma'$ and $y \in \Sigma_t$; Σ' is a complete spacelike hypersurface for y meaning that every geodesic emanating from y remains to the future of Σ' or crosses it; $\chi_{\Sigma_{t-\delta}}$ is the characteristic function on $\Sigma_{t-\delta}$. Moreover, Σ' contains actual space at time δ prior to the happening of the annihilation process as seen by the measurement apparatus and it is, moreover, assumed that $\Sigma_{t-\delta}$ can be reached by means of a geodesic starting at y at the instant y is born. Under those conditions, and possibly some slight technical details, one should be able to show that $\widetilde{\psi}_y$ can be given a unique[1] Fourier decomposition at y [2]. This expression gives the probability for some spot or trace to be found in Σ_t, whereas the calculation refers to a past state $\Sigma_{t-\delta}$. So, the determination of the probability that a trace of the particle's impact is found in a certain region is fixed at the moment of annihilation, which means that detection is by no means a simple mechanism. It is a bit like a wound on your skin which appears some time after you have been hurt. Now, we are all set for our relative probability interpretation to be defined: the relative amplitude for a particle to be detected into world tubes W_{Σ_i} on the slices Σ_{i,t_i} where $W_{\Sigma_1} \cap W_{\Sigma_2} = \emptyset$ given annihilation events $y_i \in \Sigma_{i,t_i}$, is determined by

$$\frac{d_{\Sigma_1}(\psi, y_1; \delta)}{d_{\Sigma_2}(\psi, y_2; \delta)}$$

where ψ is the spacetime state of the particle and y_1, y_2 happen at the same "psychological" time. There is a lot of new physics in here: for example, the probability that a "wound" is found in the region Σ_t depends on the amount of some processing "time" δ associated to the apparatus. This is still a very simple model and more complex detection processes can be set up, depending upon the nature of the machine.

There remains another relative amplitude to be defined which expresses the amplitude between the processes of a particle crossing a hypersurface Σ_i and then being detected by an apparatus with world tube W_{Σ_f} at the time it is annihilated at y. It is given by

$$\frac{d_{W_{\Sigma_f}}(\psi_x, y; \delta)}{w_{\Sigma_i}(\psi_x)}$$

and it is a quantity which is really never considered in standard quantum mechanics. In the next section, we shall further formalize the remarks of this section and generalize it to multi-particle theories. We now turn our head towards some interesting example confirming that our theory is the right one.

3.2 Some interesting example.

What we will show in this section is that while maintaining flatness but imposing a non-trivial topology, leading to periodicity conditions on the wave vectors associated to the plane waves as defined by the usual d'Alembertian operator, arises automatically in our framework due to an infinite winding of geodesics.

[1]Note that our Fourier waves are on mass shell, this very input is mandatory of course.

[2]For example, it may be that the kernel of the mapping between the (on shell) Fourier waves on tangent space at y and the distributional square integrable functions on Σ' is continuous (it is obviously surjective), as is the case in the next section. Nevertheless, the ordinary scalar product on the spatial part of tangent space at y provides one with a criteron of determining the orthogonal complement of the kernel leading to a unique representant of the quotient space.

Let us study the example of the timelike cylinder $\mathbb{R} \times S^1$ with coordinates (t, θ) where θ has to be taken modulo $L > 0$ and see if only the discretized modes $k^1 = \frac{2\pi n}{L}$ for some $n \in \mathbb{Z}$ play a part in the propagator. The reader has to be capable of figuring out that

$$\phi(x, k^a, y) = e^{-i(\sqrt{(k^1)^2+m^2}\delta t - k^1 \delta\theta)} \left[\sum_{n \in \mathbb{Z}} e^{-ik^1 Ln} \right]$$

where

$$y - x = (\delta t, \delta\theta)$$

in the global flat coordinate system. This function is clearly invariant under the translation $\delta\theta \to \delta\theta \pm L$ and it is therefore well defined on the cylinder. Forming now a wave packet at x

$$\psi_x(y) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int dk^1 \widehat{\psi}(k^1) \phi(x, k^a, y) = \frac{1}{(2\pi)^{\frac{1}{2}}} \int dk^1 \widehat{\psi}(k^1) e^{-i(\sqrt{(k^1)^2+m^2}\delta t - k^1\delta\theta)} \left[\sum_{n \in \mathbb{Z}} e^{-ik^1 Ln} \right]$$

and taking the Fourier transform with

$$\frac{1}{\sqrt{L}} e^{-i\frac{2\pi p \delta\theta}{L}}$$

gives

$$\psi_x(y) = \frac{1}{L} \sum_{p \in \mathbb{Z}} \left(\int_0^L \psi_x(y) e^{-i\frac{2\pi p \delta\theta}{L}} d(\delta\theta) \right) e^{i\frac{2\pi p \delta\theta}{L}}$$

and it is easy to calculate that

$$\frac{1}{L} \int_0^L \psi_x(y) e^{-i\frac{2\pi p \delta\theta}{L}} d(\delta\theta) = \frac{1}{(2\pi)^{\frac{1}{2}} L} \int dk^1 \int_{-\infty}^{+\infty} d(\delta\theta) e^{-i\left(\sqrt{(k^1)^2+m^2}\delta t + (\frac{2\pi p}{L} - k^1)\delta\theta\right)} \widehat{\psi}(k^1).$$

The latter equals

$$\frac{(2\pi)^{\frac{1}{2}}}{L} e^{-i\sqrt{\left(\frac{2\pi p}{L}\right)^2 + m^2}\delta t} \widehat{\psi}\left(\frac{2\pi p}{L}\right)$$

which results in the ordinary Fourier transform

$$\psi_x(y) = \frac{(2\pi)^{\frac{1}{2}}}{L} \sum_{p \in \mathbb{Z}} \widehat{\psi}(\frac{2\pi p}{L}) e^{-i\left(\sqrt{\left(\frac{2\pi p}{L}\right)^2+m^2}\delta t - \frac{2\pi p \delta\theta}{L}\right)}.$$

So, the winding of geodesics kills of all modes which do not satisfy the global boundary conditions. A similar result of course holds for the propagator and the reader may enjoy making that exercise. This example obviously generalizes to higher dimensional cylinders over the spatial d-dimensional torus $\mathbb{T}^d$. This concludes the shortest chapter of this book, but nevertheless an important one as it indicates very clearly the line of thought to be followed in the chapters to come.

Chapter 4

General considerations regarding psychology.

In discussing issues about the mind, we must learn to be precise; in sharp contrast to the tradition in those fields of study, I shall outline my viewpoint with mathematical precision which might be an unreasonably high standard in those fields but in my view is the only path towards discussing these matters in a truely profound way. I shall make a bold conjecture of how our perception of space and time might be related to other issues of the mind. It is up to you to agree or to disagree with those viewpoints; at least, it seems to me, there is something nontrivial to it and certainly the mathematics behind it is compelling. So, we shall start here by discussing the kind of mathematics which would be required to break the boost symmetry of the Poincaré algebra and therefore distinguish space from time. Later on, we shall discuss in depth how this issue, in my view, relates to the dynamics of our mental profile. Space, as we perceive it, appears to allow for rotations and time is completely detached from it - it does not transform in our mental perception no matter what what you think or do. In fact, the kind of rotations we should consider are active ones regarding our own body and passive ones regarding the outside world. Indeed, our body actively rotates, but nothing happens to the outside world, it is just our perspective which changes (ignoring backreaction effects on all interaction fields). The reason why we know we rotate is due to the spacetime acceleration we undergo during the process and our senses pick up on that. One more comment is in place, there is almost a natural definition of x, y, z axis associated to the symmetries of our body. The z axis is defined by means of the gravitational acceleration we are undergoing in case we are stationary and the y axis is the projection of the vector away from our eyes on the plane perpendicular to the z, t axis. The x axis is then fixed; under normal circomstances, this definition agrees with the line connecting our feet to our head, the line of incoming eyesight and finally the line connecting our shoulders. So, biology breaks all free choice of rotation and rotating therefore requires a nontrivial act. Since actions require algebra, this suggests that our experience of time commutes with our experience of space as well as with all rotations thereon. There are two beautiful realizations of the Lorentz group which embody this key idea algebraically; that is the spin $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ representation and not the Dirac representation where the "vectors" $x_a \gamma^a$ transform as Lorentz vectors. The fundamental algebraic components of both representations are given by the Pauli matrices σ^a where $\sigma^0 = 1$ the 2×2 identity matrix. Denoting by $\vec{a}, \vec{b}$ two three dimensional real vectors generating a rotation and boost respectively, then the above reprentations are implemented by means of

$$D(\vec{b} + i\vec{a}) := e^{(\vec{b}+i\vec{a})_j \sigma^j},\ E(\vec{b} + i\vec{a}) := e^{(-\vec{b}+i\vec{a})_j \sigma^j}$$

respectively and one notices that

$$D(\vec{b} + i\vec{a}) = (E(\vec{b} + i\vec{a})^{\dagger})^{-1} = \sigma_2 \overline{E(\vec{b} + i\vec{a})} \sigma_2.$$

All matrices A in this representation tranform of course under a change of basis g as gAg^{-1}. A "spacetime vector" is now given by $x := x_a \sigma^a$ and transforms as

$$D(\vec{b} + i\vec{a}) x D(\vec{b} + i\vec{a})^{-1}$$

and likewise so for $E(\vec{b} + i\vec{a})$. Hence, one notices that x_0 remains untouched but the x_i transform into complex numbers in case $\vec{b} \neq 0$. Assuming that the only allowed transformations are those which preserve the reality condition $x_a \in \mathbb{R}$, we conclude that $\vec{b} = 0$ and we obtain a mere rotation around the $\vec{a}$ axis for an angle of $|\vec{a}|$. Note that in this view, the different directions of space anticommute. It is worthwhile to mention that the matrices $D(\vec{b} + i\vec{a})$ constitute all 2×2 complex matrices with unit determinant $SL(2, \mathbb{C})$. This group is the so-called universal cover of the orthochronous Lorentz group and one can define from complex 2 vectors, real four dimensional vectors, and carry the action of $SL(2, \mathbb{C})$ on those complex 2 vectors into an action of the real orthochronous Lorentz group. For sake of completeness, I will give a brief overview of this formalism. Let W be a two dimensional complex vector space with basis e_A and volume form ϵ_{AB}. In the literature, one puts $e_1 = o$ and $e_2 = n$: $(o, o) := \epsilon_{AB} o^A o^B = (n, n) := \epsilon_{AB} n^A n^B = 0$ and $(o, n) = 1$. Clearly, $SL(2, \mathbb{C})$ leaves the volume form invariant. The complex conjugation sends W to $\overline{W}$, that is $v \in W \to \overline{v} \in \overline{W}$ which is spanned by $\overline{o}, \overline{n}$ with a volume form represented by $\overline{\epsilon}_{A'B'}$ where we agree that primed indices refer to transformations under the complex conjugate representation. $W \otimes \overline{W}$ is four dimensional over $\mathbb{C}$ and one is interested in its real subspace. The latter is spanned (over the real numbers) by

$$\begin{aligned}\sigma^0 &= \frac{1}{\sqrt{2}}\,(o\otimes\overline{o}+n\otimes\overline{n})\\ \sigma^1 &= \frac{1}{\sqrt{2}}\,(o\otimes\overline{n}+n\otimes\overline{o})\\ \sigma^2 &= \frac{i}{\sqrt{2}}\,(o\otimes\overline{n}-n\otimes\overline{o})\\ \sigma^3 &= \frac{1}{\sqrt{2}}\,(o\otimes\overline{o}-n\otimes\overline{n})\end{aligned}$$

and the reader understands that the σ^a are the usual Pauli matrices but now with indices A, A' transforming in two different representations. A smll computation yields that these vectors are orthonormal with regard to $\omega\otimes\overline{\omega}$ and obey

$$\sigma_a^{AA'}\sigma_b^{BB'}\omega_{AB}\overline{\omega}_{A'B'} = -\eta_{ab}$$

where η_{ab} is the standard Minkowski metric of signature $(+---)$. This suggests an identification with some inertial system (t, x, y, z) by means a "solder" form

$$\sigma = t\widehat{t} + x\widehat{x} + y\widehat{y} + z\widehat{z} := x^a\sigma_a^{AA'}.$$

Given all this, it is now an elementary exercise to show that an element $T \in SL(2,\mathbb{C})$ defines, by means of the solder form, an ortochronous Lorentz transformation on x^a. This action will become useful when we want to couple spinor currents to spacetime vectors. It is worthwile to mention that the Pauli matrices correspond to the complex quaternions where minus the unit is a square of each of them. The spin $(0, \frac{1}{2})$ and $(\frac{1}{2}, 0)$ representations define the so called massless Weyl particles which are, as is well known, Fermions. However, in the mental world, we do not care about the speed of light dictating our unconscious interactions and therefore we will be interested in a quantization which treats them as bosons. Let us now argue why this is interesting from the point of view of psychology in a broader sense beyond the mere issue of spacetime awareness; I will take here the point of view that regarding any issue, be it a question, a thought, a worry, a desire or anything you like, that the fundamental dichotomy which dictates our interactions is one of conservatism, that is taking some definite point of view, versus one of a transformative nature seeing change no matter what a conservative person percieves as a definite state, which is maybe not even desirable for him. Another way of saying this is that conservative people accept this issue as being settled in a particular way, whereas transformative people oppose any settlement and ask for change in a particular way. Transformative people are wanderers, searching for an anchor, but not knowing where to start or what to adhere to. In this regard, it seems natural to postulate that the dynamics of the nature of the questions one asks is one which has a personal tendency towards conservatism - ultimately, we are all happy to take a rest and settle in a particular answer regarding these questions based upon an emergent phenomenon called logic. So, the reader must understand that momentum here is not a real quantity in contrast to standard quantum mechanics of particles! People who are transformative have an imaginary reality, a dreamworld of how it could be in the future whereas for conservative people reality is what it is. This is the way in which the spirit differs from an ordinary particle whose options, in the usual framework, are fixed once and for all and we merely study transitions between those options (this is the Schrodinger point of view). I shall make a further assumption here, which is that the stable "ground state" of a community regarding an issue is one which is precisely in the middle between conservatism and progressiveness. This is a healthy assumption as it allows for a very dynamical attitude towards anything in life, you are on the one hand attached to the knowledge you have, but on the other hand you are flexible enough to change your mind when facts call upon you; in either, you are never sure but relaxed. I believe this mentality also to be connected to Darwinism as such persons are the ones who thrive in society. In my previous publications on the matter, I have called such a point of view towards a specific topic of "Schwitchoriem" type, simply because I like to play with words. Conservative types were white and transformative types black in analogy with ying and yang; we shall stick to that convention in this work. So, now the question is, how to translate this algebraically and what does it have to do with the fundamental representation of $SL(2,\mathbb{C})$? For simplicity, let us agree that the definite position or validation you can take towards a certain issue is labelled by a real number λ; this is just a way of coding things and it allows one to naturally speak about a certain distance between positions. Therefore, we postulate that the black operator X works on the state of the system $|\Psi(\lambda)\rangle$ as

$$X|\Psi(\lambda)\rangle = \lambda|\Psi(\lambda)\rangle.$$

Furthermore, a state is something indefinite and describes in a way how the world values different definite (white) points of view on the matter. This operator is clearly linear by definition. Now, the algebraic object P associated to the transformative or black type must obey the definition of change; hence,

$$[P, X] := PX - XP = 1$$

or $P = \frac{d}{d\lambda}$. All algebraic relations can be rewritten as

$$(X, P) \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X \\ P \end{pmatrix} = 1$$

where we use a shorthand

$$\omega := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

Hence, the pair

$$\begin{pmatrix} X \\ P \end{pmatrix}$$

can be seen as a two spinor. Indeed, a trivial computation reveals that the symmetry group of our defining relation is given by $A \in SL(2, \mathbb{C})$ since

$$A^T \omega A = \omega$$

where A^T is the transpose of A. It is necessary to remark here that we take an Einsteinian point of view regarding the translations $X \to X + a$, $P \to P + b$ which also preserve the algebra but merely recalibrate the very language we use to describe phenomena. Since those things are personal and never change and we shall associate both X, P with a kind of dimensionless energy, we have to disregard those. The meaning of attributes does not change, which does not mean of course that the attributes attached to a state of the world cannot change. Note that a dynamics regarding the A matrix is not sufficient in order to deal with spiritual interactions, it just says from which dichotomy you are approaching the world, but you still have to choose one of the opposites! Calling them π_1, π_2, where

$$\pi_i : \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} \to Z_i$$

we suggest that in a conversation you do not only change your point of view (dichotomy), but you also must have the freedom to choose for the *upper* (1) or *lower* (2) side; the (X, P) dichotomy is called the *canonical* dichotomy and in that case is the projector on the upper side the same as picking the conservative perspective, whereas the projector on the lower side corresponds to the progressive perspective[1]. To say in plain language what I mean, during a conversation, you may look with new glasses at the same issue, but still you can take an upper stance or a lower stance wanting to change that new viewpoint you were just considering. To model such choice function, you could introduce a real variable s and consider the step function $\theta(s)$[2], smoothen it out a bit around 0, denoted by $\tilde{\theta}$, and finally define

$$\pi(s) = \cos\left(\frac{\tilde{\theta}(s)\pi}{2}\right)\pi_1 + \sin\left(\frac{\tilde{\theta}(s)\pi}{2}\right)\pi_2$$

so that there is a fast switching between an upper and lower point of view. The dynamics for s must be coupled to everything, your physical brain, the current black state, as well as your dichotomy. We shall discuss this at greater depth later on. So, our fundamental dichotomy has the natural symmetry of relativity theory; consider now the transformation

$$A_S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \in SL(2, \mathbb{C})$$

which we dub as the Schwitchoriem transformation whose second component

$$a = \frac{1}{\sqrt{2}}(X + P)$$

[1] In a previous publication of this work, I used black an white for the upper and lower side regardless of the dichotomy chosen, which may have lead to some confusion.

[2] $\theta(s) = 0$ if $s < 0$ and one otherwise.

is precisely the "middle" between the black and white perspective which is still a lower choice to make from our point of view. As a small aside, in standard quantum theory, one represents X, P on a so-called Hilbert space with an hermitian inner product leading one to the anti-automorphism † obeying roughly[3]

$$(\alpha A + \beta B)^\dagger = \overline{\alpha} A^\dagger + \overline{\beta} B^\dagger,\ (AB)^\dagger = B^\dagger A^\dagger$$

In this account, we have that $X^\dagger = X$ and $P^\dagger = -P$ so that

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} X \\ P \end{pmatrix} = \begin{pmatrix} a^\dagger \\ a \end{pmatrix}$$

where a represents the schwitchoriem point of view and $a^\dagger$ the diametrically opposite point of view. Obviously, one has that

$$\left[a, a^\dagger\right] = 1$$

by definition. Now, we come to the definition of the ground state $|0\rangle$ (of an issue) which a free society, in which there is no interaction between issues and therefore no emergent conventions, which require mental energy, by means of a learning process, aspires to reach. It is defined by the property that the Schwitchoriem, who is the ultimate opportunist, perceives it as a cold or neutral world in which he feels relaxed. Mathematically, this reads

$$a|0\rangle = 0.$$

As is usually done in quantum field theory, one may consider the positive operator

$$H = a^\dagger a$$

and one obtains that

$$H|0\rangle = 0$$

which precisely means that the ground state has zero psychological weight for the Schwitchoriem[4]. Here I launch our second principle which is that we project the mental state function down in our mind by using the energy operator attached to the operator Z representing our viewpoint; indeed, one cannot ask about Z itself except when Z is self-adjoint (or normal); this happens only for a four (five) parameter family of profiles given by

$$\begin{pmatrix} a & ib \\ \frac{ad}{b} + i\frac{1-ac}{b} & c + id \end{pmatrix}$$

where $a, b, c, d \in \mathbb{R}$ (or a is complex and $b = ar$ with $r, c, d \in \mathbb{R}$) in case you choose the upper profile and we assume $b \neq 0$ and likewise so for the lower profile.

[3]The properties below only hold for bounded operators, for unbounded operators you have to take domain issues into account, but I shall neglect such subtleties here.

[4]One can define for every psychological type Z the intrinsic free weight attached to its profile as $Z^\dagger Z$.

The reader may utter here that our notion of a dichotomy so far is a strange one given that, classically, one thinks of a dichotomy in terms of two sharp unique opposites whereas here the opposites come in two real colours; indeed, the white profile has as opposites $P + cX$ where c is any complex number. One could eliminate this freedom and leave only for 2 complex numbers instead of 3 by demanding that $AA^\dagger$ is a diagonal matrix, in either that the profiles are perpendicular to one and another. In matrix language this reads

$$A = \begin{pmatrix} a & b \\ c & \frac{bc+1}{a} \end{pmatrix}$$

assuming a is nonzero and

$$a\overline{c} + b\overline{\frac{bc+1}{a}} = 0$$

which always has a unique solution given by $\overline{c}(|a|^2 + |b|^2) = -b$. Note that the opposite condition $A^\dagger A$ is diagonal, leads to $\overline{a}b + \overline{c}\frac{bc+1}{a} = 0$ or $(|a|^2 + |c|^2)b = -\overline{c}$. This equation is quadratic in c instead of linear and does not always have a solution; therefore, the previous condition would be the correct one. Likewise, the complex condition AA^T is diagonal leads to $(a^2 + b^2)c = -b$ which has no solution in case $a^2 = -b^2$ for $b \neq 0$. We shall not take this point of view here and allow for the Greek dichotomy to be two dimensional over the real numbers; it is just so that good and bad can have different expressions and we will allow for this. In what follows, we shall call the result of combining a profile with an upper-lower choice the decision. Obviously, the 5 parameter family of profiles leading to a definite decision has meaure zero, the 6 dimensional remainder being called normal. For the latter $ZZ^\dagger = Z^\dagger Z + r1$ where $r \neq 0$ (using the adjointness properties of X, P as well as the commutation relations). In that sense are the Schwichoriem and anti-Schwichoriem profiles given by

$$\begin{pmatrix} a^\dagger \\ a \end{pmatrix} \begin{pmatrix} a \\ -a^\dagger \end{pmatrix}$$

the most symmetrical normal ones. The former being hermitian, whereas the latter is the anti-hermitian conjugate. This requires further reflection as a white person will only require no mental energy when he sees at the universe in the distributional state with $\lambda = 0$. As mentioned before, only a three parameter family of decisions is capable of further reduction of the wave function; he or she cannot only answer "I look at it from that point of view, which is maybe complex, with such an intensity (energy)" but also state in a compatible way, "I see it like that". In this sense are almost all tests most psychologists give too limited since they force you to engage in their specific (not necessarily even white) real reality (not even a complex or imaginary one) given that they are not interested in how you feel about their task, whereas the truth is that the person simply does not want to or cannot give any answer in this way. Since, you just fill in something to please them, they draw entirely bogus conclusions on the nature of "reality". They don't even kow what reality is: it is far more complex than the world in which they operate, the latter being one of definite, real, pre-cooked answers! Now, unlike the doctrine in physics where individual systems aspire to be in the lowest energy state possible, humans do enjoy mental effort and like to spend energy in things. Our description of variables attached to an issue seems at first sight ad odds with the existence of mathematical certainties which require only binary answers. However, I shall argue later how, by means of a learning process in which classical logic is dynamically embedded, classical logic, as we practise it may be an emergent phenomenon. This will be discussed at length in part 3. Obviously, the stable ground state of our, interacting, world costs energy for the Schwitchoriem as there exist many issues which are poored into more or less definite black pointer states. Indeed, these conventions are called law, logic and truth causing for a polarization of Schwitchoriem vacuum. Before we proceed, let us write down some interesting first order dynamics a psychological type given by a matrix $A(t)$ might undergo; up to third order we have

$$\frac{d}{dt}A(t) = aYA(t) + bA^{-1}(t)ZA^2(t)$$

where a, b are arbitrary complex numbers and Y, Z arbitary traceless complex matrix fields which therefore belong to the Lie algebra of $SL(2, \mathbb{C})$. Indeed, one immediately verifies that

$$\frac{d}{dt}\det(A(t)) = \det(A(t))\mathrm{Tr}\left(\left(\frac{d}{dt}A(t)\right)A^{-1}(t)\right) = 0.$$

Another, more insightful proof (which is important later on) reads

$$\frac{d}{dt}(A(t)^T\omega A(t)) = A(t)^T\omega\left(aYA(t) + bA^{-1}(t)ZA^2(t)\right) - \text{Transpose} =$$

$$\omega\left(aA^{-1}(t)YA(t) + bA^{-2}(t)ZA^2(t)\right) - \text{Transpose} = 0$$

simply because any traceless matrix X obeys $X^T\omega = -\omega X = -(\omega X)^T$ since $\omega^T = -\omega$. So, in finding a general interaction theory for (multiple) issues, we shall look for "intertwiners" obeying these "commutation" relations. Nature has thought us that the best way to construct such an interaction theory, is by making it as symmetrical as possible, meaning in this case that it should be a local $SL(2, \mathbb{C})$ "gauge theory" with Y a connection and $Z = 0$. Now comes the real beef of the story; we already have such a connection which is the gravitational spin connection! Now, the gravitational spin connection really is not a gauge field in the sense that upon performing a Lorentz transformation, the predictions of our theory transform covariantly instead of remaining invariant; in that sense should observables in quantum gravity be diffeomorphism covariant and not invariant as certain luminaries proclaim. In that vein do we claim that a change of reference frame is the result of a mental intervention, one which changes the conditions of the psyche leading to a different evolution. In other words, it is an active transformation and not just one assossiated to the redundancy of the description. In that vein, let $J(x)$, where $g(J(x), J(x)) = 1$, denote the effective classical current describing our brain activity; every nanosecond or so, it gets updated by measuring the real quantum current, then we fix a mental vierbein $\tilde{e}_a$, whose equations of motion are just Fermi transported along the classical neural current

$$\nabla^F_{J(x)}\tilde{e}_a(x) :=$$
$$\nabla_{J(x)}\tilde{e}_a(x) + g(\tilde{e}_a(x), \nabla_{J(x)}J(x))J(x) - g(\tilde{e}_a(x), J(x))\nabla_{J(x)}J(x) = 0$$

with $\tilde{e}_0(x) = J(x)$. The reader immediately notices that Fermi transport is not covariant under local rotations of the $\tilde{e}_j(x)$. It is crucial to understand that this must be so: a change of a reference frame attached to a physical observer requires not only a conscious intervention but also the necessary energy to realize that (ignoring even backreactions on the spacetime fabric itself). As long as no such intervention occurs is there no freedom to rotate as explained in the beginning of this chapter. Boosting is very much like an adiabatic process, it takes a while and the changes per unit time are infinitesimal; hence we take the viewpoint that such process is correctly described by

$$\nabla^F_{J(x)}\tilde{e}^a(x) - \alpha^a_{\ b}(t)\tilde{e}^b(x) = 0$$

where $\alpha^b_a(t)dt$ is the boost executed at time t and t is defined with respect to a dynamical coordinate system as follows. Take Σ_0 as a spatial hypersurface, just at the moment prioir to contemplating to change your reference frame and t is then simply defined by means of $\partial_t = \tilde{e}_0$ and $t = 0$ on Σ_0. The attentive reader must have noticed that the latter equation breaks the relation $\tilde{e}_0 = J(x)$, however, an infinitesimal moment in time later $J(x)$ reallings itself with $\tilde{e}_0$ meaning that

$$J^\nu(x^\mu + J^\mu(x)\,dt) := \tilde{e}_0(x^\mu + J^\mu(x)\,dt).$$

So, there is an inherent discontinuity in the process given that $\nabla_{J(x)}J(x)$ is determined by the classical effective equations of motion; but the latter are never integrated when the mind intervenes. The accelerations merely serve as an extra initial condition in defining the Fermi derivative locally. There is no way of explicitely integrating these equations but they should be programmed on a computer taking finite time steps δt and take the limit δt to zero. The reader checks that our equation preserves the orthonormal character of the basis given that $\alpha_{ab}(t) = -\alpha_{ba}(t)$. Actually, I am using nonstandard infinitesimal analysis here, but the correct time derivative is given by

$$\nabla^C_{J(x)} := \nabla_{J(x)} + \tilde{\omega}^a_{\ b} + \frac{i}{2}\tilde{\omega}_{cd}\mathcal{J}^{cd}$$

where $\mathcal{J}^{cd}$ are the generators of the Lorentz group in the spin $(\frac{1}{2}, 0)$ representation and $\tilde{\omega}^a_{\ b} = -\tilde{e}_{b\nu}\nabla_{J(x)}\tilde{e}^{a\nu}$. A spinor $C(x)$ undergoes then an infinitesimal boost

$$C(x) \to e^{i\alpha_{ab}(t)dt\mathcal{J}^{ab}}C(x)$$

and the equations of motion are covariant with respect to this procedure. Note also that the connection $\tilde{\omega}^a_{\ b}$ does not vanish in case no change of reference frame takes place (which would have been the case if we would have defined

$\widehat{\omega}^a_{\ b} = -\tilde{e}_{b\nu}\nabla^F_{J(x)}\tilde{e}^{a\nu}$) thereby allowing for interactions between the profile fields and acceleration of the matter distribution. This must be so and later in this chapter shall we discuss couplings of a nonlinear nature. Now, before we proceed to the more general case of multiple issues, let us discuss the ramifications of this idea a bit further. In this regard, it is of crucial importance to note that the canonical dichotomy, which corresponds to the identity matrix, does not change under Lorentz boosts as it should be!! Indeed, our eyes are suddenly not taking a mixed perpective regarding the incoming electromagnetic radiation; they keep on measuring as usual and also preserve their upper perspective. Fact of the matter is, that more complex issues, even the knowledge of mathematical theorems are not approached from the canonical dichotomy (it almost is, but not quite). For example, when a mathematician walking on the street is being asked for his views on a certain mathematical statement, he might very well answer that he is just walking and you should ask this question later again when he is at home sitting at his desk. In this sense is revealing of mathematical knowledge not a canonical dichotomy, but one which is intertwined with other issues such as your state of motion. However, you might ask him a simpeler question requiring a trinary answer - which is much more easy to give - whether this particular statement is true or not? He can then quickly say, yes/no or I don't know. So, this is a very bold conjecture, that mixed issues transform under your change of reference frame; even a simple rotation might cause you to reflect differently on mixed issues; for example a professor sitting at his desk, rotating his chair for 180 degrees so that he is sitting with his back to the desk might suddenly conclude that he cannot think in this way, he needs his desk in front of him to write things down on paper and order his thoughts. So, the fact that he does not see his desk, which is encoded in the accelerations of his new brain current changes his profile on thinking.

4.1 Intermezzo.

We shall further deepen our understanding of the above regarding two different aspects. The first is that so far, we have looked upon the defining black-white relations from the point of view of complex geometry; here we develop the perspective from the Hermitian point of view and draw analogies between both. Indeed, instead of taking as defining relation

$$(X,P)\sigma_2 \begin{pmatrix} X \\ P \end{pmatrix} = -i1$$

we could as well have used

$$(X,P)^\dagger \sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} = -1$$

leading one to a group of transformations A defined by

$$A^\dagger \sigma_1 A = \sigma_1.$$

The latter is four dimensional and not six dimensional and consists out of a $U(1)$ part and something isomorphic to $SO(1,2)$ consisting out of boosts in the $2,3$ direction and a rotation around the 1 axis, realized by $K^2 := -\frac{i}{2}\sigma^2$, $K^3 := -\frac{i}{2}\sigma^3$, $J^1 := \frac{1}{2}\sigma^1$ respectively, obeying

$$\left[K^2, K^3\right] = -iJ^1,\ \left[J^1, K^2\right] = iK^3,\ \left[J^1, K^3\right] = -iK^2.$$

Basically, we also have those generators in our $SL(2,\mathbb{C})$ Lie algebra and we can identify them. We shall come back to this duality in point of view in part 3 of this book. A second issue regards a natural representation of our profile operators, in particular, consider the following operators

$$\begin{aligned}
E &= \frac{-i}{2}(X,P)\sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-1}{2}(X,P)(-i\sigma_2)(i\sigma^3) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-i}{2}(2XP+1) \\
&= (X,P)^\dagger(-i\sigma^1)(\frac{1}{2}\sigma_3) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{-1}{2}(X,P)^\dagger\sigma^2 \begin{pmatrix} X \\ P \end{pmatrix} \\
T &= i(X,P)\sigma_2 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= i(X,P)(-i\sigma_2)(i1_2) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= 1_2 \\
&= -(X,P)^\dagger\sigma_1 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma_1)(-i1_2) \begin{pmatrix} X \\ P \end{pmatrix} \\
-H &= \frac{1}{2}(X,P)\sigma_3 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(X,P)(-i\sigma^2)(-\sigma^1) \begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(-X^2+P^2) \\
&= \frac{-1}{2}(X,P)^\dagger 1_2 \begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma^1)(-\frac{i}{2}\sigma^1) \begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}$$

$$
\begin{aligned}
B &= \frac{1}{2}(X,P)1_2\begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(X,P)(-i\sigma^2)(i\sigma^2)\begin{pmatrix} X \\ P \end{pmatrix} \\
&= \frac{1}{2}(X^2+P^2) \\
&= \frac{-1}{2}(X,P)^\dagger\sigma_3\begin{pmatrix} X \\ P \end{pmatrix} \\
&= (X,P)^\dagger(-i\sigma^1)(-\frac{\sigma^2}{2})\begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}
$$

Before we proceed, let us mention that these expressions suggest a deeper relationship between $\sigma^1, -i\sigma^2, -\sigma^3, 1_2, i\sigma^2$ on the other[5]. This almost suggests that the 1-axis and time are of the same kind and likewise so for the $2-3$ axis; which is the case for the gravitational field on the earth where the 1 axis equals the z axis and is given by ∂_r and time is related to height. The only freedom which is still left is rotation around the 1 axis, which connects the $2-3$ axis in which biological creatures can move without effort. The reader also notices that there is a reflection symmetry around all axis which amounts to space-time reversal. I was very interested in this observation when I noticed it for the first time and we shall try to make sense out of it later on. One notices that E, H, B are self adjoint operators and that:

$$[H,E] = 2iB,\ [B,H] = 2iE,\ [B,E] = 2iH$$

which is the algebra of $SO(1,2)$ with $J^1 = \frac{1}{2}H, -\frac{1}{2}B = K^2, \frac{1}{2}E = K^3$ and not $SU(2)$. The reader may enjoy understanding that these insights result from quantization of a $1+0$ dimensional complex spinor field theory with as real action

$$S = i\int dt \begin{pmatrix} \Phi(t) \\ \Psi(t) \end{pmatrix}^\dagger \sigma^1 \frac{d}{dt}\begin{pmatrix} \Phi(t) \\ \Psi(t) \end{pmatrix}$$

which is clearly $U(1)\times SO(1,2)$ invariant. A small computation reveals that there exist two independent real components and two imaginary ones forming two conjugate pairs which decouple in the usual Poisson bracket. The correct bracket to quantize however is the Dirac bracket which gives half of the identity (putting $\hbar = 1$) for the conjugate variables. Restricting to one canonical pair, we have that $X = \mathrm{Re}(\Phi(t)), P = 2\mathrm{Im}(\Psi(t))$ since the canonical momentum of $\mathrm{Re}(\Phi)$ equals $\mathrm{Im}(\Psi) = \frac{iP}{2}$ and likewise the canonical momentum of $\mathrm{Im}(\Psi)$ is $-\mathrm{Re}(\Phi) = -X$ so that everything is consistent. I dropped the time dependency here because the Hamiltonian is exactly zero. The charges associated to the

[5] Note that the Hermitian geometry only provides for $i\sigma^2 = \sigma^3$ and $\sigma^1 = -1_2$; all other relations arise from the correspondance with the complex geometry which gives $i\sigma^1 = \sigma^2$ and $\sigma^3 = -1_2$

symmetry group are therefore (ignoring the other conjugate pair)

$$\begin{aligned}
1_2 &\rightarrow \frac{1}{2}\begin{pmatrix} X \\ P \end{pmatrix}^{\dagger}\sigma^1\begin{pmatrix} X \\ P \end{pmatrix} \\
\frac{1}{2}\sigma^1 &\rightarrow \frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^{\dagger}1_2\begin{pmatrix} X \\ P \end{pmatrix} \\
-i\frac{1}{2}\sigma^2 &\rightarrow \frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^{\dagger}\sigma^3\begin{pmatrix} X \\ P \end{pmatrix} \\
-i\frac{1}{2}\sigma^3 &\rightarrow -\frac{1}{4}\begin{pmatrix} X \\ P \end{pmatrix}^{\dagger}\sigma^2\begin{pmatrix} X \\ P \end{pmatrix}
\end{aligned}$$

so that it is clear that the algebra is preserved[6]. To turn it into $SU(2)$ we need to analytically continue and state that $J^1 = -\frac{1}{2}H, J^3 = -\frac{1}{2}iE, J^2 = -\frac{1}{2}iB$ which means that the rotations around the $2-3$ axis correspond to anti-Hermitean operators. That this constitutes the right point of view is exemplified by considering the adjoint actions

$$\begin{aligned}
\left[J^3, X\right] &= -\frac{1}{2}X \\
\left[J^3, P\right] &= \frac{1}{2}P \\
\left[J^2, X\right] &= -\frac{i}{2}P \\
\left[J^2, P\right] &= \frac{i}{2}X \\
\left[J^1, X\right] &= \frac{1}{2}P \\
\left[J^1, P\right] &= \frac{1}{2}X
\end{aligned}$$

Upon identifying X with $(1,0)^T$ and P with $(0,1)^T$ one sees that $J^j \equiv \frac{1}{2}\sigma^j$ which confirms our previous analysis. Notice that there is another interesting observation here; in a way, the representation of $SO(1,3)$ is broken down to one of $SO(1,2)$ given that the latter constitues the unitary part of the former. The attentive reader must have noticed that there is a slight ambiguity in the above

[6]Note that we have changed sign of $\sigma^1 = -\frac{1}{2}H$ and $\sigma^3 = -\frac{1}{2}E$ but that is inconsequential.

determination of the charges and that we could equally well have considered

$$
\begin{aligned}
1_2 &\to \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right)^\dagger \sigma^1 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right) \\
&= -\frac{1}{2} 1 \\
\frac{1}{2}\sigma^1 &\to \frac{1}{2} \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right)^\dagger 1_2 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right) \\
&= \frac{1}{2}\left(X^2 - \frac{P^2}{4} \right) \\
-i\frac{1}{2}\sigma^2 &\to \frac{1}{2} \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right)^\dagger \sigma^3 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right) \\
&= -\frac{1}{2}\left(X^2 + \frac{P^2}{4} \right) \\
-i\frac{1}{2}\sigma^3 &\to -\frac{1}{2} \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right)^\dagger \sigma^2 \left(\begin{array}{c} X \\ \frac{P}{2} \end{array} \right) \\
&= -\frac{i}{2}\left(XP + \frac{1}{2} \right).
\end{aligned}
$$

which changes the appearance on the right hand side by a factor of 2. Perhaps, there is something deeply hidden in this and that all (inverse) powers of 2 and -1 are encoded into nature.

Finally, note that there is another Hermitian way of encoding the commutation relations which is given by

$$\left(\begin{array}{c} X \\ iP \end{array} \right)^\dagger \sigma^2 \left(\begin{array}{c} X \\ iP \end{array} \right) = 1.$$

The symmetries of this relation are again given by $U(1) \times SO(1,2)$ but this time the rotation is around the 2 axis. This leads to an identification of $-\sigma^1 = \sigma^2$ which causes for the entire theory to be invariant under the symmetry $1 \to i$; this has been suggested into the work of 't Hooft and Nobbenhuis regarding the cosmological constant. Note that the associated complex geometry given by

$$\left(\begin{array}{c} X \\ iP \end{array} \right)^T \sigma^2 \left(\begin{array}{c} X \\ iP \end{array} \right) = 1$$

is redundant and does not provide for any new information: its symmetry group is given by

$$A^T \sigma_2 A = \sigma_2$$

and is again the usual $SL(2, \mathbb{C})$. Finally, note that the Schwitchoriem duality is not included in any of the two Hermitean geometries meaning there is no way, by means of the associated symmetries, to rotate the free particle into the Harmonic oscillator or into a particle with no kinetic term at all. This is why all viewpoints seem to be important. Note here that reality itself, that is the psychic wavefunction for all observers is taken to be static so far, it are the perspectives or profiles which change and reality changes only by means of by means of measurement. This is certainly a part of the game, but on the other hand do we know that psychic reality changes if material reality does and this has nothing to do with your decision, that may be perfectly black! So, your local black world evolves too: for example, when you see someone hurt, you will most likely help her so that reality changed in the sense that this became the most likely answer. Of course, this particular information regarding the image of the hurt person must be hidden into you brain currents, another reason why we must take the complex geometry of our commutation relations most seriously given it provides for a $(\frac{1}{2}, 0)$ representation of the Lorentz group and therefore allows for a quadratic coupling to those currents. Indeed, the coupling itself must be Lorentz invariant: the psyche is attached to the body, it is just so that you will have different brain currents when moving relative to one and another compared to when you are in rest to each other simply because incoming signals shift accordingly. For example, when driving a car, you are less likely to stop for someone in need on the street as when meeting this person on foot. We shall not go into this matter further in this book as we clearly lack experimental data of how our psyche couples precisely to our brain currents and how our moral values are affected by what we see from at the level of molecules and so on. A computer again can be just fed with billions of pictures or conversations of the same person so that it eventually recognizes when this person is sad or happy, if the programmer just attached these words to those pictures in the first plae, which is something very different from having an understanding of these words. Again, all of this has its limits, but perhaps nature causes complicated "machines" not to be a machine any longer, by means of something we cannot comprehend. Finally, remark that the only symmetry which is common to all viepoints is a boost around the 3 or y axis associated to the E operator; we shall come back to this in part 3.

4.2 Further exploration of the dynamics of the profile matrix.

Since any profile matrix can be identified with $A(x) = e^{\vec{a}(x).\vec{\sigma}}$ where $\vec{a}$ is a complex vector, we have a canonical isomorphism mapping it to

$$\widehat{A} = e^{\vec{a}.\vec{J}}$$

where as mentioned previously $J_1^\dagger = J_1$, $J_2^\dagger = -J_2$, $J_3^\dagger = -J_3$. This allows one to couple the profile matrices to the wavefunction describing the black reality. At this point, it is necessay to mention that all representation Hilbert spaces used in physics carry a natural complex structure, meaning that one can tell whether a vector is real or imaginary. In that vein, can we define the complex conjugate of an operator and both X, P obey

$$\overline{X} = X, \overline{P} = P$$

which leads to

$$\overline{J_1} = J_1,\ \overline{J_2} = -J_2,\ \overline{J_3} = J_3.$$

Therefore, we can relate the adjoint representation to the inverse of the complex conjugate representation if we find an operator S obeying $S^2 = 1$ such that

$$J_j^\dagger S = -S\overline{J}.$$

In the standard $su(2)$ representation, this is easily seen to be given by $\pm\sigma^2$; in our framework however, we have to focus on J^3 (which is of course fully equivalent) given that

$$J_1 S = -SJ_1,\ J_2 S = -SJ_2,\ J_3 S = SJ_3.$$

I have not seen any discussion of this "charge conjugation" in the literature but it is obvious from the theory outlined in section 5.1 that there is precisely (upon a sign) one S satisfying, moreover, $S^\dagger = S$. The latter is given by $S_{\sigma,\sigma'} = \delta_{\sigma\sigma'}(-1)^{\sigma-j}$ where σ is half integer in case j is and runs from $-j \ldots j$. In general, our operators X, P are the usual multiplication and differential operators with respect to α and are represented on the Hilbert space of square integrable functions in those psychological variables at any point in spacetime. The associated representation, by means of the J^j, of the rotation group is obviously reducible and can be written as an infinite direct sum of irreducible representations; hence the only freedom in the choice of S is a relative sign ± 1. Hence, we shall be interested in couplings of the kind

$$K(A(x), \Psi(x)) := \frac{\int ds\ \overline{\Psi}(x,s)\, S\, \overline{\widehat{A}(x)\, \Psi(x,s)}}{\int ds\ \overline{\Psi}(x,s)\, S\, \overline{\Psi(x,s)}}$$

where $\Psi(x,s)$, $\widehat{A}(x)$ transform under an (infinitesimal) Lorentz transformation $\Lambda^{\frac{1}{2}}(x) = e^{i\vec{b}(x).\vec{\sigma}}$ as

$$\frac{\widehat{\Lambda^{\frac{1}{2}}(x)}\Psi(x,s)}{\int ds\ \overline{\Psi}(x,s)\,\Psi(x,s)},\ \widehat{\Lambda^{\frac{1}{2}}(x)}\widehat{A}(x)\widehat{\Lambda^{-\frac{1}{2}}(x)}$$

such that $K(A(x), \Psi(x))$ remains an invariant. This principle reflects that the evolution of your profile regarding the mental state remains the same if you change motion. Note that $\widehat{\Lambda^{\frac{1}{2}}(x)}$ is not a unitary operator and that therefore change of reference frame changes (slightly) your notion of orthonormal basis which means that you have a different reality - a change from the traditional viewpoint upon quantum mechanics (note that we renormalized the wave and therefore had to consider the denominator in the definition of $K(A(x), \Psi(x))$

which we assume to be nonzero). Before we proceed, let us think about this a bit further; this viewpoint is exactly the same as we had for the change an external state underwent if the hypothetical observer boosted; the latter description, as we shall explain in full depth in part 3, is just a rule of thumb. What really happens is that the state of your brain changes relative to the exterior world which is equivalent to an appearant change in the outher world! We will comment further upon that at the end of section 4.3 when discussing issues wich boost into one and another when changing frame of reference, such as the different components of your brain currents themselves which just reflect that the state of your brain has altered regarding its coupling with the external world. Indeed, we see colours slighly differently when moving, lengths of objects contract and so on. Note that in part 2 of this book on quantum mechanics, we shall again be rude to the observer and not describe him or her in the fullest detail; therefore, the operators attached to energy and momentum are just effective descriptions of a similar process which goes on in the body of the observer himself! There is another change with regard to traditional quantum field theory here, which is that the profile field acts upon the black-white variables which constitute a "quantum system" themselves. Also this is a novel extension of the usual lore where everything is put on the same level; in the psyche, there are different levels, there are issues, profiles on issues, issues on profiles of issues and so on. The psyche works as such, it is a reflection of intelligence. If you want artificial intelligence to be able to partially reason as a human, you would simply have to feed it with all infinite conversations between humans to obtain a level of speech which is somewhat coherent. All AI can do up till now is give an encyclopedic overview of what is known, but it will actually never make any choice by itself and even if it would (which you can program by means of a random generator) then still you would have to program it as such in order for successive profiles to be coherent. Therefore, AI will never be able to do any original research. Let us also mention that the dynamics for the profile matrix must be as such that the identity matrix is a fixed point and attractor. It is important to notice that the action of the profile matrices (and therefore the action of the symmetry group) is not a unitary one, which leads in general to complex action principles instead of real ones and we shall come back to this topic in part 3.

4.3 Multiple issues.

In a way, this is a system of one issue; going over to N issues we obtain N operators X_i and P_j satisfying

$$[P_i, X_j] = \delta_{ij} 1$$

as well as

$$[X_i, X_j] = [P_i, P_j] = 0$$

and we look for symmetries of this algebra[7]. Those include the so-called Bogoliubov transformations which map Schwitchoriems on pure issues to Schwitchoriems on mixed issues; but since the world of issues is classical, those mixed issues are not seen as defining the ground state of society. The full matrix algebra has complex dimension $(2N)^2$ for which the quadratic forms associated to N matrices T^i are put to the identity and the remaing quadratic forms associated to $2N(N-1)$ different matrices $S^{[ij]}_{ab}$ are mapped to the zero operator. Here, $(T^i)^{ka}_{lb} = \delta^k_l \delta^i_k \omega^a_b$ and $S^{[ij]}_{ab} = E_{ij}F_{ab} - E_{ji}F_{ba}$, where E_{ij} is the $N \times N$ matrix with all zero entries except for (ij) where it equals one and F_{ab} is the two times two matrix given by $(F_{ab})_{cd} = \delta_{ac}\delta_{bd}$, are all antisymmetric matrices. The reader may enjoy proving that preservation of these constraints is equivalent to

$$\mathrm{Tr}\left(A^T T^j A \omega_N\right) = -1,\ \mathrm{Tr}\left(A^T S^{[ij]}_{ab} A \omega_N\right) = 0$$

where

$$\omega_N = 1_N \otimes \omega$$

and 1_N is the $N \times N$ identity matrix. So, this leaves us with a $4N^2 - 2N(N-1) - N = 2N^2 + N$ dimensional *complex* Lie algebra of symmetry transformations. As we knew already, for $N = 1$, we have three generators, naturally associated to space if our question regards the being or perception of space and we added "time" to this picture as the identity matrix. Note that in the exceptional case of the fundamental representation of $SL(2, \mathbb{C})$, the generators do not only constitute a Lie algebra but naturally give rise to a complex algebra with time added. There was another way spacetime arose from this representation and that was by taking the tensor product with its complex conjugate representation and taking real sections. So, in a way, there are two natural algebraic connections between the different psychological types, regarding a certain issue, and the four dimensional nature of spacetime - this might lead to a useful idea when formulating dynamical laws for psychological types. The tensor product construction is the most easy one to generalize[8]; indeed; one may consider a $2N$ dimensional complex vector space defined by

$$W \sim \oplus_{j=1}^{N} \begin{pmatrix} X_j \\ P_j \end{pmatrix}$$

then insisting that our symmetry transformations preserve the natural symplectic form ω_N imposes $\frac{N(N+1)}{2} - 1$ extra constraints. Hence, going over $W \otimes \overline{W}$ and taking real sections, we have a natural *complex* action of our symmetry group on $\otimes_N(M^4)$ where M^4 is 4 dimensional (complex) Minkowski spacetime. In more detail, consider

$$-\delta_{ij}\eta_{ab} = \delta_{ij}\sigma_a^{AA'}\sigma_b^{BB'}\omega_{AB}\overline{\omega}_{A'B'}$$

[7]Here it is worthwhile to notice that although we consider the direct sum construction in determining the distinct profiles, we shall of course take the tensorproduct of the "one issue Hilbert spaces" when representing the issue operators.

[8]The solder form here can be taken to be static, however alternative theories of gravitation may be developed using dynamical solder forms.

and

$$(\Lambda)^{ia}_{jb} := \sigma^{a}_{AA'}\sigma^{BB'}_{b}A^{iA}_{kB}\overline{A}^{kA'}_{jB'}$$

then elementary algebra reveals that they constitute elements of $SO(N, 3N; \mathbb{C})$. Indeed, it is generally not so that $(\Lambda)^{ia}_{jb}$ is a real matrix albeit this is certainly so for $N = 1$. To demand it is real is equivalent to the supplementary conditions

$$A^{iA}_{kB}\overline{A}^{kA'}_{jB'} = \overline{A}^{iA'}_{kB'}A^{kA}_{kjB}$$

or equivalently that the $N \times N$ matrices A^{A}_{B} satisfy the following 8 complex conditions

$$\overline{A}^{A'}_{B'}A^{A}_{B} = A^{A}_{B}\overline{A}^{A'}_{B'}.$$

Note that if you would insist upon a real embedding of the entire group without any additional reality conditions then you would need to embed it into $SO(N^2, 3N^2; \mathbb{R})$ and we shall leave this as an exercise to the reader. In the sequal, we shall not go over to any reduced symmetry group, but merely consider gauge transformations which do preserve ω_N which is all we really need. Now, we come to an important point, we must couple spacetime currents (which display the motion of our spirit attached to our body) with the evolution of these matrices and we can use this isomorphism between N copies of the tangent bundle and our mental habitat of N questions, denoting by $\mathcal{J}^{ab}$ the generators of the spin $(\frac{1}{2}, 0)$ representation[9], we can take

$$\mathbf{J}^{N\,ab} = \sum_{j=1}^{N} 0_2 \oplus_1 \ldots 0_2 \oplus_{j-1} \mathcal{J}^{ab} \oplus_j 0_2 \ldots \oplus_{N-1} 0_2$$

where 0_2 is the zero matrix in 2 dimensions. Clearly, this is an element of the Lie algebra of our symmetry group (the direct sum reprentation); now lets consider the spin connection

$$\nabla^{C,N}_{J(x)} := \nabla_{J(x)} + \frac{i}{2}\tilde{\omega}_{ab}\mathbf{J}^{N\,ab}$$

which is mandatory to make the dynamics of the $(2N) \times (2N)$ matrices A belonging to our symmetry group covariant under a change of reference frame. There is an interesting thing to mention here, the intertwiners governing the interactions of profiles have to be dynamical objects themselves in contrast to what happens in particle theory. From a mental point of view, this is entirely logical since the interactions between two identical profiles vary in time; I suggest that those matrices only couple to the mental variables and not the type operators themselves[10]. Here, it is of course understood that a $2N$ complex "mind vector" V transforms under a local Lorentz transformation as

$$V \to e^{i\zeta_{ab}\mathbf{J}^{N\,ab}}V$$

[9] $\mathcal{J}^{0j} = \frac{i}{2}\sigma^j$, $\mathcal{J}^{jk} = \epsilon_{jkl}\frac{1}{2}\sigma^l$

[10] Note that everything is consistent here given that upon applying a spin $\frac{1}{2}$ Lorentz transformation g to X results in $(\omega g X g^{-1})^T = -(g^{-1})^T X^T g^T \omega = \omega g X g^{-1}$ by using the properties $X^T\omega = -\omega X$ and $g^T \omega g = \omega$.

where ζ_{ab} is real an antisymmetric and generates a Lorentz transformation on the vectors by means of

$$e^{\zeta_{ab} J^{ab}}$$

where the J^{ab} are the real antisymmetric generators of the defining representation and $J^{ab} = -J^{ba}$. Likewise can the reader now construct more general intertwiners allowing for different issues to mix with one and another as well as to change your perspective upon things. We now come back to our projectors $\pi_{A,j}(s_j)$ where $A = 1, 2$ and $j : 1 \ldots N$ onto the conservative or progresive side of your dichotomy. I did not mention this previously, but the tensor $\pi_{A_k}(s_j; j = 1 \ldots N) : \mathbb{C}^{2N} \to \mathbb{C}$ simply is the identity matrix so that everything is manifestly Lorentz covariant. It is obvious that one should construct the Lie algebra of our Lie group in terms of the Pauli matrices such that the new quantum generators can be constructed by means of the identical procedure; for $N = 2$ the reader may verify that the complex Lie algebra is generated by

$$R^j := \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix} \; T := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

and

$$P^j = \begin{pmatrix} \sigma^j & 0 \\ 0 & 0 \end{pmatrix} \; Q^j := \begin{pmatrix} 0 & 0 \\ 0 & \sigma^j \end{pmatrix}.$$

The commutation relations are

$$\left[R^j, R^k\right] = 2i\epsilon_{jkl}(P^l + Q^l), \; \left[R^j, P^k\right] = -iT\delta^{jk} + i\epsilon_{jkl}R^l, \; \left[R^j, Q^k\right] = iT\delta^{jk} + i\epsilon_{jkl}R^l,$$

$$\left[P^j, Q^k\right] = 0, \; \left[R^j, T\right] = 2i(P^k - Q^j), \; \left[P^j, T\right] = -iR^j, \; \left[Q^j, T\right] = iR^j$$

and

$$\left[P^j, P^k\right] = 2i\epsilon_{jkl}P^l, \; \left[Q^j, Q^k\right] = 2i\epsilon_{jkl}Q^l.$$

Mapping each generator S to

$$S \to \widehat{S} := \frac{1}{2}(X_1, P_1, X_2, P_2) \begin{pmatrix} i\sigma^2 & 0 \\ 0 & i\sigma^2 \end{pmatrix} \begin{pmatrix} X_1 \\ P_1 \\ X_2 \\ P_2 \end{pmatrix}$$

leads to

$$R^1 \to i(-X_1X_2 + P_1P_2) \tag{4.1}$$
$$R^2 \to -(X_1X_2 + P_1P_2) \tag{4.2}$$
$$R^3 \to -i(X_1P_2 + P_1X_2) \tag{4.3}$$
$$T \to i(X_1P_2 - P_1X_2) \tag{4.4}$$
$$P^1 \to -iH_1 \tag{4.5}$$
$$P^2 \to -B_1 \tag{4.6}$$
$$P^3 \to E_1 \tag{4.7}$$

(4.8)

and likewise for Q^j with one and two interchanged. We already know we have to analytically continue $\widehat{P}^1 \to -i\widehat{P}^1,\ \widehat{P}^2 \to i\widehat{P}^2,\ \widehat{P}^3 \to -i\widehat{P}^3$ and likewise for $\widehat{Q}^j$ to get the correct Lie algebra of $SU(2)$. Assuming this has been done and upon using the same symbols for the redefined meanings, one sees that one needs to perform the following analytic continuation

$$\widehat{R}^1 \to i\widehat{R}^1,\ \widehat{R}^2 \to -i\widehat{R}^2,\ \widehat{R}^3 \to i\widehat{R}^3$$

to ensure consistency of the first commutation relations and the reader may verify that all other remaining commutation relations are satisfied. This concludes, up to this point, the very mathematical setting behind our line of thought on the fundamental dichotomy which is conjectured to largely determine the way we interact with one and another. Sociology and psychology as it has been practised so far are not even close to properly adressing those issues from a foundational point of view. It seems that the way both pseudosciences are used by policy makers is to impose a restriction upon free will and behaviour and, even if you commited no punal offence, to put you away in some asylum to "protect" yourself as well as your surroundings. Psychiatrists have become the modern inquisition aimed at taming and re-educating undesirable elements in society, and it is just horrible that they proclaim their gratuitious "deseases" have some objective scientific value. Those, who interested in those power games, please, throw this book away since this is not of my interest. I want to gain a deeper understanding of why people react in a certain way with the goal of an inclusive, modern society and not an exclusive, medieval one which judges and prosecutes. Only Italians seem to have understood this profound message and gave up upon mental asylums due to the work of a psychiatrist in Trieste: long live Italy and maybe I will go on retirement there. Of course, issus are not the only thing our psyche turns around, there is also the issue of consciousness and any theory involving communication of issues must also consider whether this happens in a conscious (c) or unconscious (u) way regarding reception (R) and sending (S) of "signals". This constitutes our second dichotomy which is also of fundamental importance in psychology, but unlike our first dichotomy, where one is free to choose ones point of view and the quantum mechanical description is the accurate one, consciousness is something which simply seems happen to us; it seems rather perverted that one would "pose the question" (unconsciously of course) rearding ones awareness about other questions of the mind. Although there is no strict logical contradiction here as far as I can understand, we will treat consciousness on a classical level of geometry. In particular, we will make a completeness assumption that every send signal is also received so that in a sending-reception process there are two parameters involved indicating the degree of awareness. We shall describe the world in the black basis; that is, we work with real variables attached to issues and study how these values as well as the issues propagate. Ultimately, the real state is a complex wavefunction in as well the issues (which cause for a discrete labelling j), the values (which are just real numbers α_j) and the spacetime location. A signal transmits information which we write down by the letters α, β (which are taken as vectorial quantities); now, it is not so that the received information equals the transmitted one, the reciever may attach a different value to a certain sentence or he might slightly store the message in a different wording in his brain (possibly also with a different appreciation) and thereby mixing the vector components. We must still, for each message, attach the degree of awareness to it; indeed, we have learned that between dichotomies there must exist a continuous spectrum of possibilities. You may not be fully aware of something or largely unaware, but have a gut feeling that you saw something vague but don't quite remember what. Therefore, each entry in the vector α is replaced by a couple (α_j, t_j) where $t_j \in (-\infty, +\infty)$ and $t = -\infty$ equals u and $t = +\infty$ equals c. Keeping this in mind, we use the symbol $\overline{\alpha}$ in this new sense (so it is a $2N$ dimensional vector instead of a N dimensional vector). Now, we must still add the flat $2N$ dimensional complex space of psychological profiles on top of this construction and then we are done. So now that we know how our total space should look like (locally), let us ask some questions regarding the metric. As said before, the bundle of types should be flat (in the bundle coordinates), but there must be some non-trivial dependency of the "profile operators" A on the psychological variables $\overline{\alpha}$ but not really upon the spacetime coordinate x (apart from the influence through the spin connection) since physical signals almost never influence our questions; it is just the level of awareness regarding those signals which get influenced, for example when you are in pain. So, to speak into the language of the next Part of this book, we must find free propagators for the material particles, psychological variables and profile operators separately and then construct intertwiners between them. This should be intertwiners (a) relating the matter propagators (in either incoming physical signals) to the propagators of the psychological variables and (b) relating the psychological variables and types amongst each other. In order to realize this one must have a "learning field" $T^{A_j}_{\alpha_j t_k}(x, \overline{\alpha})$ where $k, j : 1 \ldots N$ and $A = 1, 2$ where the latter transform under local Lorentz transformations and the α_j, t_k coordinates do not transform at all given that they are are canonically associated to the operators X_j, P_j and those are not dynamical. Note that this somewhat Newtonian stance is justified since those variables, unlike (instantaneous) physical attributes[11] of a particle do not depend upon the state of motion relative to the observer but they pertain to the inner kitchen of the observer himself! In a way, it is logical that your state of physical motion in as well the gravitational field as the other force fields interacts with your dichotomy regarding certain issues[12]. Indeed, any motion in the background scenery requires work to be done by the body so that *internal* positions are influenced by that very activity; for example, when asked "what speed you are considering in moving towards the fridge (while sitting in your armchair) to get some beer", you might at first say that you don't know, that you are currently sitting in your chair and that this question is irrelevant to you (mixed profile, it bothers you in a certain sense). Next, when you stand

[11]In classical theories thse attributes transform under local Lorentz transformations but as discussed at length before, this is not the case in quantum theory.

[12]Certainly not issues regarding your perception of the outside world; that one is always black unless you get a stroke or something like that.

up to get the beer and being asked the same question again, you might say, I consider going at 5 km an hour because that is the optimal speed to get my beer (black profile), whereas finally, upon approaching the fridge you answer to the same question again that you consider slowing down (white profile) otherwise you might walk past the fridge. Actually, what you call speed is expressed in the number of steps you make per unit time times the step length, but the only way to step is to use your muscles! Indeed, any single rigid object on the surface of the earth which cannot transform its internal energy into labour will either remain stationary or experience a varying force field (in space) due to the surface of the earth (gravity, as Einstein beautifully expressed it, is not a force and you don't feel it). This is the very crucial distinction between Einstein's view (which is the correct one) and Newton's view upon gravitation. Newton would say there are two forces working on the body, a gravitational one and one due to the surface of the earth conspiring as to keep you on the surface of the earth and there is *almost* no net force felt when you are moving. Einstein, on the other hand, would say that that such a body really is accelerating all the time because it does not move on a geodesic in the gravitational field. Our *interpretation* of speed really regards an acceleration in the Einsteinian sense and that is the reason why we do not only feel speed[13] to some extend but also our own weight! However, the variation of the force field due to the surface of the earth is *interpreted* in a different way: for example, a cylindrical body will experience that this contact force varies as a delta peak $F\delta(\theta-\alpha t)\vec{e}_\theta$, where $\vec{e}_\theta$ is naturally associated to its polar coordinate system, whereas an outside observer would say that the contact force is a delta peak $F\delta(x+R\alpha t)\vec{e}_z$, where the z axis is fixed with respect to him. It are the interpretations which are crucial for the evolution of the profile operators: indeed regarding the issue "what is the angular speed cylinder John is rolling with on the floor?", John himself might take the mixed point of view and say that he does not know wether he is rolling or not, but he feels stinges upon his mantle which continuously move over the entire mantle whereas previously, they were in one place only. Maybe, some evil deamon is playing a game with him? He doesn't know, it is complicated. An observer from the outside will of course give a definite answer. When asking then to John to which extend he would like the stinges to be in one place only, he might give a definite very high appreciation. The psychological appreciation (in the black perspective in terms of the α variables) of walking towards the fridge would roughly be the same in all three circumstances (prior to applying your profile operator). Indeed, at any instant would you have answered that 5 km an hour is the preferred speed *if* you would want to go to the fridge in the first place. Likewise, to give another example, do you look differently at abstract intellectual thought when you are walking on the street; you will most likely say that you do not engage in thought right now and that such activity is to be done when you are sitting quietly at your desk at home. To illustrate these previous thoughts with a specific calculation we attach a Newtonian frame

[13]The way we feel speed depends of course upon the means we use to move; for example, when going on foot, I have a different experience than driving a bicycle.

of reference to the center of the earth in polar coordinates (r, θ, ϕ); then upon keeping (r, ϕ) fixed, say $r = R$, $\phi = 0$ where R is the radius of the earth, then we would say that the motion $\theta(t) = \alpha t$ with α a ridiculously small number is one of constant velicity. However, in Cartesian coordinates, the resulting vector $\vec{r}(t)$ satisfies

$$\frac{d^2}{dt^2}\vec{r}(t) = -\alpha^2 \vec{r}(t)$$

resulting in a magnitude of acceleration squared of $\alpha^4 R^2$ which is a ridiculously small number. In Einsteins view, the spacetime geometry is to a good approximation given by

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 - dr^2 - r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right)$$

and upon renormalizing our velocity field we get the quantity

$$U(t) = \frac{c}{\sqrt{c^2 - R^2\alpha^2 - \frac{GM}{R}}} \left(c\partial_t + \alpha\partial_\theta\right).$$

Calculating the accelation gives, with $d\tau = \frac{cdt}{\sqrt{c^2 - R^2\alpha^2 - \frac{GM}{R}}}$,

$$\frac{D}{d\tau}U(t) = -\frac{(R\alpha^2 + \frac{GM}{R^2})c^2}{c^2 - R^2\alpha^2 - \frac{GM}{R}}\partial_r.$$

The magnitude squared of this vector is

$$\left(\frac{(R\alpha^2 + \frac{GM}{R^2})c^2}{c^2 - R^2\alpha^2 - \frac{GM}{R}}\right)^2$$

which is in order of α given by

$$\sim R^2\alpha^4 + (\frac{GM}{R^2})^2 + 2R\alpha^2\frac{GM}{R^2}$$

ignoring the denominator divided by c, since the latter is close to unity. Hence, the Newtonian velocity squared divided by R appears in this formula which of order $10R = 67000000$ larger (in standard units) than the Newtonian acceleration for $R\alpha \sim 1$. So, to wrap up, there is no feeling and no change of perspective associated to Einsteinian velocity of a free body, but what we call speed here on earth really regards an acceleration and we do feel that. These considerations are crucial when you couple our sensors to the electromagnetic field which is really responsible for the brain activity in our body; those change substantially in Einstein's view given that stationary matter (in Newton's theory) is accelerating all the time causing for electromagnetic radiation and therefore nontrivial neural activity. In our work above, regarding the evolution of the profile field, we considered a first order differential equation which means that it couples directly to the electrical currents in the brain. Regarding our geometry of our psychological space, we now conjecture that it is **locally**, meaning associated to a **classical** part of the brain, where classicality is a product of the spirit which is impossible to describe in any mathematical way, is given by

$$g_{\mu\nu}(x)dx^{\mu} \otimes dx^{\nu} + \Omega^2(\overline{\alpha}, x)\left(\delta_{\alpha_j \alpha_k} d\alpha_j \otimes d\alpha_k + \delta_{jk} dt_j \otimes dt_k\right).$$

On top of this, you have to consider the $2N$ dimensional complex profile bundle. Indeed, the point behind coupling the profile operators to our state of motion is that we can now write down interaction terms coupling possibly distinct matter currents quadratically to our profile operators[14] by means of the following intertwiners

$$\mathbf{Z}^{\mathbf{t},\mathbf{u}\,ab\,(B,l)(B',m)}_{\mathbf{r},\mathbf{s}\;(A,j)(A',k);} := \delta^t_j \delta^u_k \delta^l_r \delta^m_s \sigma^{aB'B} \sigma^b_{A'A}$$

where $\sigma^{aB'B} = \sigma^a_{C'C} \overline{\omega}^{B'C'} \omega^{BC}$. So, this allows for brain currents and their derivatives not only to guide our profile, but also to interact with it in a non-linear fashion. For example, one may consider the Lorentz tensor $g(\tilde{e}^a(x), \nabla^C_{J(x)} \tilde{e}^b(x))$ coupling to $A^{jA}_{lB}(x)\overline{A}^{kA'}_{mB'}(x)$ in the equation of motion $\nabla^{C,N}_{J(x)} A = \ldots$. There is one exceptional question or issue, which one may call the primary question and it is, "what do I think". Indeed, in a conversation, it is not enough to simply adjust your points of view on several issues, but you must also know what issue to speak about regarding what has been said before. You also have to think, to select the relevant sentence of all possible things you might be able to say at that very moment in time! Note that I claim something very different from AI which can learn how to think by digesting loads of text and seeing correlations between them; humans, on the other hand seem to bypass this issue being well capable of reasoning about things with very little or no input indeed.

The reader must reflect further here, given that language contains a certain duality; indeed, given an issue (X, P) one can wonder "what is my profile on that issue", which naively results in a pair (X^*, P^*) where X^* is the black perspective on that matter, meaning I give you a definite profile, and P^* is the white perspective meaning I want to change any profile on that matter. This is a legitimate question which also has the black-white perspective build in. So, there is a distinction between the questions "what state is your country in?" and "how do you perceive the state of your country?"; indeed, if you would anwer the second question from a from a non-black perspective, then there is no

[14]Here, we stress again that not every part of the brain has the same effective complexity, or number of degrees of freedom albeit the fundamental prescription at level zero is entirely democratic and does not divide the brain into cells, neurons and so on. Indeed, it is the mind which does that in a totally mysterious way and therefore the total bundle is dependent upon the localization in brain. This is most conveniently modelled by neural networks associated to the mental level where the nodes are associated to the "brain entities" and the lines connecting the nodes are associated to electrical circuits. Each line can be thought of as carrying an issue and a profile operator and the nodes really live in the direct sum of those lines regarding the profile operators and in the direct product regarding the Hilbert spaces on which these issues are represented. This is similar to a spin network in Loop Quantum Gravity.

way you can answer the first one in the way we have anticipated before. Indeed, you can also say, I look at it from a mixed point of view or I have no point of view on the matter, it is complex. This game of embedded questions is in principle infinite and the dynamics should be as such as to stimulate blackness at a certain level. So, we must add N new variables q_j to our description above of the geometry taking trinary values $0, 1, 2$ where 0 means "no point of view, its complex", 1 a mixed point of view and 2 a definite profile. A comment of a more technical nature is in place here: the profile space of (X, P) is a six dimensional real manifold so X^* cannot just be a linear operator which severely complicates our algebraic point of view. In this book, we shall concern ourselves with primary issues meaning (a) they do not refer to one and another and (b) anyone has a distinct point of view upon them; as explained here, this is a huge simplification of reality - even worse, most computers are simply black and give totally nonsensical answers to questions they have not been trained for. Another kind of duality, which *is* compatible with our framework, regards the question (X^*, P^*) whether "how you see some issue (X, P) changing?". In that case, the white perspective X^* is given by iP and $P^* = iX$; hence, one notices that $X^{**} = -X$, $P^{**} = -P$ meaning that "your vision upon the change of your vision of the change of a certain issue" is the same as the opposite vision on that issue. This is an interesting consequence of our language which goes beyond mere word play. As a final example of issues which do refer to one and another regarding the spacetime symmetries, one may consider the components of the energy momentum vector of a particle, here the black realities attached to each of them boost in one and another upon changing your own motion. So, albeit the operators do not change, a point of view we also took in our work upon the operational formulation of quantum field theory, and albeit the decision is white, the wavefunction itself must transform under a different representation as the one considered here. Indeed, we have only considered questions so far which do not "rotate" into one and another upon changing your reference frame, but the extension is fairly easy to make: denote by $(M^{ab})^c{}_d = \eta^{ac}\delta^b_d - \eta^{bc}\delta^a_d$ then transforming the black momentum operators X^b of your brain currents as $\Lambda^a{}_b X^b$ and the associated antihermitean "position operators" P^c as $((\Lambda^{-1}))_b{}^c P^b$ clearly preserves the algebra (note that we raise and lower indices here with respect to the Euclidean metric), given that

$$(\Lambda^{-1})_b{}^c \Lambda^a{}_d \left[P^b, X^d\right] = (\Lambda^{-1})^{bc}\Lambda^{ab}1 = \delta^{ac}1$$

Then, a Lorentz transformation is given by

$$\widehat{\Lambda} := e^{-2\alpha_{j0}M^{j0}\otimes\sigma^3 + \alpha_{jk}M^{jk}\otimes 1_2}$$

and the reader verifies that

$$\widehat{\Lambda}^T \omega_4 \widehat{\Lambda} = \omega_4$$

as well as

$$\widehat{\Lambda}^\dagger 1_4 \otimes i\sigma^1 \widehat{\Lambda} = 1_4 \otimes i\sigma^1$$

as it should. This last property implies that all associated charges are Hermitian as they should. One notices furthermore that the Lorentz transformations leave the A indices invariant and therefore it is possible to just consider the components A^{i0}_{j0} of the profile matrices which transform as a matrix in the defining representation of the Lorentz group; hence, it is possible to couple those with the classical currents $g(\tilde{e}_a(x), \nabla^C_{J(x)}\tilde{e}^b(x))$.

As a final comment, we must mention that the mental world is inherently, at its core, definite and classical; for example the consciousness parameters[15] t_j and the profile matrices A are classical whereas the choice projectors $\pi_{j,A}$ constitute a finite dimensional quantum system of $2N$ states. Regarding definitenes, we clearly argued, by means of our first duality where you question the black reality of a profile attached to an issue, that there has to be a certain level at which we are all black otherwise nothing of value can be communicated in this world. This makes the dynamics of the profile and choice variables trivial at that level. At lower levels of embedded issues, nature aspires definiteness so that the entire dynamics for one issue, which has 9 free real variables, has a 3 dimensional submanifold as attractor. That the world is inherently classical has been stressed many times by the founding fathers of quantum theory and it seems to be somewhat of a forgotten lesson for those who apire an exclusive quantum view on the world. In that regard are the brain currents guiding the equations of motion of the profile matrices as well as the choice projectors classical between two conscious observations. These classical currents must of course be related to their quantum counterparts as determined by the last observation of the latter; (undergoing a classical dynamics which is the "classical limit" of the quantum dynamics) but I shall not engage in a precise description here, that is way beyond our current level of understanding. So keep this in mind when I discuss these issues further in part 3.

4.4 Another view on the Schwitchoriem.

In what follows, we take $N = 1$; in that case, our black operator X diagonalizes with pure states given by $\delta(x - b)$ whereas white states are of the form e^{ikb}. As such, no information loss occurs and the black-white views are just different configurations of the same substance. By definition

$$g(x) = \int dyg(y)\delta(y - x)$$

and we have that

$$(\mathcal{F}g)(k) = \frac{1}{\sqrt{2\pi}} \int dyg(y)e^{iky}.$$

We now look for states for which

$$(\mathcal{F}g)(x) = g(x).$$

[15]They are also crucial for determining when a measurement of some quantal issue should occur.

Taking into account that

$$(\mathcal{F}(g))(k) = \frac{1}{\sqrt{2\pi}} \int dx e^{ikx} g(x)$$

the aforementioned class is given by the Gaussian functions

$$e^{-a(x-b)^2}$$

since

$$\frac{1}{\sqrt{2\pi}} \int dx e^{ikx} e^{-a(x-b)^2} = \frac{1}{\sqrt{2\pi}} \int dx e^{ikb} e^{-a((x-b)-\frac{ik}{2a})^2} e^{-\frac{k^2}{4a}} = \sqrt{\frac{1}{2a}} e^{-ab^2} e^{-\frac{1}{4a}(k-2iab)^2}$$

and for this function to satisfy our criterion it is necessary and sufficient that $a = \frac{1}{2}$ and $b = 0$. Hence, the "diagonal" in the white-black plane coincides precisely with the ground state of the Schitchoriem. Indeed,

$$\frac{1}{\sqrt{2}} \left(x + \frac{d}{dx} \right) e^{-\frac{x^2}{2}} = 0$$

which is the characterization we were looking for. Another useful characterizarion of this fact is that

$$iP = \mathcal{F}^\dagger X \mathcal{F}$$

so that our ground state obeys

$$\mathcal{F}(xg(x)) = ixg(x)$$

which is easily seen to be true.

4.5 On the geometry of spacetime; Ehresmann geometry.

In this section, we shall take a somewhat broader view on spacetime as we did before. The standard view is to consider spacetime and tangent space, which is related to our local perceptions, is considered as a byproduct of that. The question now is, something which people believing in paranormal phenomena usually believe in, is wheteher this perception can also dynamically influence matter. Up till now, we have been very conservative in the sense that issues were guided by matter currents as well as our perspective therupon, but the only way the spirit intervened in the state of matter was by means of measurement. We shall in this final section broaden somewhat our geometrical picture of spacetime allowing for information on tangent space to propagate to spacetime itself. This means that we shall take the tangent bundle $T\mathcal{M}$ as a manifold for the theater of the physical world[16]. The standard view on $T\mathcal{M}$ is that ordinary coordinate

[16] I acknowledge useful communication with Arkdiusz Jaczyk on this topic.

systems (x^μ) on $\mathcal{M}$ get lifted to $T\mathcal{M}$ by means of the canonical basis ∂_μ; hence every vectorfield $V(x)$ in $T\mathcal{M}$ gives rise to natural coordinates (x^μ, v^ν). This point of view is entirely kinematical and only lifts coordinate transformations on $\mathcal{M}$ to $T\mathcal{M}$. Such coordinate systems do not have any physical meaning and it is not wise in general to couple the transformation laws of the base space to that of tangent space. As stressed in the introduction of this chapter, we want the coordinates on tangent space to have physical meaning. Hence, their very definition must be coupled to dynamical objects on $T\mathcal{M}$ as vector bundle. The obvious candidate is the vierbein $e_a(x)$ and every vectorfield $V(x) = v^a(x)e_a$ gives rise to coordinates (x^μ, v^a). Jadczyk pointed out to me that such construction had been made for more general Lie groups in Munteanu. The local coordinate transformations on $\mathcal{M}$ do not propagate to $T\mathcal{M}$, since one simply has (x'^μ, v^a). Under a local Poincaré transformation $(\Lambda(x), w(x))$, however, the coordinates on the tangent bundle transform as

$$(x, v^a) \Rightarrow (x, \Lambda(x)^a_{\ b} v^b + w^a(x))$$

since $e_a(x) \Rightarrow \Lambda(x)_a^{\ b}(x)e_b(x)$. Therefore, the partial derivatives mix as follows

$$\begin{aligned}
\partial'_\mu &= \partial_\mu + v^c \Lambda^b_{\ c}(x)\partial_\mu \Lambda_b^{\ a}(x)\partial_a - \Lambda_b^{\ a}(x)\partial_\mu w^b(x)\partial_a \\
\partial'_a &= \Lambda_a^{\ b}(x)\partial_b
\end{aligned}$$

and the differential forms transform as

$$\begin{aligned}
dx'^\mu &= dx^\mu \\
dv'^a &= \partial_\mu \Lambda^a_{\ b}(x) v^b dx^\mu + \partial_\mu w^a(x) dx^\mu + \Lambda^a_{\ b}(x) dv^b.
\end{aligned}$$

This indicates that we better use a distinct notation for the tensors which transform with respect to e_a and the tensors defined by ∂_μ, ∂_a even if the basis elements e_a and ∂_a transform identically. A lesson is that we cannot simply consider ∂_μ separately in the context of $T\mathcal{M}$ and where necessary, we shall use primed indices a' to denote the distinction while unprimed indices always transform with respect to the local Lorentz group. The invariant tensors are given by δ^a_b, δ^A_B while δ^μ_ν and $\delta^{a'}_{b'}$ viewed as tensors on $T\mathcal{M}$ (the other coordinates are vanishing) are not invariant at all. Here, A is a shorthand notation for $A = (\mu, a')$, in either it is the natural index on $T\mathcal{M}$ where $x^A = (x^\mu, v^a)$. Since this is a new geometry and the notation might be a bit unusual to the reader, let me make these statements more explicit. I presume that the claim for δ^a_b is quite obvious since

$$\delta'^a_b = \Lambda^a_{\ c}(x)\Lambda_b^{\ d}(x)\delta^c_d = \delta^a_b$$

under the action of local Poincaré transformations. Since the case of δ^A_B is standard in all textbooks on geometry, let me move to δ^μ_ν. The latter is a tensor defined with respect to a preferred coordinate system (x^μ, v^a) and we have to investigate its transformation behavior under local Poincaré transformations; an easy computation reveals that

$$\begin{aligned}
\delta'^\mu_\nu &= \delta^\mu_\nu - \left(v^c \partial_\mu \Lambda^b_{\ c}(x) + \partial_\mu w^b(x)\right)\Lambda_b^{\ a}(x)\delta^\mu_{a'} \\
&= \delta^\mu_\nu
\end{aligned}$$

and

$$\delta'^{a'}_{\mu} \quad = \quad \left(\partial_\mu \Lambda^a{}_b(x) v^b + \partial_\mu w^a(x)\right).$$

All other type of coefficients are computed to vanish and the reader is invited to repeat this exercise for $\delta^{a'}_{b'}$. In the future, we shall make use of δ^{μ}_{ν} as if it were an invariant tensor, which is justified by the fact that the (coordinate dependent) "projection" on the μ indices is. The reader who thinks that this is a fluffy concept might enjoy the following definition. We call an object $T^{\alpha_1 \ldots \alpha_r}_{\beta_1 \ldots \beta_s}$ where $\alpha_j, \beta_k \in \{A, \mu, a'\}$ a partial tensor if the object transforms consistently within the limitation of its indices. That is, the μ indices only feel coordinate transformations on $\mathcal{M}$, the a' indices transform only under local Lorentz transformations and finally, the A index undergoes the whole transformation group. In this language, δ^{μ}_{ν} is a partial tensor since $\delta^{\mu}_{a'}$ cannot become nonzero under the full transformation group. However, it is not a tensor either since the above computation reveals that $\delta^{a'}_{\mu}$ becomes nonzero in different coordinate systems. Later on, we shall still define physical tensors and write down the relationship between the latter and full or partial tensors on $TT\mathcal{M}$. Now, we do something which is rather similar to what happens in Finsler geometry, we aim to define horizontal sub bundles $H_{(x,v^a)}T\mathcal{M}$ of $TT\mathcal{M}$ over $T\mathcal{M}$. Therefore, we need to introduce a new object $\mathcal{A}^B_\mu$ which compensates for the action of the local Poincaré group on ∂_μ. The latter transforms as

$$\mathcal{A}'^{B}_{\mu}(x, v'^b) = \frac{\partial x'^B}{\partial x^C}\mathcal{A}^C_\mu(x, v^b) - \left(v^c \partial_\mu \Lambda^b{}_c(x) + \partial_\mu w^b(x)\right) \Lambda_b{}^a(x) \delta^C_a \frac{\partial x'^B}{\partial x^C}$$

under local Poincaré transformations. Under spacetime transformations, it transforms covariantly in the μ and B index. The reader may wish to verify that all this is consistent since $A^B_{a'}(x, v^c)$ is defined to be zero and therefore $\mathcal{A}^B_\mu$ transforms as a partial tensor in the μ index. This new type of "gauge" theory (which mixes up spacetime and the tangent space) is studied right after all axioms are given. We change therefore our entire point of view since the relation

$$\partial_\mu = e^a_\mu(x) e_a$$

does not behave well under local Lorentz transformations and we want to extend the vierbein to $e^a_\mu(x, v^b)$ so that it lives in $T\mathcal{M}$ as a manifold[17]. Therefore, we define a set of "gauge" operators

$$\mathcal{D}_\mu(x, v^a) = \partial_\mu - \mathcal{A}^B_\mu(x, v^a)\partial_B$$

which at each point (x, v^a) span a linear space $H_{(x,v^a)}T\mathcal{M}$ isomorphic to $T\mathcal{M}_x$ by sending

$$W^\mu(x, v^a)\mathcal{D}_\mu(x, v^a) \Rightarrow W^\mu(x, v^a)\partial_\mu.$$

[17]The role of the translations might be a bit confusing here since here since all coordinate systems defined so far started from a preferred origin. However, there is no contradiction since the coordinate definition of the origin just shifts too.

The latter map is the formal definition of the bundle projection τ so that we get a formal triple $(HT\mathcal{M}, \tau, T\mathcal{M})$. The $W^{\mu}(x, v^{a})$ transform as scalars under local Lorentz transformations and as ordinary vectors under spacetime $\mathcal{M}$ coordinate transformations. Likewise, one has a vertical sub bundle $VT\mathcal{M}$ spanned by the $\partial_{a'}$ which gets projected to the zero vector in $T\mathcal{M}$. Later on, we will formulate the necessary condition so that this construction is promoted to a nonlinear connection in the standard Finsler sense. The original tetrad $e_{a}(x)$ which does not depend upon v^{a}, but which defines v^{a}, is then to be associated to $e^{a}_{\mu}(x, 0)$ by

$$e^{a}_{\mu}(x, 0)e_{a}(x) = \partial_{\mu}$$

but it can be redefined as an element of $H_{(x,v^{a})}T\mathcal{M}$ by the formula

$$e_{a}(x, v^{b}) = e^{\mu}_{a}(x, v^{b})\mathcal{D}_{\mu}(x, v^{b}).$$

A constraint invariant under local Lorentz transformations is that

$$\mathcal{A}^{B}_{\mu}(x, 0) = 0.$$

This implies that the origin of $T\mathcal{M}_{x}$ is an invariant point while the rest of tangent space dynamically positions itself in $H_{(x,v^{a})}T\mathcal{M}$. The whole construction depends upon the preferred origin of the tangent space at x but this is entirely physical since the observers still reside there. This means that in general, the translation degrees of freedom are irrelevant and we ignore them from now. The reason why I included them in the geometry anyway is motivated by the following : (a) there is nothing wrong with having a pointed P affine space with translation symmetry, it just means you have two preferred points, P and the origin of your coordinate system and there exists exactly one coordinate system in which both agree (b) the translation symmetry is a symmetry of the free theory living in $VT\mathcal{M}$ and one might impose that the gravitational theory also obeys it (c) the translation symmetry has to be broken at some point of course since the vielbein is a dynamical entity living on $T\mathcal{M}$ and the projection from $T\mathcal{M}$ to $\mathcal{M}$ is only in the initial conditions. But again, let me stress that we could have broken translation invariance already at the level of the gravitational theory and nothing in what follows would be influenced by this; only the transformation laws for $\mathcal{A}^{B}_{\mu}$ would change.

The transformation law for the "gauge" potential under local Lorentz transformations can be further simplified to

$$\mathcal{A}'^{B}_{\mu}(x, v'^{a}) = \frac{\partial x'^{B}}{\partial x^{C}}\mathcal{A}^{C}_{\mu}(x, v^{b}) - \partial_{\mu}\Lambda^{B}_{\ c}(x)v^{c}.$$

Before we proceed, let us further tell something about ordinary gauge theory; the gauge law forces us to introduce a new addition law $\oplus$ satisfying

$$\alpha_{\mu}(G) \oplus \alpha_{\mu}(H) = \alpha_{\mu}(G) + G\alpha_{\mu}(H)G^{-1}$$

where

$$\alpha_\mu(G) = \partial_\mu G G^{-1}.$$

This is necessary to make the action $\delta_\mu : G \Rightarrow \delta_\mu(G)$ where

$$\delta_\mu(G) : A_\mu \Rightarrow G A_\mu G^{-1} + \alpha_\mu(G)$$

into a regular group action. The reader is invited to find out that $\oplus$ is non-commutative, has a unit element $\alpha_\mu(1)$, and $\alpha_\mu(G)$ has as inverse $\alpha_\mu(G^{-1})$. Moreover, there is a canonical way to define multiple $\oplus$ sums by

$$\alpha_\mu(G) \oplus \alpha_\mu(H) \oplus \alpha_\mu(K) = \alpha_\mu(G) + G\alpha_\mu(H)G^{-1} + GH\alpha_\mu(K)H^{-1}G^{-1}$$

and it is easy to check that this operation is associative. Hence, we have a group structure and α_μ is a group homomorphism. The reader should notice that δ_a^A is a well defined invariant tensor and that $\Lambda^B{}_a(x)$ transforms as a scalar under M coordinate transformations (and therefore everything is well defined). Replacing $\Lambda(x)$ by $\Lambda(x)\Gamma(x)$ transforms the gauge term as

$$-\partial_\mu \Lambda^B_b(x)\Gamma^b{}_c(x)v^c - \Lambda^B_C(x)\partial_\mu \Gamma^C_d(x)v^d$$

where $\Lambda^B_C(x)$ has not an invariant meaning but the product with $\partial_\mu \Gamma^C_d(x)$ has. The extra twist here is of course the dependence of the gauge term upon the v^a, but the transformation law as written there is completely logical and gives rise to the sum

$$\left(\alpha_\mu(\Lambda(x), \Gamma^a_b(x)v^b) \oplus \alpha_\mu(\Gamma(x), v^b)\right)^B = \alpha^B_\mu(\Lambda(x), \Gamma^a_b(x)v^b) + \Lambda^B_C(x)\alpha^C_\mu(\Gamma(x), v^b)$$

where all symbols have their obvious meaning. As before $\oplus$ has the correct properties with respect to 1 and Λ^{-1}, also it is non-commutative and the sum has a clear associative extension. Therefore, $\alpha_\mu(\Lambda(x)\Gamma(x), v^a) = \alpha_\mu(\Lambda(x), \Gamma^a_b(x)v^b) \oplus \alpha_\mu(\Gamma(x), v^a)$ which extends to a homomorphism from the semi-direct product group $SO(1,3) \times \mathbb{R}^4$ to the gauge group by representing the translational part trivially. A small calculation reveals that this group structure makes δ_μ into a left action as before. The question now is how we generate local Poincaré invariant "tensors" from the gauge potential $\mathcal{A}^B_\mu(x, v^a)$? The answer is the usual one, we calculate the commutators of the "covariant" derivatives

$$[\mathcal{D}_\mu(x, v^a), \mathcal{D}_\nu(x, v^a)] = -2\left(\partial_{[\mu} A^B_{\nu]}(x, v^c) - A^C_{[\mu}(x, v^c)\partial_{|C|} A^B_{\nu]}(x, v^c)\right)\partial_B$$

from which we learn that the field strength

$$F^B_{\mu\nu}(x, v^c) = \partial_{[\mu} A^B_{\nu]}(x, v^c) - A^C_{[\mu}(x, v^c)\partial_{|C|} A^B_{\nu]}(x, v^c)$$

transforms as

$$F'^B_{\mu\nu}(x, \Lambda^a{}_b(x)v^b + w^a(x)) = \frac{\partial x'^B}{\partial x^C} F^C_{\mu\nu}(x, v^b)$$

under local Poincaré transformations. Under general coordinate transformations however a gauge term develops as a small calculation reveals

$$F'_{\mu\nu}(x,v^b) = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}F_{\alpha\beta}(x,v^b) - \mathcal{D}'_{[\mu}(x(x'),v^a)\left(\frac{\partial x^\alpha}{\partial x'^{\nu]}}\right)\mathcal{D}_\alpha(x,v^a)$$

and the reader is invited to write this transformation law out in the somewhat messy basis ∂_B. The coordinate invariant vector-fields which respect $\mathcal{M}$ as a base manifold are given by

$$W^\mu(x,v^a)\mathcal{D}_\mu(x,v^a)$$

as well as

$$V^a(x,v^b)\partial_a.$$

They span the entire $TT\mathcal{M}_{(x,v^a)}$ if and only if the matrix given by $\left(\delta^\nu_\mu - \mathcal{A}^\nu_\mu(x,v^a)\right)$ is regular. Hence, we may bring

$$[W(x,v^a),V(x,v^a)]$$

back in $H_{(x,v^a)}T\mathcal{M}\oplus V_{(x,v^a)}T\mathcal{M}$ for $W(x,v^a),V(x,v^a)\in H_{(x,v^a)}T\mathcal{M}\oplus V_{(x,v^a)}T\mathcal{M}$ but this transformation will have a complicated rational dependence upon the gauge field. Again, one might impose the invariant statement that the origin is an exception to this by requiring $F^B_{\mu\nu}(x,0)=0$. It is important that I make one point clear and comment upon the notation I shall use; we have at this moment two μ's and a's, one set mixes and transforms according to the ordinary basis ∂_A and the other doesn't mix and transforms according to our new physical basis. We shall not distinguish between them notationally and from now on we shall mostly rely upon the second concept. However, to make sure the reader understands everything is consistent, let us start from a vector in the unphysical basis

$$W(x,v^a) = W^\mu(x,v^a)\partial_\mu + W^a(x,v^a)\partial_a$$

and denote as a shorthand $B(x,v^a) = (1-\mathcal{A}(x,v^a))^{-1}$ where the reader may want to check that B is local Lorentz invariant and transforms as a matrix under coordinate transformations of $\mathcal{M}$. Then, we obtain the following decomposition

$$W(x,v^a) = W^\mu(x,v^a)B^\nu_\mu(x,v^a)\left(\partial_\nu - \mathcal{A}^B_\nu(x,v^a)\partial_B\right) +$$

$$\left(W^b(x,v^a) + W^\mu(x,v^a)B^\nu_\mu(x,v^a)\mathcal{A}^b_\nu(x,v^a)\right)\partial_b$$

and we must verify that both coefficients now transform in the new way. For the first coefficient, this is trivial, so we have to check it only for the second one. Indeed, the latter transforms as

$$W^\mu(x,v^a)\partial_\mu\Lambda^b{}_c(x)v^c + \Lambda^b{}_c(x)W^c(x,v^a) + \partial_\nu\Lambda^b{}_c(x)v^cW^\mu(x,v^a)B^\nu_\mu(x,v^a)\mathcal{A}^\mu_\nu(x,v^a)$$
$$+W^\mu(x,v^a)B^\nu_\mu(x,v^a)\Lambda^b{}_c(x)\mathcal{A}^c_\nu(x,v^a) - W^\mu(x,v^a)B^\nu_\mu(x,v^a)\partial_\nu\Lambda^b{}_c(x)v^c$$

which reduces to a Lorentz boost of the original expression as it should. The connection shall always be defined with respect to the physical basis and we perform a basis transformation such that the A in

$$F^A_{\mu\nu}(x, v^a)$$

are also with respect to the new basis. In fact, the old basis needs only to be used to solve the equations of motion but does not appear anymore in the construction of the field equations. To please the formal geometers, in the spirit of Finsler geometry, one may define a nonlinear connection by simply stating that everywhere

$$TT\mathcal{M}_{(x,v^a)} = H_{(x,v^a)}T\mathcal{M} \oplus V_{(x,v^a)}T\mathcal{M}$$

holds. At this point we define physical tensors $T^{\alpha_1\ldots\alpha_r}_{\beta_1\ldots\beta_s}$ where $\alpha_j, \beta_k \in \{\mu, a\}$ and the latter is required to transform consistently in all indices, meaning that the tensor evaluated in the complementary indices remains zero. In other words, it is an object acting upon the separate bundles $HT\mathcal{M}$, $VT\mathcal{M}$ and their duals. What we just accomplished is to write physical tensors in terms of partial and full tensors. Before we proceed, the reader might wonder how we construct a dual basis to the $\mathcal{D}_\mu$. Obviously, one imposes that

$$\begin{aligned}
\mathcal{D}x^\mu(x, v^a)\,(\mathcal{D}_\nu(x, v^a)) &= \delta^\mu_\nu \\
\mathcal{D}x^\mu(x, v^a)\,(\partial_a) &= 0 \\
\mathcal{D}v^a(x, v^a)\,(\partial_b) &= \delta^a_b \\
\mathcal{D}v^a(x, v^a)\,(\mathcal{D}_\nu(x, v^a)) &= 0
\end{aligned}$$

where it is clear that $\mathcal{D}v^a \neq dv^a$ but explicitly depends upon v^a. If we solve these equations in $T^*T\mathcal{M}$, then obviously we all assume the dx^A to work *ultra local* in contrast to the differential operators while the standard duality map allows one to define a Lie bracket which makes the dual base non commuting[18]. Clearly, the latter requires a quasi local action meaning the dx^A act differently on a function when it comes with a ∂_B or a dx^B. Also, the exterior derivative needs a correction due to the presence non-symmetric gauge terms as explained on the following page. For now, we do not care about these issues and simply define covariant tensors (and we have already used them) by imposing the appropriate transformation properties. Fine, so how should we define a connection? Since the latter is defined in a universal way depending on four basic axioms, which should all be satisfied, we have no choice but to define the connection on $TT\mathcal{M}$. This is entirely logical since the "ghost" gravitational waves should propagate as well in space-time as in tangent space. As a first calculation, we determine how $\mathcal{D}_\mu(x, v^a)W^\nu(x, v^a)$ transforms under coordinate transformations. The formula are

$$\frac{\partial x^\alpha}{\partial x'^\mu}\mathcal{D}_\alpha(x, v^a)\left(\frac{\partial x'^\nu}{\partial x^\beta}W^\beta(x, v^a)\right)$$

[18]More precise, if α is the duality map then $[\alpha(V), \alpha(W)] = \alpha([V, W])$.

which equals

$$\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial x'^{\nu}}{\partial x^{\beta}}\mathcal{D}_{\alpha}(x,v^a)W^{\beta}(x,v^a)+\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\mathcal{D}_{\alpha}(x,v^a)\left(\frac{\partial x'^{\nu}}{\partial x^{\beta}}\right)W^{\beta}(x,v^a)$$

where the "gauge" terms explicitly reads

$$\Delta\Gamma^{\nu}_{\mu\gamma}=\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\frac{\partial^2 x'^{\nu}}{\partial x^{\alpha}\partial x^{\beta}}\frac{\partial x^{\beta}}{\partial x'^{\gamma}}-\frac{\partial x^{\alpha}}{\partial x'^{\mu}}\mathcal{A}^{\kappa}_{\alpha}(x,v^a)\frac{\partial^2 x'^{\nu}}{\partial x^{\beta}\partial x^{\kappa}}\frac{\partial x^{\beta}}{\partial x'^{\gamma}}.$$

Unlike in standard relativity, the "gauge" term is not symmetric and therefore the connection must contain symmetric as well as antisymmetric terms which adds a nonzero torsion tensor. The reason here is the non commuting basis of partial differential operators $\mathcal{D}_{\mu}$ and the physical origin of this mathematical construction can be traced back to the quantum mechanical spin on $T\mathcal{M}$ which should -on average- be balanced by gravitational spin (implying an extension of Einstein-Cartan theory on $T\mathcal{M}$ instead of $\mathcal{M}$). The other "gauge" term can be read off from the following calculations

$$\mathcal{D}_{\mu}\left(\Lambda^a_{\ b}(x)V^b(x,v^c)\right)=\mathcal{D}_{\mu}\left(\Lambda^a_{\ b}(x)\right)\Lambda_c^{\ b}(x)V'^c(x,v'^d)+\Lambda^a_{\ b}(x)\mathcal{D}_{\mu}V^b(x,v^c)$$

resulting in a gauge term

$$\Delta\Gamma^a_{\mu c}(x,v^e)=\mathcal{D}_{\mu}\left(\Lambda^a_{\ b}(x)\right)\Lambda_c^{\ b}(x)$$

which satisfies

$$\Delta\Gamma^a_{\mu c}(x,v^e)=-\eta_{dc}\eta^{ab}\Delta\Gamma^d_{\mu b}(x,v^e).$$

The reader may verify that no other gauge terms arise, but for reasons which will come clear later on we do not put $\Gamma^{\nu}_{a\mu}$ to zero. Hence, we have the following equations:

$$\Gamma^c_{ab}(x,v^d)=\Gamma^{\mu}_{ab}(x,v^d)=\Gamma^b_{a\mu}(x,v^d)=\Gamma^b_{\mu\nu}(x,v^d)=\Gamma^{\nu}_{\mu b}(x,v^d)=0$$

and we have to determine the remaining 152 coefficients since $\eta_{a\{c}\Gamma^a_{|\mu|b\}}=0$ implying that $\nabla_{\mu}\eta_{ab}=0$. These degrees of freedom can be uniquely filled up by the following equations

$$\nabla_A e^{\mu}_a(x,v^c)=0$$

and

$$T^{\kappa}_{\mu\nu}(x,v^e)=-2\Gamma^{\kappa}_{[\mu\nu]}(x,v^e)+2F^{\kappa}_{\mu\nu}(x,v^e)=0$$

which implies that, in the limit for $\mathcal{A}^B_{\mu}$ to zero, $\Gamma^{\nu}_{\mu\lambda}$ reduces to the standard Levi-Civita connection. These restrictions can be uniquely solved to give

$$\Gamma^{\mu}_{a\nu}(x,v^c)=e^b_{\nu}(x,v^c)\partial_a e^{\mu}_b(x,v^c)$$

$$\Gamma^b_{\mu a}(x,v^c)=-e^b_{\nu}(x,v^c)\left(\mathcal{D}_{\mu}(x,v^c)e^{\nu}_a(x,v^c)-\Gamma^{\nu}_{\mu\kappa}(x,v^c)e^{\kappa}_a(x,v^c)\right)$$

and finally

$$\Gamma^{\mu}_{\nu\kappa}(x,v^c)=$$

$$-e^\mu_b(x,v^c)\mathcal{D}_{(\nu}(x,v^c)e^b_{\kappa)}(x,v^c) - e^\mu_a(x,v^c)e^b_{(\kappa}(x,v^c)e^{a\beta}(x,v^c)\mathcal{D}_{\nu)}(x,v^c)e_{b\beta}(x,v^c)$$

$$+\frac{1}{2}e^\mu_a(x,v^c)e^{a\beta}(x,v^c)\mathcal{D}_\beta(x,v^c)\left(e^b_\nu(x,v^c)e_{b\kappa}(x,v^c)\right) - F^\mu_{\kappa\nu}(x,v^c) + F_\kappa{}^\mu{}_\nu(x,v^c) -$$

$$F_{\nu\kappa}{}^\mu(x,v^c)$$

where the last tensor is written with respect to the physical basis. The reader may verify that the last formula is a direct consequence of the Koszul formula and moreover,

$$\Gamma_{a\mu b}(x,v^c)$$

is antisymmetric in a and b as it should. The reader notices that

$$T^a_{\mu\nu}(x,v^c) = 2F^a_{\mu\nu}(x,v^c)$$

and therefore nontrivial torsion is present. Define the Riemann tensor as usual by

$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X Z - \nabla_{[X,Y]}Z$$

and with respect to the coordinate basis this gives

$$R(\mathcal{D}_A(x,v^c),\mathcal{D}_B(x,v^c))\mathcal{D}_C(x,v^c) = \nabla_A\nabla_B\mathcal{D}_C - \nabla_B\nabla_A\mathcal{D}_C + 2\nabla_{F_{AB}}\mathcal{D}_C$$

where, obviously,

$$[\mathcal{D}_A(x,v^c),\mathcal{D}_B(x,v^c)] = -2F_{AB}(x,v^c).$$

Before we proceed, the reader may want to explicitly verify that everything works out as it should since after all, we are working in a unusual basis. For example, let us calculate the commutator between two vectorfields $V(x,v^c)$ and $W(x,v^c)$:

$$[V(x,v^c),W(x,v^c)] =$$

$$\left(V^\mu(x,v^c)\mathcal{D}_\mu(x,v^c)W^\nu(x,v^c) - W^\mu(x,v^c)\mathcal{D}_\mu(x,v^c)V^\nu(x,v^c)\right)\mathcal{D}_\nu(x,v^c) -$$

$$2V^\mu(x,v^c)W^\nu(x,v^c)F_{\mu\nu}(x,v^c)$$

To verify that this expression is well defined, we calculate the transformation behavior under coordinate transformations; the relevant terms are

$$2V^\mu(x,v^c)W^\kappa(x,v^c)\mathcal{D}_{[\mu}\frac{\partial x'^\nu}{\partial x^{\kappa]}}\frac{\partial x^\gamma}{\partial x'^\nu}\mathcal{D}_\gamma(x,v^c)$$

$$+2V^\mu(x,v^c)W^\kappa(x,v^c)\mathcal{D}_{[\mu}\frac{\partial x^\gamma}{\partial x'^\nu}\frac{\partial x'^\nu}{\partial x^{\kappa]}}\mathcal{D}_\gamma(x,v^c)$$

which sum up to zero. Unlike $F_{\mu\nu}(x,v^c)$, $F_{a\mu}(x,v^c)$ is a tensor under coordinate transformations, but under local Poincaré transformations a gauge term of the kind

$$\frac{1}{2}\mathcal{D}_\mu(x,v^c)\left(\Lambda_a{}^b(x)\right)\partial_b$$

develops. The coordinate expressions of the curvature tensor are given by

$$\begin{aligned}
R_{abc}{}^{d}(x,v^e) &= 0\\
R_{abc}{}^{\mu}(x,v^e) &= 0\\
R_{ab\mu}{}^{c}(x,v^e) &= 0\\
R_{ab\mu}{}^{\nu}(x,v^e) &= -\partial_a\Gamma^{\nu}_{b\mu}(x,v^e)+\partial_b\Gamma^{\nu}_{a\mu}(x,v^e)+\Gamma^{\nu}_{a\kappa}(x,v^e)\Gamma^{\kappa}_{b\mu}(x,v^e)-\\
&\quad \Gamma^{\nu}_{b\kappa}(x,v^e)\Gamma^{\kappa}_{a\mu}(x,v^e)=0
\end{aligned}$$

and the reader may verify that the last equation holds. This means that the tangent space is flat and curvature can at most live on spacetime or in the "intermediate" space. The remaining expressions are computed to be

$$\begin{aligned}
R_{a\mu b}{}^{\nu}(x,v^e) &= 0\\
R_{a\mu b}{}^{c}(x,v^e) &= -\partial_a\Gamma^{c}_{\mu b}(x,v^e)-2F^{\nu}_{a\mu}(x,v^e)\Gamma^{c}_{\nu b}(x,v^e)\\
R_{a\mu\nu}{}^{b}(x,v^e) &= 0\\
R_{a\mu\nu}{}^{\kappa}(x,v^e) &= -\partial_a\Gamma^{\kappa}_{\mu\nu}(x,v^e)+\mathcal{D}_\mu\Gamma^{\kappa}_{a\nu}(x,v^e)+\Gamma^{\alpha}_{\mu\nu}(x,v^e)\Gamma^{\kappa}_{a\alpha}(x,v^e)-\\
&\quad \Gamma^{\alpha}_{a\nu}(x,v^e)\Gamma^{\kappa}_{\mu\alpha}(x,v^e)-2F^{\alpha}_{a\mu}(x,v^e)\Gamma^{\kappa}_{\alpha\nu}(x,v^e)\\
&\quad -2F^{b}_{a\mu}(x,v^e)\Gamma^{\kappa}_{b\nu}(x,v^e)\\
R_{\mu\nu a}{}^{\kappa}(x,v^e) &= 0\\
R_{\mu\nu a}{}^{b}(x,v^e) &= -\mathcal{D}_\mu\Gamma^{b}_{\nu a}(x,v^e)+\mathcal{D}_\nu\Gamma^{b}_{\mu a}(x,v^e)+\Gamma^{c}_{\nu a}(x,v^e)\Gamma^{b}_{\mu c}(x,v^e)\\
&\quad -\Gamma^{c}_{\mu a}(x,v^e)\Gamma^{b}_{\nu c}(x,v^e)-2F^{\kappa}_{\mu\nu}(x,v^e)\Gamma^{b}_{\kappa a}(x,v^e)\\
R_{\mu\nu\kappa}{}^{a}(x,v^e) &= 0\\
R_{\mu\nu\kappa}{}^{\lambda}(x,v^e) &= -\mathcal{D}_\mu\Gamma^{\lambda}_{\nu\kappa}(x,v^e)+\mathcal{D}_\nu\Gamma^{\lambda}_{\mu\kappa}(x,v^e)+\Gamma^{\alpha}_{\nu\kappa}(x,v^e)\Gamma^{\lambda}_{\mu\alpha}(x,v^e)-\\
&\quad \Gamma^{\alpha}_{\mu\kappa}(x,v^e)\Gamma^{\lambda}_{\nu\alpha}(x,v^e)-2F^{a}_{\mu\nu}(x,v^e)\Gamma^{\lambda}_{a\kappa}(x,v^e)-\\
&\quad 2F^{\alpha}_{\mu\nu}(x,v^e)\Gamma^{\lambda}_{\alpha\kappa}(x,v^e).
\end{aligned}$$

Let me make some remarks regarding the remarkable structure of these equations. The expressions $R_{a\mu b}{}^{c}(x,v^e)$ and $R_{a\mu\nu}{}^{\kappa}(x,v^e)$ are all first order in the spacetime derivatives; the spacetime derivatives of the different fields $e^a_\mu(x,v^c)$ and $\mathcal{A}^B_\mu(x,v^c)$ decouple but there is some novelty in this type of equation in the sense that it may contain both derivatives of the kind $\partial_t e^a_\mu(x,v^c)$ as $\partial_t\partial_b e^a_\mu(x,v^c)$ and to uniquely solve those requires a new view on initial value problems. I believe the linearized equations to be ultra hyperbolic and shall write them out in full detail later on. The remaining two expressions $R_{\mu\nu a}{}^{b}(x,v^c)$ and $R_{\mu\nu\kappa}{}^{\lambda}(x,v^c)$ are classical second order expressions without the above mentioned curiosity. A two time and six space formalism seems here the right thing to do since we have a direct sum metric $g_{\mu\nu}\oplus\eta_{ab}$ on $TT\mathcal{M}$. This implies physically that non-local (or better non-causal) correlations in the metric tensor will build up instantaneously, the matter equations of motion of course obey the usual hyperbolic laws (with respect to $g_{\mu\nu}$). A few years ago, I thought about using a two time formalism (where one time is rolled up on a cylinder) to explain away the Bell inequalities; this formalism can be made entirely consistent by declaring that the "Kaluza-Klein" modes cannot be observed implying that no tachyons are measured but non-local correlations nevertheless build up rather quickly. In order to better understand what is the right thing to do, we study now the first and second Bianchi identities. One easily reads off that

$$\begin{aligned} R_{a\mu(bc)}(x, v^e) &= 0 \\ R_{\mu\nu(ab)}(x, v^e) &= 0 \end{aligned}$$

while the usual standard Bianchi identity in torsion less Riemannian geometry

$$R_{[\mu\nu\kappa]}{}^{\lambda}(x, v^c) = 0$$

does not hold anymore. Indeed, an elementary calculation yields

$$R_{[\mu\nu\kappa]}{}^{\lambda}(x, v^c) = -2\mathcal{D}_{[\mu}(x, v^c)F^{\lambda}_{\nu\kappa]}(x, v^c) + 4F^{\alpha}_{[\mu\nu}(x, v^c)F^{\lambda}_{\kappa]\alpha}(x, v^c)$$

$$-2F^{a}_{[\mu\nu}(x, v^c)\Gamma^{\lambda}_{|a|\kappa]}(x, v^c)$$

and it is the last term on the right hand side which makes this expression non vanishing (due to the Bianchi identities for $\mathcal{A}^{B}_{\mu}(x, v^c)$ which we work out next). The reader may also verify that

$$R_{\mu\nu(\kappa\lambda)}(x, v^c) \neq 0 \neq R_{a\mu(\nu\kappa)}(x, v^c).$$

It is helpful to first understand the second Bianchi identities for the field strength $F^{C}_{AB}(x, v^c)$. Although the latter are not tensors, the Bianchi identities are valid in any "gauge" and coordinate system. The first equality is given by

$$\begin{aligned} 0 \;=\; & [\mathcal{D}_{\mu}(x, v^c), [\mathcal{D}_{\nu}(x, v^c), \mathcal{D}_{\kappa}(x, v^c)]] + [\mathcal{D}_{\nu}(x, v^c), [\mathcal{D}_{\kappa}(x, v^c), \mathcal{D}_{\mu}(x, v^c)]] + \\ & [\mathcal{D}_{\kappa}(x, v^c), [\mathcal{D}_{\mu}(x, v^c), \mathcal{D}_{\nu}(x, v^c)]] \end{aligned}$$

which is equivalent to

$$0 = -2\mathcal{D}_{[\mu}(x, v^c)F^{\lambda}_{\nu\kappa]}(x, v^c) - 4F^{\lambda}_{\gamma[\mu}(x, v^c)F^{\gamma}_{\nu\kappa]}(x, v^c)$$

$$-4F^{\lambda}_{a[\mu}(x, v^c)F^{a}_{\nu\kappa]}(x, v^c)$$

and

$$0 = 2\mathcal{D}_{[\mu}(x, v^c)F^{a}_{\nu\kappa]}(x, v^c) + 4F^{a}_{b[\mu}(x, v^c)F^{b}_{\nu\kappa]}(x, v^c) + 4F^{a}_{\lambda[\mu}(x, v^c)F^{\lambda}_{\nu\kappa]}(x, v^c)$$

and the reader notices that writing these equations explicitly in terms of the potential $\mathcal{A}^{B}_{\mu}(x, v^c)$ is not that easy given the presence of $B^{\mu}_{\nu}(x, v^c) = (\delta^{\mu}_{\nu} - \mathcal{A}^{\mu}_{\nu}(x, v^c))^{-1}$. The second equality is given by the vanishing of

$$[\mathcal{D}_{\mu}(x, v^c), [\mathcal{D}_{\nu}(x, v^c), \partial_a]] + [\mathcal{D}_{\nu}(x, v^c), [\partial_a, \mathcal{D}_{\mu}(x, v^c)]] + [\partial_a, [\mathcal{D}_{\mu}(x, v^c), \mathcal{D}_{\nu}(x, v^c)]]$$

which is equivalent to

$$0 = \mathcal{D}_{[\mu}(x, v^c)F^{\kappa}_{\nu a]}(x, v^c) - 2F^{\alpha}_{[\nu a}(x, v^c)F^{\kappa}_{\mu]\alpha}(x, v^c) - 2F^{b}_{[\nu a}(x, v^c)F^{\kappa}_{\mu]b}(x, v^c)$$

and

$$0 = \mathcal{D}_{[\mu}(x,v^c)F^b_{\nu a]}(x,v^c) - 2F^d_{[\nu a}(x,v^c)F^b_{\mu]d}(x,v^c) - 2F^\kappa_{[\nu a}(x,v^c)F^b_{\mu]\kappa}(x,v^c)$$

and finally, the last one equals

$$\begin{aligned} 0 &= [\mathcal{D}_\mu(x,v^c),[\partial_a,\partial_b]] + [\partial_a,[\partial_b,\mathcal{D}_\mu(x,v^c)]] + [\partial_b,[\mathcal{D}_\mu(x,v^c),\partial_a]] \\ &= -2\left(\partial_{[a}F^\lambda_{b]\mu}(x,v^c) + 2F^\lambda_{\alpha[a}(x,v^c)F^\alpha_{b]\mu}(x,v^c)\right)\mathcal{D}_\lambda(x,v^c) \\ &\quad -2\left(\partial_{[a}F^d_{b]\mu}(x,v^c) + 2F^d_{\alpha[a}(x,v^c)F^\alpha_{b]\mu}(x,v^c)\right)\partial_d \end{aligned}$$

leading in total to six types of Bianchi identities. Likewise, we now compute the "ordinary" second Bianchi identities; the first one is given by

$$\begin{aligned} 0 &= \left[\nabla_{[\mu}(x,v^c),\left[\nabla_\nu(x,v^c),\nabla_{\kappa]}(x,v^c)\right]\right]W^\gamma(x,v^c) \\ &= \left(\nabla_{[\mu}(x,v^c)R_{\nu\kappa]\alpha}{}^\gamma(x,v^c) + 2\nabla_{[\mu}(x,v^c)\left(F^a_{\nu\kappa]}(x,v^c)\Gamma^\gamma_{a\alpha}(x,v^c)\right)\right)W^\alpha(x,v^c) + \\ &\quad \left(R_{[\nu\kappa\mu]}{}^\alpha(x,v^c) + 2F^a_{[\nu\kappa}(x,v^c)\Gamma^\alpha_{|a|\mu]}(x,v^c)\right)\nabla_\alpha(x,v^c)W^\gamma(x,v^c) \\ &\quad -2\nabla_{[\mu}(x,v^c)F^a_{\nu\kappa]}(x,v^c)\partial_a W^\gamma(x,v^c) \\ &\quad +2F^a_{[\nu\kappa}\left(\partial_a\nabla_{\mu]}(x,v^c) - \nabla_{\mu]}(x,v^c)\partial_a\right)W^\gamma(x,v^c) \end{aligned}$$

and the last term is computed to be

$$-2F^b_{[\nu\kappa}(x,v^c)\left(2F^a_{|b|\mu]}(x,v^c) + \Gamma^a_{\mu]b}(x,v^c)\right)\partial_a W^\gamma(x,v^c)$$

$$-4F^b_{[\nu\kappa}(x,v^c)F^\alpha_{|b|\mu]}(x,v^c)\nabla_\alpha(x,v^c)W^\gamma(x,v^c) -$$

$$2F^b_{[\nu\kappa}(x,v^c)\left(2F^\beta_{|b|\mu]}(x,v^c)\Gamma^\gamma_{\beta\alpha}(x,v^c) + \partial_b\Gamma^\gamma_{\mu]\alpha}(x,v^c)\right)W^\alpha(x,v^c)$$

and the reader is advised to explicitly check that the correct transformation laws hold. Given the above Bianchi identities, two new expressions arise; the first (second) one being a correction to the first (second) Bianchi identity:

$$\begin{aligned} 0 &= R_{[\nu\kappa\mu]}{}^\alpha(x,v^c) + 2\left(\Gamma^\alpha_{a[\mu}(x,v^c) - 2F^\alpha_{a[\mu}(x,v^c)\right)F^a_{\nu\kappa]}(x,v^c) \\ 0 &= \nabla_{[\mu}(x,v^c)R_{\nu\kappa]\alpha}{}^\gamma(x,v^c) + 2\nabla_{[\mu}(x,v^c)\left(F^a_{\nu\kappa]}(x,v^c)\Gamma^\gamma_{a\alpha}(x,v^c)\right) - \\ &\quad 2F^b_{[\nu\kappa}(x,v^c)\left(2F^\beta_{|b|\mu]}(x,v^c)\Gamma^\gamma_{\beta\alpha}(x,v^c) + \partial_b\Gamma^\gamma_{\mu]\alpha}(x,v^c)\right). \end{aligned}$$

The reader might verify that our new expression for the first Bianchi identity coincides with the old one by making use of previous identities. One can rewrite these formula in a more conventional form; indeed, inspection reveals that

$$\begin{aligned} 0 &= R_{[\nu\kappa\mu]}{}^\alpha(x,v^c) - T^\alpha_{a[\mu}(x,v^c)T^a_{\nu\kappa]}(x,v^c) \\ 0 &= \nabla_{[\mu}(x,v^c)R_{\nu\kappa]\alpha}{}^\gamma(x,v^c) - T^a_{[\nu\kappa}(x,v^c)R_{\mu]a\alpha}{}^\gamma(x,v^c) \end{aligned}$$

which is identical to the usual Bianchi identities in Einstein-Cartan theory. This was to be expected since the latter are more universal than the former: indeed, our connection is a constrained affine connection in $6+2$ dimensions written out in a non-holonomic basis. To appreciate that this is indeed the fact, one may verify that

$$0 = \left[\nabla_{[\mu}(x,v^c), \left[\nabla_{\nu}(x,v^c), \nabla_{\kappa]}(x,v^c)\right]\right] V^a(x,v^c)$$

leads to exactly one new equality

$$\begin{aligned} 0 &= \nabla_{[\kappa}(x,v^c) R_{\mu\nu]b}{}^a(x,v^c) - \\ & \quad 2\left(2F^{\alpha}_{d[\kappa}(x,v^c)\Gamma^a_{\alpha b}(x,v^c) + \partial_d \Gamma^a_{[\kappa|b|}(x,v^c)\right) F^d_{\mu\nu]}(x,v^c) \\ &= \nabla_{[\kappa}(x,v^c) R_{\mu\nu]b}{}^a(x,v^c) - T^d_{[\mu\nu}(x,v^c) R_{\kappa]db}{}^a(x,v^c). \end{aligned}$$

Therefore, without any further computation, the remaining second Bianchi identities are given by

$$\begin{aligned} 0 &= \nabla_{[a}(x,v^c) R_{\mu\nu]\kappa}{}^{\lambda}(x,v^c) - T^b_{[\mu\nu}(x,v^c) R_{a]b\kappa}{}^{\lambda}(x,v^c) - T^{\alpha}_{[\mu\nu}(x,v^c) R_{a]\alpha\kappa}{}^{\lambda}(x,v^c) \\ 0 &= \nabla_{[a}(x,v^c) R_{b\mu]\kappa}{}^{\lambda}(x,v^c) - T^{\alpha}_{[b\mu}(x,v^c) R_{a]\alpha\kappa}{}^{\lambda}(x,v^c) \\ 0 &= \nabla_{[a}(x,v^c) R_{\mu\nu]b}{}^{d}(x,v^c) - T^{\alpha}_{[\mu\nu}(x,v^c) R_{a]\alpha b}{}^{d}(x,v^c) - T^{e}_{[\mu\nu}(x,v^c) R_{a]eb}{}^{d}(x,v^c) \\ 0 &= \nabla_{[a}(x,v^c) R_{b\mu]d}{}^{e}(x,v^c) - T^{\alpha}_{[b\mu}(x,v^c) R_{a]\alpha d}{}^{e}(x,v^c) \end{aligned}$$

and the other four, first Bianchi identities are

$$\begin{aligned} 0 &= R_{[\mu\nu a]}{}^{\alpha}(x,v^c) - T^{\alpha}_{b[\mu}(x,v^c) T^b_{\nu a]}(x,v^c) - \nabla_{[\mu}(x,v^c) T^{\alpha}_{\nu a]}(x,v^c) \\ 0 &= R_{[\mu ab]}{}^{\alpha}(x,v^c) - T^{\alpha}_{\beta[\mu}(x,v^c) T^{\beta}_{ab]}(x,v^c) - \nabla_{[\mu}(x,v^c) T^{\alpha}_{ab]}(x,v^c) \\ 0 &= R_{[\mu\nu a]}{}^{b}(x,v^c) - T^{b}_{d[\mu}(x,v^c) T^{d}_{\nu a]}(x,v^c) - \\ & \quad T^{b}_{\alpha[\mu}(x,v^c) T^{\alpha}_{\nu a]}(x,v^c) - \nabla_{[\mu}(x,v^c) T^{b}_{\nu a]}(x,v^c) \\ 0 &= R_{[\mu ab]}{}^{d}(x,v^c) - T^{d}_{\alpha[\mu}(x,v^c) T^{\alpha}_{ab]}(x,v^c) - \nabla_{[\mu}(x,v^c) T^{d}_{ab]}(x,v^c). \end{aligned}$$

All this means that our geometry is an extremely subtle generalization of Riemannian geometry in four spacetime dimensions. Indeed, it is wider than ordinary geometry of the vielbein and spin connection in $3+1$ dimensions but is much more constrained than Einstein-Cartan geometry in $6+2$ dimensions. Indeed, the flatness of tangent space as well as the vanishing of many torsion coefficients show that this is the case.

One can now calculate the contracted Bianchi identities in order to generate "conservation laws"; however, Noether's theorem does not apply to geometries with a nonzero torsion and the resulting equations do not permit to extract the correct conserved tensors. Indeed, from the second Bianchi identities, one calculates that

$$
\begin{aligned}
0 &= \partial_a \Big(R_{b\mu\kappa}{}^{\mu}(x,v^c) e^{\kappa a}(x,v^c) - \delta^a_b R_{d\mu\kappa}{}^{\mu}(x,v^c) e^{\kappa d}(x,v^c) \Big) \\
&\quad - T^{\alpha}_{b\mu}(x,v^c) R_{a\alpha}{}^{a\mu}(x,v^c) + T^{\alpha}_{a\mu}(x,v^c) R_{b\alpha}{}^{a\mu}(x,v^c) \\
0 &= \partial_a \Big(R_{b\mu}{}^{ad}(x,v^c) e^{\mu}_{d}(x,v^c) - \delta^a_b R_{d\mu}{}^{df}(x,v^c) e^{\mu}_{f}(x,v^c) \Big) \\
&\quad - T^{\alpha}_{b\mu}(x,v^c) e^{\mu}_{f}(x,v^c) R_{a\alpha}{}^{af}(x,v^c) + \\
&\quad T^{\alpha}_{a\mu}(x,v^c) e^{\mu}_{f}(x,v^c) R_{b\alpha}{}^{af}(x,v^c)
\end{aligned}
$$

and the reader is invited to construct the four remaining equations (which involve derivatives ∇_μ). Regarding the matter sector, it is obvious that the transition of spiritual to ordinary matter is a cumbersome process to the extend that the person in question needs to be unconscious for spiritual information to enter in the realm of his or her physical reality and observation. Moreover, it is far more likely that addition of physical information occurs by means of infiltrant spirits attached to real poison, so called markers, resident in the body of the person in question. Only when a test person is free of such spirits could this coupling be studied.

Part II

Quantum theory.

Chapter 5

Spin, two point functions, probability and particle statistics.

In the previous chapter, we defined the two point function in a general time-orientable curved space time by means of

$$W(x,y) = \sum_{w:\exp_x(w)=y} \int_{T^\star \mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\widetilde{\phi}(x,k^a,w^a)$$

or equivalently

$$W(x,y) = \sum_{w:\exp_x(w)=y} W(x,w)$$

where $W(x,w) = \int_{T^\star \mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\widetilde{\phi}(x,k^a,w^a)$ and $\widetilde{\phi}(x,k^a,w^a) = e^{-ik_a w^a}$ with $w^a w_a = 2\sigma(x,y)$ and $\sigma(x,y)$ is Synge's function in case the GS condition holds. Equivalently,

$$\widetilde{\phi}(x,k^a,w^a) = e^{i\sigma(x,y)_{,\mu} e^{\mu}_a(x) k^a}$$

as the reader may show or $w^a = -e^{a\mu}(x)\sigma_{,\mu}(x,y)$; also, it is clear that the definition of the propagator does not depend upon the choice of the vierbein at x which justifies dropping it in our notation. To prove that the associated two point function satisfies indeed quantum causality, consider the reflection around w^a, the latter is a Lorentz transformation, preserving the sign of k^0 if k^a is a causal vector and maps $k^a w_a$ to $-k^a w_a$; hence, $W(x,y) = \overline{W(x,y)}$ which proves our assertion. One can now wonder to what extend the Klein-Gordon equation still plays a roll; consider that $W(x,y) \equiv W(\sigma_{,\mu}(x,y))$ satisfies

$$\left(\Box' + m^2\right) W(x,y) = -ig^{\alpha'\beta'}\sigma_{,\mu\beta'\alpha'} \frac{\partial}{\partial \sigma_{,\mu}} W(x,y) + m^2 W(x,y)$$

$$-g^{\alpha'\beta'}\sigma_{,\mu\alpha'}\sigma_{,\nu\beta'}\frac{\partial^2}{\partial\sigma_{,\mu}\partial\sigma_{,\nu}}W(x,y)$$

where primed indices refer to y and unprimed to x and all derivatives of σ are covariant derivatives. The reader now notices that in the coincidence limit $y \to x$, we have that the right hand side reduces to zero where we use Synge's rule $[\sigma_{,\mu\beta'}] = -g_{\mu\beta}$ and $[\sigma_{,\mu\alpha'\beta'}] = 0$ where the square brackets indicate that the limit $y \to x$ is taken. Before we proceed, let us stress that our point of view is relational in the sense that it is the way we have build the two point function, the point of view of field operators was absolute in the sense that propagation is a derived concept of composite entities whereas here, the bi function is fundamental. Notice also that the above formula gives our covariantization of the flat space time equation and as anticipated in the previous part, the right hand side is in general not zero; we will come to other, more substantial deviations later on. Our two point function is natural in the sense that it only depends upon the geodesics joining the two points which is as "local" as one may get. There is a useful information interpretation of our formula which is that the information of the creation of a particle travels on geodesics possibly exceeding the local speed of light: therefore, the interacting theory will be constructed as a theory of interacting information currents. It is obvious that the singularity structure of our two point function is of Hadamard type and therefore identical to the one of the standard Minkowski vacuum; this leads to infinite renormalizations which one would preferably avoid and this matter will be thoroughly discussed in the next chapters. Also, the most general possible definition of the Feynman propagator is provided by

$$\Delta_F(x,y) = \sum_{w:\exp_x(w)=y \text{ and w is in the future lightcone of x}} W(x,w) +$$

$$\sum_{w':\exp_y(w')=x \text{ and w' is in the future lightcone of y}} W(y,w') + \sum_{w:\exp_x(w)=y \text{ and w is spacelike at x}} W(x,w)$$

which shows that in general it cannot be derived from the total Wightman function but rather from the more fundamental ones attached to one geodesic.

5.1 General theory of spin.

The theory of spin has been developed from different points of view. Historically, Dirac rediscovered the gamma matrices and simply noticed that the generator of rotations around the j axis is of the form

$$-\frac{1}{2}\begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}$$

where σ_j is the j'th Pauli matrix which gives a double copy of the irreducible spin $\frac{1}{2}$ representation of $SU(2)$. The boosts however come with a relative minus sign and the Dirac representation is of type $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ of the homogeneous

Lorentz group, see [1] for more details. So, in this section, we shall take an approach which is closer related to the "field" point of view constructed by Weinberg, rather than what he calls the particle point of view which is based upon abstract reasoning regarding transformation laws of the eigenstates of the energy momentum operators and some rotation operator in an irreducible unitary representation of the Poincaré group. In the latter view, he first derives transformation properties of those states, which upon an investigation of statistics, induce a transformation on particle creation an annihilation operators which in turn induce transformation laws on the particle-antiparticle vectors associated to a certain momentum and spin. This is all very universal and nice and we shall shamelessly copy his results in the next section when I discuss our novel insights from the operational point of view. In this section, we directly copy his transformation laws for vectors associated to a certain momentum and spin and take that as the basis for our understanding. This requires no fields or anything like it, albeit it boils down to the same mathematics, but we simply ask for a *natural* definition of vectors associated with a state of definite momentum and spin. We shall only treat the theory for particles with a mass here and make some comments for massless spin one particles. Notice that we have not said anything yet about statistics in general, those properties have to be "read off" from symmetry properties of the propagator which we did already for spin zero particles. They have to be bosons, since you can switch the locations of particle creation and annihilation without modifying the Wightman function for events connected exclusively by spacelike geodesics. This approach has certain advantages because in the operational approach, it is for example unknown why massless particles do not have a continuous spectrum of internal degrees of freedom whereas in the "vector" or "field" representation, this is utterly clear. So beware, I am going to use some results in the book of Weinberg, but I shall *directly* motivate why these transformation laws should hold without undergoing the entire analysis from particles to fields.

So, this section contains no new material, albeit I do provide for a different point of view than Weinberg does. To make the presentation as clear as possible, we organize the discussion as follows: first, we introduce general properties of massive particles with spin and then impose a constraint on the Lorentz tranformations connecting particles with different momenta and the same spin. This is also something Weinberg does and he does not fully motivate why this constraint has to hold apart from the fact that the relativistic notion of spin should coincide with the Euclidean one. I provide here no new input and I will use the same *convention* as he does. Nothing physical depends upon that convention as we shall show expliciteły for Dirac particles given that the propagators are insensitve to such recalibrations. Next, we go over to Dirac particles and flesh out that theory; you will see that there are naturally two kinds of particles in this representation corresponding to vectors and covectors, the former which we call particles and the latter anti-particles. So, the transformation laws for those vectors and covectors have to be the same regarding the representation of the spin group $SU(2)$ but of course a covector transforms covariantly under the (nonunitary) irreducible representation of the Lorentz group whereas a vector transforms contravariantly. Now, in the Dirac representation, there is a natural correspondence between vectors and covectors so that the transformation law for the anti-particle covector can be turned into vector form. These are the general transformation laws we are interested in and they are precisely the ones obtained by Weinberg in the general case of arbitrary spin. Consider a free particle with mass $m > 0$ such that its wave vector k is timelike, $k^{\perp}$ is therefore a three dimensional Euclidean space, with inner product defined by $-\eta_{ab}$, and carrying the defining representation of the little group[1] of k, which is $SO(3)$. To start with, we can consider the case $k = (m, 0, 0, 0)$ and one notices that, for example, that the rotation around the z axis belongs to the little group. In fact, any rotation belongs to the little group, but we are picking one generator because you caqn only diagonalize one generator in any irreducible representation of the Lie algebra of $SO(3)$; the latter being given by

$$[L_j, L_k] = i\epsilon_{jkl} L_l$$

where the L_j are hermitean matrices, the square brackets denote the commutator and ϵ_{jkl} is the usual totally antisymmetric tensor. A little bit of algebra reveal that

$$(L_1)^2 + (L_2)^2 + (L_3)^2 = L^2$$

commutes with every generator; therefore in an irreducible representation, it should be a multiple of the identity operator. As is well known, all finite dimensional irreducible representations are characterized by a half integer number j, and has dimension $2j + 1$. Therefore, let σ, σ' be indices running from $-j \ldots j$, then the generators take the following standard form

$$(L_3)_{\sigma\sigma'} = \sigma\delta_{\sigma\sigma'} \quad (5.1)$$

$$(L_1 \pm iL_2)_{\sigma'\sigma} = \delta_{\sigma'\sigma\pm 1}\sqrt{(j \mp \sigma)(j \pm \sigma + 1)} \quad (5.2)$$

and as is well known

$$[L_1 + iL_2, L_1 - iL_2] = 2L_3$$

and therefore serve as lowering and raising operators. We are also interested in tensors mixing different representations of this Lie algebra, since those serva as natural intertwiners in a rotationally invariant theory. Such tensors are constructed from taking a tensor product representation

$$A \otimes B$$

with A, B half integers and noticing that this can be decomposed into a direct sum of spin j representations where j varies between $|A - B|$ an $A + B$ with multiplicity one. The Clebsch Gordan coeffiecients are then the natuaral projectors on those representations, meaning that a vector Ψ_{ab} where $a : -A \ldots A$ and $b : -B \ldots B$ gets projected to

$$\Psi_j(m) := \sum_{ab} C(jm, ab)\Psi_{ab}$$

[1]The little group of k^a is defined as the subgroup of the Lorentz group leaving k^a invariant.

where $m : -j \ldots j$ and $\Psi_j(m)$ transforms irreducible undr the spin j transformation. These coefficients have several symmetries and you can find a more general analysis of what followq in Weinberg [1]. I am not going to repeat all this material here, since it has been explained before. Now, so far for spin; what we are interested in now are representations of the homogeneous Lorentz group which intertwine properly with spin j representations of $SO(3)$. We know this already to be the case for the Dirac representation, since the rotation around the three axis is a double copy of the spin $\frac{1}{2}$ rotation matrix. Now, I shall give you the right answe straight away and then show that this fully coincides with my view on vectors and covectors in the Dirac representation to be related to particle and anti-particle creation. Before we proceed, let me explain one further thing: we start out with the canonical four vector k and we attach thereon vectors $u(k,\sigma), v(k,\sigma)$ where the $u's$ are the particle vectors and the $v's$ the raised covectors (by means of the appropriate map); then we are going to look for Lorentz transformations $\Lambda(p)$ such that $D(L(p))$ brings $u(k,\sigma), v(k,\sigma)$ into $u(p,\sigma), v(p,\sigma)$ where D is our irreducible representation of the Lorentz group. Notice also that the litte group $D(R)$ where R is a spatial representation has to induce a spin j transformation $S^j(R)$ on these base vectors, that is

$$\begin{aligned} D(R)u(k,\sigma) &= S^j(R)_{\sigma'\sigma}u(k,\sigma') \quad (5.3)\\ D(R)v(k,\sigma) &= \overline{S^j(R)_{\sigma'\sigma}}v(k,\sigma') \quad (5.4)\end{aligned}$$

and the only mysterious thing here is the complex conjugation in the second formula. We shall explain where that comes from if we go to the Dirac theory. Consider now $\Lambda(p)$ to be chosen and take an arbitrary Lorentz transformation Γ then the action of $D(\Gamma)$ on, say, $u(p,\sigma)$ reads

$$D(\Gamma)u(p,\sigma) = D(\Lambda(\Gamma(p))D(\Lambda(\Gamma(p))^{-1}\Gamma\Lambda(p))u(k,\sigma)$$

Obviously, $\Gamma(p))^{-1}\Gamma\Lambda(p)$ belongs to the little group of k and therefore we denote it by $W(\Gamma,p)$. Thus,

$$D(\Gamma)u(p,\sigma) = S^j(W(\Gamma,p))_{\sigma'\sigma}u(\Gamma(p),\sigma').$$

Now, what we demand is that the $\Lambda(p)$ are chosen as such that for any spatial rotation R holds that $W(R,p) = R$. Weinberg defines $\Lambda(p) = R(\hat{p})B(|p|)R^{-1}(\hat{p})$ where $B(|p|)$ is given by the standard boost around the z axis bringing k into $(\sqrt{|\vec{p}|^2 + m^2}, 0, 0, |\vec{p}|)$:

$$B(|p|) = \left(\begin{array}{cccc} \gamma & 0 & 0 & \sqrt{\gamma^2 - 1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sqrt{\gamma^2 - 1} & 0 & 0| & \gamma \end{array} \right)$$

where $\gamma = \frac{\sqrt{|\vec{p}|^2 + m^2}}{m}$. Upon writing $\frac{\vec{p}}{|\vec{p}|} = \hat{p} = (0, \sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta))$ we can write

$$R(\hat{p}) = e^{i\phi L_3}e^{i\theta L_2}.$$

With those conventions one arrives at

$$\Lambda(p) = \left(\begin{array}{cccc} \gamma & (\gamma^2-1)\hat{p}_1 & (\gamma^2-1)\hat{p}_2 & (\gamma^2-1)\hat{p}_3 \\ (\gamma^2-1)\hat{p}_1 & 1 & (\gamma-1)\hat{p}_1\hat{p}_2 & (\gamma-1)\hat{p}_1\hat{p}_3 \\ (\gamma^2-1)\hat{p}_2 & (\gamma-1)\hat{p}_1\hat{p}_2 & 1 & (\gamma-1)\hat{p}_2\hat{p}_3 \\ (\gamma^2-1)\hat{p}_3 & (\gamma-1)\hat{p}_1\hat{p}_3 & (\gamma-1)\hat{p}_2\hat{p}_3 & 1 \end{array} \right).$$

Now, Weinberg checks that this boost satisfies our convention; that is, take any rotation R then

$$W(R,p) = R(\hat{R(p)})B^{-1}(|p|)R^{-1}(\hat{R(p)})RR(\hat{p})B(|p|)R^{-1}(\hat{p}$$

and notice that $R^{-1}(\hat{R(p)})RR(\hat{p})$ brings the z axis into itself. Therefore, it commutes with $B^{-1}(|p|)$ and the entire expression reduces to R as it should. The reader understands from this computation that the only ambiguity consists of a redefintion of $R(\hat{p})$ by $T(\hat{p})R(\hat{p})$ where $T(\hat{p})$ is a rotation around the $\hat{p}$ axis. Now, the general transformation law is given by

$$u(p,\sigma) = \sqrt{\frac{m}{p^0}}D(\Lambda(p))u(k,\sigma),\ v(p,\sigma) = \sqrt{\frac{m}{p^0}}D(\Lambda(p))v(k,\sigma).$$

The factor $\sqrt{\frac{m}{p^0}}$ stems from the fact that you still have to multiply $u(p,\sigma)$ with a wave $e^{-ip.w}$ and that the latter has a norm squared p^0 with regard to the Klein Gordon inner product for waves. Hence, $\sqrt{\frac{m}{p^0}}$ serves as a normalization factor. The reader who is interested into the kind of irreducible representations (A,B) of the homogeneous Lorentz group which allow for vectors transforming as above under the spin j representation of the rotation group, is advised to consult [1]. There, a full classification and explicit formulae in terms of the Clebsch Gordan coefficients is given. We now see how this realizes into the Dirac picture and show there where the "strange" transformation properties under the spin $\frac{1}{2}$ representation comes from.

The Dirac representation is defined by means of the γ^a matrices satisfying

$$\gamma^a\gamma^b + \gamma^b\gamma^a = 2\eta^{ab}1$$

and $(\gamma^a)^\dagger = \eta^{aa}\gamma^a$ with a special role for γ^0 since

$$\gamma^0(\gamma^a)^\dagger\gamma^0 = \gamma^a.$$

Note that we take the opposite convention to Weinberg who took the spacetime signature to be $-+++$. In matrix form

$$\gamma^0 = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \gamma^j = \gamma^0 = \left(\begin{array}{cc} 0 & -\sigma_j \\ \sigma_j & 0 \end{array} \right).$$

As is well known, the generators of the Lorentz group are given by six tensor valued operators J^{ab} where $a,b : 0 \ldots 3$ and $J^{ab} = -J^{ba}$ which obey

$$\left[J^{ab}, J^{cd}\right] = -i\left(\eta^{bc}J^{ad} - \eta^{ac}J^{bd} + \eta^{ad}J^{bc} - \eta^{bd}J^{ac}\right)$$

and the reader verifies that

$$\mathcal{J}^{ab} = \frac{-i}{4}\gamma^{[a}\gamma^{b]}$$

where the brackets denote anti-symmetrization, satisfy this commutator algebra. Usually, we denote the rotations by

$$J^i := \epsilon_{ijk} J^{jk}$$

and the reader verifies that they obey

$$\left[J^j, J^k\right] = -i\epsilon_{jkl} J_l$$

which is the "rotation" algebra in a space[2] signature $---$. In matrix notation, they read

$$J^j = -\frac{1}{2}\begin{pmatrix} \sigma_j & 0 \\ 0 & \sigma_j \end{pmatrix}$$

which shows that the Dirac representation contains two irreducible spin-$\frac{1}{2}$ unitary representations of $SU(2)$. Note that the third Pauli matrix is real and self adjoint, so it equals its complex conjgate. We will now find the correct vectors $u(k,\sigma), v(k,\sigma)$ from a *different* point of view than Weinberg, he neeeds the parity transformation as well as relativistic causality to fix those vectors given that their transformation proprties leaves for three ambiguities. Unlike in the operational approach of the next section, we do not dispose of a parity transformation or a time reversal and neither do I know what relativistic causality is since we are not working with operator fields. We shall eliminate all three of these ambiguities at once without using these supplemenary assumptions. The split we are looking for arises naturally if one makes the following observations: notice that

$$\left[\gamma^0, (J^{ab})^\dagger\right] = J^{ab}$$

and that therefore $D^{\frac{1}{2}}(\Gamma) := \Gamma^{\frac{1}{2}}$ obeys

$$(\Gamma^{\frac{1}{2}})^\dagger\gamma^0 = \gamma^0\Gamma^{-\frac{1}{2}}$$

where $\Gamma^{-\frac{1}{2}}$ is the inverse of $\gamma^{\frac{1}{2}}$. The reader may want to check the covariance property

$$\Gamma^a_b\Gamma^{\frac{1}{2}}\gamma^b\Gamma^{-\frac{1}{2}} = \gamma^a.$$

We are interested in finding canonical projection operators $P(k)$ such that $\Gamma^{\frac{1}{2}}P(k)\Gamma^{-\frac{1}{2}} = P(\Gamma(k))$. The reader verifies that

$$P^\pm(k) = \frac{1}{2m}(k_a\gamma^a \pm m).$$

Trivially,

$$\Lambda^{\frac{1}{2}}P_\pm(k)\Lambda^{-\frac{1}{2}} = P_\pm(k)$$

[2]Here, we differ a bit with Weinberg, who uses the convention $+++$ for space; hence, we have to reverse the sign of the generators of the spin $\frac{1}{2}$ representation too.

for Λ in the little group of k and

$$P_+(k)P_-(k) = 0$$

as well as the "hermiticity" properties with respect to the indefinite scalar product

$$\langle v|w\rangle = \overline{v}^T \gamma^0 w.$$

Now, it remains to find a preferred basis for those two dimensional subspaces: for this purpose, we introduce commuting operators with $P_\pm(k)$ which are defined by means of an infinitesimal rotation in a two plane perpendicular to k; more in particular, let m, n denote two unit spacelike vectors perpendicular to k and one and another, then a generator of rotations in the n, m plane is given by by

$$R(n, m) = n_{[a}m_{b]}J^{ab}$$

which constitutes an hermitean operator with respect to the indefinite scalar product and defines two hermitean projection operators

$$P_\pm(n, m) = \frac{1}{2}\left(\mp 2R(n, m) + 1\right)$$

satisfying

$$P_+(n, m)P_-(n, m) = 0.$$

Therefore, we can *define* four canonical, normalized, wave vectors $u_{n,m,k;\pm}, v_{n,m,k;\pm}$ as solutions to

$$P_+(k)P_\pm(n, m)u_{n,m,k;\pm} = u_{n,m,k;\pm}$$

and

$$P_-(k)P_\pm(n, m)v_{n,m,k;\pm} = v_{n,m,k;\pm}.$$

We study these vectors now in somewhat more detail; under a general Lorentz transformation, we have that

$$u_{\Lambda n,\Lambda m,\Lambda k;\pm} = \Lambda^{\frac{1}{2}} u_{n,m,k;\pm}$$

and likewise for $v_{n,m,k;\pm}$. We now choose a Lorentz frame such that $k = me_0, n = e_1, m = e_2$; in that case $P_\pm(e_0)$ and $P_\pm(e_1, e_2)$ are *also* hermitian operators with respect to the standard Euclidean inner product so that the $u_{e_1,e_2,me_0;\pm}, v_{e_1,e_2,me_0;\pm}$ constitute both an orthonormal basis with respect to the Lorentzian as well as the Euclidean inner product. In particular, we have that

$$\frac{1}{4}\left(\gamma^0 + 1\right)\left(\pm i\gamma^1\gamma^2 + 1\right) u_{e_1,e_2,me_0;\pm} = u_{e_1,e_2,me_0;\pm}$$

which reduces to

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \pm\sigma_3 + 1 & 0 \\ 0 & \pm\sigma_3 + 1 \end{pmatrix} u_{e_1,e_2,me_0;\pm} = 4u_{e_1,e_2,me_0;\pm}$$

and therefore

$$u_{e_1,e_2,me_0;\pm} = \frac{\epsilon_{\pm}1}{\sqrt{2}} \begin{pmatrix} \chi_{\pm} \\ \chi_{\pm} \end{pmatrix}$$

and likewise

$$v_{e_1,e_2,me_0;\pm} = \frac{\kappa_{\pm}1}{\sqrt{2}} \begin{pmatrix} \chi_{\pm} \\ -\chi_{\pm} \end{pmatrix}$$

where $\sigma_3 \chi_{\pm} = \pm \chi_{\pm}$ and $\chi_{\pm}^{\dagger}\chi_{\pm} = 1$; $\kappa_{\pm}, \epsilon_{\pm}$ are for now unknown unitary numbers. Now, we wish to identify the $u_{e_1,e_2,me_0;\pm}$, $v_{e_1,e_2,me_0;\pm}$ with the $u(k,\pm), v(k,\pm)$. The latter have to obey

$$\theta_i J^i u(k,\alpha) = \frac{1}{2}\sigma^i_{\beta\alpha}\theta_i u(k,\beta),\ \theta_i J^i v(k,\alpha) = -\frac{1}{2}\overline{\sigma^i}_{\beta\alpha}\theta_i v(k,\beta).$$

Taking only rotations around the z-axis, we have arrived at the natural candidates

$$u(k,\pm) = u_{e_1,e_2,me_0;\pm},\ v(k,\alpha) = v_{e_1,e_2,me_0;\mp}.$$

Insisting upon the full rotation conditions fixes all those vectors, see [1], up to an overall unitary number which has no influence on the physics and can be set to one. In particular, we have in standard form

$$u(k,\pm) = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_{\pm} \\ \chi_{\pm} \end{pmatrix},\ v(k,\pm) = \pm\frac{1}{\sqrt{2}} \begin{pmatrix} \chi_{\mp} \\ -\chi_{\mp} \end{pmatrix}.$$

This fixes our theory of spin-momentum vectors associated to particles; remains to clarify the transformation laws for $v(p,\sigma)$. Here, we start from our philosophy that vectors are associated to particles whereas covectors to anti-particles. As said previously, the anti-particle co-vectors should transform in the same way under spin rotations as particle vectors do. It therefore suffices to say that the natural mapping between vectors and covectors is given by

$$v \to \overline{v}^T \gamma^0$$

which explains our last concern. We shall later on derive the propagator from first principles and all formulae in Weinberg automatically fall out which suggests further generalizations towards particles of higher spin, but this is a concern for later.

Let me make some brief comments about massless spin 1 particles in the vector, or defining, representation of the Lorentz group. Here the little group of a null vector k is the Euclidean group in two dimensions $E(2)$, from which only the rotation part, with respect to any timelike vectorfield e_0 and spatial axis e_3, is unitarily represented. It is well known that the acction of the translations does not leave the helicity vectors $e(\pm)_\alpha$ invariant but the bilinear $k_{[\alpha}e(\pm)_{\beta]}$ is an invariant under the little group. This results in the well known stance that only antisymmetric tensors in two indices can carry a spin one massless particle and transform covariantly. Notice further that the represention of the rotation group is real so that the $u's$ and $v's$ all transform in the same way and hence no distinction between particle and anti-particle shows up. Indeed, the canonical raising or lowering operation of indices simply happens with the (inverse) spacetime metric and no complex conjugation is involved. Moreover, in traditional QFT on flat Minkowski, the Ward identities show that all ambiguities in the definition of the helicity states and propagators proportional to k vanish. This is not expected to hold on a general curved spacetime and one might indeed question the relevance of gauge invariance here.

Spin-$\frac{1}{2}$ particles.

Now that we have set the preliminaries for the discussion, let us return now the definition of the straightforward generalization of our "Schrodinger" equation for particles with internal degrees of freedom living in an irreducible representation of the Lorentz group. In particular, we shall treat the case of spin $\frac{1}{2}$ particles first. We completely abandon the "quantum field" viewpoint here and derive the entire theory from a novel implementation of well known physical principles. That is, we aim to generalize the entire framework and derive all well known results of the free theory in flat Minkowski from novel principles without ever speaking about Hamiltonians, field operators, action principles and so on. So, what I propose is a "nouvelle cuisine" for quantum theory: a purely geometrical framework with a realist ontology. Since we work in a general curved space time, we need a Lorentz connection $\omega_{\mu\,b}^{a}$ and the reader may verify that the associated spin connection is given by

$$\omega_{\mu j}^{k} = \frac{i}{2}\omega_{\mu ab}(\mathcal{J}^{ab})_{j}^{k}$$

where the $k, j : 0 \ldots 3$ denote spinor indices and the generator of spin rotations J^{ab} has been introduced before. Therefore, the spin covariant derivative looks like

$$\nabla_{\mu}^{s} = \nabla_{\mu} + \omega_{\mu b}^{a} + \frac{i}{2}\omega_{\mu ab}(\mathcal{J}^{ab})_{l}^{k}$$

where $\omega_{\mu b}^{a}$ is given by

$$\omega_{\mu b}^{a} = -e_{b}^{\nu}\nabla_{\mu}e_{\nu}^{a}$$

and one may directly verify the antisymmetry property

$$\omega_{\mu ab} = -\omega_{\mu ba}.$$

Coming back to the main line of our story, we would like to introduce a function $\phi_{m}(x, k^{a}, y)_{j'}^{i}$ where primed indices again refer to y and m is the mass of the particle such that

$$W(x,y)_{j'}^{i} = \int_{T^{\star}\mathcal{M}_{x}} \frac{d^{4}k}{(2\pi)^{3}}\delta(k^{2} - m^{2})\theta(k^{0})\phi_{m}(x, k^{a}, y)_{j'}^{i}$$

denotes some "propagator". Upper indices refer to spin properties of a vector while lower indices to those of a covector and moreover, annihilation and creation always go in a vector-covector pair. We maintain the convention that particle creation in the propagator corresponds to a covector (indeed, it has to contract with a vector) in the propagator while anti-particle creation corresponds to a vector index in the propagator. Note also that for simplicity of notation, we did not include for a sum over all geodesics but it is of course understood that you should do that. So, the above propagator signifies the amplitude for an anti-particle to be created at x, with spin component i, and be annihilated at y with spin component j'. Likewise, we should have an amplitude $\psi_m(x, k^a, y)_i^{j'}$ to denote the "propagation" of a particle from x, with spin i towards y with spin j'. To fix the propagator, we will proceed in the same way as for the particle of zero spin, arguing what the coincidence limit $\phi_m(x, k^a, x)$ should look like and then solve for the entire spacetime by using the Schrodinger equation associated to (geodesic) paths γ:

$$\frac{D'^s}{dt}\phi(x, k^a, \gamma(t))^i_{j'} = -i\dot{\gamma}^\mu(t)k_\mu(t)\phi(x, k^a, \gamma(t))^i_{j'}.$$

Indeed, the latter is our replacement for the Dirac equation and we will study its solution later on. Let us start by the most straightforward principles of which the first does not necessarily need to be satisfied in a general curved space time but it is for sure true in Minkowski due to spatial homogeneity. That is, the coincidence limit $\phi_m(x, k^a, x)^i_j$ does not depend upon x and it transforms in the adjoint representation of $SL(2, \mathbb{C})$ meaning that

$$\phi_m(x, (\Lambda k)^a, x) = \Lambda^{\frac{1}{2}}\phi_m(x, k^a, x)\Lambda^{-\frac{1}{2}}.$$

The latter requirement, taken together with our generalized Schrodinger equation, ensures that the definition of the propagator shall be independent of the Lorentz frame chosen. Both conditions, taken together, imply that our only building blocks are $k_a\gamma^a$ and $m1$ and since we only work with on shell momenta, $\phi_m(x, k^a, x)$ may be chosen of the form $\alpha(k_a\gamma^a + \beta m1)$ where α and β are complex numbers: the mass dimension should be zero so that the limit of zero mass gives a nonvanishing result. Now, we arrive at our third and most important principle which says that the creation and annihilation of both a particle and antiparticle with the same four momentum should give a vanishing amplitude on shell when summing over all internal degrees of freedom, that is:

$$\phi_m(x, k^a, x)\psi_m(x, k^a, x) = \psi_m(x, k^a, x)\phi_m(x, k^a, x) \sim (k^2 - m^2).$$

This gives that $\phi_m(x, k^a, x) = \alpha(k_a\gamma^a \pm m1)$ and $\psi_m(x, k^a, x) = \alpha'(k_a\gamma^a \mp m1)$. Finally, we have our fourth condition which I call the positive energy condition, which says that

$$\frac{1}{4}\mathrm{Tr}(\gamma^0\phi_m(x, k^a, x)) = k^0 = \frac{1}{4}\mathrm{Tr}(\gamma^0\psi_m(x, k^a, x))$$

which states that the energy of a particle equals the zero'th component of its momentum vector. This further limits $\alpha = \alpha' = 1$; so we are left with

$$\phi_m(x, k^a, x) = (k_a\gamma^a \pm m1) = \pm 2mP_\pm(k), \quad \psi_m(x, k^a, x) = (k_a\gamma^a \mp m1) = \mp 2mP_\mp(k)$$

and given our previous analysis, it is clear that $\psi_m(x, k^a, x) = k_a\gamma^a + m1)$ and the other way around for $\phi_m(x, k^a, x)$. This ends our discussion of the coincidence limit; our novel principles have brought us to matrices which equal $\pm 2mP_{\pm}(k)$ giving the propagator a dimension of mass^3 in contrast to the propagator for a spin-0 particle.

Now, we come to the integration of the Schrodinger equation: the latter is easy and natural and before giving its solution, denote by $(\Lambda^{\frac{1}{2}}(x, w))_i^{j'}$ the *spin* holonomy attached[3] to the geodesic from x to $y = \exp_x(w)$ determined by tangent vector w and similarly for $(\Lambda(x, w))_a^{b'}$ the associated Lorentz holonomy. Thus given our initial conditions, the solutions to the "equation of motion" read

$$\tilde{\phi}_m(x, k^a, w)^i_{j'} = (k_a(\gamma^a)^i_r - m\delta^i_r)(\Lambda^{-\frac{1}{2}}(x, w))^r_{j'}\tilde{\phi}(x, k^a, w)$$

and

$$\tilde{\psi}_m(x, k^a, w)^{j'}_i = (\Lambda^{\frac{1}{2}}(x, w))^{j'}_r(k_a(\gamma^a)^r_i + m\delta^r_i)\tilde{\phi}(x, k^a, w).$$

We will now prove a remarkable property which shows that quantum causality, as it is usually understood, holds for this propagator. Indeed, the very structure of our formulae suggests that there may be a relationship between $\tilde{\psi}_m(x, k^a, w)$ and $\tilde{\phi}_m(y, k^{a'}_{\star w}, -w_{\star w})$ where, as before, $k^{a'}_{\star w} = (\Lambda(x, w))^{a'}_b k^b$. Indeed, a small calculation reveals that

$$\begin{aligned}
\tilde{\phi}_m(y, k^{a'}_{\star w}, -w_{\star w})^{j'}_i &= (k_b((\Lambda(x, w))^{-1})^b_{a'}(\gamma^{a'})^{j'}_{k'} - m\delta^{j'}_{k'})(\Lambda(x, w)^{\frac{1}{2}})^{k'}_i\tilde{\phi}(y, k^{a'}_{\star}, -w_{\star w}) \\
&= (\Lambda^{\frac{1}{2}}(x, w))^{j'}_l\left(k_b(\gamma^b)^l_i - m\delta^l_i\right)\overline{\tilde{\phi}(x, k^a, w)}
\end{aligned}$$

where we have used on the first line that $\Lambda^{\frac{1}{2}}(x, w) = (\Lambda^{\frac{1}{2}}(y, -w_{\star w}))^{-1}$; in the second line, we used covariance of the gamma matrices under joint spin and Lorentz transformations as well as the previous established formula for $\tilde{\phi}(x, k^a, w)$. Now, the way in which this formula becomes useful is by means of the partial particle and antiparticle propagators:

$$W_p(x, y = \exp_x(w), w)^{j'}_i = \int_{T^\star\mathcal{M}_x}\frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)\tilde{\psi}_m(x, k^a, w^a)^{j'}_i$$

and

$$W_a(x, y = \exp_x(w), w)^i_{j'} = \int_{T^\star\mathcal{M}_x}\frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)\tilde{\phi}_m(x, k^a, w^a)^i_{j'}.$$

Indeed,

$$\begin{aligned}
W_a(y, \exp_x(w) = y, -\star w)^{j'}_i &= \int_{T^\star\mathcal{M}_y}\frac{d^4k_{\star w}}{(2\pi)^3}\delta(k^2_{\star w} - m^2)\theta(k^0_{\star w})\tilde{\phi}_m(y, k^{a'}_{\star w}, -w_{\star w})^{j'}_i \\
&= (\Lambda^{\frac{1}{2}}(x, w))^{j'}_l\int_{T^\star\mathcal{M}_x}\frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)\left(k_b(\gamma^b)^l_i - m\delta^l_i\right)\overline{\tilde{\phi}(x, k^a, w)}
\end{aligned}$$

[3]The reader should be aware that I have used a surpressed notation where I should really write $\tilde{\phi}_m(x, k^a, w, e_b(x), e_c(y))^i_{j'}$ instead of $\tilde{\phi}_m(x, k^a, w)^i_{j'}$ but it is understood that all a, i, j indices are taken with respect to $e_b(x)$ and the j' index with respect to $e_c(y)$. Of course, such information is mandatory to define the spin and Lorentz holonomy.

and we concentrate now on points $x \sim y$ which are exclusively connected by spacelike geodesics. In that case, we could write

$$\tilde{\phi}(x, k^a, w) = e^{-ik_a w^a}$$

where w^a is the spacelike tangent at x to the geodesic connecting x with y. Choosing now for each term a different Lorentz frame at x such that the vector w is parallel to the three axis e_3; we perform, as before, a reflection around w given by $k^3 \to -k^3$ to obtain

$$(\Lambda^{\frac{1}{2}}(x,w))_l^{j'} \int_{T^\star \mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0) \left(k_b(\gamma^b)_i^l - 2k_3(\gamma^3)_i^l - m\delta_i^l\right) e^{-ik_3 w^3}$$

where $e^{ik_3 w^3} = \tilde{\phi}(x, k^a, w)$. Summing this formula with the corresponding part of $W_p(x,y)_i^{j'}$ in the same frame gives

$$(\Lambda^{\frac{1}{2}}(x,w))_l^{j'} \int_{T^\star \mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0) \left(2 \sum_{j=0\ldots 2} k_j (\gamma^j)_i^l\right) e^{-ik_3 w^3}$$

which is immediately seen, due to the antisymmetry of some part of the integrand under $k_1, k_2 \to -k_1, -k_2$, to reduce to

$$(\Lambda^{\frac{1}{2}}(x,w))_l^{j'} (\gamma^0)_i^l i \int_{T^\star \mathcal{M}_x} \frac{d^3k}{(2\pi)^3} e^{-ik_3 w^3}$$

where the last integral equals $\delta^3(w^a)$ which proves that

$$W_p(x,y,w)_i^{j'} + W_a(y,x,-\star w)_i^{j'} = 0$$

in any local Lorentz frame. This constitutes a proof of the well known statement that the amplitude for a particle with spin i to travel from x to y and be annihilated with spin j' equals the amplitude for an antiparticle with spin j' to travel from y to x where it is annihilated with spin i. The very minus sign reveals that spin-$\frac{1}{2}$ particles are fermions, meaning that exchanging two particles comes with a minus sign; this constitutes the proof of the spin statistics theorem in our setting at least for spin-0 and spin-$\frac{1}{2}$ particles. I should really mention that the standard approach towards fermions on a general curved spacetime did not even come close in obtaining such a general result. As before, we can now define the Feynman propagator for particle propagation $\Delta_{F,p}(x,y)_i^{j'}$ as we did for for scalar particles by summing over all geodesics between x and y and insisting upon propagation towards the future possibly replacing a particle propagatot by minus the anti-particle propagator attached to that geodesic. We also could define a Feynman propagator for anti-particle propagation as $\Delta_{F,a}(x,y)^i_{j'}$ as before by replacing p with a and the reader immediately notices that $\Delta_{F,a}(x,y)^i_{j'} = -\Delta_{F,p}(y,x)^i_{j'}$. This concludes our discussion of the free Fermi theory and the reader notices that all salient features of the standard

Minkowski theory have been saved. We can now, as in the previous case suggest gravitational modifications of the two point function for causally related points such that causality remains valid but the singularity structure of the propagator changes. The way to do this is exactly identical to the one suggested before for the scalar two point function and therefore, we do not have to discuss this further on here. Evidently, our propagator does not satisfy the Dirac equation anymore and the reader is invited to investigate if the latter would still hold in the coincidence limit $y \to x$ just as the Klein Gordon equation did for the scalar two point function.

Spin 1 "gauge" particles.

In contrast to what one may expect, the two point function for massless spin-1 particles is extremely easy to guess, even when they carry another charge such as is the case for non-abelian gauge theories. We do not speak anymore in terms of gauge transformations which were necessitated by the quantum field viewpoint but we derive the main formula for the two point function and the Feynman propagator from two simple demands. The reader should appreciate the plain simplicity of the construction as the computation of the two point function for non-abelian gauge fields in standard quantum field theory is a matter of laborious work, the proof that gauge particles satisfy bosonic statistics being evident. Hence, we are interested in computing a quantitity

$$W^{\alpha\beta'}_{\mu\nu'}(x,y) = \int_{T^{\star}\mathcal{M}_x} \frac{d^4k}{(2\pi)^3}\delta(k^2)\theta(k^0)\psi(x,k^a,y)^{\alpha\beta'}_{\mu\nu'}$$

and again, we derive the correct form of the two point function. Note here that our group transformations are global transformations and therefore do *not* depend upon the space time point; so, the indices α, β' stands for the adjoint representation of the compact simple Lie group whose algebra is defined by

$$[t_\alpha, t_\beta] = i f^{\gamma}_{\alpha\beta} t_\gamma$$

where $f_{\alpha\beta\gamma} = f^{\delta}_{\alpha\beta} g_{\delta\gamma}$ is totally antisymmetric and the positive definite invariant Cartan metric is given by $g_{\alpha\beta}$. The fact that we do not make any distinction between covariant and contravariant vectors is due to the possibility to raise and lower indices with both metrics $g_{\mu\nu}$ and $g_{\alpha\beta}$. Let us study the coincidence limit $y \to x$ of $\psi(x,k^a,y)^{\alpha\beta'}_{\mu\nu'}$ first. Since there is no mass parameter, the only object of mass dimension zero which we can write down is a multiple of $g_{\mu\nu}g^{\alpha\beta}$, the only other term one can write down on shell has mass dimension squared and is given by a multiple of $k_\mu k_\nu g^{\alpha\beta}$. So here, we make our first law, $\psi(x,k^a,y)^{\alpha\beta'}_{\mu\nu'}$ has mass dimension zero and we can absorb any positive, real constant in the definition of the Cartan metric; so we obtain that

$$\psi(x,k^a,x)^{\alpha\beta}_{\mu\nu} = -g_{\mu\nu}g^{\alpha\beta}$$

where the minus sign originates from the fact that the vectors of helicity ± 1 should come with a plus sign. Writing out our Schrodinger equation is extremely easy

$$\frac{D'}{dt}\psi(x,k^a,\gamma(t))^{\alpha\beta'}_{\mu\nu'} = -i\left[\dot{\gamma}(k)\right](t)\psi(x,k^a,\gamma(t))^{\alpha\beta'}_{\mu\nu'}$$

and when $\gamma(t)$ is a geodesic, the solution is given by

$$\psi(x,k^a,y)^{\alpha\beta'}_{\mu\nu'} = -g_{\mu\nu'}(x,y)\phi(x,k^a,y)g^{\alpha\beta'}$$

where $g_{\mu\nu'}(x,y)$ denotes the parallel transport of the metric along the geodesic. The latter can be written as a composition of the Van Vleck matrix with Synge's function and since the metric is covariantly constant one has that $g_{\mu\nu'}(x,y) = g_{\nu'\mu}(y,x)$. In case multiple geodesics join x and y, we obtain that

$$W^{\alpha\beta'}_{\mu\nu'}(x,y) = -\sum_{w:\exp_x(w)=y} g_{\mu\nu'}(x,w)g^{\alpha\beta'}W(x,w)$$

where $W(x,w) = \int \frac{d^3k}{(2\pi)^3}\delta(k^2-m^2)\theta(k^0)e^{-ik_a w^a}$, which shows that the two point[4] function for spin-1 particles transforming under a global, compact symmetry group is determined by the two point function of the scalar theory, a transporter and the Cartan metric. From our previous results and the symmetry of the transporter as well as the Cartan metric it follows that

$$W^{\alpha\beta'}_{\mu\nu'}(x,y) = W^{\beta'\alpha}_{\nu'\mu}(y,x)$$

for $x \sim y$ so that our theory satifies quantum causality and has bosonic exchange properties. Clearly, massless spin-1 particles are their own antiparticles as there exists only one two point function and not two. Let us better understand the magic which happened here: instead of following the quantization procedure of a theory with a local gauge symmetry and impose a gauge, we simply took the transformation group of the quantum numbers to be a *global* one. This is a meaningful point of view since those numbers themselves do not correspond to any force field, they are attributes of particles which is something different. Here, we obtain on one sheet of paper a result which can be found in every textbook and which requires a long introduction to derive. As mentioned in the previous section, the structure constants $f_{\alpha\beta\gamma}$ and Cartan metric $g_{\alpha\beta}$ will be used to build interactions, everything is perfectly consistent with quantum chromo dynamics and quantum electrodynamics. The Feynman propagator $\Delta^{\alpha\beta'}_{F\,\mu\nu'}(x,y)$ has precisely the same prescription as is the case for spin-0 particles, which concludes the discussion for spin-1 particles. We now come to the discussion of Faddeev-Popov ghosts; first, let us ask ourselves why we insist upon spin-1 particles to transform in the adjoint representation and spin-$\frac{1}{2}$ in the defining one. The general reason is that it allows us to write down intertwiners of the kind

$$(\gamma^a)^i_j e^\mu_a(x)(t_\alpha)^m_n$$

[4]The fact that we need the Cartan metric for the construction of the two point function is precisely the reason why the Lie group had to be compact and simple in the first place.

and as the reader may verify, this is the only way to couple spin-1 and spin-$\frac{1}{2}$ particles. This leaves us with the question of coupling spin-0 particles to spin-1, the relevant intertwiner is given by

$$f_{\alpha\beta\gamma}\nabla^{\mu}$$

where the derivative acts on the ghost propator and therefore these spin-0 particles should transform as a vector in the adjoint representation; moreover they should have fermionic exchange properties since $f_{\alpha\beta\gamma}$ is totally anti-symmetric. and it is very easy to derive the correct propagator

$$W_p(x,y) = \theta(x)\overline{\theta(y)}g^{\alpha\beta}W(x,y)$$

where we have used Grassman numbers $\theta(x), \theta(y)$.

From our Schroedinger equation, it follows that particles born at x are determined by

$$\phi_{m,k;\nu'}(x,y) = \sum_{w:\exp_x(w)=y} (\Lambda(x,w)^{-1})^{\mu}_{\nu'} m_{\mu} e^{-ik_a w^a}$$

where $k^{\mu}m_{\mu} = 0$. Here, the reader notices that the longitudinal modes k_{μ} can come to life if there are different geodesics connecting x with y since

$$-g_{\mu'\nu'}(y)\Lambda(x,w)^{\mu'}_{\mu}k^{\mu}\Lambda(x,v)^{\nu'}_{\nu}k^{\nu} < 0.$$

This is fairly problematic as the resulting norm becomes of indefinite nature, something we should wish to avoid. We have a similar problem as in the case of spin-$\frac{1}{2}$ particles since there is no reason *why* the restriction of a particle wave, born at x, to some Σ' could be written as the restriction of a particle wave annihilated at z. In the case of spin-$\frac{1}{2}$ particles, we could understand this situation by means of the anti-particles *created* at z but there is no such luxury at hand here. We shall be ruthless here and eliminate the null modes as well as our problem of non-compatible particle notions by employing the $SO(3)$ class of vierbeins associated to the surface Σ'. Indeed, the latter determines a preferred timelike vector field e_0 and therefore a preferred notion of helicity vectors belonging to $T^{\star}\Sigma'$; more precisely, we define the equivalent of the T_{x,e_0} mapping in the spin-0 case by means of

$$T_{x,e_0}(\Lambda(x,w)^{-1})^{\mu}_{\nu'}m_{\mu}e^{-ik_a w^a} = \sqrt{k^{0'}}P_{k^{a'}=\Lambda(x,w)^{a'}_{b}k^b;e_0}((\Lambda(x,w)^{-1})^{\mu}_{\nu'}m_{\mu})e^{-ik_a w^a}$$

where $k^{\mu'}m_{\mu'} = 0$, $k^{0'}$ is the component of $k^{a'}$ with respect to e_0 and $P_{k^{a'};e_0}$ projects a covector on the space orthogonal to $k_{\mu'}$ and $e_{0\mu'}$. Here,

$$(\Lambda(x,w)^{-1})^{\mu}_{\nu'}m_{\mu}e^{-ik_a w^a}$$

is to be seen as a covector-valued function in the complexification of $T_{\star}\mathcal{M}_y$, where $y = \exp_x(w)$, in the vector variable $w^a \in T^{\star}\mathcal{M}_x$. Hence we define,

$$P_{x,e_0}\left(\sum_{w:\exp_x(w)=y}\int d^3k\widehat{\psi}(k)\sum_{m^i:m^i_{\mu}k^{\mu}=0,i=1\ldots 3}(\Lambda(x,w)^{-1})^{\mu}_{\nu'}m_{\mu}e^{-ik_a w^a}\right) =$$

$$\sum_{w:\exp_x(w)=y} \int d^3k\, \widehat{\psi}(k) \sum_{m^i : m^i_\mu k^\mu = 0, i=1\ldots3} P_{\Lambda(x,w)^{a'}_b k^b; e_0}((\Lambda(x,w)^{-1})^\mu_{\nu'} m_\mu) e^{-ik_a w^a}$$

and we suggest now that under reasonable conditions the function space

$$\mathcal{S}_x(\Sigma') = \{y \in \Sigma' \to P_{x,e_0 \perp \Sigma'} \Phi_x(y) | \Phi_x \text{ is a spin one wave defined at } x\}$$

is independent of x. That is,

$$\mathcal{S}_x(\Sigma') = \mathcal{S}_z(\Sigma').$$

Under these conditions, we find some Φ_z such that $y \in \Sigma' \to P_{z,e_0 \perp \Sigma'} \Phi_z(y)$ equals $y \in \Sigma' \to P_{x,e_0 \perp \Sigma'} \Phi_x(y)$. The corresponding probability interpretation then being given by the scalar product

$$\langle \Psi_z(y) | \Phi_z(y) \rangle = - \int_{\Sigma'} d^3y \sqrt{h(y)} g^{\mu'\nu'}(y) \, \overline{(T_{z,e_0 \perp \Sigma'} \Psi_z(y))_{\mu'}} (T_{z,e_0 \perp \Sigma'} \Phi_z(y))_{\nu'}$$

which coincides with the version in Minkowski. The reader may check from here that the formula for propagation is given by

$$\left(P_\Sigma(\Psi_x)\right)_{\alpha''}(z) = - \int_\Sigma d^3y \sqrt{h(y)} g^{\mu'\nu'}(y) \, (T_{x,e_0 \perp \Sigma} \Psi_x(y))_{\mu'} \overline{(T_{z,e_0 \perp \Sigma} W(z,y)_{\alpha''\nu'})}$$

and we shall have more to say about this in chapter eight; the treatment of gravitons also being postponed to that chapter.

Chapter 6

Old problems requiring new physics.

In this chapter, we shall work our way towards an appropriate definition of the interacting theory, everything we said so far relating to the free theory. In the best quantum field theory books, one formally derives constraints on the possible interactions which leads one to the field picture and the *dogma* of relativistic causality. The latter, which says that physically realistic observables, located at spatially separated events, must commute is however totally unnecessary: the commuting of the field operators for Bosonic particles, which is mandatory for a Lorentz covariant scattering matrix, is by no means a sign that all realistic observables should commute. In particular, it would imply that the projection operator on the distributional state of a particle created at x is not an observable and neither is the propagator. However, the field picture has many more problems given that its defining constituents are not well defined as is the case in our approach so far which matches field theory exactly on a Minkowski background. So far, we have argued that the "correct" two point function for a spin-0 particle in a general curved background space time is given by

$$W(x,y) = \int_{T^{\star}\mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\phi(x, k^a, y)$$

where

$$\phi(x, k^a, y) = \sum_{w^a \in T\mathcal{M}_x : \exp_x(w) = y} e^{-ik_a w^a}$$

where the exponential map is defined as usual. In Minkowski space time, this expression is given by

$$W(x,y) = \int_{T^{\star}\mathcal{M}_x} \frac{d^3k}{2(2\pi)^3\sqrt{\vec{k}^2 + m^2}} e^{-ik_a(y^a - x^a)}$$

which may be computed further by making a distinction between the spacelike, null, and timelike case. For spacelike $y^a - x^a$, one may choose the Lorentz frame such that $y^a - x^a = \sqrt{(y-x)^2} e_3$ resulting in

$$W(x,y) = \int_{T^\star \mathcal{M}_x} \frac{d^3k}{2(2\pi)^3 \sqrt{\vec{k}^2 + m^2}} e^{-ik^3 \sqrt{-(y-x)^2}}$$

$$= \frac{1}{8\pi^2} \int_0^\infty r\, dr \int_{-\infty}^{+\infty} dk \frac{1}{\sqrt{k^2 + r^2 + m^2}} e^{ik\sqrt{-(y-x)^2}}$$

an integral which simply does not exist! Indeed, *no* momentum integral in standard field theory exists in the sense of Lebesgue as one considers integration of widely fluctuating functions which do not go sufficiently fast to zero at infinity so that the positive and negative, real and imaginary parts of the integrand do not give finite integrals by themselves. It does exist as a bi-distribution however:

$$W(f,g) := \int_{\mathcal{M}} dx \int_{T^\star \mathcal{M}_x} \frac{d^3k}{2(2\pi)^3 \sqrt{\vec{k}^2 + m^2}} \int_{\mathcal{M}} dy\, e^{-ik_a(y^a - x^a)} f(x) g(y)$$

or

$$W(f,g) := \int_{\mathbb{R}^3} \frac{d^3k}{2(2\pi)^3 \sqrt{\vec{k}^2 + m^2}} \int_{\mathcal{M}\times\mathcal{M}} dx\, dy\, e^{-ik_a(y^a - x^a)} f(x) g(y)$$

since all tangent spaces are isomorphic and the smooth test functions f, g are of compact support; it is the order of the integrals in the last expression which counts. In the literature $W(x,y)$ is often presented as a smooth function $\tilde{W}(x,y)$ with a delta distribution on the light-cone; in either

$$W(f,g) = \int_{\mathcal{M}\times\mathcal{M}} f(x) g(y) \tilde{W}(x,y)$$

and the reader may easily find out that $\tilde{W}(x,y)$ is given by special Bessel functions for $x \sim y$. Indeed, in that case, we have that

$$\tilde{W}(x,y) := \frac{m}{\sqrt{-(x-y)^2} 4\pi^2} \int_0^\infty \frac{dk}{\sqrt{k^2+1}}\, k\, \sin(km\sqrt{-(x-y)^2}) e^{-\epsilon k^2} =$$

$$\frac{m}{\sqrt{-(x-y)^2} 4\pi^2} K_1(m\sqrt{-(x-y)^2})$$

as a formal expression. Indeed, it is fairly easy to check by means of partial integration that $K_1(z)$ satisfies Bessel's equation

$$z^2 \ddot{K}_1(z) + z\dot{K}_1(z) - (z^2+1) K_1(z) = 0$$

with appropriate boundary conditions. However, $\tilde{W}(x,y)$ is not absolutely integrable given that it does not vanish at infinity (it remains constant on spacelike hyperbola). Therefore, one cannot extend the definition of $\tilde{W}(x,y)$ from Schwartz functions to smooth L^2 functions of non-compact support as one would expect of realistic wave packages. However, it is worthwhile to mention that

$K_1(z)$ diverges as $\frac{1}{z}$ at $z = 0$ and goes to zero as e^{-z} at $z = +\infty$. Indeed, coming back to the formal integral representation of $K_1(z)$ one may consider the effect of smoothening out with a Schwartz function of compact support as cutting off the integral at high momenta so that only the lower momenta count; this cutoff can be computed by means of a square contour in the complex plane which goes form 0 to R to $R+i\frac{\pi}{2}$ to $i\frac{\pi}{2}$ to 0 in the variable α where $k = \sinh(\alpha)$. The large vertical integral oscillates in a bounded way for large R but becomes irrelevant in the limit for R to infinity when smeared out with test functions while the vertical integral from 0 to $\frac{\pi}{2}$ is irrelevant. In this way, it can be shown that the Schwartz kernel $K_1(z)$ corresponds to the integral

$$K_1(z) = \int_0^\infty dt\, \cosh(t) e^{-\cosh(t) z}$$

and it is easy to see that this expression diverges as $\frac{1}{z}$ if z approaches zero. Hence, $K_1(z)$ is not uniformly bounded and therefore the best kind of duality one may set up is one of L^1_{loc} which are the absolutely integrable functions of compact support disjoint from the lightcone. To construct interactions, we need to calculate the Feynman propagator, which has been defined in full generality before, and has a formal integral representation on Minkowski as

$$\Delta_F(x, y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik_a(y^a - x^a)}}{k^2 - m^2 + i\epsilon}$$

where ϵ is a positive infinitesimal which may be taken to zero after all computations have been performed. Hence, integrals of the kind

$$\int dx\, dv\, dy\, dz \tilde{\Delta}_F(v, x) \tilde{\Delta}_F(w, x) \tilde{\Delta}_F(y, x) \tilde{\Delta}_F(z, x) f(v, w, y, z)$$

are well defined for an entire complex analytic f with exponential falloff on the real section towards infinity. Loops, however, are not well digested since one cannot give direct meaning to

$$\int_{\mathcal{M} \times \mathcal{M}} dx\, dy\, \tilde{\Delta}_F(x, y)^2 f(x, y)$$

with $f(x, y)$ an absolutely integrable function, not necessarily of compact support. In standard calculations physicists make, they do not even care about those issues, they take the usual momentum integral representation of the Feynman propagator which makes no mathematical sense at all, combine such integrals and use Fubini's theorem, change variables without further ado. Of course, this nonsense doesn't work at all; to get some finite answer, which is manifestly Lorentz covariant,the standard procedure is to take infinite "particular sums" of such nonsensical expressions, shamelessly interchange integration and summation, identify divergent geometrical series with their finite expressions correponding to an analytic continuation of where the series is well defined, and finally make the result (partially) finite by means of a redefinition of the bare parameters of the theory with an infinite amount. This bullhit has become culture in the particle physics community and you get away with doing so. Of course, there exist different schemes giving different answers so that nothing in fact remains. Moreover, this procedure splits theories into two categories: those to which *some* procedure of this kind can be applied, called the renormalizable theories, and those to which it cannot, the non renormalizable ones. The shear arbitrariness of the *infinite* renormalization procedure as well as the lack of a deep physical motivation behind it resulted in my thesis that interacting QFT on Minkowski does not exist and that gravitation had to play a fundamental role in making each Feynman diagram finite to the dismay of many field theorists I know of. To make myself cler, renormalization is a necessary element of any theory which discerns interactions from a free world; given that the self energy corrections must be adsorbed in the free theory. We shall not adress this issue in this work as we do not dispose yet of a clear guideline for a physical renormalization scheme in contrast to some formal (but strictly speaking ill defined) arguments one can make on Minkowski.

In this work, we try to restore sanity to the mathematical description of reality and advocate the point of view that there is new physics to be found, effectively deforming the definition of the free propagator, such that active Lorentz invariance is given up, but passive "local Lorentz invariance" not. This regularization procedure will ensure that the propagator is a well defined and smooth function with suitable falloff properties towards infinity. We shall propose here a first step in making the integrals well defined whereas care regarding smoothness and sufficiently fast falloff towards infinity is dealt with later on. We want to keep the definition of $\tilde{\phi}(x, k^a, w^a)$ for now, but we shall provide every exponential $e^{-ik_a w^a}$ with an exponential suppression factor which is *local* at x and y; these factors may be interpreted as a kind of "resistance" spacetime offers to the sending and receiving of geodesic signals. If w^a is causal, then this suppression factor *might* be defined by

$$\alpha(x, k^a, w^b) = R_{\alpha\beta}(x)k^\alpha k^\beta + R_{\alpha'\beta'}(y)k^{\alpha'}_{\star w^b}k^{\beta'}_{\star w^b} + \gamma(k_a w^a)^2$$

where $R_{\alpha\beta}$ is the Ricci tensor and $\star w^b : T^\star\mathcal{M}_x \to T^\star\mathcal{M}_y : k^a e^\mu_a(x) \to k^{a'}_{\star w^b} e^{\mu'}_{a'}(y)$ denotes as usual parallel transport along the geodesic defined by $w^b e^\alpha_b(x)$. The latter induces an orthochronous Lorentz transformation and (un)primed indices do refer to y (x). Here, we require the weak energy condition that $R_{\alpha\beta}V^\alpha V^\beta > 0$ for all timelike vectors V^α. This certainly does the job for a timelike w^a, however for a null w^a this formula may be insufficient to get convergence. In case w^b is spacelike, then denote by $R(w^b)^\alpha_\beta$ the reflection around w^b: the latter is an idempotent isometry on the future pointing causal vectors. One could now define

$$\alpha(x, k^a, w^b) = R_{\alpha\beta}(x)k^\alpha k^\beta + R_{\alpha'\beta'}(y)k^{\alpha'}_{\star w^b}k^{\beta'}_{\star w^b} +$$

$$R_{\alpha\beta}(x)R(w^b)^\alpha_\kappa k^\kappa R(w^b)^\beta_\gamma k^\gamma + R_{\alpha'\beta'}(y)R(w^{b'}_{\star w^b})^{\alpha'}_{\kappa'}k^{\kappa'}_{\star w^b}R(w^{b'}_{\star w^b})^{\beta'}_{\gamma'}k^{\gamma'}_{\star w^b} + \gamma(k_a w^a)^2$$

and by using that $R(\lambda w^b)^{\alpha}_{\beta}$ is independent of λ for $\lambda \neq 0$ (a reflection is defined by an axis, not an orientation), we have that

$$\alpha(x, k^a, w^b) = \alpha(y, k^{a'}_{\star\, w^b}, -w^{b'}_{\star\, w^b})$$

and

$$\alpha(x, k^a, w^b) = \alpha(x, R(w^b)^a_b k^b, w^c).$$

The distinction between the spacelike and causal case is obvious since null w^a do not canonically define a reflection and the reflection around timelike vectors swaps the future and past lightcones. The reason why you need to reflect those terms is because you want the usual property to hold that the propagator is a symmetric function for spacelike separated events. In flat Minkowski, such a regularization scheme would still maintains global Lorentz invariance of the propagator, indeed consider

$$\phi_\mu(x, k^a, y) = \sum_{w^a \in T^\star \mathcal{M}_x : \exp_x(w) = y} e^{-ik_a w^a} e^{-\mu\alpha(x, k^a, w^b)}$$

and as before

$$W_\mu(x, y) = \int_{T^\star \mathcal{M}_x} \frac{d^4 k}{(2\pi)^3} \delta(k^2 - m^2)\theta(k^0)\phi_\mu(x, k^a, y).$$

Then, obviously, we obtain that

$$\overline{W_\mu(x, y)} = W_\mu(y, x)$$

and

$$W_\mu(x, y) = W_\mu(y, x)$$

for $x \sim y$. It is kind of obvious that this propagator on a de-Sitter space time is *not* finite for $\mu, \lambda > 0$ given that the Ricci tensor is proportional to the metric and therefore all curvature terms are constant due to the on shell condition. More precisely, for timelike w^a we do have exponential suppression due to the $(k_a w^a)^2$ term, but the latter does not do a proper job in case w^a is spacelike. Thus, in a maximally symmetric space time, where the Riemann tensor is fully equivalent to the metric itself, there is no way to get a theory out satisfying our finiteness criteria. One can easily save the day by relying on geometries which do locally define a *preferred* timelike unit vectorfield V^μ; as is well known, such geometries are *generic* and may even be algebraically special; Wylleman has recently given an explicit construction hereof. Hence, one could simply replace the $(k_a w^a)^2$ term by a $(k_\mu V^\mu)^2$ in case w is timelike and $(k_\mu V^\mu)^2 + (R(w)^\mu_\nu k^\nu V_\mu)^2$ in case w is spacelike evaluated in as well x as y replacing there k by $k_{\star w}$. In the case w is spacelike, this prescription would provide one with the necessary falloff and symmetry properties. The physical message here is plain and simple, in the non-relativistic theory, one had that the two point function is well defined unlike in the Minkowski case; to restore these salient properties, we need a physical arrow of time which is realized by a generic (classical) matter distribution. All maximally symmetric space times are pathological in the sense that no realistic matter propagates on them; now, people would argue that such timelike vectorfield is not observed in nature as it might suggest a violation of "active lorentz invariance" although everything is locally Lorentz covariant, simply because the measure is. To insist upon active Lorentz invariance is of course rather nonsensical; what our suppression terms do is to incorporate a kind of "resistance" of the spacetime fabric to the creation and annihilation of a signal of a particular type. The gravitational field is such an aether and Minkowski's idealization is just fictitious; I have no idea whether it is sensible to say that these suppression terms have to be small in some or not as they do not pertain to the *propagation* aspect of the signal but merely to the creation and annihilation thereof. It is still possible to work in a spatially homogeneous and isotropic cosmology, such as the one given by the usual Friedman universes, even if the Ricci tensor there is also a multiple of the metric tensor, but at least such a universe provides for a dynamical arrow of time. It will turn out, however, that a little friction on the propagation of the signal is also required in order to obtain suitable convergence properties.

The reader might infer at this point that the local weight factors seem rather ad-hoc, an attitude which I can agree with to some extend. Let us first comment that every more general framework for physics always allows for more possibilities: we can know the principles of nature but not its representation! Einstein's theory enlarged our vision on the universe by many orders of magnitude and likewise does our principle of local Lorentz covariance do regarding the possible quantum laws. The *local* weights attached to the creation and annihilation process are however of a different nature: one interpretation is that they are attached to an action occurring *outside* the framework of four dimensional space time. Constales suggested to me that one could regard spacetime as being made out of atoms and that $\sqrt{\mu}(V^a k_a)^2$ could be interpreted as a kind of self energy the wave has relative to the space time gas. Here, the length scale $\sqrt{\mu}$ could serve as an inverse temperature and one could therefore uphold a thermodynamic interpretation. This is certainly an interesting point of view but in my eyes no "weakening" of our continuum formalism: I accepted already for a long time that our theories come with motivated representations and that there are in general no real reasons to prefer one representation over another. This is the beauty of science, we are never able to tell to the fullest extend how things *are* but we can gain insights about what kind of ideas are necessary and moreover, we are able to refute certain world images.

6.1 First steps with modified propagators.

We will now start to investigate, by means of a couple of examples, the consequences of the local suppression terms added above. The reader immediately notices that we have a different prescription for causal geodesics than for spacelike geodesics so that ultimately one may expect discontinuities on the lightcone. Indeed, everywhere else, our regularization scheme leads to a C^{∞} scalar propagator with uniformly bounded covariant derivatives, except at the lightcone where we will have to perform a supplementary regularization. This issue is *not* important for theories regarding interacting spin-0 particles or quantum electrodynamics, the theory of charged spin-$\frac{1}{2}$ particles interacting by means of a massless spin-1 particle since here, the derivatives of the propagators do not play any role. They become only important in non-abelian gauge theory or the theory of gravitons. The reader should very well understand that the lightcone regularization falls within our principle of local Lorentz invariance so that there is nothing strange about it; however, it is always interesting to see how it becomes necessary by means of more "elementary" computations. I have decided to present this chapter at a pedestrian level, showing step by step by examples what one needs in order to obtain a well defined theory: ultimately, the reader should understand the presence of the supplementary parameters as a mere possibility allowed by nature and there is *no* reason why supplementary constraints should be imposed upon the theory. On the contrary, such limitations often make the theory ill defined and this is what we want to avoid.

To start with, let us study our regularization scheme in Minkowski space time where ∂_t has to be associated to the timelike vectorfield V^{μ} defined by some physical observer making the quantum particle feel an aether due to him or herself, and see if our integral has all desired properties. As is evident from the previous discussion, the only problem with the two point function really resides near the null cone and for this purpose it is sufficient to take the massless limit $m \to 0$. With these reservations

$$W_{\mu}(x,x') = \frac{1}{2(2\pi)^3}\int \frac{d^3\vec{k}}{\left|\vec{k}\right|} e^{-i\left(|\vec{k}|(t'-t)+\vec{k}.(\vec{x}'-\vec{x})\right)} e^{-2\mu\left|\vec{k}\right|^2}$$

for points x, x' which are causally related. We will not explicitly calculate the regularization for spacelike separated events and leave this as an exercise for the reader. One may further calculate the propagator to be

$$\begin{aligned} W_{\mu}(x,x') &= \frac{1}{(2\pi)^3\,|\vec{x}'-\vec{x}|}\int_0^{\infty} dk \sin(k\,|\vec{x}'-\vec{x}|) e^{-ik(t'-t)-2\mu k^2} \\ &= \frac{1}{2i(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|} e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{8\mu}} \int_0^{\infty} dk e^{-\left(k+i\frac{(t'-t+|\vec{x}'-\vec{x}|)}{2\sqrt{2\mu}}\right)^2} - \\ &\quad \frac{1}{2i(2\pi)^3\sqrt{2\mu}\,|\vec{x}'-\vec{x}|} e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{8\mu}} \int_0^{\infty} dk e^{-\left(k+i\frac{(t'-t-|\vec{x}'-\vec{x}|)}{2\sqrt{2\mu}}\right)^2} \end{aligned}$$

and to study the limit $\mu \to 0$ is a rather subtle issue since, albeit the real part of both integrals equals $\frac{\sqrt{\pi}}{2}$ independent of the arguments $t'-t \pm \left|\vec{x'}-\vec{x}\right|$, the complex part is diverging this limit. More precisely, we note that both integrals are of the form

$$I(c) = \int_0^\infty dk e^{-(k+ic)^2}$$

and the integrand is complex analytic in k and c. For real c, we may compute the integral by considering the limit of a contour in the complex plane from 0 to R to $R - ic$ to $-ic$ and finally back to 0. As usual, the integral over the large vertical part vanishes in the limit for R to infinity while the remainder gives

$$I(c) = \int_0^\infty dk e^{-k^2} - i \int_0^c dk e^{k^2}.$$

This shows that the imaginary part of $W_\mu(x, x')$ equals

$$\frac{\sqrt{\pi}}{4(2\pi)^3 \sqrt{2\mu}\, |\vec{x}' - \vec{x}|} \left(e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{8\mu}} - e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{8\mu}} \right)$$

which converges in the limit for μ to zero to the usual delta functions on the lightcone. The real part however is given by

$$-\frac{1}{2(2\pi)^3 \sqrt{2\mu}\, |\vec{x}' - \vec{x}|}$$

$$\left(e^{-\frac{(t'-t+|\vec{x}'-\vec{x}|)^2}{8\mu}} \int_0^{\frac{t'-t+|\vec{x}'-\vec{x}|}{2\sqrt{2\mu}}} dk e^{k^2} - e^{-\frac{(t'-t-|\vec{x}'-\vec{x}|)^2}{8\mu}} \int_0^{\frac{t'-t-|\vec{x}'-\vec{x}|}{2\sqrt{2\mu}}} dk e^{k^2} \right)$$

and the An easy computation shows that

$$\left(\int_0^c e^{k^2} dk \right)^2 = \frac{\pi}{2} \int_0^{\sqrt{2}|c|} dr\, r e^{r^2} = \frac{\pi}{4}(e^{2c^2} - 1)$$

which implies that

$$\int_0^c e^{k^2} dk = \frac{1}{g(c)c} \left(e^{c^2} - 1 \right)$$

where $0 \leq g(c) \leq \sqrt{\frac{2}{\pi}}$ and $g(0) = \sqrt{\frac{2}{\pi}}$ and $g(+\infty) = 0$ with as asymptotic behaviour $\frac{2}{\sqrt{\pi}|c|}$. Hence, the real part of the two point function behaves as

$$\begin{aligned}
\mathcal{R}e\, W_\mu(x, x') &= \frac{1}{2(2\pi)^3 \sqrt{2\mu}\, |\vec{x}' - \vec{x}|} \frac{1}{c_+(x, x', \mu) g(c_+(x, x', \mu))} \\
&\quad \left(1 - e^{-c_+(x,x',\mu)^2} \right) \\
&= -\frac{1}{2(2\pi)^3 \sqrt{2\mu}\, |\vec{x}' - \vec{x}|} \frac{1}{c_-(x, x', \mu) g(c_-(x, x', \mu))} \\
&\quad \left(1 - e^{-c_-(x,x',\mu)^2} \right)
\end{aligned}$$

and

$$c_\pm(x, x', \mu)) = \frac{t' - t \pm |\vec{x}' - \vec{x}|}{2\sqrt{2\mu}}.$$

It is easy to see that the limit of μ to zero of $\mathcal{R}e\, W_\mu(x, x')$ is infinite for all x, x' and must be interpreted in the distributional sense as we did before. The convergence properties to infinity along a branch of $t' - t - r\,|\vec{x}' - \vec{x}| = 0$, where $r \geq 1$ goes proportional to

$$\frac{1}{|\vec{x}' - \vec{x}|}$$

since for $r > 1$ both $c_\pm(x, x', \mu)$ blow up to infinity for $|\vec{x}' - \vec{x}|$ to infinity, which is not quadratically integrable in $\vec{x}'$. It is obviously so that in Minkowski space time, it will never be possible to get the integral

$$\int |\Delta_{F,\mu}(x,y)|^2\, dxdy = \int |W_\mu(x,y)|^2\, dxdy$$

finite due to the translation symmetry. However, this is not something we should be ambitious of as such integrals have nothing to do with real physics. We shall examine now whether this weak asymptotic behaviour is sufficient to get finite loop diagrams by studying some cases which usually give infinite results. Before we proceed, let us notice that, under the agreement that the coincidence limit is defined by the causal prescription, we have

$$W_\mu(x,x) = \frac{1}{4\pi^2} \int_0^\infty dk\, k e^{-2\mu k^2} = \frac{1}{16\pi^2 \mu}$$

which is a finite number usually much larger than one since μ is taken to be small.

Before we proceed, let us think about potential trouble regarding our general definition so far. The reader must have wondered what should happen to our definition when there exists a continuum of geodesics joining x to y such as is the case in a closed universe where the spatial metric is the one of a three sphere embedded in a flat four dimensional Euclidean space and time is perpendicular to that[1]. In that case, one obtains, just like on a two sphere endowed with the standard Riemannian metric, a continuum of geodesics joining any point to its antipodal point after a certain time lapse. Hence, the prescription is in this case mathematically meaningless. However, these points constitute the only exception, all other points in spacetime being connected by means of a countable number of geodesics, just as was the case for the timelike cylinder. Therefore, the message is to simply take the continuous extensions towards those points of our regularized propagator and use that as its definition. This is not too bad given that one would expect, physically, such thing to happen: there is just too much information flowing to the antipodal point and our prescription, which is again of distributional nature on the sphere, does not know how to deal with it. This example brings us to an important property, namely that it is desirable to consider only those tangent vectors w in $T^\star\mathcal{M}_x$ such that $\exp_x$ is a local diffeomorphism in a neighborhood of w. This is important when we want

[1] So the metric equals $dt^2 - (ds_3)^2$.

derivatives of the propagator to be defined and we will come back to this later on. Finally, let us study what happens regarding the regularization scheme for spacelike geodesics defined by $w \in T^{\star}\mathcal{M}_x$ when w approaches the null cone in x so that the corresponding sequence of points $\exp_x(w)$ converges to a point on the null cone. We will show that the w contribution to the propagator vanishes in this limit which means in Minkowski that the entire propagator vanishes. Therefore, we obtain a jump on the lightcone given our previous computations. To get an idea why this is true, consider four dimensional Minkowski spacetime, x as the origin, and a sequence of space like vectors $w_\alpha = (\tanh(\alpha), 1, 0, 0)$, where we are interested in the limit for α to $+\infty$. The reflection around w is defined by

$$R(w) = 1 + 2\frac{1}{1 - \tanh^2(\alpha)}(\tanh(\alpha), 1, 0, 0)^T(\tanh(\alpha), -1, 0, 0)$$

which in matrix form reads

$$R(w) = \frac{1}{1 - \tanh^2(\alpha)}$$

$$\begin{pmatrix} 1 + \tanh^2(\alpha) & -2\tanh(\alpha) & 0 & 0 \\ 2\tanh(\alpha) & -(\tanh^2(\alpha) + 1) & 0 & 0 \\ 0 & 0 & (1 - \tanh^2(\alpha)) & 0 \\ 0 & 0 & 0 & (1 - \tanh^2(\alpha)) \end{pmatrix}$$

as the reader may verify. Therefore,

$$(V^a k_a)^2 + (V^a(R(w)k)_a)^2 =$$

$$(k^0)^2 + \left(\frac{1}{1 - \tanh^2(\alpha)}((1 + \tanh^2(\alpha))k^0 - 2\tanh(\alpha)k^1)\right)^2$$

and this expression diverges to $+\infty$ for any k^a in the limit for $\alpha \to +\infty$ in case $m > 0$. Only in case $m = 0$ and $k^0 = k^1$ do we obtain a finite answer $(k^0)^2$ but the set of such wave vectors has measure zero so that we may conclude that $e^{-2\mu((V^a k_a)^2 + (V^a(R(w)k)_a)^2)} \to 0$ in the limit for $\alpha \to \infty$ almost everywhere for $m \geq 0$. This shows that the regularized propagator for space like separated points has vanishing limit towards the lightcone in Minkowski space time. I leave it up to the reader to show that this is generically the case, at least for every w mode. Hence, we have shown the need for an extra regularization scheme near the lightcone.

We shall now argue what kind of "extensions" one can make regarding the Schroedinger equations we have written down to determine the generalized Fourier waves while keeping in mind the nature of the suppression terms we have to build in. In principle, one can write down an infinite number of terms commensurable with local Lorentz covariance: this is not really a surprise given that general covariance allows for a similar extension of Newton's gravitational theory. We can constrain, however, the extend to which this principle should be applied by restricting to data which is locally determined by the second derivatives of the metric tensor in the same way as Einstein's theory follows uniquely from action principles in local densities containing at most two derivatives of the metric tensor but there is no such a-priori need to do so. So, we shall mainly discuss a few examples of "deformations" which I deem interesting but the reader may invent plenty more of them. We start by giving the example of an "energy" term which could be added to the Schroedinger equation and which respects Lorentz covariance on the propagation part. As mentioned already, we assume that our geometry provides for a unit timelike vectorfield V^μ causing friction in the creation and annihilation of particles at definite space time points: as is well known, a unit timelike vectorfield determines a unique Riemannian metric tensor $h_{\mu\nu}(x)$ as

$$h_{\mu\nu} = 2V_\mu V_\nu - g_{\mu\nu}$$

given our signature convention $(+---)$. The reader should keep in mind that all indices are raised and lowered with the Lorentzian metric and associated vierbein; so $h_{ab} = e^\mu_a e^\nu_b h_{\mu\nu}$ with the standard vielbein e^μ_a. With these lessons in mind, we can now write down another covariant energy term given by

$$\sqrt{h_{ab}(x_{w^c}(s))w^a(s)w^b(s)}$$

where $w^\mu(s) = \frac{dx^\mu(s)}{ds}$. So, our differential equation becomes

$$\frac{d}{ds}\tilde{\phi}_\kappa(x,k^a,w^b,s) = \left(-iw^\mu(s)k_\mu(s) - \kappa\sqrt{h_{\mu\nu}(x_{w^b}(s))w^\mu(s)w^\nu(s)}\right)\tilde{\phi}_\kappa(x,k^a,w^b,s)$$

giving rise to the solution

$$\tilde{\phi}_\kappa(x,k^a,w^b) = e^{-ik^a w_a}e^{-\kappa\int_0^1\sqrt{h_{\mu\nu}(x_{w^b}(s))w^\mu(s)w^\nu(s)}}ds.$$

In our case of Minkowski space time, and some vielbein with $e_0 = \partial_t$, $h_{ab} = \delta_{ab}$ and

$$\phi_\kappa(x,k^a,y) = e^{-ik^a(y_a - x_a)}e^{-\kappa|y-x|}.$$

For sake of convergence, it is assumed that the real part of κ is greater than zero. It turns out that this suppression mechanism is interesting as integrals of the kind

$$\int \Delta_{F,\mu,\kappa}(x,y)\Delta_{F,\mu,\kappa}(y,z)$$

are in the same "function class" as $\Delta_{F,\mu,\kappa}$ meaning they have similar falloff properties towards infinity so that the proof of perturbative renormalizability of the theory becomes self evident as we shall see later on[2]. Roughly speaking, all cases are covered if integrals of the kind

$$\int dy e^{-\kappa|x-y|-\rho|y-z|}$$

[2]Although the proof of convergence of the series is more involved as we will figure out later.

where $\kappa, \rho > 0$ belong to the same function class as $e^{-\zeta|x-z|}$ for some other $\zeta > 0$. From a simple triangle inequality estimate, one obtains that

$$|x-z| + \left|y - \frac{x+z}{2}\right| \leq -|x-z| + 2\left|y - \frac{x+z}{2}\right| \leq |x-y| + |z-y|$$

for $\left|y - \frac{x+z}{2}\right| \geq 2|x-z|$. This splits the integral into two parts as follows

$$e^{-\min\{\kappa,\rho\}|x-z|} \int_{\left|y-\frac{x+z}{2}\right| \geq 2|x-z|} e^{-\min\{\kappa,\rho\}\left|y-\frac{x+z}{2}\right|} dy$$

$$+ e^{-\min\{\kappa,\rho\}|x-z|} \int_{\left|y-\frac{x+z}{2}\right| \leq 2|x-z|} dy$$

and this may further be bounded by

$$2\pi^2 \left(\frac{6}{(\min\{\kappa,\rho\})^4} e^{-\min\{\kappa,\rho\}|x-z|} + 4e^{-\min\{\kappa,\rho\}|x-z|} |x-z|^4 \right)$$

where $2\pi^2$ equals the volume of the three dimensional unit sphere with radius one. These functions obviously belong to the same class as $x^n e^{-\kappa x} \leq a e^{-\zeta x}$ for some $a > 0$ and $0 < \zeta < \kappa$ for all n. The same technique can be applied to an arbitrary number of points $x, z, \ldots$ in the integral as the reader may easily verify for himself. This shows that our idea is an interesting one regarding theories which do not need derivatives of the propagators given that the behavior on the light cone is still anomalous.

We will restrict the situation to be considered to geometries such that for every $x, y \in \mathcal{M}$ there exist open environments $\mathcal{V}_x, \mathcal{W}_y$ of x and y respectively, as well as disjoint opens $\mathcal{O}_{x,w} \subset T\mathcal{M}$, containing (x,w) for each $w : \exp_x(w) = y$, projecting down to $\mathcal{V}_x$ such that

$$\phi(x', k^{a'}, y') = \sum_{w' : exp_{x'}(w') = y'} \tilde{\phi}(x', k^{a'}, w')$$

for all $x' \in \mathcal{V}_x$ and $y' \in \mathcal{W}_y$ where every w' lies in exactly one $\mathcal{O}_{x,w}$ and vice versa. In other words, we assume $\exp_x$ to be a local diffeomorphism around each w such that $\exp_x(w) = y$ and moreover, the different $w's$ are "regularly" separated so that we can find open neighborhoods around them diffeomorphically mapping to $\mathcal{W}_y$ without intersecting. It would be worthwhile to investigate this condition in closer detail but I suspect it to be true generically; from this, one can locally construct Synge functions $\sigma(x', y'; w)$ which are defined for each w on $\mathcal{V}_x \times \mathcal{W}_y$ so that $\tilde{\phi}(x', k^{a'}, w')$ contains the exponential $e^{ik^{a'}\sigma_{a'}(x', y'; w)}$ where $w' \in \mathcal{O}_{x,w}$. Now, we modify for example the latter by a prefactor of

$$e^{-\frac{L^2}{\sigma^2(x', y'; w)}}$$

which the reader immediately recognizes as the insertion of a factor

$$-\frac{2L^2}{\sigma^3(x',\gamma(s);w)}\sigma_{,\alpha'}(x',\gamma(s);w)\dot{\gamma}^{\alpha'}(s)$$

in the Schroedinger equation for the potential. Now, it is a well known property of the function $e^{-\frac{1}{z^2}}$ that the limit for $z \to 0$ of $z^{-n}e^{-\frac{1}{z^2}}$ vanishes for any $n \in \mathbb{N}$. Therefore, under reasonable *uniform* boundedness properties with respect to $h_{\mu\nu}$, of the covariant differentials, given by $g_{\mu\nu}$, of $\sigma(x,y;w)$ regarding w, the reader should be able to verify that not only

$$\lim_{w' \to \mathcal{N}_{x'}} \tilde{\phi}_{\mu,\kappa,L;\alpha\ldots\beta'\ldots}(x',k^{a'},w') = 0$$

where $\mathcal{N}_{x'}$ denotes the lightcone at x' in $T^{\star}\mathcal{M}_{x'}$, but also that

$$\phi_{\mu,\kappa,L;\alpha\ldots\beta'\ldots}(x',k^{a'},y')$$

is well defined as a function. Again, I suspect this to be true for generic space times and we shall make use of these results when developing non-abelian gauge theory and as well as the graviton theory. L is a *length* squared scale; conventionally, one might choose $L = \mu$ but we don't have to.

We finish this discussion by providing for the correct definition of the Feynman propagator *directly*

$$\Delta_{F,\mu,\kappa,L}(x,y) = \sum_{w:\exp_x(w)=y \text{ and w is in the future lightcone of x}} W_{\mu,\kappa,L}(x,w) +$$

$$\sum_{w':\exp_y(w')=x \text{ and w' is in the future lightcone of y}} W_{\mu,\kappa,L}(y,w') +$$

$$\sum_{w:\exp_x(w)=y \text{ and w is space like at x}} W_{\mu,\kappa,L}(x,w).$$

The reader must understand how this definition differs from the previous one and that $\Delta_{F,\mu,\kappa,L}(x,y)$ is everywhere differentiable.

6.2 Physical remarks regarding the construction.

The reader not familiar with Feynman diagrams will understand that the kind of integrals considered in this chapter are mandatory for any spin-0 theory to be well defined. We have not given any attention so far to the regularization of the spin-$\frac{1}{2}$ or spin-1 propagator since that would merely have obfuscated the presentation and would not have brought any essential point on the table. The reasons for doing so are, however, somewhat different: I am not aware of any known physical theory which contains the derivatives of the Fermi-propagator so that the fine details of the spin-0 regularization scheme near the lightcone seem somewhat unnecessary to implement albeit everything proceeds in a straightforward way. The regularization scheme for any integer spin propagator, on the other hand, is identical to the spin-0 case so that there is nothing lost in presenting that case only. We shall come back to estimates for regularized propagators of higher spin in a later chapter since, at this point, the global constraints imposed on such regularization would seem to be rather ad-hoc and not so important. It is however in chapter nine, while studying an alternative vacuum cosmology of the hyperbolic type that the relevant constraints will become physically clear and intuitive. For this reason, I have decided to postpone the issue of regularization for higher spin propagators to chapter ten, where they will serve as the basis for a very general proof of finiteness of Feynman diagrams.

So, we are not completely done yet and the cosmological $SO(3)$-class of common vierbeins for $g_{\mu\nu}$ and $h_{\mu\nu}$ shall have an important role to play in that construction as the reader can guess immediately. Upper bounds on spin components have to happen in such cosmological class of reference frames as arbitrary local Lorentz transformations are in the position to violate any inequality. Therefore, we shall just finish this chapter by making some further physical comments; our regularization scheme near the lightcone imposed that the latter is a forbidden place for a particle to be found, even a massless one. Whereas in the traditional theory, the lightcone comes with a delta singularity, the latter has been smeared in a certain band in the space like and timelike region near the lightcone where the width is measured in the cosmological class of reference frames determined by a preferred timelike vectorfield. This is a sensible thing to do as the lightcone is a kind of "unbreakable" wall in the classical theory which has now been softened in the quantum theory. Instead of taking the negative attitude that the regularization scheme has many liberties and therefore, the canonical character of our theory is destroyed, one should cherish the very fact that our computations show that such regularization is necessary. Moreover, it falls within the class of Lorentz covariant theories and therefore, this is the very best we can do, no further determination can reasonably be expected. Only religious bigots with no understanding of physics whatsoever could keep on complaining about this very point but to them I say: go and study some elementary relativity my "friend". Therefore, let me stress that the only aspect of our construction which appears to call for a "deeper" picture regards the insertion of the weight factors associated to the creation and annihilation process of particles. Again, our construction showed that such idea is *necessary* but it might find a "prettier" origin in a different representation of the same physics. Here, we must make the deep remark that the property of particle statistics hinges upon the reflection symmetry of the weight factors associated to the creation and annihilation processes; therefore, we *need* a new principle of nature from which statistics follows, which is the one that such processes are indeed reflection symmetric. This was to be expected as the entire, standard, argumentation behind particle statistics hinges upon properties of flat space time and there is no a-priori reason for standard or any kind of statistics to hold in curved space time. In this book, we shall not study theories of that kind as they would require very novel and deep ideas regarding its very formulation.

Chapter 7

Interactions for (non-abelian) gauge theory and gravitons.

This chapter will show why the integrals in the previous chapter were important: we will systematically explain all the necessary ingredients prior to defining the interacting theory. We shall work in the utmost generality and explain the genesis of gauge invariance, a principle necessitated by the operational Minkowski theory from a different point of view. Therefore, gauge invariance comes in a different guise and indeed, our derivation is very different but gives completely isomorphic results in the aforementioned unphysical limit. The intention of this chapter is to give a *formal* definition meaning that all questions regarding the "well-definedness" of the theory are postponed until chapters ten and eleven, where we shall provide for a rather general answer. Indeed, our theory is given by a so-called perturbation series and there need to be shown two things: (a) finiteness of and appropriate bounds on the constituents of the series (b) convergence of the series in some well chosen domain of the interaction parameters. In the literature on standard quantum field theory, regarding point (a), one has a control over finiteness (after an illegitimate infinite substraction) for the so-called renormalizable theories but no bound whatsoever so that addressing (b) is far out of reach. We shall progress systematically in this chapter by including particles of higher spin one at the time; also, we shall define simplified scattering amplitudes first, which resemble expressions found in the literature. Only at a later stage do we define the real physical amplitudes and weights associated to more complex processes. All proofs in chapters ten and eleven refer to the simplified situation; the same results regarding the more complex physical amplitudes are however quickly obtained by means of the same methods.

7.1 Interactions for spin-0 particles.

In order to describe realistic theories such as QED and QCD, we should include spin degrees of freedom by means of the gamma-matrices; however, we will content ourselves for now with the description of relativistic ϕ^4 theory. We have so far defined the two point function and the associated Feynman propagator for a free particle born or created at a space time event x; in the definition of the two point function, geodesic paths were allowed to travel into the relativistic past whereas this is explicitly forbidden in the definition of the Feynman propagator. So what is interaction? It is nothing but the process of scattering of information currents at an intermediate spacetime events z called an internal vertex of the Feynman diagram. Depending upon the type of interaction vertices do we have different theories and we shall see that local symmetry properties impose severe constraints on the possible types of interaction vertices. Every vertex comes with a coupling constant λ which in non-gravitational theories has been assumed to be dimensionless; given that the mass dimension of every interaction vertex has to be four, we conclude that the only possible such theory is one with interaction vertices having four legs corresponding to one endpoint of the Feynman propagator given that the propagator has dimension mass squared. We are almost there now, it turns out to be that in standard quantum mechanics, λ has to be multiplied by $-i$ which constitutes one aspect of "unitarity"; furthermore, it is logical that the contribution of each unlabbeled scattering diagram, which is just a multigraph, has to be divided by its symmetry factor[1] $s(D)$ and that all diagrams have to be ssummed over and we demand each interaction vertex to be connected to an IN or OUT boundary vertex[2]. Hence, we want to calculate an amplitude such as $\langle \mathrm{OUT}\, y_1, \ldots, y_m | \mathrm{IN}\, x_1, \ldots, x_n \rangle$ regarding the creation process of n particles at x_i and the subsequent detection of m particles at y_j where the y_j come after the x_i in psychological time. Here, the order of the spacetime labels x_i, y_j is irrelevant given that our particles obey bosonic statistics; with those reservations

$$\langle \mathrm{OUT}\, y_1, \ldots, y_m | \mathrm{IN}\, x_1, \ldots, x_n \rangle = \sum_D \frac{(-i\lambda)^V}{s(D)} \left(\prod_{j=1}^{V} \int_{\mathcal{M}} dz_j \sqrt{g(z_j)} \right)$$
$$\prod_{\text{edges }(\alpha_i,\alpha_j)} \Delta_F(\alpha_i, \alpha_j)$$

where $\alpha_k \in \{z_l, x_i, y_j\}$ and V stands for the number of internal vertices of the diagram. Moreover, the IN vertices are never directly connected to themselves

[1]We call a transformation $\phi : G \to G$, where G is an unlabelled multigraph a symmetry if it permutes the interaction vertices a well as the lines (of indentical type) preserving adjacency. The latter means that if E is an edge between v and w then $\alpha(E)$ is an edge between $\alpha(v)$ and $\alpha(w)$.

[2]In the standard formulation, one also considers diagrams which contain vertices which are not connected to an IN or OUT vertex, the so called vacuum bubbles. However, all such diagrams are ill defined and decouple from the dynamics, meaning they just provide for a constant amplitude (which is infinite) in calculating matrix elements between IN and OUT states; hence, they are not physically relevant and we ignore them in our discussion.

and the same holds for the OUT vertices; this definition also holds in case IN or OUT are empty. In case IN and OUT are empty, the amplitude equals one; this constitutes the *definition* of the theory and we notice that the only unexplained factor so far concerns the domain of integration $\mathcal{M}$. We shall give a physical motivation for this definition in the comments section of this chapter: there is very little, if almost nothing, ad-hoc about it as the reader will understand.

We will now specify three distinct choices of $\mathcal{M}$ one can, in principle, make; the reader familiar with quantum field theory will recognize that the issue we are discussing here is related to an "instantaneous" notion of vacuum state which in Minkowski spacetime is determined by means of global Poincaré covariance in the (interacting) theory. To set the stage for the discussion, let us remark that relativity theory does not contain anything like a psychological "now" and leads to a block universe view where you can communicate with one and another without the other person actually "being" there. It is predestined that he or she will be there and that is all that matters; there is no such thing as conscious perception in relativity. This, at least, is a good feature of quantum theory. A ramification of this viewpoint is that you can actually ask "localized" questions about the universe which reside in the actual psychological past and future! This is something quantum theory also forbids, all your personal questions are projected on the psychological now which is reasonable. On the other hand, relativity allows you in principle to take any observer test line (which ideally has no gravitational back reaction effects) and that will not change anything to the observations made in the rest of the universe. That is clearly a physical statement which is necessary to do science. So here, we need to introduce a global "npw" and that is really everything which is left from the uual notion of time in uantum mechanics in our theory. The dynamical predictions of the theory depend upon this now; since there is no way of experimentally verifying such idea, we conclude that the predictions made by any healthy physical theory should not depend much upon such a ntion of absolute space. In either quantum theory is not that unlocal as most people would be willing to believe. I have in the past discussed this notion of now as being the consequence of the growth of a four dimensional universe to the future, very much in the same spirit as the Rideout-Sorkin growth process in causal set theory, where the growth is associated to an external time. Note, that this NOW has nothing to do with some Newtonian character of the interactions but reveals the healthy point of view that all interactions from IN to OUT cannot travel to the *realized* past of IN and nor to the potential future of OUT. Therefore, we have to complement the setup explained so far with an initial S_I and final S_F spatial hypersurface associated to the IN state and OUT state, meaning that they contain x_i and y_j respectively and are disjoint. Associated to two hypersurfaces, one can define the sandwiched region $R(S_I, S_F)$ as the set of events x such that every curve emanating from x either remains within $R(S_I, S_F)$ or leaves it by crossing $S_I \cup S_F$; hereby, it is assumed that any inextensible past oriented causal curve leaves $R(S_I, S_F)$ at S_I and any inextensible future oriented causal curve leaves $R(S_I, S_F)$ at S_F. Note that this definition is framed as such that closed timelike curves are allowed for given that we did not demand the hypersurfaces to be achronal; moreover, S_I, S_F are chosen such that $R(S_I, S_F)$ is nonempty. When the domain of integration of the interaction vertices is given by $R(S_I, S_F)$, we say that our quantum theory is of TYPE I and this picture fully coincides with the usual Schrodinger evolution expressed in the interaction picture (the reader may want to verify explicitely that this is indeed the case). In a *classical* theory of the universe, one can speak about the realized past as a classical spacetime to the past of S_I; this is *not* so in a quantum theory where the past consists out of measurements made and those do not constitute a classical spacetime at all since spacetime is rather unknown when no measurement occurs. In that regard, for classical spacetime theories, we define a quantum theory to be of type II when all events past to S_F have to be taken into account in the computation of the transition amplitude $\langle \mathrm{OUT}\, y_j, j = 1 \ldots m | \mathrm{IN}\, x_i, i = 1 \ldots n \rangle$. In a sense, this would mean that the recorded spacetime history plays a role in the behavior of elementary particles when evolving to the future: this is not a silly idea but one reminiscent of Einstein causality. Type I is the most logical one in the sense that elementary particles do not care about the future nor about the past and all computations have to occur within $R(S_I, S_F)$. Type III is the opposite of Type II meaning that the potential (deterministic) future of S_I beyond S_F plays a role in the determination of the relevant amplitudes; the computations in quantum field theory regarding the S matrix are of Type II and III in the sense that the entire space time is taken into account, this may have been motivated by the very definition of asymtotic states but they simply don't exist in the interaction picture and such viewpoint is worthles anyway in an evolving (quantum) cosmology. We now turn our heads towards the right setup for a spin-1 theory, again with no dimensionful coupling constants.

7.2 Interactions for general (non-abelian) gauge theories.

In this section, we describe some part of the known relevant physics regarding interactions between spin-$\frac{1}{2}$ particles by means of massless spin-1 bosons. In doing so, we assume that the theory has some global symmetry group giving rise to charges for the fermionic as well as the bosonic particles in case the group is non-abelian. Standard non-abelian gauge theory is constructed in a way where the transformation laws of the gauge potential, or particle polarization, $A^{\alpha}_{\mu}(x)$ are *induced* from the transformation laws of the multiplets on representation space. This means, in particular, that all interactions are constructed from the basic object

$$\mathbf{A}_{\mu} = A^{\alpha}_{\mu}(t_{\alpha})^{m}_{n}$$

by means of Lie-algebra operations as well as the trace operation between two Lie-algebra elements, where the t_{α} constitute the generators of the Lie-algebra

$$[t_{\alpha}, t_{\beta}] = i f^{\gamma}_{\ \alpha\beta} t_{\gamma}$$

and $\mathrm{Tr}(t_\alpha t_\beta) = g_{\alpha\beta}$. Here,

$$f_{\gamma\alpha\beta} = g_{\gamma\delta} f^{\delta}_{\ \alpha\beta}$$

is totally anti-symmetric in its three covariant indices and $g_{\alpha\beta}$ is positive definite. The former condition follows from the latter as one may show and the latter is required for a finite dimensional positive probability interpretation. Moreover, we do not take into account interactions requiring a length scale which implies all our interaction vertices are of mass dimension four. Moreover, by the very definition of interaction, the respective vertices need to be tri- or four-valent since gauge fields contribute a mass dimension of 1, while spinorial particles a mass dimension of $\frac{3}{2}$. All these considerations leave us with the following intertwiners

$$\begin{aligned} f_{\alpha\beta\gamma} \left(\nabla_\kappa A^\alpha_\mu\right) A^\beta_\nu A^\gamma_\lambda g^{\kappa\nu} g^{\mu\lambda} &= -i\mathrm{Tr}\left(\nabla_\kappa \mathbf{A}_\mu \left[\mathbf{A}_\nu, \mathbf{A}_\lambda\right]\right) g^{\kappa\nu} g^{\mu\lambda} \\ f_{\alpha\beta\gamma} f^{\alpha}_{\ \beta'\gamma'} A^\beta_\mu A^\gamma_\nu A^{\beta'}_{\mu'} A^{\gamma'}_{\nu'} g^{\mu\nu} g^{\mu'\nu'} &= -\mathrm{Tr}\ \left(\left[\mathbf{A}_\mu, \mathbf{A}_\nu\right] \left[\mathbf{A}_{\mu'}, \mathbf{A}_{\nu'}\right]\right) g^{\mu\mu'} g^{\nu\nu'} \end{aligned}$$

concerning the self interaction of the gauge particles[3]. There remain the following two vertices

$$(\mathbf{A}_\nu)^m_n (\gamma^a)^i_j e^\nu_a(x) \Psi_{im} \overline{\Psi}^{jn}, \;\; f_{\alpha\beta\gamma} \nabla^\mu v^\beta \overline{v}^\gamma A^\alpha_\mu$$

where the last vertex is constructed from

$$\mathbf{v} = v^\alpha t_\alpha$$

as

$$-i\mathrm{Tr}\left(\left[\nabla^\mu \mathbf{v}, \overline{\mathbf{v}}\right] \mathbf{A}_\mu\right).$$

Therefore, just out of completeness, we should supplement our theory with a spin zero particle and anti-particle transforming in the adjoint representation of the symmetry group with Fermionic statistics due to the anti-symmetry of the commutator. In chapter six, we argued that the relevant two point functions for such particle had to be given by

$$W^{\alpha\beta}_a(x,y) = \overline{\theta}(x)\theta(y)W(x,y)g^{\alpha\beta}, \;\; W^{\alpha\beta}_p(x,y) = \theta(x)\overline{\theta}(y)W(x,y)g^{\alpha\beta}$$

and in calculating Feynman diagrams, integration over the Grassmann coordinates should occur.

Hence, we are left with precisely the same four interaction vertices as in standard non-abelian gauge theory. Moreover, by rescaling the Lie algebra generators $t_\alpha \to \lambda t_\alpha$, suitably defining the interaction constant $\tilde{g}$ of the theory and by redefining the Grasmann numbers $\theta \to \lambda' \theta$ we obtain that they are of standard textbook form; that is, derived from a gauge covariant closed two form field

[3]The only other two remaining options $\mathrm{Tr}\left(\nabla_\kappa \mathbf{A}_\mu \left[\mathbf{A}_\nu, \mathbf{A}_\lambda\right]\right) g^{\kappa\mu} g^{\nu\lambda}$ and $\mathrm{Tr}\ \left(\left[\mathbf{B}_{\mu\nu\lambda}, \mathbf{A}_\kappa\right]\right) Z^{\mu\nu\lambda\kappa}$ vanish by means of symmetry. Types such as $\mathrm{Tr}\ \left(\left[\left[\mathbf{A}_\mu, \mathbf{A}_\nu\right], \mathbf{A}_\lambda\right] \mathbf{A}_\kappa\right) Z^{\mu\nu\lambda\kappa}$ can be expressed in terms of the previous cases.

strength.

We have just finished the discussion of the structure of the interaction vertices; now, we turn our head towards the definition of the interacting theory akin to what we have accomplished in the previous chapter. As before, we define the interacting theory as a sum over connected Feynman diagrams between in IN and OUT states $|\mathrm{IN}\,(x_1, a_1), \ldots, (x_n, a_n)\rangle$ and $|\mathrm{OUT}\,(y_1, b_1), \ldots, (y_m, b_m)\rangle$ respectively where a_i, b_j is associated to A^{α}_{μ} where α is a group index in the adjoint representation or linked to v^{im} corresponding to a particle in the IN state and an anti-particle in the OUT state, or a covariant spinor and group index in the defining representation, associated to $\overline{v}_{im}$, with the opposite interpretational conventions[4]. Here, it is understood that all x_i (y_j) belong to non-intersecting spacelike, but not necessarily achronal, hypersurfaces S_I (S_F) such that S_F is in the future of S_I as before. The diagrams we consider are such that any internal vertex is connected to an IN or OUT vertex, no IN (OUT) vertices are connected by a single propagator to an IN (OUT) vertex since otherwise there would exist an IN (OUT) vertex where a particle would arrive (leave) in contrast to the meaning of IN and OUT. What we state is that the correct interpretation is given by putting the IN vertices as first argument in the Feynman propagator and the OUT vertices as last argument; we don't care about a unique interpretation for the internal vertices.

We will proceed by writing things down in case the gauge group is $U(1)$ since that simplifies notation given that there is no charge attached to photon lines and ghost particles are absent; the general case, including ghosts, following immediately from the restricted one. Therefore, as explained before, the only interaction vertex or intertwiner is given by

$$e^{\mu}_{a}(x)(\gamma^{a})^{i}_{j}$$

which has no internal *vertex* symmetries since all lines are distinguished and no interchange of lins can occur between two vertices. An internal vertex with label k is therefore represented by a triple (μ_k, i_k, j_k) where the index j_k is covariant and the remaining two contravariant in that *order*[5]. Take then the series $(b_m, \ldots, b_1, (\mu_1, i_1, j_1), \ldots, (\mu_V, i_V, j_V), a_1, \ldots, a_n)$ where V represents the number of internal vertices and define the rule that the transposition of a space time index with any other index corresponds to plus one, while the transpositionof a spinor index with another spinor index corresponds to minus one. Moreover, only covariant and contravariant spinor indices of different vertices can couple to one and another; then, we define the sign of a diagram as the sign which results from the reordering of the labels such that for all internal lines the covariant

[4] To be precise, a lower spinor index for the a_j corresponds to a lower index in the *particle* Feynman propagator, which corresponds therefore to the propagation of a particle coupling it to v^{im} whereas an upper index refers to the propagation of an anti-particle coupling it to $\overline{v}_{im}$ and the other way around for the out vertices b_j.

[5] Which is important since the particles of spin 1/2 are Fermions and exchanging them causes for a minus sign to arise.

index is just one place to the left of the corresponding contravariant one and all indices corresponding to internal vertices coupling to an IN (OUT) index are precisely one place to the left (right) of it (no matter whether it regards a covariant or contravariant index). With all this in mind, we write formally

$$\langle \mathrm{OUT}\,(y_1,b_1),\ldots,(y_m,b_m)|\mathrm{IN}\,(x_1,a_1),\ldots,(x_n,a_n)\rangle = \sum_D \frac{(-ie)^V \epsilon(D)}{s(D)}$$

$$\int dz_1 \sqrt{h(z_1)} \ldots \int dz_V \sqrt{h(z_V)} \prod \Delta_{F;\, c_l c_{p(l)}}(\alpha_l, \alpha_{p(l)})$$

$$\prod \Delta_{F,p}(\alpha_k, \alpha_{r(k)})^{j_{r(k)}}_{i_k} \prod (\gamma^{c_q})^{i_q}_{j_q}$$

where $\epsilon(D) = \pm 1$ is the sign of the diagram and $\alpha \in \{z_k, x_i, y_j\}$. I say formally, since experience has shown that the series does not converge albeit every diagram gives a finite contribution which we will show explicitly in chapter ten where we shall estimate the magnitude of a diagram. Corrections to unitarity should therefore occur and we will comment upon that in chapter eleven.

7.3 Gravitons.

We have not treated spin two particles so far yet neither from the point of abstract spin nor from the side of the two point function or Feynman propagator. We shall not treat this second issue in this chapter and we postpone it until chapter ten when proving finiteness of the respective Feynman diagrams. Therefore, we first treat spin and derive consequently the symmetry group of the interacting theory. Here, Newton's constant will come into play giving rise to a coupling constant of the dimension of length, the so-called Planck length l_p. Given that a (massless) spin-one particle is described by means of a Lorentz vector, it is natural to look for a tensor product representation of the Lorentz group

$$\Lambda^a_b \Lambda^c_d h^{bd}$$

in order to isolate massless spin two particles invariant under an irreducible representation of the little group $E(2)$ associated to the lightlike momentum vector k. Regarding the entire Lorentz group, there exist two irreducible components, the symmetric and anti-symmetric tensors and the massless spin two particle resides in the symmetric part. Indeed, as is well known, we should look for symmetric states carrying helicity ± 2, There are exactly two of them $e_i \otimes e_i$ where e_i denotes the state of helicity $(-1)^i$ for $i = 1, 2$; furthermore $k \otimes e_i + e_i \otimes k$ denotes a zero norm particle of helicity $(-1)^i$ and likewise so for $l \otimes e_i + e_i \otimes l$. Finally, there are four states of helicity zero: one of positive norm given by $e_1 \otimes e_2 + e_2 \otimes e_1$ where the norm is given by $-\eta_{ab}$, two zero norm particles given by $k \otimes k$ and $l \otimes l$ and finally one of negative norm given by $k \otimes l + l \otimes k$. The little group of k leaves a six dimensional space invariant which is given by the symmetrization of k, e_i a space of two positive norm particles of helicity ± 2, two zero norm particles of helicity ± 1 and finally two particles of helicity zero, one of positive norm and the other of zero norm. In contrast to the probability theory for a massless spin-1 particle, where the longitudinal mode could be ignored because it is of zero norm, there is no reason to ignore the massless helicity zero particle given by $e_1 \otimes e_2 + e_2 \otimes e_1$. Therefore, we conclude that any theory for a massless particle of helicity ± 2 comes with a massless particle of helicity 0, in sharp contrast to the standard view upon spin.

Given this new result, we now come to the determination of the symmetry group of the graviton theory. The big distinction with gauge theory is that the generators of the diffeomorphism Lie-algebra act quasi-locally, instead of ultra-locally, on the "gauge potential" $h_{\mu\nu}$, where we have gotten from h_{ab} to $h_{\mu\nu}$ by means of the vierbein e^a_μ, associated to the Lorentzian space time metric $g_{\mu\nu}$. Indeed, the Lie algebra of the diffeomorphism group is given by the vectorfields $\mathbf{V}$ which are realized by means of the Lie-derivative

$$\delta_{\mathbf{V}} = \mathcal{L}_{\mathbf{V}}.$$

The Lie algebra is preserved given that

$$[\mathcal{L}_{\mathbf{V}}, \mathcal{L}_{\mathbf{W}}] = \mathcal{L}_{[\mathbf{V},\mathbf{W}]}.$$

The Lie derivative on a general tensor field $T^{\mu_1 \ldots \mu_r}_{\nu_1 \ldots \nu_s}$ is given by

$$\mathcal{L}_{\mathbf{V}} T^{\mu_1 \ldots \mu_r}_{\nu_1 \ldots \nu_s} = T^{\mu_1 \ldots \mu_r}_{\nu_1 \ldots \nu_s ; \alpha} V^\alpha - T^{\beta \ldots \mu_r}_{\nu_1 \ldots \nu_s} V^{\mu_1}_{;\beta} - \ldots + T^{\mu_1 \ldots \mu_r}_{\beta \ldots \nu_s} V^{\beta}_{;\nu_1} + \ldots$$

where we have used the Levi-Civita connection associated any space time metric. We now come to the definition of what we mean with a generally covariant theory: under the usual action of space time diffeomorphisms, the space time metric $g_{\mu\nu}$ as well as the graviton polarization $h_{\mu\nu}$ transform as

$$g \to g + \mathcal{L}_{\epsilon \mathbf{V}} g, \;\; h \to h + \mathcal{L}_{\epsilon \mathbf{V}} h.$$

Subsequent application gives

$$(g + \mathcal{L}_{\epsilon \mathbf{V}} g) + \mathcal{L}_{\epsilon \mathbf{W}} (g + \mathcal{L}_{\epsilon \mathbf{V}} g) = g + \mathcal{L}_{\epsilon(\mathbf{V}+\mathbf{W})} g + \mathcal{L}_{\epsilon \mathbf{W}} \mathcal{L}_{\epsilon \mathbf{V}} g$$

and the property

$$[\delta_{\epsilon \mathbf{V}}, \delta_{\epsilon \mathbf{W}}] = \delta_{\epsilon^2 [\mathbf{V},\mathbf{W}]}$$

is needed for this to be an action. In order for $g_{\mu\nu}$ to remain *stationary* we therefore form the combination

$$g_{\mu\nu} + l_p h_{\mu\nu}$$

and define

$$\delta'_{\epsilon \mathbf{V}} h = \delta_{\epsilon \mathbf{V}} h + (l_p)^{-1} \delta_{\epsilon \mathbf{V}} g, \;\; \delta'_{\epsilon \mathbf{V}} g = 0$$

where the Plank length has been inserted because the graviton propagator has dimension mass^2. It is readily verified that

$$\delta'_{\epsilon(\mathbf{V}+\mathbf{W})} = \delta'_{\epsilon \mathbf{V}} + \delta'_{\epsilon \mathbf{W}}$$

and

$$[\delta'_{\epsilon\mathbf{V}}, \delta'_{\epsilon\mathbf{W}}] = \delta'_{\epsilon^2[\mathbf{V},\mathbf{W}]}$$

given that

$$\delta'_{\epsilon\mathbf{V}}\delta'_{\epsilon\mathbf{W}} = \delta_{\epsilon\mathbf{V}}\delta'_{\epsilon\mathbf{W}}.$$

The symmetries of a graviton theory require that internal interaction vertices between gravitons are constructed from scalar densities under the action δ' while interactions with ghost particles are constructed from tensor densities under the action δ. The rationale is the same as the one in non-abelian gauge theory where one adds all *covariant* interaction terms which do not stem from a local gauge symmetric scalar density to the theory and couples them to ghost particles.

As is well known, the interaction vertices and two point function are all we need to define a generally covariant quantum theory; we do not have any problems regarding the definition of a covariant measure.

7.4 Comments.

The picture we arrived at in this chapter is one of colliding *information* currents where, at each instant, the point of collision is uncertain and, therefore, should be integrated over. Moreover, one should sum over all diagrams given that any collision pattern should contribute. Theories do not come in any simpler form than this and the construction in this chapter is therefore almost self-evident. The only non-obvious part being the imaginary nature of the coupling constants associated to the internal vertices as well as the uniform measure attached to the diagrams. Why should the constant in front of the contribution of a Feynman diagram not depend upon the number of internal vertices as well on the size of the latter? The principle that they don't is called "unitarity" and we shall test this assumption in chapter eleven, where we will study convergence or analycity properties of the series. I realize of course that this constitutes a deviation away from traditional quantum mechanics; but one should simply accept *this* formulation as the proper one and forget about the attempts made by Dirac, Schroedinger and Heisenberg.

So, the content of this chapter is very much like the presentation in chapters five and six: preliminary and in need of closer inspection. In chapter seven, we already saw what was needed to get finite Feynman diagrams out for spin-0 particles; in the next section, we will understand what kind of global constraints on the geometry are needed to make everything well defined for spin-$\frac{1}{2}, 1, 2$ particles as well. Albeit we shall work by means of a simple prototype cosmology, a lot is to be learned from this example and it will provide us with the crucial ingredients and insights.

The reader must wonder, given that we have computed an amplitude between a process of birth of n-particles at separated locations x_i and annihilation of

m-particles later on, in the process view, also at separated locations z_j, how to calculate the correct weights serving in a relative probability interpretation. The reader will immediately understand that this concerns a natural extension of the theory laid out in chapters five and six; all particles in the IN state are created in a certain state $\sum_k \otimes_{i=1}^n \Psi^k_{x_i}$ where each $\Psi^k_{x_i}$ is defined relative to the space time point of creation x_i. All IN particles are "measured" leaving the source on spacelike hypersurfaces $\Sigma_i \subset R(S_I, S_F)$ which are spatially separated from one and another in the causal relationship restricted to $R(S_I, S_F)$. Moreover, they are measured by m "irreducible" measurement apparati given by world tubes $W_{\Sigma'_j}$ such that $z_j \in W_{\Sigma'_j}$. Hence, we have to calculate the modified amplitudes

$$\langle \mathrm{OUT}\,(y'_j, b_j), j = 1 \ldots m | \mathrm{IN}\,(y_i, a_i), i = 1 \ldots n \rangle_{\mathrm{phys}} =$$

$$\sum_D \prod_{k=1}^V \int_{\mathcal{M}} dz_k \sqrt{g(z_k)} \left(\prod_{\text{bosonic IN lines}\, i} T_{\alpha(i), e_0 \perp \Sigma_i} \right) A(D, (y'_j, b_j), (y_i, a_i), (z_k, c^r_k))$$

where r is a degeneracy index allowing an internal vertex to appear more than once and $\alpha(i) \in \{(y'_j, b_j), (z_k, c^r_k)\}$ denoting the endpoint of the IN-line associated to the boson born with parameters (y_i, a_i). Also, $y_i \in \Sigma_i$ and all IN quantum numbers are defined with respect to the $SO(3)$-class of reference frames given be $e_0 \perp \Sigma_i$; moreover, the definition of the $T_{\alpha(i), e_0 \perp \Sigma_i}$ operator has been canonically extended to accommodate for the Feynman propagator Δ_F and the reader should fill in these details. Moreover $y'_j \in \Sigma'_{j;t-\delta}$ where $z_j \in \Sigma'_{j;t} \subset S_F$; A is defined in such a way that if we drop the product

$$\prod_{\text{bosonic IN lines}\, i} T_{\alpha(i), e_0 \perp \Sigma_i}$$

then the expression reduces to the usual one. We are now in a position to define

$$\langle \mathrm{OUT}\,(y'_j, b_j), j = 1 \ldots m | \mathrm{IN} \sum_k \otimes_{i=1}^n \Psi^k_{x_i},\, \Sigma_i, i = 1 \ldots n \rangle_{\mathrm{phys}} =$$

$$\sum_k \left(\prod_{\text{all IN lines}\, i} \int_{\Sigma_i} dy_i \sqrt{h(y_i)} \right)$$

$$\text{PropagOperator} \left[\otimes_{i=1}^n \Psi^k_{x_i}(y_i); \langle \mathrm{OUT}\,(y'_j, b_j), j = 1 \ldots m | \mathrm{IN}\,(y_i, a_i), i = 1 \ldots n \rangle_{\mathrm{phys}} \right]$$

where this "propagation operator" has been defined before in chapters five and six. Given the definition of the y'_j regarding z_j, it is now clear from previous considerations how to compute the weight of detection at $\Sigma'_{j;t} \subset S_F$; the latter should be computed in the tensor product of the one particle spaces associated to z_j. For fermions, we recall that z_j is not necessarily interpreted as a point of annihilation which concludes the discussion of physical weights.

I realize that I have hidden some details in the notion above, but it should be clear what the expressions mean and how they should be calculated. For example, the definition of "propagation operator" differs for particles of spin-$0, \frac{1}{2}, 1, 2$ in such a way that I cannot use a unified notation. Nevertheless, we have treated the issue of propagation for single particle waves in full detail in all these cases; therefore, it should be clear what the definition is. The only point which might need some clarification is the extension of the T_x operator to the Feynman propagator $\Delta_F(x, y)$ for integer spin particles; the rule is that one has to multiply plane waves starting at x or y with the square root of the component of the four momentum with respect to e_0 as defined in y; also, all projection operators for waves of higher spin have to be executed in y. This finishes this chapter; ultimately, it are these physical quantities one has to compute and not the naive ones defined by

$$\langle \mathrm{OUT}\,(z_j, b_j), j = 1 \ldots m | \mathrm{IN}\,(x_i, a_i), i = 1 \ldots n \rangle.$$

However, the analysis regarding bounds on these amplitudes as performed in chapter ten remains identical and we proceed with the "naive" quantities in the remainder of this book.

Chapter 8

Study of an alternative vacuum cosmology.

We have shown so far in chapter seven on the regularization of the Feynman propagator that Minkowski space time is not suited to define a relativistic quantum theory in, given that it does not determine a dynamical notion of time and therefore does not allow for the necessary friction terms to be defined in a way which does not directly depend upon the observer. Before we proceed, some words of physical significance are in place, in a Schwarzschild and Kerr-Newman rotationally symmetric black hole solution we can speak of a null Killing horizon, which coincides with the union of black hole surfaces defined by Hawking, where our preferred timelike vectorfield, or gravitational arrow of time, becomes null and therefore quantum theory becomes ill defined again. It may be clear that generic perturbations in the initial data, even smooth ones of compact support, will destroy the Killing Horizon *and* most likely, also the strongly future asymptotically predictable character of the space time. Indeed, to my knowledge, the issue of stability regarding the very definition of an event horizon by means of the past of the boundary of the asymptotic future in some conformal space time has not been properly examined. I really do not care much about it, as I have always found this definition rather contrarian and "unphysical" to some extend (given that in quantum gravity the future is not known at all). What our thoughts above reveal is that Kerr-Newman space times also cannot serve as a background for quantum theory as the Lebesgue well-definedness of the propagator goes havoc on the horizon and also within. One might again want to resort to weaker, dual, interpretations as before but it could be that the old problems of Minkowski come back in some different jacket. With those words of caution, we now proceed to the definition of the two-point function on the $k = 0$ or spatially flat Friedmann universe, which in the case of interest serves as an alternative vacuum.

8.1 A cosmological vacuum.

The metric is given by

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$$

and the Einstein equations with cosmological constant $\Lambda' = 3\Lambda$ and homogeneous isotropic fluid reduce to

$$3\frac{\dot{a}^2}{a^2} = 8\pi\rho + 3\Lambda$$

and

$$\frac{3\ddot{a}}{a} = -4\pi(\rho + 3p) + 3\Lambda.$$

The energy momentum conservation law reads

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

while the geodesic equation equals

$$\frac{d^2t}{ds^2} + \dot{a}a\left|\frac{d\vec{x}}{ds}\right|^2 = 0, \ \frac{d^2\vec{x}}{ds^2} + 2\frac{\dot{a}}{a}\frac{dt}{ds}\frac{d\vec{x}}{ds} = 0.$$

In this section, we shall be interested in the cosmological vacuum defined by $\rho = p = 0$; in that case, the scale factor reads

$$a(t) = \alpha e^{\sqrt{\Lambda}t}$$

with $\alpha > 0$ and the Ricci tensor is given by

$$R_{\alpha\beta} = 3\Lambda g_{\alpha\beta}$$

in other words, our cosmology is an Einstein space and obeys the weak energy condition. Performing the coordinate transformation $\tilde{t} = \frac{e^{-\sqrt{\Lambda}t}}{\alpha\sqrt{\Lambda}}$ leads to the expression

$$ds^2 = \frac{1}{\tilde{t}^2\Lambda}(d\tilde{t}^2 - dx^2 - dy^2 - dz^2)$$

which shows that our Einstein space is conformally flat. It is also a space of constant positive *sectional* curvature as the Riemann tensor takes on the form

$$R_{\alpha\beta\mu\nu} = \Lambda\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\nu}g_{\beta\mu}\right)$$

a property which will be most convenient later on when performing our Wick rotation. It is nevertheless not a maximally symmetric space time such as is the case for a de-Sitter space time. Taking $\tilde{t}$ as a time coordinate suggests a big crunch while the t coordinate hints to an exponentially expanding universe.

They both determine the same unit norm timelike vectorfield up to a time orientation, which explains the qualitative difference; in the sequel, we will keep on working in the t, x, y, z instead of in the $\tilde{t}, x, y, z$ system. Further specialization of the geodesic equation leads to

$$\frac{d^2t}{ds^2} + \sqrt{\Lambda}\alpha^2 e^{2\sqrt{\Lambda}t} \left| \frac{d\vec{x}}{ds} \right|^2 = 0$$

and

$$\frac{d^2\vec{x}}{ds^2} + 2\sqrt{\Lambda}\frac{dt}{ds}\frac{d\vec{x}}{ds} = 0$$

from which it can be deduced that

$$\left| \frac{d\vec{x}}{ds} \right| = \beta e^{-2\sqrt{\Lambda}t}$$

with $\beta \geq 0$. These equations show that the affine time derivative slows down so that one may wonder wether it is possible to get at $t = +\infty$ in the first place. As we will show, this is the case for future oriented timelike geodesics but *not* so for spacelike geodesics for which the $\frac{dt}{ds} > 0$ part of the solution has a finite future t and s extend. One obtains the Newtonian law

$$\frac{d^2t}{ds^2} + \sqrt{\Lambda}(\alpha\beta)^2 e^{-2\sqrt{\Lambda}t} = 0$$

which can be integrated to give

$$\frac{e^{-\sqrt{\Lambda}t}}{\sqrt{\frac{\delta}{\alpha^2\beta^2} + e^{-2\sqrt{\Lambda}t}} + \frac{\sqrt{\delta}}{\alpha\beta}} = e^{-\sqrt{\delta\Lambda}(s+\gamma)}$$

where $\alpha, \beta, \delta \geq 0$ and $\gamma \in \mathbb{R}$. This, again, leads to

$$t(s) = -\frac{1}{\sqrt{\Lambda}} \ln\left(\sqrt{\frac{4\delta}{\alpha^2\beta^2}} \frac{e^{-\sqrt{\delta\Lambda}(s+\gamma)}}{1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)}} \right)$$

and $\gamma > 0$. It is clear that for $s < -\gamma$ the space time is past geodesically incomplete, unless $\gamma = +\infty$, while for s to plus infinity, we obtain again an approximate linear relation between t and s. The geodesic equation for the spatial part then becomes

$$\frac{d^2\vec{x}}{ds^2} + 2\sqrt{\delta\Lambda}\frac{1 + e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}{1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}\frac{d\vec{x}}{ds} = 0$$

which leads to

$$\frac{d\vec{x}}{ds} = \vec{\beta}\frac{4\delta}{\alpha^2\beta^2}\frac{e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}{(1 - e^{-2\sqrt{\delta\Lambda}(s+\gamma)})^2}$$

where $\left|\vec{\beta}\right|^2 = \beta^2$. This last formula may again be integrated to yield

$$\vec{x}(s) = \vec{r}_0 - 2\vec{\beta}\sqrt{\frac{\delta}{\Lambda}}\frac{1}{\alpha^2\beta^2}\frac{1}{1-e^{-2\sqrt{\delta\Lambda}(s+\gamma)}}$$

where, in the limit for β to 0, $\vec{r}_0$ has to renormalize by an infinite constant too. As it turns out, we have only given a parametrization for future oriented causal geodesics; in terms of the initial values x and $v = (\frac{dt}{ds})_{s=0}$, $\vec{v} = (\frac{d\vec{x}}{ds})_{s=0}$ the original parameters read

$$\begin{aligned}
\vec{\beta} &= \vec{v}e^{2\sqrt{\Lambda}t} \\
e^{-\sqrt{\delta\Lambda}\gamma} &= \frac{1}{\alpha e^{\sqrt{\Lambda}t}\left|\vec{v}\right|}\left(v - \sqrt{v^2 - \alpha^2 e^{2\sqrt{\Lambda}t}\left|\vec{v}\right|^2}\right) \\
\delta &= v^2 - \alpha^2 e^{2\sqrt{\Lambda}t}\left|\vec{v}\right|^2 \\
\vec{r}_0 &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v - \sqrt{v^2 - \alpha^2 e^{2\sqrt{\Lambda}t}\left|\vec{v}\right|^2})}
\end{aligned}$$

so in the limit of Λ to zero $\vec{r}_0$ renormalizes $\vec{x}_0$ by an infinite amount. One notices that δ has the geometric significance of the length squared of the tangent vector of the geodesic at x which we may put to one since we deal with timelike geodesics. This further simplifies our formula to

$$\begin{aligned}
e^{-\sqrt{\delta\Lambda}\gamma} &= \sqrt{\frac{v-1}{v+1}} \\
\vec{r}_0 &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v-1)}
\end{aligned}$$

and with these reservations, we obtain that

$$\begin{aligned}
t(s) &= -\frac{1}{\sqrt{\Lambda}}\ln\left(\frac{2e^{-\sqrt{\Lambda}(t+s)}}{v+1-(v-1)e^{-2\sqrt{\Lambda}s}}\right) \\
\vec{x}(s) &= \vec{x} + \frac{\vec{v}}{\sqrt{\Lambda}(v-1)} - \frac{2\vec{v}}{\sqrt{\Lambda}(v-1)\left(v+1-(v-1)e^{-2\sqrt{\Lambda}s}\right)}.
\end{aligned}$$

From the first equation, one can solve v in function of $z = e^{-\sqrt{\Lambda}s}$; the formula is given by

$$v = \frac{2ze^{\sqrt{\Lambda}(t'-t)} - 1 - z^2}{1-z^2}$$

with $z > e^{-\sqrt{\Lambda}(t'-t)}$. Insertion into the second equation fixes z by the polynomial

$$z^2 + 1 - \left(2\cosh(\sqrt{\Lambda}(t'-t)) - \Lambda\left|\vec{x'} - \vec{x}\right|^2 \alpha^2 e^{\sqrt{\Lambda}(t'+t)}\right) z = 0$$

where the evaluation holds for $(t', \vec{x}')$ future timelike related to $(t, \vec{x})$. Notice that we have an asymptotic region of radius $\frac{1}{\sqrt{\Lambda}\alpha e^{\sqrt{\Lambda} t}}$, so unlike Minkowski space time, in our vacuum cosmology, it is impossible for $\vec{x}'$ to become infinite and therefore any observer has a nontrivial horizon. It is easy to solve our equation to

$$s = -\frac{1}{\sqrt{\Lambda}} \ln \left(g(x, x'; \Lambda, \alpha) - \sqrt{g(x, x'; \Lambda, \alpha)^2 - 1} \right)$$

where

$$g(x, x'; \Lambda, \alpha) = \cosh(\sqrt{\Lambda}(t' - t)) - \Lambda \left| \vec{x'} - \vec{x} \right|^2 \frac{\alpha^2 e^{\sqrt{\Lambda}(t'+t)}}{2}.$$

In the limit for $\sqrt{\Lambda}$ to zero, this expression becomes

$$s_0^2 = \lim_{\sqrt{\Lambda} \to 0} \frac{\left((t' - t) \sinh(\sqrt{\Lambda}(t' - t)) - \sqrt{\Lambda} \left| \vec{x'} - \vec{x} \right|^2 \alpha^2 e^{\sqrt{\Lambda}(t'+t)} + O(\lambda) \right)^2}{g(x, x'; \Lambda, \alpha)^2 - 1} =$$

$$(t' - t)^2 - \alpha^2 \left| \vec{x}' - \vec{x} \right|^2$$

as it should be. This formula can be easily analytically continued to the region

$$-1 < g(x, x'; \Lambda, \alpha) < 1$$

by

$$is' = -\frac{1}{\sqrt{\Lambda}} \ln \left(g(x, x'; \Lambda, \alpha) - i\sqrt{1 - g(x, x'; \Lambda, \alpha)^2} \right)$$

where we have made the branch cut for the complex square root in the upper half plane at for example $\frac{\pi}{2}$. It is then easily computed that

$$-s'(x, x'; \Lambda, \alpha)^2 = -\frac{1}{\Lambda} \left(\arccos \left(g(x, x'; \Lambda, \alpha) \right) \right)^2$$

and one can again check that the $\sqrt{\Lambda}$ to zero limit is given by

$$-s_0'(x, x'; \alpha)^2 = (t' - t)^2 - \left| \vec{x}' - \vec{x} \right|^2 \alpha^2$$

as it should, so our formula is entirely correct. One can easily see that this result comes by considering the case $\delta < 0$ which corresponds to spacelike geodesics which live a finite amount of time t in the future as well as a finite amount of affine parameter time s in the past and the future. This is again a distinction with Minkowski which is geodesically complete and where spacelike geodesics reach out to infinite values of time in the future. The relevant formulae are deduced by performing the analytic continuation to $\delta < 0$ and putting $\delta = -1$:

$$\begin{aligned} t(s) &= -\frac{1}{\sqrt{\Lambda}} \ln \left(\frac{\sqrt{-\delta}}{\alpha\beta \sin(\sqrt{-\delta\Lambda}(s + \gamma))} \right) \\ \vec{x}(s) &= \vec{r}_0 - \sqrt{\frac{-\delta}{\Lambda}} \frac{\vec{\beta}}{\alpha^2 \beta^2 \tan(\sqrt{-\delta\Lambda}(s + \gamma))}. \end{aligned}$$

As before

$$\begin{aligned}
\vec{\beta} &= \vec{v}e^{2\sqrt{\Lambda}t} \\
e^{i\sqrt{\Lambda}\gamma} &= \frac{v+i}{\sqrt{v^2+1}} \\
\vec{x} &= \vec{r}_0 - \frac{v\vec{v}}{\sqrt{\Lambda}(v^2+1)}.
\end{aligned}$$

This reshapes our solutions as

$$\begin{aligned}
t(s) &= -\frac{1}{\sqrt{\Lambda}}\ln\left(\frac{e^{-\sqrt{\Lambda}t}}{\sin(\sqrt{\Lambda}s)v+\cos(\sqrt{\Lambda}s)}\right) \\
\vec{x}(s) &= \vec{x} + \frac{v\vec{v}}{\sqrt{\Lambda}(v^2+1)} - \frac{\vec{v}(v-\tan(\sqrt{\Lambda}s))}{\sqrt{\Lambda}(v^2+1)(1+v\tan(\sqrt{\Lambda}s))}
\end{aligned}$$

and the reader notices that in the limit $\tan(\sqrt{\Lambda}s) = v$, our assumption $\frac{dt}{ds} \geq 0$ no longer holds. Nevertheless, this solution is past incomplete in the sense that for $s = \frac{1}{\sqrt{\Lambda}}\arctan(-\frac{1}{v})$ it diverges to $t = -\infty$ and $|\vec{x}| \to \infty$. We notice that for $\tan(\sqrt{\Lambda}s) = v$ one has that $\frac{dt}{ds} = 0$ and for later times s, the geodesic evolves again towards lower $t(s)$ values. Our parameter domain goes beyond $s = \frac{\pi}{2\sqrt{\Lambda}}$ at which point $\tan(\sqrt{\Lambda}s)$ blows up to infinity but nothing special happens given that the limit of $\vec{x}$ as well as its derivatives are well defined. Hence, our parameter domain goes between $\frac{1}{\sqrt{\Lambda}}\arctan(-\frac{1}{v})$ and $\frac{\pi}{\sqrt{\Lambda}} + \frac{1}{\sqrt{\Lambda}}\arctan(-\frac{1}{v})$. We proceed first by determining the world function for the above parametrization, giving the following formulae

$$\begin{aligned}
v &= \frac{e^{\sqrt{\Lambda}(t'-t)} - \cos(\sqrt{\Lambda}s)}{\sin(\sqrt{\Lambda}s)} \\
\Lambda\alpha^2\left|\vec{x}' - \vec{x}\right|^2 e^{2\sqrt{\Lambda}t} &= \frac{(v^2+1)\tan^2(\sqrt{\Lambda}s)}{(1+v\tan(\sqrt{\Lambda}s))^2}
\end{aligned}$$

which leads to

$$s'(x,x';\Lambda,\alpha) = \frac{1}{\sqrt{\Lambda}}\arccos(g(x,x;\Lambda,\alpha))$$

a result which we obtained previously by means of analytic continuation; this formula covers the full spacelike region as the maximal length of a spacelike geodesic equals $\frac{\pi}{\sqrt{\Lambda}}$ which is precisely the range of that function. It is interesting to study the limit for $v \to +\infty$ of our solution; from any starting point in space time one arrives at $t = +\infty$ in a parameter time $s = \frac{\pi}{2\sqrt{\Lambda}}$ at which $\frac{dt}{ds} = 0$ and still the limit of the tangent vectors has unit norm. This means, in particular, that in any direction of space one can trace back these data for smaller t values providing one with a null hypersurface of events in space time demarcating, within the region of events which can be connected by means of a spacelike curve to the initial point, those events which can be reached by a spacelike *geodesic* starting at x. In particular, this horizon is given by

$$|\vec{x}' - \vec{x}| = \frac{1}{\alpha\sqrt{\Lambda}e^{\sqrt{\Lambda}t}} + \frac{1}{\alpha\sqrt{\Lambda}e^{\sqrt{\Lambda}t'}}$$

and it obviously lies fully in the region

$$-1 < g(x, x'; \Lambda, \alpha) < 1.$$

This leads us to the following definition: given a space time point x, the spacelike geodesic horizon $HS(x)$ is the boundary of the region which can be reached by means of a spacelike geodesic. Likewise, we define the future timelike horizon $HT(x)$ at x as the boundary of the region of space time which can be reached by means of timelike geodesics. $HS(x)$ is not necessarily a null hypersurface as it the case for our cosmology and neither does $HT(x)$ need to coincide with the boundary of $J^+(x)$. Note that the outer part of $HS(x)$ coincides, in our case, with the boundary of $J^-(I^+(x))$ which is the standard horizon for timelike signals in a general cosmology. Hence, there is a region of space time which cannot be reached by any geodesic starting at x; this is a novel feature to be taken into account in the quantum theory which we shall do later on. We finish this section by making a comment upon the way the vectorfield e_0 is chosen from local physical considerations. The most obvious criterion is a *quasi*-local one which says that the Riemann curvature squared (or the Ricci curvature squared) of the Riemannian metric on the orthogonal spacelike hypersurface attains an absolute minimum 0. It may be that there exists some *ultra*-local criterium by looking for minima of some function in the space time Riemann tensor components evaluated in a tetrad with timelike vector given by ∂_t. The latter characterization would be preferred in my mind but we leave such fine points for the future.

8.2 The modified propagator on the new vacuum cosmology.

Before we come to the calculation of the two point function, we need to determine the parallel transporter $\Lambda_{\beta}^{\alpha'}(x, y)$ between two points; the latter is defined, as before, by means of transport of a vector along the unique geodesic connecting x with y. Before we come to the explicit computations, let us try to guess the structure of the result based upon symmetry considerations. As is well known $-\sigma_{\mu}(x, y)$ gives the tangent co-vector at x to the geodesic connecting x with y of length equal to the geodesic length; that is

$$g^{\mu\nu}(x)\sigma_{\mu}(x, y)\sigma_{\nu}(x, y) = 2\sigma(x, y)$$

where we have suppressed Λ, α in the notation of Synge's function $\sigma(x, y)$. For future convenience, let us denote by $e_0 = \partial_t$, $e_i = \frac{e^{-\sqrt{\Lambda}t}}{\alpha}\partial_i$ the standard tetrad which is constant under parallel transport on timelike geodesics of constant $\vec{x}$. Hence, the transporter expressed with respect to this tetrad $\Lambda_b^{a'}(x,y)$ is the unit matrix if y has the same space coordinate than x. More in general, one would expect $\Lambda_b^{a'}(x,y)$ to be a Lorentz boost determined by the $e_0, e_a\sigma^a(x,y)$ plane with a magnitude proportional to $\sqrt{\sum_i \sigma_i(x,y)^2}, \sigma^0(x,y)$ where $\sigma^a(x,y) = e^{a\mu}(x)\sigma_\mu(x,y)$ and it has been understood that the a index has been raised with the flat Minkowski metric η^{ab}. Let us now make the explicit computations; the transport equation is given by

$$\begin{aligned} \frac{d}{ds}Z^0(s) + \alpha^2\sqrt{\Lambda}e^{2\sqrt{\Lambda}t}\vec{v}(s).\vec{Z}(s) &= 0 \\ \frac{d}{ds}\vec{Z}(s) + \sqrt{\Lambda}\left(\vec{v}(s)Z^0(s) + \vec{Z}(s)v(s)\right) &= 0 \end{aligned}$$

where $v^\alpha(s)$ is the unit tangent to the geodesic in affine parametrization. From our solutions for timelike and spacelike geodesics, it is easy to see that initial vectors Z perpendicular to e_0 and $\vec{v}$ remain so which confirms our claim that unit vectors perpendicular to e_0 and $e_a\sigma^a(x,y)$ are left invariant for as well spacelike as timelike geodesics[1]. Remains to figure out the boost parameter; here we study the transport of $Z = e_0$. The fact that parallel transport preserves the norm allows us to write

$$Z(s) = (\cosh(\gamma(s)), \sinh(\gamma(s))\frac{\vec{v}(s)}{\sqrt{v(s)^2-1}})$$

for timelike geodesics with $\gamma(0) = x$. Hence, we obtain that the first transport equation reduces to

$$\frac{d\gamma(s)}{ds} = -\sqrt{(v(s)^2-1)\Lambda}$$

and taking the explicit formula for

$$v(s) = \frac{v+1+(v-1)e^{-2\sqrt{\Lambda}s}}{v+1-(v-1)e^{-2\sqrt{\Lambda}s}}$$

results in

$$\gamma(s) = \left(\ln\left(\frac{1+\sqrt{\frac{v-1}{v+1}}e^{-\sqrt{\Lambda}s}}{1-\sqrt{\frac{v-1}{v+1}}e^{-\sqrt{\Lambda}s}}\right) - \ln\left(\frac{1+\sqrt{\frac{v-1}{v+1}}}{1-\sqrt{\frac{v-1}{v+1}}}\right)\right).$$

Upon substitution by the well known formula for v in function of t, t', s and s in function of $g(x,x';\Lambda,\alpha)$, we arrive after some algebra at

$$\sqrt{\frac{v-1}{v+1}} = \sqrt{\frac{e^{\sqrt{\Lambda}(t'-t)} - e^{\sqrt{\Lambda}s}}{e^{\sqrt{\Lambda}(t'-t)} - e^{-\sqrt{\Lambda}s}}}$$

[1]Invariant in the sense that the components only undergo a rescaling as to preserve the local norm.

and some rather complicated formula

$$\gamma(s) = \ln\left(\frac{1-z^2}{\left(\sqrt{1-ze^{-\sqrt{\Lambda}(t'-t)}} - \sqrt{z^2 - ze^{-\sqrt{\Lambda}(t'-t)}}\right)^2}\right)$$

$$-\ln\left(\frac{2ze^{\sqrt{\Lambda}(t'-t)} - 1 - z^2}{2ze^{\sqrt{\Lambda}(t'-t)} - 1 - z^2 - 2\sqrt{\left(z^2(e^{2\sqrt{\Lambda}(t'-t)}+1) - z^3e^{\sqrt{\Lambda}(t'-t)} - ze^{\sqrt{\Lambda}(t'-t)}\right)}}\right)$$

where $z = g(x,x';\Lambda,\alpha) - \sqrt{g(x,x';\Lambda,\alpha)^2 - 1}$. A similar result holds for spacelike geodesics and the above calculations show already that exact calculations for the two point function will look rather messy. However, regarding the issue of convergence, we can make useful estimates and it is important to notice that

$$-\ln\left(\frac{1+\sqrt{\frac{v-1}{v+1}}}{1-\sqrt{\frac{v-1}{v+1}}}\right) \leq \gamma(s) \leq 0$$

for $s \geq 0$ meaning that in the limit for the affine parameter towards future infinity, the boost parameter converges to a finite negative value. Only in the limit for v towards infinity does $\gamma(s)$ converge to infinity too. Towards the past, $\gamma(s) \to +\infty$ if $t(s) \to -\infty$; for spacelike geodesics, one obtains a different qualitative result which is that in the limit for the affine time towards its finite negative and positive values (with a difference of $\frac{\pi}{\sqrt{\Lambda}}$), $\gamma(s)$ blows up towards minus infinity in the limit towards the positive value and to plus infinity in the limit towards the negative value.

We now come to the determination of the two point function and will denote the relevant formula in terms of first derivatives of Synge's function $\sigma_a(x,x';\Lambda,\alpha)$ and the boost parameter

$$\gamma(x,x';\Lambda,\alpha).$$

There is no need to use their explicit expressions to arrive at the desired results and if the reader wants to, he or she can manipulate the final expressions by substituting for the above obtained formula. The two point function we shall study is given by

$$W_\mu(x,x';\Lambda,\alpha) = e^{-6\mu\Lambda m^2}\int \frac{d^4k}{(2\pi)^3}\delta(k^2-m^2)\theta(k^0)e^{ik^a\sigma_a(x,x';\Lambda,\alpha)}e^{-\mu(k^0)^2-\mu(\Lambda_a^{0'}(x,x';\Lambda,\alpha)k^a)^2}$$

where x' is causally related to x, since otherwise we would have to include reflection symmetric terms, and $\Lambda_a^{0'}(x,x';\Lambda,\alpha)$ is given by

$$\Lambda_a^{0'}(x,x';\Lambda,\alpha)k^a = \cosh(\gamma(x,x';\Lambda,\alpha))k^0 + \sinh(\gamma(x,x';\Lambda,\alpha))\frac{\vec{k}.(\vec{x}'-\vec{x})}{|\vec{x}'-\vec{x}|}$$

and x' is supposed to lie within the total geodesic horizon of x (here the total geodesic horizon is defined as the boundary of the set of events which can be reached from x by means of a geodesic). In chapter seven, we studied this integral in Minkowski space time where ∂_t has to be associated to the timelike vectorfield defined by some physical observer making the quantum particle feel an aether due to the him or herself. We shall compute here *explicitly* that precisely the same issues show up and go somewhat deeper into the nature of the Wick transformation. Denoting by $\vec{\sigma}(x, x'; \Lambda, \alpha) = (\sigma_i(x, x'; \Lambda, \alpha))$, where the i index refers to the spatial part of the vierbein and not to the space components of σ_μ, and correspondingly

$$|\vec{\sigma}(x, x'; \Lambda, \alpha)| = \sqrt{\sum_i \sigma_i(x, x'; \Lambda, \alpha)^2}$$

we arrive, after some algebra, to

$$\begin{aligned} W_\mu(x, x'; \Lambda, \alpha) \quad = \quad & \frac{e^{-6\mu\Lambda m^2}}{8\pi^2} \int_0^\infty dk \frac{k}{\sqrt{k^2+m^2}} \int_{-k}^{k} dz e^{i\sqrt{k^2+m^2}\sigma_0 - \mu(1+\cosh^2(\gamma))(k^2+m^2)} \\ & e^{-\mu \sinh^2(\gamma)\left(z + \left(\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2} - i\frac{|\vec{\sigma}|}{2\mu\sinh^2(\gamma)}\right)\right)^2} \\ & e^{\mu \sinh^2(\gamma)\left(\frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2} - i\frac{|\vec{\sigma}|}{2\mu\sinh^2(\gamma)}\right)^2} \end{aligned}$$

where we have suppressed all dependencies upon x, x', Λ, α in the right hand side and used that $\vec{\sigma}(x, x'; \Lambda, \alpha) \sim \vec{x}' - \vec{x}$ in evaluating the integral. At this point, it is instructive to give some comment about the general structure of the integral. The μ suppression terms we included are sufficient for our purposes just as it is the case for Minkowki. This property is rather independent of the behavior of γ which we have shown to converge to an asymptotic, finite negative value in the limit of the parameter time towards plus infinity for future timelike related events. It may be better to replace the $(V_a k^a)^2$ suppression term by a $h_{ab}k^a k^b$ suppression where h_{ab} is, as before, the Riemannian metric determined by the timelike vectorfield. It is immediately seen that the absolute value of $W_\mu(x, x'; \Lambda, \alpha)$ is bounded by a universal constant proportional to $\frac{1}{\mu}$, which is actually sufficient for our proof of finiteness since we have to take into account the Riemannian surpression term due to κ. However, we are interested in more detailed properties of this function and carry on.

Coming back to the calculation of $W_\mu(x, x'; \Lambda, \alpha)$, the integral over z is a Gaussian one which cannot be exactly done, but to which we can find a useful upper bound. In particular, we estimate integrals of the type

$$F(k, c) = \int_{a(k)}^{b(k)} dz e^{-a(z+ic)^2}$$

for $c \geq 0$. Taking the differential of $F(k, c)$ with regards to c results in

$$\frac{d}{dc}F(k, c) = i \int_{a(k)}^{b(k)} \frac{d}{dz} e^{-a(z+ic)^2} = i\left(e^{-a(b(k)+ic)^2} - e^{-a(a(k)+ic)^2}\right).$$

Therefore we obtain that

$$|F(k,c)| \leq \int_0^c dz e^{az^2} \left(e^{-a\,b(k)^2} + e^{-a\,a(k)^2} \right) + \frac{\sqrt{\pi}}{\sqrt{a}}$$

and upon using our previous results, the latter expression reduces to

$$|F(k,c)| \leq \frac{1}{acg(\sqrt{a}c)} \left(e^{ac^2} - 1 \right) \left(e^{-a\,b(k)^2} + e^{-a\,a(k)^2} \right) + \frac{\sqrt{\pi}}{\sqrt{a}}.$$

For the purpose of asymptotic analysis, we may clearly ignore the constant on the right hand side, since the resulting expressions converge exponentially fast in the limit for $|\vec{\sigma}|$ towards infinity, and we obtain that

$$|W_\mu(x,x';\Lambda,\alpha)| \sim \frac{1}{4\pi^2} \left(1 - e - \frac{|\vec{\sigma}|^2}{4\mu \sinh^2(\gamma)} \right) \int_0^\infty dk \frac{k}{\sqrt{k^2+m^2}}$$

$$e^{-\mu(k^2+m^2)} \left(e^{-\mu \sinh^2(\gamma)\left(k + \frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2}\right)^2} + e^{-\mu \sinh^2(\gamma)\left(k - \frac{\cosh(\gamma)}{\sinh(\gamma)}\sqrt{k^2+m^2}\right)^2} \right).$$

which shows that $W_\mu(x,x';\Lambda,\alpha)$ does not converge to zero in the limit for $|\vec{\sigma}|$ to infinity, keeping γ fixed, for x' future causally related to x. It is much harder to obtain an estimate in case $|\vec{\sigma}|$ remains finite but σ_0 blows up to plus infinity. The only result I am able to obtain is that the propagator converges to a nonzero value in σ_0 along $|\vec{x}' - \vec{x}| = 0 = \vec{\sigma}$ and $\gamma = 0$.

We now turn our head towards the study of the impact of κ on $W_{\mu,\kappa}(x,x';\Lambda,\alpha)$. Denote by

$$E(x,x';\Lambda,\alpha,\kappa) = e^{-\kappa \int_0^{\tilde{s}} \sqrt{h_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds}}}$$

the exponentiated energy along the timelike geodesic connecting x with x', then

$$W_{\mu,\kappa}(x,x';\Lambda,\alpha) = E(x,x';\Lambda,\alpha,\kappa) W_\mu(x,x';\Lambda,\alpha)$$

and, in case $|\vec{x}' - \vec{x}| = 0$, then one has

$$E(x,x';\Lambda,\alpha,\kappa) = e^{-\kappa |t'-t|}.$$

In order for every sub integral of

$$\alpha^3 \int dx' e^{3\sqrt{\Lambda}t'} \left| \Delta_{F,\mu,\kappa}(x,x';\Lambda,\alpha) \right|^n$$

to be finite, it is therefore necessary that $\kappa > 3\sqrt{\Lambda}$, a condition which did not appear in Minkowski space time. Regarding the proof of perturbative finiteness, we will require some other bound to which we will come back to in a short while. Actually, without any further computation, the reader should realize that our cosmology behaves very different from ordinary Minkowski; on one side, one has the existence of all horizons and on the other, one notices that Minkowski can be conformally compactified while the Friedmann cosmology can't. The latter feature causes scattering processes in the future to occur with a higher amplitude which might ultimately not be suppressed anymore by our geodesic energy terms $E(x, x'; \Lambda, \alpha, \kappa)$. This would forbid Type III quantum theories but not Type II or Type I; in Minkowski space time, there is no such distinction between the past and the future and therefore, such behavior is not to be expected. As it will turn out, Type III quantum theories are allowed for as long as κ is sufficiently large. Coming back to our computation, one immediately sees that

$$\int ds \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} = \int_0^{\sqrt{2\sigma(x,y)}} ds \sqrt{2\left(\frac{dt}{ds}\right)^2 - 1}$$

where

$$\frac{dt}{ds} = \frac{v + 1 + (v-1)e^{-2\sqrt{\Lambda}s}}{v + 1 - (v-1)e^{-2\sqrt{\Lambda}s}}$$

an expression which decreases from v to 1 at $s = \infty$. In Minkowski $\Lambda = 0, \alpha = 1$ and this expression equals $\sqrt{2(\sigma^0(x,y))^2 - 2\sigma(x,y)} = |x - y|$; for a cosmological space time this is very different. In general, we have that,

$$\int_0^{\sqrt{2\sigma}} ds \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \geq \sqrt{2\sigma} \sim |t' - t|$$

for $|t' - t|$ large and $|\vec{x}' - \vec{x}| < \frac{e^{-\sqrt{\Lambda}t}}{\alpha\sqrt{\Lambda}}$ fixed. Moreover, the inequalities and similarities become equalities in the limit for σ to infinity. Note that σ is infinite within the lightcone and zero on the lightcone in the limit for t' towards ∞, but the pathology on the lightcone needs to be studied further. Actually, one obtains that the energy increases from the symmetrical point $|\vec{x}' - \vec{x}| = 0$ towards the boundary of the lightcone along the "hyperbola" of constant σ which is contained within a domain of compact $\vec{x}'$. We need a finer estimate in order to obtain conclusive results on convergence; some algebra shows that

$$\int_0^{\sqrt{2\sigma}} ds \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \geq \frac{1}{\sqrt{\Lambda}}\sqrt{\frac{v+1}{v-1}}$$

$$\left(\ln\left(\frac{1 + \sqrt{\frac{v-1}{v+1}}}{1 + \sqrt{\frac{v-1}{v+1}}e^{-2\sqrt{2\Lambda\sigma}}}\right) + \ln\left(\frac{1 - \sqrt{\frac{v-1}{v+1}}e^{-2\sqrt{2\Lambda\sigma}}}{1 - \sqrt{\frac{v-1}{v+1}}}\right)\right)$$

upon substitution of v by

$$v = \frac{2e^{-\sqrt{\Lambda}(\sqrt{2\sigma} - (t'-t))} - 1 - e^{-2\sqrt{2\Lambda\sigma}}}{1 - e^{-2\sqrt{2\Lambda\sigma}}}.$$

In order to study the σ to zero limit, we only need to take into account the second term; this one reduces in leading order to

$$\frac{1}{\sqrt{\Lambda}}\ln\left(\frac{3 + \frac{1}{1 - e^{-\sqrt{\Lambda}(t'-t)}} + \frac{1}{e^{\sqrt{\Lambda}(t'-t)} - 1}}{\frac{1}{1 - e^{-\sqrt{\Lambda}(t'-t)}} + \frac{1}{e^{\sqrt{\Lambda}(t'-t)} - 1} - 1}\right)$$

meaning that for large $|t' - t|$ this expression behaves approximately as $|t' - t| + \frac{\ln(4)}{\sqrt{\Lambda}}$ which is all we need. Actually, due to the nature of the Riemannian metric, we immediately have a lower bound of $|t' - t|$ on the (Lorentzian) energy and an upper bound on the *Riemannian* distance of $|t' - t| + \frac{1}{\sqrt{\Lambda}}$; the constant of $\frac{\ln(4)}{\sqrt{\Lambda}}$ is the only nontrivial thing in the above formula and the reader can easily see that this estimate is very accurate. This means that in the limit for σ equal to zero and $|t' - t|$ towards infinity, the exponentiated energy goes as

$$E(x, x'; \Lambda, \alpha, \kappa) = \frac{1}{(\sigma^0)^{\frac{\kappa}{\sqrt{\Lambda}}}}$$

something which falls quicker off than $\frac{1}{(\sigma^0)^3}$ given our previous bound on κ.

Towards the past, we have that the local energy is an increasing quantity and

$$\infty > \sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \geq \sqrt{2v^2 - 1}$$

which means that the energy is larger than

$$\sqrt{2(\sigma^0)^2 - 2\sigma}.$$

Akin to the future timelike case, this lower bound is actually insufficient as in the limit for $t'(s)$ to minus infinity, one obtains that

$$\sigma^0 = \sqrt{2\sigma}\frac{1 + e^{2\sqrt{2\Lambda\sigma}}}{e^{2\sqrt{2\Lambda\sigma}} - 1}$$

which converges to $\frac{1}{\sqrt{\Lambda}}$ in the limit for σ to zero. Just like in the previous case, one could perform the full integration,

$$\int_{-\sqrt{2\sigma}}^{0} ds\sqrt{h_{\alpha\beta}\frac{dx^\alpha}{ds}\frac{dx^\beta}{ds}} \geq \frac{1}{\sqrt{\Lambda}}\sqrt{\frac{v+1}{v-1}}$$

$$\left(\ln\left(\frac{1 + \sqrt{\frac{v-1}{v+1}}e^{2\sqrt{2\Lambda\sigma}}}{1 + \sqrt{\frac{v-1}{v+1}}}\right) + \ln\left(\frac{1 - \sqrt{\frac{v-1}{v+1}}}{1 - \sqrt{\frac{v-1}{v+1}}e^{2\sqrt{2\Lambda\sigma}}}\right)\right)$$

where

$$v = \frac{1 + e^{2\sqrt{2\Lambda\sigma}} - 2e^{\sqrt{\Lambda}(t' - t + \sqrt{2\sigma})}}{e^{2\sqrt{2\Lambda\sigma}} - 1}$$

or simply remark that the energy is always greater or equal to $|t' - t|$, which is all we actually need.

Similar convergence properties apply for spacelike geodesics, as the reader may want to verify for himself which finishes the discussion of this section. The only important conclusion is that the energy is always larger that the t' distance traveled which is sufficient to obtain convergent integrals. There remains something to be said about the Riemannian metric $h_{\alpha\beta}$ associated to our cosmological space time: it is a metric of constant negative sectional curvature $-\Lambda$ and therefore, balls in this metric have a volume which blows up at most exponentially fast in the radius due to a well known theorem in Riemannian geometry. Our Riemannian space has constant sectional curvature but is again not maximally symmetric; this behavior of balls in the Riemannian metric poses however no problem for our Type II quantum theory as the volume of the past lightcone blows up linearly in $-t'$ for t' towards minus infinity in opposition to the volume of the future lightcone which blows up exponentially in $\left|\tilde{t} - t\right|$ and the $t < t' < \tilde{t}$ slice of the lightcone contains the intersection of the future lightcone with the $\left|\tilde{t} - t\right|$ ball which reaches above the $\tilde{t} - t - \frac{1}{\sqrt{\Lambda}}$ slice and therefore has a volume scaling as $e^{3\sqrt{\Lambda}(t'-t)}$ which indeed shows exponential scaling of the balls for late times t'.

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All considerations in this chapter reveal that the hyperbolic behavior of the Wick rotation $h_{\mu\nu}$ of $g_{\mu\nu}$ has an effect on the quantum theory regarding the behavior at late times t. The latter impact is important and regards many fine details in the next chapter, but it is not insurmountable as we shall see and a Type III quantum theory can be defined on the cosmological vacuum. The reader understands by now that the entire analysis regarding finiteness of Feynman diagrams is going to rely upon the following property: a Riemannian geometry is called exponentially finite if and only if for any x, we have that

$$\int_{\mathcal{M}} P(d(x,y))e^{-\kappa d(x,y)}\sqrt{h(y)}dy < R(P,\kappa)$$

for any $\kappa > 0$, polynomial P and some $R(P,\kappa) > 0$. Here $R(P,\kappa)$ is supposed to go to zero in the limit for κ to plus infinity. Euclidean space time, the Wick rotation of Minkowski, is exponentially finite but the Wick rotated Friedmann cosmology is *not* so when considering the entire asymptotic future. It is however exponentially finite towards the geodesic region of every point x restricted to the sub-space time $t \leq \tilde{t}$ and we have worked our way towards this. In other words, the exponential blow up in the radius r for Riemannian balls $B(x, r)$ poses no problem when considering the intersection with the region contained within the (Lorentzian) geodesic horizon of x restricted to $t \leq \tilde{t}$ given that for large r, this intersection blows up *linearly* in r as opposed to the short scale r^4 behavior. It may be clear that we can nevertheless accommodate for the entire cosmological vacuum by means of the following notion: a Riemannian geometry is called exponentially finite on a scale $\zeta > 0$ if and only if for any x we have that

$$\int_{\mathcal{M}} P(d(x,y)) e^{-\kappa d(x,y)} \sqrt{h(y)} dy < R(P, \kappa)$$

for any $\kappa > \zeta > 0$, polynomial P and some $R(P, \kappa) > 0$. This remark concludes this chapter.

Chapter 9

Perturbative finiteness.

In this chapter, we gather all our insights obtained so far and prove that the interacting theory is well defined at the perturbative level, meaning that every Feynman diagram is finite, and we aspire to obtain useful bounds. We proceed step by step and start by investigating the regularized propagators for spin-$\frac{1}{2}, 1, 2$ particles and obtain the required bounds on the propagator as well as on the spin-derivatives theirof with respect to the preferred $SO(3)$-class of vierbeins. From hereon, we eliminate all non-trivial structure of the interaction vertices so that we are left with ordinary integrals over space time of the function

$$e^{-\kappa d(x,y)}$$

which allows one to obtain several bounds on the respective Feynman diagrams. Our bounds on the propagators reveal that we have to work in space times such that the physical Wick rotation provides for an exponentially finite Riemannian geometry. It is with respect to that class that all our results pertain.

However, before coming to all that, it is somewhat amusing to quiet the mind of the impatient physicist who might have misunderstood quite some details of semi-classical gravitational physics and related to that, the so-called cosmological constant problem. Indeed, the suggested fascination of some physicists with this problem, amongst which 't Hooft, has always baffled me since the CC-problem is really no stranger than the infinite renormalizations occurring in ordinary quantum field theory. This is something 't Hooft can live with, probably because he got a Nobel prize for that piece of mathematical "art", but on the other hand, he "feels" that there is something deep behind the CC-issue probably requiring a deterministic quantum mechanics. So, as I told you, there are still those who aspire to become electron psychologists, an ambition which is correlated to the juvenile delusion that one can become a man "who knows everything". I will make it very clear now that in our setup, there is no cosmological constant problem and the reader may appreciate this at several levels: indeed, there is even no theory of semi-classical gravitation.

9.1 No CC-problem.

One might at this point reflect if one can still couple geometry semiclas

$$\langle 0|T_{\mu\nu}(x)|0\rangle =$$

$$\lim_{y\to x}\left(\partial_\mu \partial_{\nu'} W(x,y) - \frac{1}{2} g_{\mu\nu'}(x,y)\left(g^{\alpha\beta'}(x,y)\partial_\alpha \partial_{\beta'} W(x,y) - m^2 W(x,y)\right)\right)$$

and we will now spawn some comments hereupon. In our *full* regularization scheme involving μ, κ, L we obtain that

$$\langle 0|T_{\mu\nu}(x)|0\rangle = 0$$

being an equality which is of course covariantly conserved. Therefore, in the approximation of no interactions, the state with no electrons present does not contribute as a source for the gravitational field which is obviously the only sensible answer. Again, in case interactions are included or nontrivial states are considered no such expression can be found in our theory. We will see now what happens in the μ, κ regularization scheme: in this case, the expression does not exist because the limit differs when y approaches x from the spacelike or timelike side. The fundamental reason herefore is to be found in the "reflection symmetry" in the suppression terms for spacelike geodesics, something which only depends upon an axis and not a magnitude, nor a specific orientation. We recall that this symmetry was needed to obtain bose statistics which crucially determined the definition of the Feynman propagator. Now, it may very well be that bose statistics is something which does not survive in a curved space time, but then the Feynman propagator would depend upon a frame of reference as there is no canonical way to define it. This is an avenue which we shall not take here; the reader, moreover, notices that the limit taken for y in the future lightcone of x gives an expression which is not covariantly conserved at all. This can be easily seen by noticing that for $y \in I^{\pm}(x)$ sufficiently close to x one has that

$$W_\mu(x,y) = \int \frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)e^{ik^a \sigma_a(x,y)}$$

$$e^{-\mu\left(K_{ab}(x)k^a k^b + K_{a'b'}(y)k^{a'}_{\star\, -\sigma^c(x,y)} k^{b'}_{\star\, -\sigma^c(x,y)}\right)}$$

where $\sigma(x,y)$ denotes as usual Synge's world function and the index a refers to the operation $e^\mu_a(x)\partial_\mu$ applied to it. The quadratic form $K_{ab}k^a k^b$ satisfies the property that it blows up quadratically in any Lorentz frame towards infinity if k^0 goes to infinity. In this limit $y \to x$, $W_\mu(x,x)$ becomes a smooth function of $K_{ab}(x)$ only since $\sigma_a(x,x) = 0$. The latter, however, does not satisfy a conservation law since generically $K_{ab;\nu}(x) \neq 0$ and the same reasoning applies to the whole energy momentum tensor where second covariant derivatives of $K_{ab}(x)$ come into play and the expression becomes much more complicated. More abstract and from first principles, there is a-priori no good reason why the coincidence limit of derivatives applied to an amplitude for particle propagation should have something to do with a *vacuum* expectation value of some energy momentum tensor. The way geometry is influenced by quantum particles must therefore be encoded in a new theory which requires a super metric, a universal, and therefore background independent, metric on the space of all Lorentzian geometries (and matter configurations thereupon). This author has written ideas regarding this super-metric up in his PhD thesis.

It is nevertheless interesting to note that the vanishing of the vacuum expectation value of the "energy momentum tensor" or equivalently, dark energy, has nothing to do with *measuring* a zero cosmological constant. Indeed, given a world line γ of a detector in eigentime parametrization, the effective vacuum energy at t *might* be given by the formula

$$\dot{\gamma}^{\mu}(t-\epsilon-\delta)\dot{\gamma}^{\nu'}(t-\epsilon)S_{\mu\nu'}(\gamma(t-\epsilon-\delta),\gamma(t-\epsilon))$$

where

$$S_{\mu\nu'}(x,y)=\partial_{\mu}\partial_{\nu'}W(x,y)-\frac{1}{2}g_{\mu\nu'}(x,y)\left(g^{\alpha\beta'}(x,y)\partial_{\alpha}\partial_{\beta'}W(x,y)-m^2W(x,y)\right).$$

The latter is dependent upon the worldline and retardation times ϵ,δ and might give the impression of an accelerating cosmology even when the pressure terms are not the same. The point however, is that this inequality cannot be measured due to the non-commutation of the respective energy momentum components.

9.2 Bounds on regularized spin-zero Feynman propagators.

This section will be brief and technical, but the underlying physical and mathematical ideas should be clear. We will regularize the (Feynman) propagator in such a way that all norms of covariant derivatives of the latter, where the norm is defined with respect to the $SO(3)$-class of vierbeins, are bounded as $Ce^{-\kappa' d(x,y)}$ where C is some constant. This "universal" property, which is possible due to the nature of our regularization, is sufficient to arrive at a universal proof for finiteness of Feynman diagrams for any interacting theory with *any* kind of interaction vertices, at least in space times such that the Wick rotation is exponentially finite on some high scale. To summarize our results so far, we obtained for spin-0 particles that

$$W_{\mu,\kappa,L}(x,y)=\int\frac{d^4k}{(2\pi)^3}\delta(k^2-m^2)\theta(k^0)\sum_{w:\exp_x(w)=y}e^{-\frac{L^2}{(w^a w_a)^2}}e^{-ik^a w_a}$$

$$e^{-\kappa\int_0^1\sqrt{h(w(s),w(s))}ds}e^{-\mu(V^{\alpha}k_{\alpha})^2-\mu(V_{\beta'}k^{\beta'}_{\star w})^2+R(w)}\text{ symmetric if } w \text{ is spacelike.}$$

on the condition that there exist open neighborhoods $\mathcal{V}_x$ and $\mathcal{W}_y$, as well as $\mathcal{O}_{x,w}\subset T^{\star}\mathcal{M}$ such that: (a) $\mathcal{O}_{x,w}\cap\mathcal{O}_{x,w'}=\emptyset$ (b) for any $x'\in\mathcal{V}_x, y'\in\mathcal{W}_y$

there is exactly one $w' \in \mathcal{O}_{x,w}$ such that $\exp_{x'}(w') = y'$ and

$$\phi_{\mu,\kappa,L}(x', k^{a'}, y') = \sum_{w' : \exp_{x'}(w') = y'} \tilde{\phi}_{\mu,\kappa,L}(x', k^{a'}, w').$$

This regularization procedure does clearly not depend upon any local Lorentz frame and the integral is Lorentz invariant as it should. Before we come to the higher spin operators, whose analysis is only slightly more complicated than the one for spin-0 particles, let me indicate the bounds we are going to construct and are needed for the subsequent analysis. First, let us show under which generic conditions a bound of the type

$$|W_{\mu,\kappa,L}(x,y)| < C(\mu, \kappa, g, V, \epsilon) e^{-(\kappa - \epsilon) d(x,y)}$$

where C is a constant depending upon the geometry and μ, $0 < \epsilon \ll \kappa$ and d the Riemannian distance defined by h, can be constructed prior to showing that the same can be done for higher derivatives given that we smoothened the lightcone.

Sometimes, it happens that an infinite number of geodesics between two points exists on a space time with a trivial first homotopy group such as is the case for a closed Friedmann universe which has topology $S^3 \times \mathbb{R}$. When Wick rotating this space time, there exists a minimal length on the closed spacelike geodesics, which are also geodesics in the Wick rotated metric; albeit every such geodesic can be deformed to a point. Therefore, in this particular example, a closed geodesic that winds around n-times has at least (exactly) n-times this minimal length. This is the main feature we are interested in; suppose now that the closed geodesics are due to a nontrivial first homotopy group and that arbitrary winding numbers occur. Under rather generic conditions, we may associate to each homotopy generator a minimal length squared $M(h) > 0$ (Gromov) such that the energy of a curve with winding number $n > n_0 > 0$ between[1] x and y is greater than $d(x,y) + n\frac{M(h)}{d(x,y)+1}$ which is another expression of the fact that higher winding numbers come with a multiple of a fixed length. These considerations lead one to

$$|W_{\mu,\kappa,L}(x,y)| \leq e^{-\kappa d(x,y)} \sum_{w : \exp_x(w) = y} C_{\mu,\kappa}(x,w) e^{-\frac{L^2}{(w_a w^a)^2}} e^{-\kappa n(w) \frac{M(h)}{d(x,y)+1}}$$

where $0 < C_{\mu,\kappa}(x,w) < C_{\mu,\kappa}$. Here, further estimates regarding

$$\sum_n e^{-\kappa \frac{nM(h)}{d(x,y)+1}} = \frac{1}{1 - e^{-\kappa \frac{M(h)}{d(x,y)+1}}} \leq C(\mu, \kappa, h)(d(x,y) + 1)$$

can be made and the division through $d(x,y) + 1$ stems from infinitely large homotopy classes and can be ignored when all nontrivial topology resides in a

[1]The n_0 serves here to avoid the pathological cases where, for example, the length of two h-geodesics equals the minimal distance d and, moreover, they have a relative winding number of one.

compact region of space time. These conditions are not always true on non-compact space times in case singularities are present, giving rise to topology change and $M(h) = 0$.

Therefore, under rather generic conditions, we obtain that

$$|W_{\mu,\kappa,L}(x,y)| < D(\mu,\kappa,h)(1 + d(x,y))e^{-\kappa d(x,y)}$$

which leads to a bound of the type

$$|W_{\mu,\kappa,L}(x,y)| < C(\mu,\kappa,h,\epsilon)e^{-(\kappa-\epsilon)d(x,y)}$$

for any $0 < \epsilon \ll \kappa$. Bounding the derivatives is a far more difficult task to perform given that in the above, the specific details of Synge's function or the energy functional didn't matter. Proving the assertions following below is a task in global analysis which has never been made before as far as I know. Therefore, the reader should take them as assumptions which are most likely true for our class of cosmological vacua; the formal proof of which constitutes a gap in our knowledge. More in particular, we shall assume that, given some $N \in \mathbb{N}$ there exists some $0 < \epsilon < \kappa$ such that

$$\sqrt{W_{\mu,\kappa,L;\beta_1,\ldots,\beta_i}(x,y)h^{\alpha_1\beta_1}\ldots h^{\alpha_i\beta_i}W_{\mu,\kappa,L;\alpha_1,\ldots,\alpha_i}(x,y)}$$

$$< C_i(\mu,\kappa,L,g,h,\epsilon)e^{-(\kappa-\epsilon)d(x,y)}$$

for $i : 0 \ldots N$ and α_k, β_k any index-pair referring to x or y respectively. The same assumption will hold regarding the Feynman propagator: I am unaware under what circumstances one can strengthen this assumption for an arbitrary number of derivatives and research of such fine points is left for the future.

9.3 Bounds on regularized spin-$\frac{1}{2}$ propagators.

It was shown in chapter six that the correct frictionless two point function for particles and anti-particles of spin-$\frac{1}{2}$ are given by

$$W_p(x,y)_i^{j'} = \int_{T^\star\mathcal{M}_x} \frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)$$

$$\sum_{w:\exp_x(w)=y} (\Lambda^{\frac{1}{2}}(x,w))_r^{j'}(k_a(\gamma^a)_i^r + m\delta_i^r)e^{-ik^a w_a}$$

and

$$W_a(x,y)_{j'}^{i} = \int_{T^\star\mathcal{M}_x} \frac{d^4k}{(2\pi)^3}\delta(k^2 - m^2)\theta(k^0)$$

$$\sum_{w:\exp_x(w)=y} (k_a(\gamma^a)_r^i - m\delta_r^i)((\Lambda^{\frac{1}{2}}(x,w))^{-1})_{j'}^r e^{-ik^a w_a}$$

where $\Lambda^{\frac{1}{2}}(x,w)$ is the spin transformation associated to parallel transport of a spinor along the geodesic between x and y determined by w. It has been shown that

$$W_p(x,y)_i^{j'} + W_a(y,x)_i^{j'} = 0$$

for $x \sim y$ using reflection symmetry so that spin-$\frac{1}{2}$ particles exhibit Fermi-statistics. We now intend to regularize this propagator in the same way as it occurred for the spin-0 particle while still maintaining the Fermi property: again, this is a *condition* posed on the regularization scheme and by no means a proof. This relates to my previous comment that particle statistics relates to Minkowski space time and that there is no a-priori reason why particle statistics should hold in a curved space time with local geometric excitations. The viewpoint of the generalized Schroedinger equation suggests that

$$\widetilde{W}^{\mu,\kappa,L}_{p,a}(x,y) = \sum_{w:\exp_x(w)=y \text{ and w is causal.}} W^{\mu,\kappa,L}_{p,a}(x,w) +$$

$$\frac{1}{2} \sum_{w:\exp_x(w)=y \text{ and w is spacelike.}} \left(W^{\mu,\kappa,L}_{p,a}(x,w) - W^{\mu,\kappa,L}_{a,p}(y,-w_{\star w})\right)$$

a definition which differs in a slight but not unimportant way from the original one in my publications. There, we did not bother about smoothening the light-cone since no derivatives of Fermi-propagators are ever taken, which resulted in the alternative definition

$$\widetilde{W}^{\mu,\kappa,L}_{p,a}(x,y) = \frac{1}{2}\left(W^{\mu,\kappa,L}_{p,a}(x,y) - W^{\mu,\kappa,L}_{a,p}(y,x)\right)$$

if $x \sim y$ and $W^{\mu,\kappa,L}_{p,a}(x,y)$ otherwise. The difference regards the treatment in both prescriptions of the spacelike geodesics connecting x with y which remain spacelike once y crosses the boundary of $J^{\pm}(x)$. This difference has been taken away in the novel definition which allows for a correct smoothening procedure near the lightcone. Both definitions do not depend upon any local Lorentz frame and the propagator has the correct transformation properties under combined Lorentz and spin transformations. Again, the reader may infer that on a general class of backgrounds

$$\widetilde{W}^{\mu,\kappa,L}_{p}(x,y) = e^{-\kappa d(x,y)} \sum_{w:\exp_x(w)=y} \Lambda^{\frac{1}{2}}(x,w)\left(C^{\mu,\kappa,L}_{p;b}(x,w)\gamma^b + C^{\mu,\kappa,L}_{p}(x,w)1\right)$$

and

$$\widetilde{W}^{\mu,\kappa,L}_{a}(x,y) = e^{-\kappa d(x,y)} \sum_{w:\exp_x(w)=y} \left(C^{\mu,\kappa,L}_{a;b}(x,w)\gamma^b + C^{\mu,\kappa,L}_{a}(x,w)1\right)(\Lambda^{\frac{1}{2}}(x,w))^{-1}$$

where we have used that $\Lambda^{-\frac{1}{2}}(y,-w_{\star w}) = \Lambda^{\frac{1}{2}}(x,w)$. Just like in the previous case do we obtain from elementary considerations that

$$\sum_{w:\exp_x(w)=y} \left|C^{\mu,\kappa,L}_{\alpha;b}(x,w)\right| < C(\mu,\kappa,L,g,h)(d(x,y)+1)$$

and likewise for $\sum_{w:\exp_x(w)=y} \left| C^{\mu,\kappa,L}_{\alpha}(x,w) \right|$ where $\alpha \in \{a,p\}$. Here, the above inequalities are taken with respect to the preferred $SO(3)$-class of cosmological vierbeins which is evident given that $C^{\mu,\kappa,L}_{a;b}(x,w)$ behaves as a Lorentz vector and likewise do we need a norm estimate, with respect to the same vierbein, of the propagator. The relevant matrix norm is given by

$$||A|| = \left(\mathrm{Tr}(A^{\dagger}A)\right)^{\frac{1}{2}}$$

and an elementary computation yields

$$||W^{\mu,\kappa,L}_{p}(x,y)|| =$$

$$e^{-\kappa d(x,y)} || \sum_{w:\exp_x(w)=y} (\Lambda^{\frac{1}{2}}(x,w)) \left(C^{\mu,\kappa,L}_{p;b}(x,w)\gamma^b + C^{\mu,\kappa,L}_{p}(x,w)1 \right) || \leq 2e^{-\kappa d(x,y)}$$

$$\sum_{w:\exp_x(w)=y} \sqrt{\mathrm{Tr}\left(\Lambda^{\frac{1}{2}}(x,w)^{\dagger}\Lambda^{\frac{1}{2}}(x,w)\right)} \sqrt{\sum_b \left| C^{\mu,\kappa,L}_{p;b}(x,w) \right|^2 + \left| C^{\mu,\kappa,L}_{p}(x,w) \right|^2}$$

which can be further bounded to

$$2e^{-\kappa d(x,y)} \left(\sup_{w:\exp_x(w)=y} \sqrt{\mathrm{Tr}\left(\Lambda^{\frac{1}{2}}(x,w)^{\dagger}\Lambda^{\frac{1}{2}}(x,w)\right)} \right)$$

$$\left(\sum_{w:\exp_x(w)=y} \left(\sum_b \left| C^{\mu,\kappa,L}_{p;b}(x,w) \right| + \left| C^{\mu,\kappa,L}_{p}(x,w) \right| \right) \right).$$

Finally, this is majorated by

$$D(\mu,\kappa,g,V)e^{-\kappa d(x,y)} \left(\sup_{w:\exp_x(w)=y} \sqrt{\mathrm{Tr}\left(\Lambda^{\frac{1}{2}}(x,w)^{\dagger}\Lambda^{\frac{1}{2}}(x,w)\right)} \right) (1 + d(x,y))$$

which puts us into the position to make the following global definition. We say that the tuple (g,V) defines a spin-transport which is $0 < \delta$ exponentially finite if and only if

$$\sup_{w:\exp_x(w)=y} \sqrt{\mathrm{Tr}\left(\Lambda^{\frac{1}{2}}(x,w)^{\dagger}\Lambda^{\frac{1}{2}}(x,w)\right)} \leq F(g,V)e^{\delta d(x,y)}.$$

Obviously, we assume $\delta < \kappa$ so that

$$||W^{\mu,\kappa,L}_{p}(x,y)|| < E(\mu,\kappa,g,V,\epsilon)e^{-(\kappa-\delta-\epsilon)d(x,y)}$$

for any $0 < \epsilon \ll \kappa - \delta$ which finishes our discussion of the necessary bounds on the spin-$\frac{1}{2}$ propagator.

As mentioned previously, it is not costumary to develop theories in which derivatives of the Fermi-propagator are taken but if the reader wishes to, he or she may assume a similar bound to hold on the derivatives of the propagator. It would be worthwhile to study all these issues in far greater detail and try to say something about it from the point of view of global analysis. However, I am unaware of any such result and this book is just a temporary, organic reflection of what I know at this instant in time about this topic. As mentioned in the introduction, the reader is more than welcome to work on the remaining open issues which need further attention.

9.4 Spin-one and two propagators.

Qualitatively, all important details about regularized propagators have been revealed, but we shall nevertheless present the case of spin-$1,2$ given that those constitute the only propagators from which derivatives have to be taken in interacting theories such as non-abelian gauge theory and gravity. For a massless spin-one particle associated to a compact symmetry group, the propagator is given by

$$W^{\mu,\kappa,L}_{\gamma\nu';\alpha,\beta'}(x,y) =$$

$$-g_{\alpha\beta'} \sum_{w:\exp_x(w)=y} g_{\gamma\nu'}(x,w) W_{\mu,\kappa,L}(x,w)$$

where

$$g_{\gamma\nu'}(x,w) = (\Lambda^{-1}(x,w))^{\mu}_{\nu'} g_{\gamma\mu}(x)$$

is the parallel transport of the bi-tensor along the geodesic determined by w. Here, $W_{\mu,\kappa,L}(x,w)$ constitutes the usual spin-0 expression and the reader remembers that in interactions this propagator is contracted with the vierbein $e_a(x)$. Therefore, with respect to our $SO(3)$-class of cosmological vierbeins, we demand that

$$||\Lambda(x,w)^{a'}_{b}|| \leq F(g,V) e^{\delta d(x,y)}$$

a condition of δ exponential finiteness. In the same way as before, we obtain that

$$||W^{\mu,\kappa,L;a'}_{b;\alpha\beta'}(x,y)|| \leq |g_{\alpha\beta'}| \, E(\mu,\kappa,L,g,V,\epsilon) e^{-(\kappa-\delta-\epsilon)d(x,y)}$$

for all $0 < \epsilon \ll \kappa - \delta$. We shall assume similar bounds to hold on the first N spin derivatives of this expression so that all tricky aspects of non-abelian gauge theory are covered for.

We haven't said too much about spin two particles yet and we shall make up this deficit to some extend here; from symmetry considerations, one can derive that the two point function is given by

$$W^{\mu,\kappa,L}(x,y)_{\alpha\beta,\alpha'\beta'} = W_{\mu,\kappa,L}(x,w)$$

$$\sum_{w:\exp_x(w)=y} \left(g_{\alpha\alpha'}(x,w) g_{\beta\beta'}(x,w) + g_{\beta\alpha'}(x,w) g_{\alpha\beta'}(x,w) - \frac{1}{2} g_{\alpha\beta}(x) g_{\alpha'\beta'}(y) \right).$$

The factor $\frac{1}{2}$ has been chosen such that

$$W^{\mu,\kappa,L}(x,y)_{\alpha\beta,\alpha'\beta'}g^{\alpha\beta}(x) = W^{\mu,\kappa,L}(x,y)_{\alpha\beta,\alpha'\beta'}g^{\alpha'\beta'}(y) = 0$$

thereby eliminating all trace degrees of freedom. As before, we evaluate this propagator with respect to our $SO(3)$-class of vierbeins leading to the tensor

$$W^{\mu,\kappa,L}(x,y)_{ab,a'b'} = \sum_{w:\exp_x(w)=y}$$

$$\left(\Lambda(x,w)_{a'a}\Lambda(x,w)_{b'b} + \Lambda(x,w)_{a'b}\Lambda(x,w)_{b'a} - \frac{1}{2}\eta_{ab}\eta_{a'b'}\right) W^{\mu,\kappa,L}(x,w)$$

where a norm estimate of the "coefficients"

$$C(x,w)^{ba'}_{ab'} = \Lambda(x,w)^{a'}_{a}\Lambda(x,w)^{b}_{b'} + \Lambda(x,w)^{a'b}\Lambda(x,w)_{b'a} - \frac{1}{2}\delta^{b}_{a}\delta^{a'}_{b'}$$

can be made due to the δ-exponentially finite character of the Lorentz transporters. In general, we therefore have that

$$||W^{\mu,\kappa,L}(x,y)^{ba'}_{ab'}|| \leq E(\kappa,\mu,L,g,V,\epsilon)e^{-(\kappa-\epsilon-2\delta)d(x,y)}$$

where the trace-norm has been taken with respect to the usual $SO(3)$-class of vierbeins. Now, we shall assume that a similar bound holds for the first N spin-derivatives of the two point function, a condition which should again be further examined.

This finishes our treatment of propagators and quite evidently, we assume all the above to be valid for the Feynman propagator even if the presentation has been made for the two point function. This is quite legitimate as no essential points in the above analysis change by doing so; we are all set and ready now to prove perturbative finiteness under the geometrical constraints mentioned so far in this and previous chapters. I deem these constraints to be quite mild and again, they should be further investigated in the future.

9.5 Bounds on Feynman diagrams.

In this section, we will proceed in a few steps by treating first the so-called ϕ^4 theory and give two different types of bounds on the Feynman diagrams. The first one is specific to the theory while the second one is of a more universal nature: we shall illustrate that point of view by reducing the calculations for non-abelian gauge theory to the ones of ϕ^4 theory. Next, we move to the theory of massless spin two particles or gravitons, which can also be reduced in this way, but contains a few details relating to nonperturbative aspects which are not present in the latter theories. In particular, the friction parameter μ shall be related to the Planck length squared l_p^2; everything which will be said below pertains to Lorentzian geometries g having an exponentially finite Wick transform h at some scale $\zeta > 0$.

Let us start by mentioning an obvious equation for general Feynman diagrams, interpreted as graphs, which is that

$$V - I = C - L$$

where V is the total number of *internal* vertices of a Feynman diagram, I its number of internal lines, hereby excluding the legs towards the external points, and L is the number of loops. Finally, C is the number of components of the graph; for ϕ^4 theory and connected diagrams C is bounded by

$$C \leq \frac{n+m}{2}$$

where n, m are the number of IN and OUT vertices respectively. With these conventions, we have that the absolute value of every Feynman diagram is bounded by

$$c(m,\mu)^{I + \frac{n+m-n'-m'}{2}} \int dz_1 \sqrt{h(z_1)} \ldots \int dz_V \sqrt{h(z_V)}$$
$$\prod_{\text{all lines } (\alpha_i,\alpha_j)} e^{-\kappa d(\alpha_i,\alpha_j)}$$

where we have used that the spin-0 Feynman propagator is bounded by

$$c(m,\mu) e^{-\kappa d(x,y)}$$

and $\alpha_i \in \{z_k, x_i, y_j\}$. Moreover, for ϕ^4 theory, one has that

$$I + \frac{n' + m'}{2} = 2V$$

where $0 \leq n' \leq n$ and $0 \leq m' \leq m$ so that the prefactor may be exactly written as

$$c(m,\mu)^{2V + \frac{n+m}{2} - n' - m'}$$

. Here n', m' denote the number of IN or OUT vertices which are connected to an internal vertex. Before we proceed, let us mention some easy to see fact about the Friedmann cosmology; if z is within the geodesic horizon of x and y, then it is in the geodesic horizon of the midpoint of x and y in the Riemannian metric[2]. This observation is most convenient in the following estimates which constitute a straightforward generalization of our previous inequalities. We start by deducing a universal and simple bound which does not depend at all on the details of the interaction vertices as well as on the distances between the exterior vertices. It is simply given by

$$c(m,\mu)^{2V + \frac{n+m}{2}} R(1,\kappa)^V$$

[2]This follows most easily from the convexity of the horizon of z in the Riemannian metric d which the reader may prove as an exercise.

which is most easily proved by induction on the number of internal vertices V. Here,

$$\int_{\mathcal{M}} d^4y \,\sqrt{h(y)} e^{-\kappa d(x,y)} < R(1,\kappa)$$

as defined at the end of chapter nine. If $V = 0$, then the bound is easily seen to hold since $e^{-\kappa d(\alpha,\beta)} \leq 1$ for every leg joining two external vertices. Suppose now that the bound is true for $V \geq 0$, we will prove it for $V + 1$. Take any internal vertex v connected by at least one edge to an exterior vertex α and remove it; the effect is that we obtain a diagram with four extra external vertices (we copied four times the internal vertex) but with one internal vertex less. Remove all external legs to v from the new diagram, then the remaining part is bounded by

$$R(1,\kappa)^{V-1}.$$

Now, there remains to identify the four vertices again and perform the remaining integration over this vertex; the latter gives an extra factor of $R(1,\kappa)$ because we still have at least one external leg which proves the result. This shows that the diagram blows up in a suitable way, but there remains of course the "entropy" factor associated to all Feynman diagrams with V internal vertices and n IN and m OUT vertices. The latter remains to be investigated in the next chapter but it is very well possible that unitarity may have to be given up to make the series analytic.

This is by far the easiest proof that the Feynman diagrams are finite; in case of ϕ^4 theory, it is possible to make another, useful, estimate in case the geometry is spherical; in either, we assume that our Riemannian manifold h satisfies a volume bound for a ball of radius r around x by

$$\mathrm{Vol}_4(B(x,r)) \leq K r^4$$

for some metric dependent constant K. This includes a Type II theory for the cosmological vacuum as mentioned previously. Consider n points z_i and take the integral

$$\int_{\mathcal{M}} dz \sqrt{h(z)}\, e^{-\kappa \sum_{i=1}^{n} d(z_i,z)}$$

then, as previous, this may be bounded by

$$e^{-\frac{\kappa}{n-1}\sum_{i<j} d(z_i,z_j)} \int_{y;\, \exists z_i, z_j : d(y,\frac{z_i+z_j}{2}) < \frac{4}{3} d(z_i,z_j)} \sqrt{h(y)} dy +$$

$$e^{-\frac{\kappa}{n-1}\sum_{i<j} d(z_i,z_j)} \int_{y;\, \forall i,j\; d(y,\frac{z_i+z_j}{2}) \geq \frac{4}{3} d(z_i,z_j)} dy\, e^{-\frac{\kappa}{2(n-1)}\left(\sum_{i<j} d(y,\frac{z_i+z_j}{2})\right)} \sqrt{h(y)}.$$

Note here the factor of 2 in the denominator of the exponential in second integral; this originates from the fact that in a general Riemannian space

$$d(x,y) + d(y,z) \geq d(x,z) + \frac{1}{2} d(y, \frac{x+z}{2})$$

for $d(y, \frac{x+z}{2}) \geq \frac{4}{3} d(x, z)$. The latter formula can again be estimated by

$$e^{-\frac{\kappa}{n-1} \sum_{i<j} d(z_i, z_j)} \left(\left(\frac{4}{3} \right)^4 K \sum_{i<j} d(z_i, z_j)^4 + R\left(1, \frac{\kappa}{2(n-1)}\right) \right)$$

and the only thing the reader should notice is the division of κ through $n-1$ which lowers convergence for diagrams with multiple internal vertices. We will not apply the above estimate consistently but look for a finer estimate which will provide one with better convergence properties. Actually, we will be set with a Kirchoff diagram where the flow is given by some rational proportion of $\kappa d(x_i, z_j)$ or $\kappa d(y_j, z_k)$; at any instant of the computation, these proportions add up to one. The optimal way of spreading around is by ensuring that the you do not subdivide into smaller portions; in that way, the suppression factor at the vertex remains constant κ. Homogeneous fractalizing is the worst that can happen since it lowers κ substantially after a few vertices have been run through. Loops make no difference whatsoever, in case we have a loop and there are three external vertices, two with current κ and one with current 2κ then we obtain that κ does not get renormalized, nor at the vertex nor at the legs. Also, in case we have a loop with only two external points each with current κ, there is no lowering of κ neither at the vertex nor at the legs.

Let us reason why homogeneous spreading is a bad idea; in case any of the currents associated to a leg consists out of several pieces, then a lowering of κ will occur, but such lowering will always be less than is the case for a vertex with four external currents associated to four distinct graph points. We will now determine the maximal contribution of homogeneous fractalizing: start at any vertex z_i, then the most severe contribution regarding the integral comes when no loop is present and likewise, this situation divides κ through the largest number three. Pick now any neighboring vertex, then again, the largest division occurs again when there are three other external legs, dividing the $\frac{1}{3}$ leg into 3 times $\frac{1}{9}$ and the remaining $\frac{2}{3}$ per other leg by two which gives $\frac{1}{3}$ and yields the suppression factor of $\frac{\kappa}{6}$ on the second vertex. In the third step, the worst that can happen is that a leg of the first and second vertex meet since that would cause maximal diversification. The leg from the first vertex contains two factors $\frac{1}{3}$ and 3 factors $\frac{1}{9}$ and the same for the leg coming from the second vertex. Therefore, diversification would lead to 4 times $\frac{1}{6}$ and 6 times $\frac{1}{18}$ on the other two legs, giving a suppression of $\frac{\kappa}{12}$ at the third vertex. Clearly, this reasoning is catastrophic and we now turn our head towards no fractalizing.

This case is easy and one can partition the set $S = \{x_i, y_j\}$ into pairs $(\alpha_{2i-1}, \alpha_{2i})$; with these reservations, the quantitative result reads

$$c(m, \mu)^{2V + \frac{n+m}{2}} P(d(\alpha_{2i-1}, \alpha_{2i})\,; i = 1 \ldots \frac{n' + m'}{2}) e^{-\kappa \sum_{i=1}^{\frac{n+m}{2}} d(\alpha_{2i-1}, \alpha_{2i})}$$

where P is a polynomial of degree $4V$ and the highest order coefficient is bounded by

$$\left(\frac{4}{3}\right)^{4V} K^V \left(2^4\right)^{\frac{V(V-1)}{2}} .$$

It is the behavior of this last coefficient which makes our bound on the series non analytic. The above formula is always true for any diagram as the reader may wish to show by induction on the number of internal vertices, by integrating out a vertex without altering the connectivity properties[3], and does not hinge upon special features of the diagram such as the property that there exists a partition of the edges into paths, connecting the exterior points, and loops such that no internal vertex belongs to two loops. It is always possible to cover a graph by means of curves connecting the exterior points and loops but sometimes it is the case that two loops always intersect[4]. The reader might wonder whether the above estimate is not too crude given that we do not rely upon the details of $W_\mu(x, x')$ at all. Also, we replaced the Lorentzian geodesic energy by the inferior Riemannian distance, which is an approximation as well. My answer is a resounding no: these approximations will not significantly influence the result for the following reasons. Regarding W_μ, only very slight falloff behavior towards infinity can be shown which effectively can be minorized by means of a slight renormalization of κ (increasing its value a bit). Concerning the replacement of the energy term by the Riemannian distance; not much is to be expected here since they coincide in Minkowski given that the geodesics of both metrics are the same. Therefore, in a general analysis, these details should not matter.

The reader notices that both bounds have their advantages but that the first one is universal in nature and did not depend at all upon the four-valency of the interaction vertex. We shall now turn our head towards the perturbative renormalizability of non-abelian gauge theory; the proof of which reduces fully to the one above. The proof is almost self-evident given that every Feynman diagram consists out nothing but a product of spin-one propagators and at most second derivatives, one in each end vertex, thereof as well as Fermi and ghost propagators. The intertwiners $f_{\alpha\beta\gamma}, g_{\alpha\beta}$ and $(\gamma^a)^i_j, \eta_{ab}$ are all uniformly bounded so that the total Feynman diagram reduces to V-integrals of exponential factors

$$e^{-(\kappa-\delta-\epsilon)d(x,y)}$$

associated to all, up to second order, derivatives of *any* propagator. A fully analogous reasoning as before then shows that the contribution of any Feynman

[3]Such a vertex always exists as the following reasoning shows: start at an exterior vertex and go in the diagram by means of the edge e. On the first vertex v one meets, there is another edge f which can be connected to a different exterior vertex without coming back to v given that every vertex is connected to at least two different exterior vertices. If the other two edges of v are identified and therefore form a loop, then connect e with f and integrate out v. Otherwise connect e and f with one of the remaining edges each and integrate out v, which preserves the connectivity properties of the diagram.

[4]The reader may easily find an example of such diagram.

diagram is bounded by

$$C(D)E(\mu, \kappa, L, g, V, \epsilon)^{I+\frac{n+m+n'+m'}{2}} R(1, \kappa - \delta - \epsilon)^V$$

where $C(D)$ is a factor associated to the specific diagram D and function of the relevant intertwiners; all further symbols have the same meaning as before. I want to stress again that this bound holds in our special $SO(3)$-class of reference frames and that local boosts at the end vertices can make this number as large as one wants to.

We now finish this section by further fleshing out the graviton theory, at least at the perturbative level; comments regarding non-perturbative aspects will follow. At the perturbative level, we will need supplementary bounds on the Riemann tensor of g such as

$$R_{abcd}(x)\delta^{aa'}\delta^{bb'}\delta^{cc'}\delta^{dd'}R_{a'b'c'd'}(x) < C$$

where the Lorentz indices are taken with respect our special $SO(3)$-class of tetrads. This implies that all interaction intertwiners $Z(x)$ are uniformly bounded in these Lorentz frames and therefore, the contribution of any Feynman diagram is estimated by

$$\prod_{i=1}^{V} C(Z_i)\left(C(\kappa, \mu, L, g, V, \epsilon)\right)^E \int dz_1 \sqrt{h(z_1)} \ldots \int dz_V \sqrt{h(z_V)} \prod_{\text{edges } (\alpha,\beta)}$$

$$e^{-(\kappa-\epsilon-2\delta)d(\alpha,\beta)}$$

where $C(Z_i)$ is a constant depending upon the intertwiners Z_i, V is the number of internal vertices, E the number of edges (internal and external) and α, β are the coordinates of an internal or external vertex respectively. The bound on the propagators is valid up till the fourth covariant derivatives of the graviton propagator, with maximally two covariant derivatives per vertex each, and the same for the ghost propagator

$$\Delta^{\mu,\kappa,L}_{F;\nu\nu'}(x, y) =$$

$$- \sum_{w:\exp_x(w)=y \text{ and w is future causal or spacelike at x}} \theta(x)\overline{\theta(y)}g_{\nu\nu'}(x, w)W_{\mu,\kappa,L}(x, w) +$$

$$\sum_{w:\exp_x(w)=y \text{ and w is past causal at x}} \theta(x)\overline{\theta(y)}g_{\nu'\nu}(y, -w_{\star w})W_{\mu,\kappa,L}(y, -w_{\star w}).$$

As we will show in the next chapter, there are some peculiar nonperturbative aspects of the graviton theory which are not present in any other interaction theory considered so far. We will show that the friction parameter μ has a bound depending upon the Planck length squared putting therefore a lower bound on the "nonlocal range" of a creation or annihilation process.

9.6 Comments.

The reader might probably wonder if a replica of the second bound on ϕ^4 theory is possible for the other theories; the answer is *no* as he may verify for himself in the case of quantum electrodynamics. Therefore, what we have done represents a kind of universal optimum and the reader should cherish the cheer simplicity of the proof enabled by the Riemannian nature of the problem. The simplicity as well as universality of the obtained results are in sharp contrast to the poverty displayed by means of the standard way of doing things: this mathematical monster has only caused the illusion that there was something deep hiding behind it and has diverted attention from the simple fact that the principle of general covariance would lead one to a class of natural solutions. This is the real lesson of this book that when people like Kallen, Weinberg, 't Hooft and many others are shouting that mathematical rigor has to be abandoned permanently, that one must logically think that these grandmasters of illegitimate mathematical manipulations are missing an essential part of physics.

In this book, we shall not adress the issue of finite renormalization which in a general curved spacetime is a complicated problem. Notice that in Minkowski spacetime, the renormalization procedure, guided by the principle of conservation of energy momentum, a luxury one does not dispose of in curved spacetime, is not sufficient to get sensible physics out. This is most rigorously proven in classical electrodynamics where in spite of (also infinite) renormalization, the theory remains completely unphysical in the sense that an electron will spontaneously accelerate under its self-field in absence of external force fields, something which totally corrupts the notion of a free particle. Likewise can such a thing happen in quantum field theory in Minkowski even after all the illegitimate manipulations appear to give a better result. We have to do better than this, but that remains a research topic for the future.

Chapter 10

Entropy and analycity.

From the previous chapter, we learned that for any Feynman diagram, under the assumptions made, its value can be estimated by

$$C^V D^E R(1,\kappa')^V$$

where C is a constant arising from the intertwiners and can be uniformly bounded if the theory contains a finite number of them, whereas D is a constant coming from the propagators and $R(1,\kappa')$ has been defined before. This reveals that, in order to get a grasp on the entire perturbation series, we have to investigate the number of connected Feynman diagrams of the type (n,m,V,E) where every symbol has the same meaning as in the previous chapter. More precisely, we shall investigate the sum

$$A(n,m,V,E) = \sum_{D \text{ of type } (n,m,V,E)} \frac{1}{s(D)}$$

and this task is the easiest to perform for ϕ^4 theory while by far the most difficult for the graviton theory which contains an infinite number of interaction vertices; therefore, in this chapter, we shall proceed by making the exercise for ϕ^4 theory, next for non-abelian gauge theory and finally for the graviton theory. Insights regarding "unitarity" are obtained and further discussed.

10.1 Non-perturbative aspects of ϕ^4 theory.

The reason why I have postponed this issue in my papers towards this book project is because it is rather obvious to perform, at least for ϕ^4 and non-abelian gauge theory. The conclusions we shall reach are, on the other hand, interesting and this fact motivates the existence of this chapter. Let me stress from the outset that no approach to relativistic quantum theory has even reached the stage where one is in a position to address these questions; our bound on the contribution of one Feynman diagram to the series is however sufficient for us to address the issue. We shall show now that an upper bound for $A(n, m, V, E)$ is given by

$$A(n, m, V, E = 2V + \frac{n+m}{2}) \leq \sum_{0 \leq n' \leq n; 0 \leq m' \leq m : n-n'=m-m', n'+m'>0} \frac{(4V)!(n-n')!}{(2V - \frac{n'+m'}{2})! 2^{2V - \frac{n'+m'}{2}} V!}$$

where n', m' have the same meaning as before. Given that any exterior vertex under consideration is connected to an interior vertex by means of an edge E, there are $4V(4V-1)\ldots(4V-n'-m'+1)$ possible choices whereas the remaining $4V - n' - m'$ lines emanating from the V internal vertices have to be identified internally. This leads to a factor

$$\frac{(4V - n' - m')!}{2^I I!}$$

where I is the number of internal lines and we know from previous considerations that $2I + n' + m' = 4V$. Clearly the $I!$ stands for the number of permutations of the internal lines whereas the factor 2^I is associated to the swapping of orientation of them. Finally, the $V!$ in the denominator stems from the permutation freedom of the internal vertices and the $(n-n')!$ denotes the number of propagators between the remaining $n - n' = m - m'$ IN and OUT vertices. This upper bound is pretty tight and clearly provides one with the right kind of asymptotics in terms of V; also, I believe it would be hard to obtain a better one given that symmetry properties of individual diagrams would become important.

We shall now estimate its asymptotic behavior for large V and hence large $E = 2V + \frac{n+m}{2}$ keeping n, m fixed. Clearly $n' + m'$ can be ignored when it comes together with V so that

$$A(n, m, V) \leq \frac{(4V)!}{(2V - \frac{n+m}{2})! 2^{2V} V!} 2^{\frac{m-n}{2}} \left(\sum_{0 \leq n' \leq n; 0 \leq m' \leq m : n-n'=m-m', n'+m'>0} (n-n')! 2^{n'} \right).$$

Therefore, our scattering amplitudes are bounded by

$$|\langle \mathrm{OUT}\, m | \mathrm{IN}\, n \rangle| \leq 2^{\frac{m-n}{2}} D^{\frac{n+m}{2}} \left(\sum_{0 \leq n' \leq n; 0 \leq m' \leq m : n-n'=m-m', n'+m'>0} (n-n')! 2^{n'} \right) \sum_{V=0}^{\infty} \frac{|\lambda|^V (4V)!}{(2V - \frac{n+m}{2})! 2^{2V} V!} (CD^2 R(1, \kappa'))^V$$

and the right hand side is easily seen to diverge for any λ. This brings me back to comments I have previously made in my papers as well as the introduction which boil down to the fact that "unitarity" or the structure of the coefficients

$$\frac{(-i\lambda)^V}{s(D)}$$

will have to be changed for diagrams with a large number of internal vertices and we have just shown that this needs to be the case. We had of course anticipated already in chapter four, on general covariance, that unitarity was incompatible with it, but now we are forced to investigate deeper implications of this fact.

Unfortunately, I have at the moment no obvious substitute for the principle of "unitarity" which had no natural place in our theory anyway. It is a remnant from the old quantum theory on flat space time which leads to all kinds of inconsistencies mentioned previously in this book. The reader must again understand that this is not a weakness in my viewpoint but a liberty which is enforced upon the theory by means of our broader perspective on microscopic physics. The latter turned out to be necessary to tame the divergences in the Feynman diagrams and to make the theory well defined; the principle of general covariance had similar implications for gravitational physics and so does it have for quantum theory. Therefore, I am *not* going to propose any specific coefficients which might make ϕ^4 theory well defined nonperturbatively but which could fail miserably for the graviton theory. Only experiment should guide us herein; the freedom associated with those coefficients should not be mistaken with a choice of an infinite number of "coupling constants". Indeed, in practice, only the first few terms of the perturbation series, regarding diagrams with a low number of internal vertices, are important and the rest can be safely ignored which tells you something about the *effect* of the remaining coefficients. To my feeling, this is as thight as the jacket can reasonably be and going over to higher values of the coupling constant opens up an infinite new world which remains unseen in ours.

10.2 Non-perturbative aspects of non-abelian gauge theory.

A similar qualitative result as the one just obtained for ϕ^4 is expected to hold in non-abelian gauge theory albeit the counting is somewhat more difficult since one disposes of four types of interaction vertices: a tri and four valent gauge boson vertex, a trivalent ghost-gauge boson vertex as well as a trivalent particle-gauge boson vertex. Since all details of the interaction vertices are washed out in the constant C and likewise so for all details of the propagators regarding the constant D, we are left with diagrams having tri and four valent vertices as well as a consistent labelling with p, a, b where p stands for particle, a for antiparticle and b for gauge boson, on the edges adjacent to the exterior vertices. This labeling should be extensible, in a non-unique way, to the interior edges when supplemented with a ghost and anti-ghost g, ag label. Therefore, we are interested in estimating amplitudes written down abstractly as

$$\langle \alpha_i, i = 1 \ldots n | \beta_j, j = 1 \ldots m \rangle$$

where $\alpha_i, \beta_j \in \{p, a, b\}$ and the ordering in the states is from one to n and one to m respectively. It is clear that we cannot provide for the exact number of labellings since that depends from graph to graph in the sum and we shall therefore provide for a reasonable upper bound for a graph with V_4 four-valent vertices and V_3 three-valent ones, ignoring the number of edges as well as n, m. An obvious upper bound is given by 6^{V_3} given that there is only one type of four-valent vertex with identical particle lines and every tri-valent vertex, together with isolation of the v, Ψ-line, fixes all other lines. There are in general three types of trivalent vertices and three lines per vertex to place the v, Ψ, hence 6^{V_3}; of course, this constitutes an overestimation of the state of affairs given that a tri-valent vertex connected to a four-valent one only leaves for two possibilities but all such details are graph dependent.

The estimates we are interested in here concern

$$D^{\frac{n+m}{2}} \sum_{0 \leq n' \leq n, 0 \leq m' \leq m, n'+m' > 0; V_3, V_4 \geq 0; 2I + n' + m' = 3V_3 + 4V_4} A(n', n, m', m, V_3, V_4) 6^{V_3}$$

$$|\tilde{g}|^{2V_4 + V_3} \, C^{V_3 + V_4} D^{\frac{3}{2}V_3 + 2V_4} R(1, \kappa')^{V_3 + V_4}$$

where $A(n', n, m', m, V_3, V_4)$ equals the number of diagrams with V_3 and V_4 trivalent, respectively four valent, vertices and all other symbols have the same meaning as before. $\tilde{g}$ is the coupling constant of the theory in the standard representation, see chapter eight. The reader understands that this is a fairly substantial overestimation of the state of affairs given that we do not take the nature of the exterior vertices into account in the determination of $A(n', n, m', m, V_3, V_4)$ implying that a particle can be connected to a four-valent vertex. However, one would expect the "real" number to be of the same magnitude which means we probably capture the right asymptotics in terms of V_3, V_4 and this is our only point of concern. $A(n', n', m', m', V_3, V_4)$ can again be estimated by

$$A(n', n', m', m', V_3, V_4) \leq \frac{(3V_3 + 4V_4)!}{2^I I! (V_3 + V_4)!}$$

where $2I + n' + m' = 3V_3 + 4V_4$; hence,

$$A(n', n, m', m, V_3, V_4) \leq \frac{(3V_3 + 4V_4)!(n - n')!}{2^{\frac{3}{2}V_3 + 2V_4 - \frac{n'+m'}{2}} (\frac{3}{2}V_3 + 2V_4 - \frac{n'+m'}{2})!(V_3 + V_4)!}.$$

This reduces our original sum to

$$D^{\frac{n+m}{2}} \sum_{V_3, V_4 \geq 0} 6^{V_3} \, |\tilde{g}|^{2V_4 + V_3} \, C^{V_3 + V_4} D^{\frac{3}{2}V_3 + 2V_4} R(1, \kappa')^{V_3 + V_4} \frac{(3V_3 + 4V_4)!}{2^{\frac{3}{2}V_3 + 2V_4} (V_3 + V_4)!}$$

$$\sum_{0 \leq n' \leq n, 0 \leq m' \leq m, n'+m' > 0, n-n' = m-m'} \frac{(n-n')! 2^{\frac{n'+m'}{2}}}{(\frac{3}{2}V_3 + 2V_4 - \frac{n'+m'}{2})!(V_3+V_4)!}$$

which diverges again due to the super exponential factor

$$\frac{(3V_3 + 4V_4)!}{(\frac{3}{2}V_3 + 2V_4 - \frac{n+m}{2})!}.$$

Therefore, we reach again the conclusion that unitarity cannot hold for a quantal gauge theory to be well defined; in particular, diagrams with a high number of internal vertices need to be super exponentially suppressed in these parameters.

10.3 Gravitons.

Until now, we have received the lesson that diagrams with a large number of internal vertices should be super-exponentially suppressed; in a graviton theory we anticipate another lesson which is that diagrams with large vertices should be super-exponentially suppressed too. More in particular, one meets interaction vertices with coefficient l_p^{2n} having $2n$ legs such that one obviously needs a factor $a(n)$ such that $a(n)(2n)! \to 1$ in the limit for n to infinity. This would lead to a bound of the kind

$$1 > l_p^2 \frac{C(g, V, \kappa, \epsilon)}{\mu} ||W(x,y)_{ab'}^{ba'}|| > 0$$

given that a Feynman diagram contributes the n'th power of that, which implies that

$$\mu > l_p^2 \alpha(g, V, \kappa, \epsilon)$$

is the kind of bound on the friction term μ one should anticipate in a graviton theory on a generic background. This is all I have to say about this for now, these results require deep reflection as they destroy the traditional structure of quantum field theory.

10.4 Conclusions.

We already knew that in traditional quantum field theory, the value of a Feynman diagram was not uniquely defined and moreover, that any regularization scheme is rather ad-hoc and lacks physical motivation. We rectified that by looking for modified propagators falling in the class delineated by our physical principles; in this chapter we moreover figured out that the traditional expansion series does not converge either requiring equally drastic modifications to the theory. The level of precision obtained in this book is unparalled in the literature and should constitute enough motivation for the reader to further investigate these matters. These notes conclude the main body of this book, the remaining chapter being merely an exposition about some thoughts of mine of how to formulate a physical principle giving rise to a free quantum theory for the background metric field. The particular proposal I will suggest properly reinstates "time" and strongly criticizes and departs from the timeless physics of Einstein. In particular, we will work again towards a generalized Fourier transformation and use this to define a free theory; interactions between different universes shall not be discussed.

Part III

The mental world.

Chapter 11

General discussion.

In this part, we shall further engage in the kinematical setup explained in the introduction meaning we shall try to formulate some constraints upon the dynamics; ultimately, the goal is that you should be able to program this theory on a computer such that the different persona in your programme engage in a meaningful conversation. This what I call psychology and I can assure you that the standards are way higher as those of accredited psychologists. This work is a result of the reflection of someone who studied Jung and Freud's works at the age of 13 and later went on to study exact sciences, more in particular physics and mathematics. Jung and Freud's writings are muddled, mystical and lack any grounding in a more fundamental way of reflection about the world. Moreover, there is no clear separation between morality, sociology and psychology and one experiences as well a profound lack of understanding regarding the biophysical underpinning of their "science". I thought of Freud's ideas as banal, way too simple to be even considered; this was just story telling, there was no process of falsification, I mean this was "not even wrong" to state Wolfgang Pauli. Jung was far more interesting what concerned his observations; for example, he would find out several examples of the same symbols, paintings, artwork in different cultures which lived on distinct continents and never had any contact with each other! This suggests that there are many things we have in common which go beyond our perceptions and even our history. Randomness of the Darwinian process would suggest a wild variety of different traits, but that did not appear to be the case. He gave a place in our psyche which should "explain" this phenomenon, which is our collective unconsciousness. Now Jung did propagate a lot of ideas which are, in my view total nonsense. One of them is that the goal of life is to discover and become your true self. It seems obvious to me that you are always your true self even if you tell lies to others or hide your ideas where you would prefer them to be in the open. What the grandmaster suggested is that we should engage in our unconsciousness; a totally ridiculous thing to do. My unconsciousnss regards all fine processes in my brain or even in a separate spiritual world which are simply not communicated to me on a level I am unaware of. I don't care about such things as any decent scientist should!

Only mystics and mentally troubled people (including many professional mental healthcare workers), who have even not the slightest understanding of the miraculous ways our physical world operates, indulge themselves in that kind of "armchair" philosophy. In this book, I make a serious effort to be precise, so you can agree or disagree with me; but, at least, we can discuss about something! This is not to say that their descriptive approach is not worthwhile studying but one is left with very little if no understanding at all as well as with a myriad of epistemological adventures which belong to Alice in Wonderland. In other words, the approach is not scientific, just as botany and anthropology are not. The aim of this part, is a modest attempt to fill in that gap; to provide for very accurate definitions and to *explain* why things are the way they are from very simple principles. In other words, we enter the area of *predictive* psychology based upon very few observations which are usually not behavioristic in nature.

The limitations sett upon our kinematical framework, as explained in detail in part 1 imply that we shall study mentality at a level of poor "intelligence" albeit I shall, as promised previously discuss a bit of how logical principles embedded into the dynamics might lead to emergent rationality; this is for now the best we can do. An approach to higher intelligence will require new *principles* of language formation, something which is still beyond our grasp at this moment in time. Further ideas regarding this topic will follow at the end of this book but are by no means complete nor at the stage where proper quantitative, but nevertheless qualitative, investigation becomes feasible. It is my philosophy that any person in society deserves an optimal satisfaction as long as gratuitous murder and world domination do not belong to the personal desirata; indeed, this might be part of the ultimate goal of societal life and could very well be encrypted in the dynamics. It is my hope that at least the viewpoints put forwards in this book will constitute a ground for reflection. Discussions about morality and ideology, in my opinion, belong to the lofty saloons where big men can enjoy cocky woman and Cuban cigars.

Up till now, it must be clear for the reader that I completely negate the delusion that one can know the intend behind someones actions, that it is possible to know someones emotions and certainly that it is pointless to contradict a person speaking openly about his or her intentions or emotions. There is no way to know these things and therefore it is pointless to discuss it from a scientific point of view. People should just stop thinking in this way regarding societal interactions which is the well known foundation for religious murder. Long live Copenhagen quantum theory in this regard, that its pragmatism may serve as a lesson for peaceful and respectful communication. On the other hand, a person yearns for epistemology, for an explanation why we are the way we are and where our thoughts originate from even if this subject is dead from the scientific point of view. That is, an irrational urge for an explanation behind human rationality is a firm part of our being and it needs to be dealt with too. Privately please and not by general policy makers! Historically, the church fulfilled that part and nowadays meditation centers as well as private psychologists, as intelligent conversation partners, are there to fill in that part of our lives if mandatory. In this regard, total privacy as well as absence of any reporting should be guaranteed. The psychologist is no doctor and in case of serious worries about a client should do everything in his or her power to send him off to a medical doctor; by no means should he directly contact a physician himself. This book is not about learning how to be a wicked conversational partner but about the basic physico-spiritual laws behind low intelligence psychic interactions. I will explain why these laws are the way they are and discuss the basic observational ramifications.

11.1 Dynamics of questions.

In this chapter, we shall put forwards some further principles any suitable theory behind basal psychological interactions should satisfy. Intelligent conversations usually require something as creativity and insight and it appears to be difficult to find out a theory about that. Indeed, current AI is limited to finding statistical distributions associated to standard answers to certain types of questions by feeding the system with a lot of text. Once you would ask it something which is weakly correlated to the texts it has devoured, the probability of getting garbage is pretty large. This is not how the human mind works given that AI reads many more books as humans do; we also know how to deal with conflicting information and critically make up our own mind, I presume AI is nowhere close to that level. Up till now, we have dealt with positions and dichotomies regarding issues, but we have not suggested any basic theory behind the very nature of the formulation of those issues. Here, I believe that computer scientists have added a valuable point of view by means of binary codation of data as well as commands (questions, actions). Binary numbers are 1 and 0, whereas words are of the form $1001011\ldots$, sequences which are shaped in time, where at each instant, a letter is chosen. Quantum mechanically, we consider quibits

$$\cos(\theta)|0\rangle + e^{i\beta}\sin(\theta)|1\rangle$$

indicating the probabilities for $|0\rangle, |1\rangle$ to be chosen as well as the interference between both. This means that at each instant, both 0 and 1 are allowed for and that interference between 0 and 1 is possible with measurement giving 0 with probability $\cos(\theta)^2$ and 1 with probability $\sin(\theta)^2$. In standard probability theory, one would only dispose of two positive real numbers which sum up to one and not dispose of an angle β which is "forgotten" but plays a dynamical role for sure. Indeed, β can be seen as mystery, an unknown factor in our ways of communication which can only be measured if we know exactly how to replicate the state

$$\Psi = \cos(\theta)|0\rangle + e^{i\beta}\sin(\theta)|1\rangle.$$

This is approximately true in simple experiments in physics with an infinite number of degrees of freedom where the circumstances are so rough that the details of the state do not really matter in the outcome of the experiment; for example, variations on tiny length scales do not matter if the experiment probes for the behavior on scales far larger than those. Indeed, the behavior of humans in the desert does not really differ from one and another whereas interactions with a beautiful companion of the "opposite" sex might differ substantially. This unknown factor indicates also that different realities co-exist at the same time; in the binary system above, two distinct questions do suffice.

In physics, we call such a simple quantum-system a quibit: it is the fundamental ingredient behind quantum computing, where a system can only be in two quantum states. Putting N quibits after one and another, we have the potentiality to form 2^N words of length N with $2^N - 1$ real components of mystery. So, the degree of disorder in such system is $N + 1$ which is nothing but $\log_2(2^{N+1})$ which is the Shannon-Von Neumann entropy associated to this system. Indeed, it is meaningful to regard disorder in this way as a word is equivalent to one message no matter how long it is. However, the complexity of the message usually increases with the length of the word or the number of words and therefore N could equally stand for that. So in a way, the higher the disorder, the more complex it becomes and this is also how we experience society. So, a language is therefore always embedded in

$$W := \oplus^{S\,\infty}_{j=1} \otimes_j V$$

where V is the one quibit space, $\otimes_j V$ is the j-quibit space and $\oplus^S$ means that we sum up over words of different length in any order[1]. It is reasonable to assume that at any instant in time, a person has at least one element out of quantal word space W in mind. In case this is not so, then the person is totally dead; otherwise, depending upon the complexity of the quantal word, it is gradually more (un)consciously alive. Usually, what we call a dead person, is still alive in a way; it is just so that the spirit of the body is totally dead, meaning no quantal words are formed anymore at the highest operational level of the person, but the atoms and molecules making up the person are certainly still alive. This is the most accurate definition possible of being dead or alive. Notice that alive does not imply conscious, so this goes beyond the usual "*je pense donc j'existe*" if thought is being restricted to conscious acts.

In this regard, the language formation process has to be interpreted as a process where more complex text $T \in D$, where

$$D := \oplus^{\infty}_{j=0} W$$

is a possibly infinite ordered collection of sentences, can be formulated. In this book, we mainly study the dynamics of V which we define as the lowest level of complexity possible; speculation about higher language formation and principles valid therein shall be postponed for the future. Here, a comment is in place, I am talking about language formation and not language recognition,

[1]Note that W also contains the empty sentence.

something which Chatgpt is very good at; the latter is no miracle indeed, if you feed enough text (to learn it how to build sentences) to the system as well as the complete Oxford dictionary, where you define an equivalence between words and indicate which ones are more posh than others, then you can effectively learn it how to reformulate a text in a posh language. Here, we are interested if one could describe the origin of language as arising from a simple dynamical process, a (Darwinian) evolution. If this were not possible, then we have to conclude that our speech is in the hands of the creator, just as we create the ability of computers to recognize patterns, that it is a gift which cannot be understood. Likewise, with the development of language, comes the development of attributes α_j to words, sentences and text. So, in a dynamical picture of evolution where not the entire Platonic space of ideas has been encoded upfront (in principle of course) , we must conclude that the manifold as well as the vector bundle grows in dimension through the intervention of intelligence and not measurement (which sticks to questions already known); so, if such an evolution could at all be described in a mathematical language, then this needs to be of a historical origin which goes way beyond our own lifespan and needs to be passed on to our siblings who are totally unconscious of it; to be a bit provocative here, even elementary particles should have some innate property of collectiveness build in, leading to atom and molecule formation as they project down often enough on the pure energy atom state. Indeed, why should an electron be in a stationary arbit around the nucleus? It could "free" itself from the tyranny of the photon field by spontanous localization and then slowely drifting away by being in a superposition of higher energy states which are farther removed from the nucleus.

Most scientists, including myself, are interested in finding biological markers for our mental capacities, which, as mentioned previously, is only part of the explanation; but it is for sure an imortant topic to study to what extend our physical constitution "lifts" towards the spirit, meaning that we dissect the person as much as possible and see how far our reductionist point of view on the world carries. It could simply be, in a way, that spirits attached to N binary composite entities cannot give "meaning" to the full space of 2^N classical words; in either complexity or disorder does not simply add up, in our definition it is sub-additive; indeed, not every N letter word has a meaningful interpretation such as **cdkz** does not make any sense in english. Therefore, the complexity of an N-bit spirit is less or equal to the sum of the complexities of the individual ones; and in practice it is much-much less as we know that a gas in equilibrium forgets about all the small details of the colliding atoms and can be effectively described in terms of 3 intensive (T,p,μ) and 3 extensive (S,V,N) variables. The problem with collective spirits usually is that its complexity might be less than the one of its "members" due to destructive interference processes, a well known phenomenon in societal life where the community is usually much less refined than its most complex individuals. Complex life forms require basic laws of nature which offer room for stability on sufficiently long time-scales; only gravitation and electromagnetism, which, in a way, make life posssible, are also in position to destroy the universe in the long run. Therefore, some scientists say, that the mere existence of humans, with their complex form of interaction, who create societies whose ingenuity may oscillate in time and not even have a mean positive growth factor, is equivalent to evidence that, when it really becomes necessary, the human endeavour is a divine one (cosmologists think our universe is a fix and physicists are entertaining the anthropic principle in these days). I, on the other hand, still want to advocate a kind of Darwinian universe where spirits with extreme complexity come and go periodically; given our current poor understanding of such issues, I proclaim that happy indifference is the best way to live with this uncertainty. If the Gods play it as such, then they are for sure compassionate with me; otherwise, in absence of their existence, I am for sure more devilish than I know of.

Given a lack of a natural bio-physical understanding at this point, albeit recent research has correlated brain activity to certain "presumed conditions of the mind", we shall confine ourselves in the sequal in discussing the interactions between the vectors determining the probabilities of which choice projector $\pi_{A,j}$ to apply (for any j) on one side and the perspective dichotomy matrices on the other. More direct interactions between matter currents and those matrices/vectors must exist but these issues are far too complex to consider at this point. We shall discuss this issue in detail in the next chapter.

A practical question is how to find out the natural values α_j corresponding to black perception of issues? In physics, we are pretty lucky that the gravitational field (as well as the external electromagnetic field) is weak and as good as time independent so that a metal bar does not change in length in our perception. The natural measure stick therefore is one where meters are expressed in fractions of a metal bar and where the kilogram is defined in a similar way. Again, we are lucky here; in principle Einstein's theory about the relationship between spacetime geometry and the stress energy tensor of matter, assuming the independence of the gravitational motion of an object regarding its internal constitution, except for its rest-mass[2] leads to an infinite mass renormalization and a slight warping of spacetime around that object. In quasi-static gravitational fields, this renormalized mass is almost constant which allows again for the introduction of a unit of mass. Likewise, do atomic clocks determine a natural unit of time and it is a miracle that in those units, which are associated to physical processes, the local speed of light is constant in all directions of space! Regarding our mental variables, we are by far not that lucky; first of all, quantities such as length and time are easily associated to real numbers, but how about feelings or perceptions? Even if they could be modelled by a real number (which we assumed so far), what would be the natural unit, the divine reference frame? We can only guess and it would for sure be helpful if one could

[2]Actually he started from the assumption that the motion of such particle should *not* depend upon its mass.

measure the physical brain energy consumed by a mental thought!

Given a finite set of N issues, as well as natural flat inertial coordinates in psychological space associated to rigid local space-time measure sticks (since we are searching for a physico-spiritual correspondence), we shall now look for technical generalizations of this idea. Up till now, we have assumed that the psychological variables covered the entire real line, which is easy from a technical point of view since it provides for a unique representation of our Lie algebra; in case the domain would be a finite interval, then we have ambiguities originating from the boundary condions and technically, it is impossible for one of the black-white operators to have a finite spectrum. The psychological space (we just concentrate on the α_j variables here) at hand is empirically (as most scales in psychological tests are) determined as an N-dimensional convex space with the barycenter as origin. A convex space of dimension N is a part of $\mathbb{R}^N$ such that the line element connecting any two points belonging to it, also belongs to it. So, in one dimension, there is exactly one line element whereas in two dimensions we have a polygon. A convex space is bounded by subspaces of lower dimension. Those of dimension zero, in either points, are called extremal elements; that is, they cannot be the midpoint of any nontrivial line segment within the body. A piecewise flat simplicial manifold is a space which is formed by means of gluing convex spaces together along the boundaries. The flat space metric is given by

$$s^2(x_1, x_2) = \sum_{i=1}^{N} (x_1^i - x_2^i)^2.$$

It is important here to ensure consistency of this procedure by ensuring that axes with a different dimension (unit) cannot rotate into one and another and that scaling always has to occur with respect to space-time units. This means, that if we take *rigid* objects determining mass and length, then scaling of the length or time bby λ induces a scaling of the by λ^{-1} (holding $c, \hbar$ fixed) which means that a metric of the type

$$s^2(p_1, p_2) = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

will transform as

$$s^2(p_1, p_2) = \lambda^2 (x_1 - x_2)^2 + \lambda^{-2} (y_1 - y_2)^2$$

in case x denotes length and y mass. In general relativity, such scalings also renormalize Newton's constant by λ^2 so that on very small scales (small λ) masses blow up, but G goes to zero. In our setup, it does not make sense to consider scale invariance of the black operators X by a factor of λ and P by the inverse of λ since such a transformation coincides precisely with a boost around the z axis; there is no need to treat the z-axis different from any of the other spatial directions. At first sight, you might object and say that meter and kilogram are mere conventions and we should be able to redefine them at will; you are free to do that in the material world and the mental world will automatically follow the new conventions and rescale its appreciations (which of course coincide with measurement) appropriately.

As mentioned previously, an important part of the dynamics of an individual's profile and choice regards its interaction with others, even elementary particles seem to have this trait at a very basic level. So, the way communities organize themselves must be the consquence of a balance equation between personal desirata of the spirit on one hand and the will to socialize on the other. If you want to convince others about your profile and choice (as well as the particular state of another (related) issue in case you take the conservative viewpoint there, but that appears to be of secondary importance[3]) regarding a certain issue in order to reach more harmony, then you will have to be extremely wicked and cautious in order to succeed. This appears to be another balance equation which is that the urge to change others in their way of approaching an issue, comes with great care and effort. I believe our innate profile is full of balance equations as such and it would definitely be interesting to find a more basic description behind those.

In general, each mental characteristic must manifest itself in physical reality by means of actions; the trouble is that most of our actions are an effective product of many distinct "traits" and that it is generally impossible for an outside observer to disentangle those (albeit they would like to believe they can). Mathematically, actions form a closed system, a mathematical group which must translate as a projection of our brain state. It is maybe useful to comment upon why we choose to work with the real number system in quantifying states of issues, emotions and so on. The idea is again an operational one of to unite and divide, that is plus and quotient; if we choose one nontrivial unit, then the addition leads to the natural numbers whereas division leads to the positive rational numbers. Introducing the neutral element as well as an antagonist or the opposite includes the negative rational numbers. Closing those in the difference metric gives one the real numbers. The fact that division and addition seperately do not depend upon the order of its arguments are called the principles of commutativity and associativity and mathematicians have played around with non-commutative and non-associative systems such as the octonions which may express a higher awareness. Given that we think elementary particles are rather silly and simple, the real number system is more as sufficient for these purposes and different dimensions are assumed to commute. However, elementary particles in the quantum world have shown to add a slight complication to this idea be somewhat more complex being that your current manifestation does not commute with the current impetus (change of manifestation). In a way, we are forced into the Einsteinian view (regarding the generality of the basic

[3]For example, in an attempt to convince others to be progressive regarding the state of their country and focus on its change instead of on their fixed perception, you might convince them that it is reasonable to rebel and eventually reach a consensus on where to go in the long run. It is usually of primary importance to make them rebellious, the rest are details to be discussed later on.

laws) as too strict theories, with little or no internal symmetries, who attempt to predict someones profile and conservative choices without allowing for any liberty usually run into conflict with reality. For example, too strict constraints and balance equations could lead to a humanity where everyone is the same: a society without great leaders or scientists. As a final comment of a technical nature, we discuss the particular origination of the metric, as well on space-time as on the appreciation-consciousness space, from a scalar product. The latter is completely determined by the requirement that the act of projection preserves the addition on the smallest scales; that is, the projection of a sum of two quantities is given by the sum of the projections. It is an expression of the fact that God loves pieceful recognition at the shortest possible scales. In that vein, chaotic or fractal geometries are not considered.

Chapter 12

Dynamics of the choice projectors.

In our discussion in part 1 we related the symmetries of the local profile matrices to the spacetime action of the local Lorentz group. I believe that the mental attraction or repulsion between two minds is not in the first place dominated by the way we look at things (dichotomy) but by the (upper or lower) choice we make regarding those issues. The combination of the dichotomy and the profile choice is what we call the decision; the reader must understand that two distinct profiles can make the same decision, even a realistic (either self adjoint) one, but they may differ in upper-lower choice. In other words, the angle (choice) from which you sell your decision is more important than the decision itself and can lead to repulsion either attraction. For example, realistic profiles on the lower side are given by

$$A = \begin{pmatrix} \frac{-i(ca+1+ida)}{b} & c+id \\ a & ib \end{pmatrix}$$

assuming $b \neq 0$. This is in my experience a true fact of life, often it happens that two persons say the same thing but from different angles and one person gets accepted whereas the other one rejected. To initiate the discussion, notice that in the theory of Dirac particles, all local symmetries of the scalar product

$$\overline{\psi}(x)\psi(x)$$

are given by $SO(3,1) \times_T U(1) \times U(1)$ where T stands for twisted. Indeed, one $U(1)$ is associated to the 4×4 identity matrix, and another one is associated to γ^0, but this generator anti-commutes with all boost matrices $\mathcal{J}^{0j}$ so that the action gets a twist. The twist has not been accounted for yet in physics, but the remaining part has; the effective local symmetry group of the Dirac particle being $SO(3,1) \times U(1)$ where the connection associated to the first group is delivered automatically by the vierbein and the $U(1)$ part is the usual electromagnetic 4 vector field. In this vein, one can always choose a gauge where the

Dirac field consists out of one real spacetime field only. The choice variables for one issue form a complex two vector satisfying

$$C(x) = \begin{pmatrix} \Phi(x) \\ \Psi(x) \end{pmatrix}$$

and the only thing that we require is that

$$C^{\dagger}(x)C(x) = 1.$$

The full symmetry of this scalar product is of course $U(1) \times SU(2)$ which is precisely the same as the internal symmetry group of the electroweak interactions. In contrast to the case of the profile operators, it does not make sense to associate the $SU(2)$ part with rotations in some eigenspace given that a rotation around 90 degrees would turn an upper choice into a lower choice, which is of course utter nonsense. So, the whole group is an internal group; but just as happens in the electroweak interactions, where one has a proper definition of an electron and neutrino (breaking the $SU(2)$ gauge invariance), likewise do we have here a proper distinction between upper and lower choices. This suggests for a pretty identical application of the Higgs mechanism by adding upper and lower $SU(2)$ singlets (as a replacement for the right handed electrons and neutrino's) to the theory and coupling those to the doublet and Higgs spinor in order to create different masses for the upper and lower profiles. I leave it up to the reader to decide whether he insists upon this implementation of the Higgs mechanism to be classical or quantum and therefore looking upon the $C(x)$ spinor as a classical or quantum constrained entity satisfying $C(x)^{\dagger}C(x) = 1$. From a classical point of view, which is somewhat more unusual, one can understand the symmetry breaking[1] at several levels: (a) the physical Higgs field H should have suitable falloff conditions towards spatial infinity such that there exists no realistic operation which can turn one ground state value v for the Higgs into another one (boundary conditions) (b) the physical Higgs field H is much smaller in absolute value as v is (in case the universe is spatially compact) so that you limit the possible space of solutions. There are some distinctions with the electroweak theory however, some of which are certain and others maybe uncertain meaning we don't have enough data here to make the decision. To illustrate those, let us start by making the folowing observations:

- lower profiles regarding an issue flock more more together in large groups with high density (that is another reason why I called lower profiles in the

[1] Actually, there does not exist any agreement in the literature upon what it means for a dynamical symmetry to be broken classically; some simply say that if a particular solution breaks this symmetry, meaning that the orbit is not invariant under this symmetry, then the symmetry is spontaneously broken. This is not a very useful point of view, another definition would be that you define the class of a configuration as all configurations which can be reached by means of a physical (exterior) operation. One then says that the symmetry is spontaneously broken if such class is not invariant under the symmetry transformations. Of course, a moot point here is how to define preciesly what you mean by a physical operation; usually, this is thought of as being associated to an observer, but how to change the orbit of planets around the sun (which are not circular)?

canonical dichotomy black, since they cause for “spiritual black holes”),

- upper profiles are more solitary amongst one and another and attract each other more on larger distances

- lower and upper profiles repel one and another on short distance scales to the effect that the conversation shifts in topic where reconsilliation can be achieved

- lower people act faster regarding this particular issue as lower profiles do; acting to sustain something requires more time and effort than actions wanting to preserve a definite point of view.

These mere observations would suggest, from a traditional point of view, that the interaction fields have as source term minus the charge density of the choice field, so that alike charged particles attract one and another; more in particular, the unbroken $U(1)$ part must lead to universal attraction between both types, meaning they have charges of the same sign; whereas the broken $SU(2)$ part would lead to a dominant short range repulsion between the two opposites. So, the spiritual world seems to be the opposite in that regard of the physical world, possibly creating instabilities, whereas the latter has a steady lowest energy state. Another remark is that one would like to couple the choice field to the profile field as well as to the psychic reality (the wave function). Regarding the latter, it is clear that the choice field has no canonical action (since it is no group for instance) on the mental state and therefore cannot couple to it directly; we have to use the coupling to the profile field instead. But there is still another possibility, we can use our unitary representation of $SO(1,2)$ which is a non-unitary representation of $SU(2)$ as well as $SO(1,3) \sim SL(2,\mathbb{C})$ to provide for a nonunitary action of the $su(2)$ gauge field strength on the mental state of the universe. Indeed, for our $U(1) \times SU(2)$ gauge field $A^a_\mu(x)$ where a runs from 0 to 3, we could take the operator

$$F^a_{\mu\nu}(x)J_a(x)$$

where $J^0(x) = 1$ and the $J^j(x)$ have been defined iin section 4.1 and the generalization thereof to multiple issues is canonical. Note that those operators are spacetime dependent; indeed the X_j, P_k operators, pertaining to distinct issues, were also thought of as belonging to a single mind localized at a spacetime point which we assumed to be somewhat smeared out so that one has effectively a countable number of minds only. The mind was assumed to evolve according to the classical currents $J(x)$, so that our issue operators (but not the wavefunction itself) are effectively dragged along those currents. Note that we really use two different times here: on one hand you have the psychological time t_p which is associated to the wavefunction and the latter is supposed to collapse in this time, a notion we used in part 2 of this book in order to define physical amplitudes of the interacting theory, whereas the time t_j we aspire here is the "personal mental time" which is associated to the classical body of the j' th observer. So, beware, this is not traditional quantum field theory! Of course, given an initial hypersurface Σ of constant time t_p, we can all set our times t_j such that $t_j = t_p$ and the position of our body is given by $\vec{x}_j$. Hence, for later times t_j, t_p there is a unique mapping from all spacetime points y reached by an observer, as long as two observers do not occupy the same point, something which we exclude, to the index j of the observer. So, we really should have denoted X^j_k, P^j_k where j denotes the observer and k the issue at hand. Regarding the profile matrices, choice fields and gauge fields, which do not depend upon the psychological variables, we already did that since those depended upon the spacetime location y and the profile and choice fields were attached to the observer: they do not evolve according to a partial differential equation, but merely obey an ODE with respect to the personal currents $J(x)$. The gauge fields on the other hand do obey a partial differential equation which causes for propagation of profiles and choices. To make these ideas concrete, one must reflect that a local gauge transformation correspondingly affects psychic reality; indeed, if the probabilities for an upper-lower choice shift, then also reality shifts nontrivially! This is an extension of what is usually meant by spontaneous symmetry breaking, that the state on which the theory acts does not possess the symmetries of the dynamics and, moreover, that the questions we ask are not gauge invariant themselves. Since the generators J^2, J^3 are anti-Hermitean, we must again import an operator T, like before which commutes with J^1 but anti-commutes with J^2, J^3 (there S did the same thing but then with respect to J^3 instead of J^1), so that we can construct scalar products of the kind

$$\langle \Psi | T F^a_{\mu\nu}(x) J_a(x) | \Psi \rangle, \quad \langle \Psi | T F^{\mu\nu\, b}(x) F^a_{\mu\nu}(x) J_b(x) J_a(x) | \Psi \rangle$$

where the first term can be coupled to something anti-symmetric such as

$$J^{[\mu}(x) \nabla_{J(x)} J^{\nu]}(x).$$

Under a gauge transformation $U(x) = e^{i\alpha^a \frac{\sigma_a}{2}}$ which acts upon the wavefunction as $\widehat{U}(x) := e^{i\alpha^a J_a(x)}$ one has that $|\Psi\rangle \to \widehat{U}(x)|\Psi\rangle$ and $(F^a_{\mu\nu}(x)\frac{\sigma_a}{2}) \to U(x)(F^a_{\mu\nu}(x)\frac{\sigma_a}{2})U^\dagger(x)$ where the latter yields for infinitesimal α^b that $F^a_{\mu\nu}(x) \to (\delta^a_c + f_{cb}{}^a \alpha^b) F^c_{\mu\nu}(x)$ whereas $\langle \Psi | T J_a(x) | \Psi \rangle$ transforms as

$$(\delta^d_a - f_{ae}{}^d \alpha^e) \langle \Psi | T J_d(x) | \Psi \rangle$$

so that both combined give

$$(\delta^a_c + f_{cb}{}^a \alpha^b)(\delta^d_a - f_{ae}{}^d \alpha^e) = \delta^d_c - f_{cb}{}^d \alpha^b + f_{cb}{}^d \alpha^b = \delta^d_c$$

as it should. Note that ideally, we would give explicit formulae for T, S but it appears that this requires further study of the particular (non-unitary) representation of those operators on Hilbert space and we shall leave such investigations for the future. It is my mere intention here to show that everything is consistent and that three different kind of groups are represented on the wave function by means of the same operators. Finally, in the above, it is understood that $\Psi(\alpha^j_k)$ and that $J_a(x)$ acts according to X^j_k, P^j_k for one particular j and that the scalar product is taken over all α^k_l, thus over all issues and all minds (observers). It must be understood here that we work of course in the unitary gauge and that gauge transformations are never performed; they are just an aspect of the dynamics and not of the interpretation thereof. Given that $\widehat{U}(x)$ is non unitary, we have to renormalize and therefore divide the interaction terms through

$$\langle \Psi | T | \Psi \rangle$$

which we assume to be nonzero.

The above view opens the door for psychic interactions given that the state is entangled which, in a way, are very real. Sometimes people are attracted towards one and another without any good reason or prior communication. Concerning the choice field, we shall give a brief qualitative view here and resort to the so-called dipole "Coulomb" approximation, ignoring mass terms and Yukawa type corrections to the potential due to the broken $SU(2)$ part. Regarding the situation of N spatially separated persons and one issue only, we obtain that the spatially integrated densities

$$\Phi_i := \frac{\int_{\mathcal{B}_i} \Phi(x)\sqrt{h}(x) d^3\vec{x}}{\sqrt{\int_{\mathcal{B}_i} \sqrt{h}(x) d^3\vec{x}}}$$

with upper-lower components integrated over the spatial bodies of the person and with $\Phi(x)$ of slow variation over the body. Then, one could consider corrections to the "perspective mass" of the i'th individual, due to interactions, as follows

$$M_i = a_i ||\Phi_i||^2 + a_{ij} \sum_{j \neq i} |\langle \Phi_i | \Phi_j \rangle| + b_{ij} \sum_{j \neq i} ||\Phi_i|| ||\Phi_j||$$

is the most general formula possible where the a_{ij}, b_{ij}, c_{ij} are coupling functions depending upon other physico-spiritual entities as well as an average distance between the bodies using the length scales set by the coupling constants of the theory. In a way, those are needed to include the last term which does not depend upon Φ_i and $b_{ij} > |a_{ij}| > 0$ given that otherwise M_i can always become negative which is forbidden. The coupling functions vanish in the limit for distances r_{ij}experience dictates that sexuality plays an important role in the interactions. One may consider what kind of other issues would mix with the dynamical law for this particular single issue. In the next chapter, I shall discuss some issues which I believe to be a foundational importance, in a way resembling the holy trinity in religion, meaning that they interfere with any issue and clothe our communication. One such variable is sexuality, modeling it by means of a binary variable S_i where $S_i = -1$ if and only if the subject is male and $+1$ if it is female, then a simple expansion gives

$$a_{ij}(r_{ij}, S_i, S_j) = \frac{\widetilde{a}_{ij} + \widehat{a}_{ij} S_i S_j + \ldots}{r_{ij}}$$

and likewise so for b_{ij}. Resorting terms gives

$$M_i = a_i||\Phi_i||^2 + \sum_{i \neq j} \frac{\widetilde{a}_{ij}|\langle \Phi_i|\Phi_j\rangle| + \widetilde{b}_{ij}||\Phi_i||||\Phi_j||}{r_{ij}} +$$

$$S_i \sum_{i \neq j} S_j \frac{\widehat{a}_{ij}|\langle \Phi_i|\Phi_j\rangle| + \widehat{b}_{ij}||\Phi_i||||\Phi_j||}{r_{ij}}$$

leading to the conclusion that $\widetilde{b}_{ij} > 0$ (the potential energy of upper-lower communication is positive, leading to repulsion) and $\widetilde{a}_{ij} < -\widetilde{b}_{ij}$ since identical choices should overall attract and therefore lower the energy. There are corrections to this depending upon the sexuality; in this regard, we take the viewpoint that interactions between opposite sexes with identical choices have negative contribution (causing for more attraction) meaning that

$$-(\widetilde{a}_{ij} + \widetilde{b}_{ij}) > \widehat{a}_{ij} + \widehat{b}_{ij} > 0.$$

Furthermore, we assume that opposite sexes with the opposite choice leads to less repulsion (and maybe even attraction), leading to $\widehat{b}_{ij} > 0$ ($\widehat{b}_{ij} > \widetilde{b}_{ij}$). This formula then suggests the following observations:

- interactions between upper-upper (lower - lower) choices result in overall decrease of the mental energy of each individual leading to attraction and a feeling of lightness (this effect is stronger between opposite sexes as between the same sexes)

- interactions between upper-lower choices lead to repulsion in the case of opposite sexes (increase of individual mass, a heavy feeling), but still might cause for attraction between the opposite sexes.

12.1 Further symmetries.

There exist plenty of issues which we can think about, and wonder about our profile. Next, we may consider taking an action (which contains as well information regarding an issue and possibly your profile thereupon) of expressing yourself. Now, there exist several possibilities here; either this new issue (consideration) refers to your previous thought, or it is only tangential to it. In case it refers to your previous thought and profile thereupon, would you also express your profile and if you would express a profile, would it be the same as the one you just considered? For example, I can wonder about punching someone on his face and think I definetly have a straight answer in mind (black profile) regarding this issue, but I could just communicate that I was thinking about it and, in case I decide to express my profile, I might utter that it is complicated, that there there are pro's and con's. We shall now concentrate on this very last possibility, that you faithfully express the issue you were considering but you may lie a bit about your profile. Actually, what I claim really matters is not the conceiling of your true thoughts, everyone does that to some extend, but the way you alter your expressions accordingly when your thoughts change. To be precise, we have an action profile matrix q_a and a thought profile matrix q_t, the latter which can undergo a change by means of an *action* q_i and we must ask how this change affects the action profile matrix. For example,

$$q_a \to q_i q_a,\, q_t \to q_i q_t$$

defines straight types, meaning the action responds in the same way as the mind does, and

$$q_a \to q_a q_i^{\dagger},\, q_t \to q_i q_t$$

defines the maximally twisted types, meaning the action is just the opposite. Mixed types are those who twist themselves to some degree, meaning for example that

$$(\beta_\lambda(e^q))(q_a) = e^{(1-\lambda)q} q_a e^{\lambda q^{\dagger}}$$

where $\lambda \in [0, 1]$ and the reader notices that

$$\beta_\lambda(e^q e^w) \neq \beta_\lambda(e^q)\beta_\lambda(e^w)$$

due to non-commutativity of q, w except in the cases $\lambda = 0, 1$. Mathematically, the reader should get used to the terminology that β_0, β_1 are called the vector and conjugate vector representations, $(\beta_{\frac{1}{2}})^2$ is the usual conjugate representation which is equivalent to a Lorentz transformation in the defining representation. It must be said that it is possible to just consider those types as actions of a change e^q on action profiles not necessarily referring to a corresponding change in the thought profile by means of the vector representation. There exist two distinct natural conjugations on the profile operators q, which are the complex conjugation $\overline{q}$ and the charge conjugation $q^c := \sigma_2 \overline{q} \sigma_2$ and

$$q^{\dagger} = -q^c$$

in case q has vanishing trace. Now, one may wonder to what extend they should relate to symmetries of interactions between two distinct spiritual beings regarding this particular issue. We define w to be self-dual in case $w^c = w$ (or $\overline{w} = w$) or anti self-dual in case $w^c = -w$ (or $\overline{w} = -w$). The first condition means that w has no charge whereas the second one says that its charge conjugate is minus itself. We say that w and w^c transform accordingly if and only if

$$(\beta^c_\lambda(e^q))w^c = ((\beta_\lambda(e^q))w)^c$$

and likewise so for the complex conjugate. We can now consider a pair of action profiles, located at nearby spatial locations and consider the joint profile as a single profile by means of the following

$$w \otimes v^c \to q = wv^c; \text{ with action } (\beta_\lambda(e^x) \otimes \beta^c_\mu(e^y))(w \otimes v^c) \to (\beta_\lambda(e^x)w)(\beta^c_\mu(e^y)v^c)$$

as well as

$$w \otimes v \to q = wv; \text{ with action } (\beta_\lambda(e^x) \otimes \beta_\mu(e^y))(w \otimes v) \to (\beta_\lambda(e^x)w)(\beta_\mu(e^y)v)$$

and

$$w \otimes \overline{v} \to q = w\overline{v}; \text{ with action } (\beta_\lambda(e^x) \otimes \overline{\beta_\mu}(e^y))(w \otimes \overline{v}) \to (\beta_\lambda(e^x)w)(\overline{\beta_\mu}(e^y)\overline{v}).$$

In some exceptional cases, these projections do define actions themselves; for example

$$(\beta_\lambda(e^x)w)\,(\beta^c_{1-\lambda}(e^{-(x^\dagger)^c})v^c) = e^{\lambda x}\,(wv^c)\,e^{-\lambda x} := \gamma(e^{\lambda x})(wv^c)$$

and the reader should notice that $(x^\dagger)^c = -x$ since x must be traceless. Similar results hold for the other two choices:

$$(\beta_\lambda(e^x)w)\,(\beta_{1-\lambda}(e^{-(x^\dagger)})v) = e^{\lambda x}\,(wv)\,e^{-\lambda x} := \gamma(e^{\lambda x})(wv)$$

and

$$(\beta_\lambda(e^x)w)\,(\overline{\beta_{1-\lambda}}(e^{-\overline{x^\dagger}}\overline{v}) = e^{\lambda x}\,(w\overline{v})\,e^{-\lambda x} := \gamma(e^{\lambda x})(w\overline{v}).$$

One notices that $\lambda = \frac{1}{2}$ is special and in all cases, we would first look at the associated transformations e^x that preserve duality meaning respectively that $x^\dagger = -x^c$ (which is identically satisfied) $x^\dagger = -x$ and finally $x^\dagger = -\overline{x}$. One notices furthermore that such "alligned actions" on action profiles naturally lead to invariants (conservation laws) such as are given by $\mathrm{Tr}(wv^c), \det(wv^c)$ in the first case. They are for sure useful in everyday conversations where people are adaptive to one and another. It remains to be seen how these mathematical symmetries should further reflect in the dynamics.

Chapter 13

Psychic symbols.

This chapter is by far the most outlandish one in this book; we have argued so far that there are very close parallelisms between the physical and mental world with one huge exception which was that mental energy appears to be negative and that therefore, the spirit has an unstable ground state, craving for action and physical energy consumption. Ultimately, the body runs out of energy by means of the mind-matter correspondence and tames the spirit in its tendency to consume. Indeed, to further elaborate on this, a human eats, drinks, thinks, moves around by itself very much in contrast to, say, a (steam) engine where the burning of fuel forces the cylinders to compress. If you would not couple an engine to a heat bath, it wouldn't do anything at all and remain at rest. The mind therefore is distinguished by a will to live, to be active; I don't claim that in the future highly advanced artificial intelligence would not be able to look for its own energy resources to be active and therefore also to have an effective will to live, even if not programmed to do so, given that I see no rational basis for the claim that such aspects of life should be limited to organic structures only. But what I do claim is that such a thing would require a change upon the traditional viewpoint of the lowest energy state in physics. It is for the moment, as far as I know, an open question as to where the electrical signals in the brain come from; for example, in a computer, a vibrating crystal is responsible for keeping track of time even if the power has been switched off. Maybe, our brain also contains such vibrating structures with a certain lifetime, sending impulses to the heart to pump and to the lungs to breathe. Even if that were the case, then it would still not explain why we feed ourselves or even why we know such a thing as hunger or appetite in order to survive. We are "programmed" as such and it costs energy to maintain all those functions; physical systems don't feed themselves, they simply undergo and survive most comfortably in the common lowest energy state possible. They harmonize in this sense, whereas the spirit does not. When I first wrote the text below, I was very much attracted to the idea of mental energy centers in the physical body, correlated to our state of mind and behavior, which closely resemble the chakra's in the Indian literature. I will without any shame use this terminology and speak about it in a realist sense beyond mere philosophy even if I cannot pinpoint as such a physiological grounding at this moment in time. Aside from that, I also tend to think that there must have been an (evolutionary?) mechanism in our psyche based upon mental images and concepts, which I shall call archetypes. As usual, I will try to be as mathematical as possible as this opens new perspectives to old ideas.

I shall use words here which are mostly used by mystics, such as astral eye, which basically refers to the divine light in you which you can actually see. Many schizophrenic people see shadows and hear things which most people don't; instead of calling it a desease or disorder, which explains nothing, I will take a more scientific point of view here and try to explain why we would see such things in the first place! From the point of view of physics, this is entirely possible, we are blind and deaf as hell; we only see a tiny bandwith of the electromagnetic spectrum and our ears likewise have a limited range. My tolerance here towards such a viewpoint stems from the fact that I have once in my life experienced such a thing for a couple of months myself, something which lawmakers and psychiatrists call psychosis. I can confess to you that my experience was nothing like what DSM V describes; these images are very real, I could see the most complex three dimensional figures in my mind spnning and oscillating at random in the utmost detail. My perception was much more sharp as it now is, you start to see connections everywhere, nothing is random any longer and likewise is this so for the voices which definetly seem to come from outside of your brain. I have no mercy, neither any affiliation with those butchers of the mind who look only at the most superficial and irrelevant things. I know of people who claim they can see aura's attached to physical bodies; it is not so that they want to be interesting or heard and who would I be to judge that this is delusional. Even if I would say it is, that is still no explanation of why they see such things in the first place and it is a downright insult to those cultures in the world who do recognize the existence of such a thing. As explained in part 1, there is definetly something connecting us all which goes beyond our observations, so let us carry this idea a bit further and maybe develop some theory. Other mental "reference subjects" which I shall use are the belly and the heart, where the former refers to both food and gut feeling whereas the latter is the source of life and compassion in the symbolic sense. What I want to suggest here is that those physical and metaphysical meanings are not formed by accident and deeply ingrained into the dynamics which shapes humanity as we know it. Indeed, the association of life with mercy is deeply embedded in any religion and we all recognize this as valuable and something to aspire. Finally, I will invoque such terms as marriage and sexuality as being central to the human endeavour. As always, our fundamental dichotomy remains the upper-lower choice one can take; let us now further introduce what I want to speak about.

Since we have rationally identified mental interactions with gauge theories, we must take the concept of psychic or mental radiation seriously. Indeed, sometimes you are in awe (or just the opposite) for a certain person without knowing why; he or she seems to radiate something quite mysterious and it is not that there is any rational ground for it regarding our basic senses such as smell and eyesight. Indeed, the girl in question may even not be physically very appealing or having a sweaty odeur, she is nevertheless breathtaking for a completely unknown reason. She radiates! I believe that we subconsciously detect such thing all the time and that we are drown by such mechanisms to alikes; you just knew there was a click from the very beginning. You did not have to talk to her, you did not have to experience her naked body, you just felt that she was fine for you. I think there is no rational ground to dismiss this as an illusion, quite the contrary! Another ramification of our findings so far is that upper-lower fields destroy the positivity of mass (so the mind anti-gravitates) whereas the gauge bosons obey it and therefore gravitate. Hence, people with a huge amount of (negative) psychic energy within themselves have the experience of being lighter in the head (floating in the air). This is also very real, I feel light in my head when thinking deeply, but after a while the body protests due to the enormous amount of fuel I am burning and I have to take a nap and some food. In the subsequent discussion, special attention will be paid to the Switchoriem as he was the reference person of the free theory without facts, logic and so on which all favour an upper profile in the end as evidence builds up. I will further define the astral (or third) eye here as being able to see this psychic radiation, allowing one to probe the "soul" of another person (of course from your perspective, so there is little or no objective value to that). The mental belly reflects then how someone feels in a given context; these are two distinct things, you can be drawn to a person by her beauty but still it might not feel all right. Last but not least, you have the heart which shows mercy or empathy; a heart can be so called warm or cold and likewise so for the belly and eye. These three symbols or functions are not issues, there is no profile neither (upper-lower) choice in them, but they constitute mere (quantum) variables which are not only functions of the psychic variables but also act upon them as to define a closed algebra[1]; just as consciousness was a (classical) variable and not an issue. Another few comments are in place here, first of all physicist's don't speak about issues but about variables. I illustrated this by means of our eyes who seem to have no freedom to take a profile, neither choice: for them, everything can effectively be described at the level of the conservative variable operator (unless you turn blind or so). Second, so far we have assumed that all our conservative/progressive issue operators were commuting and that you could mix them. This is not true in general either, in nature, it is very well possible

[1]By this physicists usually mean that some function of the dynamical variables $F(z)$ defines an action on any other function $g(z)$ of the dynamical variables by means of, for example, $F(z) \star g(z) - g(z) \star F(z)$ where $\star$ is the Moyal product (which amounts to the usual definition of the Poisson bracket by means of $\{f(z), g(z)\} = \lim_{\hbar \to 0} \frac{1}{i\hbar}(f(z) \star g(z) - g(z) \star f(z))$.). The requirement that several functions $F_i(z)$ constitute a Lie algebra amounts then to the condition that $F_i(z) \star F_j(z) - F_j(z) \star F_i(z) = \sum_k c_{ijk} F_k(z)$ with c_{ijk} a complex number. One could even strenghten this for $\hbar \neq 0$ by demanding that $F_i(z) \star F_j(z) = \sum_k d_{ijk} F_k(z) + d_{ij} 1$ where 1 denotes the canonical central extension of your algebra. In any way, in a system with the operation a Lie algebra, one can always look of course for representations, which possess a natural product underlying the Lie bracket, such that the resulting algebra is a mere central extension.

to construct variables (conservative issues) which do not commute and do not constitute our fundamental dichotomy either; moreover it often does not make sense to mix arbitrary issues as that amounts to comparing apples with pears. The really daring thing I propose here is that the way you primarily interact with people by preferring either your perception of the heart, the eye, or the belly, will have ramifications on your physical constitution; the way you look, whether or not you get fat during a marriage and so on. I believe that distinct cultures also have different traditions of approaching the way they prefer to interact with others in the sense to what element of the above trinity they find the most important one; that is a second conjecture if you want.

A parameter which is important in the way people interact, and can be seen as a black appreciation of the issue "to what degree am I energized mentally?" (so there is no need to enlarge our language here, we just have to include one extra issue), regards the mental energy they use (radiate?). If a person mentally engaged in a conversation speaks to someone who uses way less energy, then either the other person can upgrade and engage in more activity or, on the other hand, try to downgrade the speaker either by shifting the topic of the conversation or serving some food which lowers mental activity. As said, the astral eye can see the psychic person but still then, it remains to determine wether this information results in an attraction or repulsion as the two extreme opposites; the former being called the hot eye and the latter the cold one meaning that (minus) the average energy of the eye is high or extremely low. This coincides with our previous philosophy which was that the mind loves to increase (minus) its energy and is therefore attracted to anything which amounts to this effect and repulsed towards anything which causes for the opposite. Life is not that simple and we involve in a much more complicated way with one and another mixing the eye, belly and heart. It might be worthwhile to quantify this more as it appears to me that nature has forseen that, in engaging with one and another, there usually is a minimal "temperature" associated to the trinity (which in physics would amount to a choice of density matrix, a kinematical and not dynamical constraint); this implies that, for example, even if the expectation values of the individual energies attached to the eye, heart and belly are close to zero, we still engage ourselves with a minimal amount of psychic energy leading to an overall satisfaction in engaging with others even if not very gratifying on several points (so, we use a different "Hamiltonian" as just the three separate individual "Hamiltonians"). I say, usually the case, because some people are cold as hell and still thrive in society at a formal high level; very dangerous I must add. A third conjecture I make is that this overall temperature (an effective parameter describing the more detailed and complicated upper-lower interactions) constitutes the main foundation of societal interactions which goes way beyond things such as intellect and money. For example, you may have a deep friendship with a person who does not resonate with you on an intellectual level, whereas you might want to restrict the number of contact hours with a person with whom you intellectually thrive but otherwise are rather cold of. Let me also mention that the association of the belly to "gut feeling" is by no means a gratuitious play of language, given that eating and drinking together while talking leads in general to a better feeling (and therefore more patience) regarding the person/conversation. Indeed, family gatherings over dinner or meetings between partners in the restaurant are aspects of fostering community life in many cultures. I claim even more than this which is that your psyche (upper-lower choice, profile, white reality parameters, consciousness parameters) is correlated to the kind of food you like and which in a way is the best for you.

In a way, the trinity expresses how we see, feel and relate; three cornerstones of human interactions. But we already had four cornerstones of upper-lower interactions, those were given by the generators of the classical $U(1) \times SU(2)$ gauge theory (the profile operators did not define a new gauge field as explained in part 1). Now comes the real beef, we have discussed in part 1 upon how we can associate the Pauli matrices with (anti) hermitean operators on the Hilbert space of square integrable functions in the black variables so that classical variables can couple in a nontrivial way to a quantum system; what we proclaim now is (you may call that conjecture four) that the trinity coincides with the $SO(1,2)$ part and its associated local charges (which are globally conserved)[2]. This implies that overall, in the entire society, global empathy, spiritual perception and feeling are conserved quantities; they do not evolve in time. At the level of a single individual, which we shall discuss first in full detail, this implies that the correct evolution operator in the psychic variables is given by a multiple of the identity operator if one takes quantum corrrections into acount (classically the Hamiltonian vanishes)! Pretty boring indeed, but a reflection of the fact that without social engagements or any exterior world, our conceptions of what some of us think what is and what is not (note that the notions of back and white are collective ones, since any individual can choose its own profile) do not alter. If we were only to interact with "simple" matter, then there is little or no engagement and the reality, as seen by some of us, remains the same. In a way, this expresses conservation of three distinct types of energies; this is a pretty damning constraint upon humanity which I believe to have been experimentally verified already over history. Humanity does not change fundamentally, we still go to war for the most silly reasons, most of us have very limited empathy and are mostly interested in telling about themselves, enlightened minds keep on having difficulties with authorities, life does not feel any different now as it did in the middle ages and so on. The only way humanity progresses is by means of paradigm shifts; meaning an old lunatic now has become sane, but at the same time a new category of lunatics is born providing for a countereffect. Never ever has society reached the only valid conclusion, after so many generations, which is that lunatics don't exist, only enlightened spirits think as such and there are

[2]Note that the algebra holds at a local level of charges for local gauge symmetries, for example as is the case for the $su(2)$ gauge transformations in loop quantum gravity by means of the Ashtekar variables or in non abelian gauge theory by means of the time component of the Noether charges. For global symmetries, the situation is not far worse given that integrated charges will only deviate from the correct algebra by means of boundary terms.

not too many of them. Concretely, the heart corresponds to $H \sim -\sigma^1$, the belly to $B \sim i\sigma^2$ and the eye to $E \sim i\sigma^3$. How those operators were connected to the other Pauli matrices and what suggestive relationships exist between them has been discussed in part 1. The heart is nothing but the Schitchoriem energy plus $\frac{1}{2}$, which is related to time not only by means of the Schrodinger equation but also by means of its very definition as a Hermitean quadratic form assoicated to the identity matrix! The eye preserves black and white and is therefore associated to E whereas the belly B mixes them; the curious thing is that for any operator O, which is a linear combination of X, P holds that

$$[B, [E, O]] = i\,[H, O]$$

and more in general

$$[B, [E, V]] - [E, [B, V]] = 2i\,[H, V]$$

for any operator V. The spiritual heart is therefore associated to the "awareness-impetus" or conservative-progressive. Hence, it is meaninful to consider thermal states with respect to the heart or Schwitchoriem Hamiltonian and calculate expectation values of all other operators, such as

$$\frac{\mathrm{Tr}(Ee^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = \epsilon$$

where β is the inverse temperature of the state and ϵ the mean energy corresponding to the eye. Regarding the spiritual belly operator, $i\sigma^2 \sim B$, we notice the following conjugation

$$([B, O])^c := \left[iE, [B, O]^\dagger\right] = \left[iE, \left[B, -O^\dagger\right]\right] = \left[B, \left[iE, O^\dagger\right]\right] = [B, O]$$

for O a real linear combination of X, P and the superscript c denotes the charge conjugate by means of (i) the eye operator. This is completely equivalent to

$$\left(\sigma^2 v\right)^c := -\sigma^3\overline{(\sigma^2 v)} = \sigma^3\sigma^2\overline{v} = -\sigma^2\sigma^3\overline{v} = \sigma^2 v$$

where $v = (aX, bP)^T$ with $a, b \in \mathbb{R}$ and $\overline{v} = (\overline{a}X^\dagger, \overline{b}P^\dagger)^T$. So this means that the conjugate action of iE on the adjoint action of B equals the adjoint action of B and therefore charge conjugation intertwines between both actions. A similar result holds for the heart operator. The heart operator is positive definite and has a discrete spectrum of the form $n + \frac{1}{2}$ where $n \in \mathbb{N}$ meaning it is always activated. B, E on the other hand have a continuous spectrum which covers the entire real line; for example, the eigenstates associated to E are given by

$$(-is\frac{d}{ds} - \frac{i}{2})\Psi_\alpha(s) = \alpha\Psi_\alpha(s)$$

resulting in

$$\Psi_\alpha(s) = \frac{1}{\sqrt{2}}e^{\frac{i}{4}(2\alpha+i)\ln(s^2)}$$

and the reader may verify that all orthogonality properties are satisied meaning

$$\int ds\, \overline{\Psi_\beta(s)}\Psi_\alpha(s) = \delta(\alpha - \beta).$$

Regarding the belly

$$B = e^{-\frac{i\pi}{4}[H,\cdot]}E$$

which is a unitary transformation and hence preserves the spectral decomposition. So therefore, either the belly or the eye can be off, meaning having zero eigenvalue. It is furthermore possible to consider states which are invariant under $e^{i\frac{\pi}{4}H}$ up to a unitary factor; these regard superpositions of the form

$$\Phi_k := \sum_{n=0}^{\infty} a_{k,n}|8n + k\rangle$$

where $k \in \mathbb{N}$. In this case,

$$e^{i\frac{\pi}{4}H}\Phi_k = e^{i\frac{\pi(2k+1)}{8}}\Phi_k$$

so that

$$\langle\Phi_k|E|\Phi_k\rangle = \langle\Phi_k|B|\Phi_k\rangle.$$

In particular do we have that for thermal density matrices

$$\frac{\mathrm{Tr}(Ee^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = \frac{\mathrm{Tr}(Be^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})}$$

so that both have the same energy and

$$\frac{\mathrm{Tr}(He^{-\beta H})}{\mathrm{Tr}(e^{-\beta H})} = -\frac{d}{d\beta}\ln\left(\mathrm{Tr}\, e^{-\beta H}\right) = \frac{\beta}{2}\coth\left(\frac{\beta}{2}\right)$$

as a small computation reveals. Another interesting observation is that $H - B = -P^2 \geq 0$ and therefore $H \geq B$; by means of our rotation, we likewise obtain that $H \geq E$ meaning the heart is dominant over the eye and belly. As mentioned in part 1, when thinking about an issue, we use the dichotomy and upper-lower choice to get an operator $Z = aX + bP$ and we can only measure its energy defined by

$$Z^\dagger Z = |a|^2X^2 - |b|^2P^2 + (\overline{a}b - \overline{b}a)(XP + \frac{1}{2}) - \frac{1}{2}(\overline{a}b + \overline{b}a)1 =$$

$$(|a|^2 - |b|^2)B + (|a|^2 + |b|^2)H + i(\overline{a}b - \overline{b}a)E - \frac{1}{2}(\overline{a}b + \overline{b}a)1.$$

Hence, any decision operator canonically defines the way we communicate with one and another: a remarkable conclusion! For example, Schwitchoriems communicate by the heart, black people by $H + B$ and white people by $H - B$. Also note that the heart is the only pure (non mixed) choice regarding the trinity you can make. To communicate with the eye, you need $b = i\lambda a$ with $\lambda \in \mathbb{R}$, this results in

$$|a|^2(-2\lambda E + (1-\lambda^2)B + (1+\lambda^2)H).$$

Now, before we come to the fascinating issue of mental superposition between two partners, where they forget about anything whatsoever and are consumed by a blissfull feeling, something which happens when you are quietly sitting with your wife in the couch and think about nothing, time passes quickly and you just experience "temperature". To make the analogy with Wagner's Tristan und Isolde when she is going to serve him on the ship the love potion which will make him forget about who he is and join in a blissfull union. First, we shall treat distinctions between several nationalities which I have observed in the past and which approximately seem to hold. To be precise, the Belgians and the Dutch communicate maximally by mans of the belly, meaning they are either black or white and there is no grey zone; the result is that those people either have an opinion about everything or they revolt the system. Indeed, the Dutch are known for their outspoken opinions even if there is no rational, compelling reason to be as such: this leads to a vibrant debate culture with lots of specific suggestions being made; the downside is that this leads to a society where everything is classified and subject to social conventions and the Dutch are as such indeed, leading to a serious embedding of psychiatric institutions to cure those who are different. Belgians are somewhat more black-white mixed leading to less rules but more conflicts and social anxiety. Both cultures have the tradition of heavy food, beer and a complete absence of spirituality, something which resides in the eye. The Catholic church in Belgium is almost dead and the Lutheran church in the Netherlands is not one of spiritual beauty and contemplation, but one of strict adherence to rules and debates about biblical interpretations. They just don't live through a religious ceremony, and I even guess they do not understand what this is supposed to mean. The Polish and the Italians mainly go by the eye, they are very spiritual and kind hearted people; the Polish also seem to involve the belly to a higher degree as the Italians do. Indeed, their food is also certainly more healthy as the one in Belgium and Holland, but very basic and certainly not as light and delicious as Italians cook. Polish also drink beer, but by far not as much as the Belgians and the Dutch and they consume more spiritus which, as the name says it, stimulates the eye and not as much the belly. Italians on the other hand drink more wine and liquor which are drinks of the heart and are somewhat a bit more spiritual as the Polish. Indeed, Italians and Polish are known for story telling, not debating; this results in somewhat a less dynamical atmosphere as the one in Belgium and Holland, but opens the avenue for long and thoughtful conversations and loads of creativity. Indeed, Italy has long been the cultural centre of the world and food consumption there merely accompanies a long conversation and social gathering, just as this is the case in Poland, whereas Belgians and certainly the Dutch are "functional eaters" spending little time at the table. The French and Swiss in my opinion almost equally share the belly and eye and the heart, leading to healthy, solid food with a good culture of wine and a bit of beer and a nice spirituality: those operators are given by $s \leq r \leq (\sqrt{2}+1)s$ or $r \leq s \leq (\sqrt{2}+1)r$ such that $\frac{r^2-s^2}{2rs} = \sin(\theta - \psi)$ and $a = re^{\theta}$, $b = se^{i\psi}$ leading to

$$(r^2+s^2)H + (r^2-s^2)B + (r^2-s^2)E$$

and the reader verifies that the ratio $\frac{r^2-s^2}{r^2+s^2}$ is optimal at $r = (\sqrt{2}+1)s$ resulting in $\frac{2(1+\sqrt{2})}{2(2+\sqrt{2})}$ leading to $H \geq \pm\frac{(1+\sqrt{2})}{(2+\sqrt{2})}(B+E) \sim \pm 0.707(B+E)$. It must be clear here that I am speaking of issues for which no logical settlement has been established yet; but there is more than that, in conversations people will always have the tendency to select the issue as such that such communication mode is justified. For example, when talking to a Dutch person about a mathematical theorem, he will simply give an answer and that's it. Italians, on the other hand, might start discussing the underlying assumptions, the beauty or the ugliness of the result, and novel ideas regarding counterexamples if some assumptions are dropped or generalizations thereof. In my opinion, as far as experience with both cultures reaches, this is indeed the case.

13.1 Spiritual bounds.

Now, we come to the discussion of joint states or spiritual marriages if one wants to. Many of us have had the strange feeling of synchronicity by which I mean that highly correlated events occur without an obvious causal explanation from known mechanisms. Now, science tends to dismiss those as pure coincidences, but I think the occurances are just too high the be explained as such as is the case for the speed of evolution of the human species, suggesting for something way beyond random selection. Now, of course, I am realistic enough to set bounds to which this an be realized in practise, just like Schrodinger's cat will probably never materialize. So, channeling of conscious thoughts does seem to require too much information for such mechanisms to carry; but I am speaking of phenomena people often experience, like for instance me calling my ex wife or vice versa and she saying, “I was just thinking about you and thought you maybe you felt like this”. Or, people travelling around the globe going to places where they have never been before and meeting people as if it were predistined to be as such, as if they knew these persons for their entire life. I am fed up with the priests of modern science who wish to impose upon you (even by law) that this is an illusion of some kind; that we are all classical separated individuals who can be detached from society with razor blade precision. Some of them even try to ridicule the idea by so called designing “objective” tests without realizing that, by this very act, they fall within the same category as all those people, which they call crackpots themselves, denying quantum mechanics because they never have seen a Schrodinger's cat. I hear you already say, surely mister Noldus, I am sane of mind, I also have noticed those things but I do not see why they could not be ascribed to a classical mechanism. I can only say, why bother? I mean, ultimately, each mystery of quantum theory may be ascribed to a classical underlying mechanism. But the point is that if this were the case, then our view upon what a particle really is and how it behaves energetically should be radically overthrown! For example, you can explain the Schrodinger equation as being the result of a kind of self-field interaction of the particle, causing it to spontaneously accelerate and self-interfere; nobody has ever constructed such equations but it may very well be possible. Likewise could entanglment be explained by the opening of wormholes which are stable for a while; but this requires negative (mental?) energy, so is a particle mentally active? Does it microscopically change our very perspective upon spacetime; that we are blind to such things. In the previous section, we explained that the full wave function lives in an infinite tensor product

$$\mathcal{H}_T := \otimes_{i=1}^{\infty} \mathcal{H}_i$$

where $\mathcal{H}_i$ denotes the Hilbert space of square integrable functions with respect to the flat metric attached to the mental issues defining spirit i. These Hilbert spaces all have a representation on spacetime which is evolving towards the future. Hence, we already have that spirits are entangled, but here we shall go one step further by considering entangled operators or so called "contracts", as I prefer to call them, where individual observers take expectation values of the individual heart, belly and eye operators coupled to other eye, heart and belly operators after projecting down the communal state by means of their own mental energy profile operator (and possibly the profile itself if the latter is definite). This is the weak form of contracts, where people still think for themselves but care about the feelings of others. The stronger form is when you effectively do not project down by means of an individual operator, but society projects down on a complex entanglement of individual operators, so that you effectively stop thinking and feel the resulting state from other product operators attached to the heart, belly and eye respectively. They continue to live in a complicated superposition of product states attached to common values of the society of which they are not aware at the level of (maybe unconcious) thought, which presupposes individuality, but merely experience a certain temperature regarding an interaction operator which they freely choose (in either they feel the response of others regarding things they care about). The strongest form of collectiveness happens when you feel those things precisely in the way society wants you to feel them, that is when the individual heart, belly and eye operators are canonically defined by means of the communal operator used to perform the reduction of the communal wavefunction. This viewpoint is the mathematical realization of the dichotomy of individualism versus collectiveness at four possible levels (the fourth one being that you only consider your own isolated feelings and not how they interact with others); as mentioned previously, an isolated heart always feels (luke) warm but individual hearts in a collective gathering can be either very warm or cruel (negative temperature) as happens in bad marriages or oppressive regimes. Let us therefore refer to the isolated heart as the bored heart; there is a lot to say for it, but ultimately we all crave for variation and not necessarily in the positive way (more empathy, or hotter heart) but also in the negative way where people enjoy being cruel towards one and another and have destructive tendencies.

Let me stress that this is a rather unusual extension of quantum theory which always pressuposes that each observer must independently ask him or herself an isolated question and that different observers are asking questions at slightly different times. There are two issues here, if two distinct local observers would ask two different questions at the same time then there is only one collapse of the wavefunction and albeit the order in which the questions are asked does not matter, there is only given one answer which is a communal one, in either the product of both eigenvalues. This is the reflection of the fact that in quantum theory, there is only one global observer really who controls the entire universe, so in order to correct that you would have to change the usual interpretation in the sense that both observers should be aware of the result of their individual questions and this is indeed a well defined notion in relativistic (but not Euclidian) Quantum Field Theory due to causality of the local observables. In our approach to quantum theory, we refined this issue even more: we did not care about coordinates, so when a measurement apparatus measures the so called position of a particle, it does not produce any number, the latter is an interpretation a local human gives regarding the change of state of the measurment apparatus. There are really two different processes here which are usually identified as one and all details are swept under the carpet which causes for a terrible confusion. The first thing is that in first instance, nobody cares about the eigenvalue of some global product operator (which is a meaningless quantity), but they only deal with the projection itself which changes the state of each individual measurement apparatus and this is the only thing which is "felt" by the apparatus in some way. This is a radical change of perspective on quantum theory where observables pertaining to the relation between the outside world and inner world of the observers, a position Bohr very much defended, are not confined any longer to one local observer only and therefore one may forget about the eigenvalue (that only serves God). The real measured eigenvalue by the local observers regards some operator pertaining to the inside world of the observer who is measuring himself accordingly without further changing the outside world. In this vein is the meaurement apparatus "aware" of something hitting it by observing the currents of its constituting variables. We, as humans, come second in line, we observe photons scattering on the changed measurement apparatus and we presume that the measurement apparatus is faithful or static towards us in the sense that it did not alter its appearance between the moment it responded to the interaction with the particle and us interacting with the masurement apparatus by means of photons. Likewise, the observation our eyes make is one of local electrical currents which are coupled to the photon field and not the photon field itself; we just assume the photons project down accordingly and interpret the feeling (which we express as a number) attached to the state of our eyes, after all eyes in the universe have measured photons at the same time, as corresponding to an objective property of the photon. The measurement apparatus does exactly the same thing when so called measuring the position of a particle hitting it, I really measures “unusual” internal currents due to the impact of the particle arriving there, it only “knows” about the particle through perception of its own state and comparing it with its bias of what is it and what is the rest! So, an eye is getting classical at least every nanosecond and its state is what we ascribe to light with all it colours attached. For the mathematicians under us, you would say that this can be described within category theory where true observation happens on the level of objects and the impact of it reflects on the level of functions. Indeed, I claim that we cannot observe the outside world really but we can only know the response of our own body regarding the interactions with the outside world. We just have to assume that all photon measurement apparati are the same and producing commensurable results otherwise there would be no possibility for us to deduce any laws. I did not discuss energy-momentum observations of photons as yet in this book, but the reader may very well take a simplist approach and forget about the electrical currents inside the eye and just do Fourier analysis as usual of the photons as defined with regards to the local observers as if one were projecting down the photon field in first instance. This is something we do all the time in science to the extend that most even fail to comprehend that they are doing it. The observer is always swept under the carpet and people wrongly attach meaning to the values some abstract global operator attaches to the interaction of the observer with the outside world. As said, this is not what we can measure directly, we are only aware of our own body and ascribe some aspects of it as due to some unknown interaction with the environment, we are just guessing basically. The fact that we are guessing reflects in distinct opinions regarding what is internal or holistic: indeed, stomach pain could be thought of as extraeneous due to deamons pinching needles there. In the middle ages, this could have been the standard view, whereas nowadays we think of either bacteria or too much acid being responsible for the sensation. In the first case, it is still an “exterior” cause whereas no doctor would say that the universe is leaking extra acids into you stomach by means of wormholes. He would just say that you produce yourself these acids without even being able to understand the mechanism of how this happens: it is just a convention. This is what I would call a mental inertia principle, that we presume something happens when we experience something internally which we do not ascribe to ourselves, but this differs from person to person! Psychiatrists have the illusion that their idea of the interior-exterior world is the true objective one and that they can decide whether your senses are appropriate or not given that they presume their observation of the outside world and interpretation theirof to be the holy one; meanwhile, he or she is just having the same trust in this very principle. The reader must understand here that there is no truth in all of this: the reason why we value the doctors opinion regarding the acids in the stomach is because he prescribes calcium based medication or zantac and this helps. So, he has free will and can act to cure the evil. But nothing prevents you to deny that this is all just a matter of biochemistry, but that there is a deeper underlying reality in which the calcium serves as an offer to the spirits so that they stop putting acids in you. The reason why western society does not uphold such view anymore is because it is redundant in many ways; now, unlike zantac, psychiatric drugs have no effect whatsoever but just put you asleep so that the "issue" does not occur. It is very much like putting someone in prison in very many different meanings.

In this section, we go another step further, as mentioned previously, an observer in a community stops asking individualized questions (all questions themselves are entangled), it does not engage into independent thought any longer but merely feels, just like we feel temperature which is also not an observable quantity, the impact of society upon its emotions without adding anything to it. Indeed, when being indivualized, you just take expectation values, which is a form of higher awarenes, of your individual heart, belly and eye operators with respect to an (approximate) eigenstate of your decision operator; in a way, this is how you feel about yourself when dealing with your thoughts without thinking about others. Everyone, who is individualist, feels he is a compassionate person. In the previous edition of this book, I described the joint situation, which I want to discuss, as being the result of a measurement done by a higher joint spirit who sees us as quantum; I changed my mind of presenting it in that way because, albeit such a point of view is holistic in nature, there is nothing higher about this spirit at all in the sense explained in part 1 since you can always break free out of it at a higher level and return to individualism (consciously or unconsciously) at a lower level of issues. For example, in a two person society, the relevant operator may be

$$H \otimes (aE + bH + cB) + B \otimes (dH + eE)$$

where we have done the canonical decomposition with respect to the first individual. In case the two person spiritual state is $|\Psi\rangle$ then I define the "feeling" of the heart of the first observer as being given by

$$\langle \Psi | H \otimes (aE + bH + cB) | \Psi \rangle$$

and that may very well be a negative number indeed. Note that at this point, as mentioned before, you are not dealing with this issue in your mind at that level, it is not that it is mixed with other issues, which is a very different thing, but it is just not even definite. It pertains to questions about how others relate to your individual decisions regarding the decisions they have taken themselves. As I said, you basically stop thinking about your current profile on these matters (consciously or unconsciously) but you just feel the communal response at a higher level. Now, there is another higher awareness regarding this feeling in the communal spirit and that pertains to the issue of wether you like this feeling or not; for example, you have people who crave on complete adversity towards others. As mentioned in part one, we have excluded so far these higher questions referring to more basic questions (of possibly different observers) since in general, you cannot ask for a decision and heart at the same time unless you are a Schwitchoriem where all those notions coincide. So the best you can do is ask for an expectation value regarding those issues. In that way is individualism the safest strategy to go through life, you are inherently peaceful, mercy is the dominant force whereas in society you might become a killer such as happened, for example, to the Germans in the 1930 ties when peaceful people got in the grip of fascism and started to haunt Jews, something they would never done on an individual basis. People, who have a bad feeling when immersed in the collective "reality" might want to withdraw and isolate themselves becoming peaceful again; usually, this is seen as a danger to society as, especially prominent, members wish to cancel their engagement. In that way is the curch of immense importance since it urges us to be, in the first place, one with God and Jesus and they are warm, constructive in nature and hence offer a, perhaps imaginary, reinforcement of an individualist kind of attitude towards others. Indeed, religious life is a solitary one, far removed from the hustle and bustle of society. The importance of this cannot be stressed enough for example by means of the Polish people, were religion was the state enemey during communist times and people remained friendly overall due to the immersion in the beauty (spiritual eye) of the creation. Poles have finally stood up against communism and fascism during for example, the Warsaw uprisal. It makes them into an extremely resilient, creative and warm nation where I was happy to reside for a while. I will just give away some further examples of compound operators and try to formulate some principles behind them; not all compound operators are realized, actually very few of them, and one ultimately has to come up with some selection principle.

Before we proceed to that, let us introduce the notion of sex conjugation (another dichotomy, woman versus man) interchanging men and woman as well as their profiles/choices and all that: a Hermitian operator S which acts on $\mathcal{H}_T$ and for which the state of the universe is neutral (that is has eigenvalue close to zero). Here, it must be taken into account that there exist different dominant notions of rationality (logic) between the different sexes who aspire precisely the opposite. This has nothing to do with black and white as we all aspire to become black eventually and alikes still attract. Let me explain what I mean by means of some examples: men who aspire classical logic, which has been the driving seat of progress in humanity, study sciences, engineering ... whereas female logic usually leads to specialization into the social sciences, psychology, nursing, psychiatry and even medicine (which operates according to intuitionistic logic). The men who are in the middle between classical and intuitionistic logic usually study economics or law, whereas the intuitionistic ones choose for medicine, psychiatry and psychology. Indeed, doctors, psyhiatrists and psychologists are wizards from a classical point of view; this reflects in the fact that, especially psychiatry, changes of theory or point of view every couple of decades, whereas physicists and mathematicians never ever change their mind, they just refine and generalize their knowledge. Indeed, classical logic is stable, God given as to speak, whereas intuitionistic logic is temporal and suggestive instead of having a precise insight. Medical doctors also classify somewhat in this category but to a lesser extend; they would argue that it is the very nature of their field that it is as such, but this is total nonsense according to me.

The great progress in medicine has not come from trained doctors, but from physicists, engineers and chemists (hard core boys), who have constructed microscopes, RX and MRI apparati and offered a deeper insight into the working and development of pharamaceutical drugs. Woman who want to break free out of their traditional role study precisely mathematics, physics and engineering, resonating very much with those men at least on the intellectual side. Remember that I have said that such resonance is secondary to the more primitive way of approaching relationships regarding the upper-lower perspectives. For example, you may meet intellectuals of the same kind who take different sides and profiles on things which are unknown; with a slight abuse of language, you may say that in this case the lower perspective is the more rational one (but that is a matter of taste). Such distinctions may lead to severe clashes between equally qualified collegues and cause for personal frictions whereas you might sympathize with someone who has a different kind of logic but who makes the same choice as you do. Ultimately, where intuitionistic logic and classical logic meet one and another, fruitful results start to emerge. Therefore, since men and woman procreate, there is always a mixture of different rationalities making room for real feelings such as given by the heart, eye and belly; things which are not completely rational but seem to stabilize society. In that sense might a civilization which is entirely based upon one choice of logic be wiped out in a very short while. Therefore, any civilization which is low on spiritual investment and thrives primarily on logic (of some kind) as the primary force, is bound to lead to psychiatric patients, psychopaths seizing power and huge divorce rates. This is why science and religion should coexist.

In order to find a principle limiting the possible joint operators which occur in the world, I constructed the notion of a marriage contract which constitutes the very basis by which your decisions couple to one and another: indeed, in a marriage it is important that you take commensurable decisions even though you might dislike the profile your partner attaches to it. Point is that you rarely discuss such profile and just deal with the practicality of doing similar things. So even though there may be a repulsion regarding the choice field, there are paradoxically enough warm feelings attached to the partnership which leads to marriages where both partners function together very well but dislike one and another. In analogy with the single individual where your decision determined your way of dealing with the trinity, likewise will we consider here that the way a couple deals with a certain issue (we shall restrict here to $N = 1$) determines how the couple projects itself towards society and how they feel themselves in the union. The big difference here is that, when people engage, they loose their own opinion and take a common position which is an entangled one. This reflects that when a couple speaks out, they never reveal their own thoughts but present a compromise in which different individual opinions live together in superposition. A further point is that you want to express yourself in the union as faithful as possible regarding your own values and that you want your partner to appreciate your decision even if he or she dislikes the side from which you approach it. In case of a mixture of the belly and heart, the marital contract

MC could take the form

$$MC = \begin{pmatrix} 0 & e^{i\theta} \\ e^{-i\theta} & 0 \end{pmatrix}$$

which agrees for $\theta = \pi$ with the heart $-\sigma^1$ and $\theta = \frac{\pi}{2}$ with σ^2 which is the belly. One would expect such theta angle to be a dynamical variable varying from culture to culture: in Belgium for example, people seem to love others primarily based on whether their partner feels good in their company or not (the marriage contract is one of the belly). In Poland, on the contrary, it seems (I am no expert in this) important that your heart is connected to the spiritual beauty of your partner. This is of course a direct consequence of the fact that Poles usually express themselves by means of the eye and Belgians by means of the belly as discussed before. Before we discuss this in somewhat more detail, notice that

$$\left[\ln(\sqrt{-P^2+\epsilon^2}), XP\right] = \frac{-P^2}{(-P^2+\epsilon^2)} \sim 1$$

is an approximate Heisenberg conjugate to the operator $XP + a1$. It is not an exact one which is logical due to fringe effects. Therefore, a "Polish marriage" is given by the operator, given that $(i\sigma^1)(\sigma^3) = \sigma^2$,

$$a(X,P)^\dagger(X,P) \otimes (X,P)^\dagger\sigma^2(X,P) b(X,P)^\dagger\sigma^2(X,P) \otimes (X,P)^\dagger(X,P)$$

$$= -4aH \otimes E - 4bE \otimes H.$$

I will leave further exploration of those ideas for a differentt book of mine on psychology which basically contains the same material as in this book but with further comments and ramifications.

Chapter 14

Some final thoughts.

We have explored in this book some non-obvious connections between the physical and mental world which basically all resulted from the mathematical formalism by which we described dichotomies. The last chapter was by far the most suggestive one and you are of course free to agree or disagree on some of those matters. I am firmly convinced that the future will lead us to such perspective upon sciences of the mind although we shall never have a complete understanding of these issues. In this final chapter on the mind, I want to be descriptive of some sort and explain how thes findings connect with real existing philosophies regarding spiritual enlightenment and how we relate to our ancestors shaping us partially as well as the future of humanity, a collective historical memory as to speak. In thay regard, I was particularly drawn to Buddhist teachings and Salafism, although I never took the full effort to understand the latter. Although science is a great gift from the West, we must ensure that it does never ever become an arbiter of truth and thereby commit the same sin as some religious institutions did in the past. True enlightenment consists in the deep realization that we shall never ever know what is true or not: all our knowledge is contextual. Moreover, this almost forces you to have an open window towards the world and have a mild judgement, if any at all, about others. In that vein would I encourage policy makers to follow the Italian example and dispose of involountary psychiatric treatment which is a danger coming from the West which is highly correlated with "scientific correctness". I would further stimulate psychiatrists to look deeply into their own soul and dispose of all psychiatric "deseases" and return to the old fashioned idea of a sanatorium, where people are welcome to rest for a while and be released from the burden of society. Our western world is filled with such pseudoscientific bullshit coming especially from the UK and US involving concepts such as human resources, people management, psychological assessment, IQ tests and so on, whereas you simply must take the time to engage with other people and really speak to them. I am digusted with all those useless consultancy agencies, making you pay big time for their "professional advice" on how to prepare for an interview, how to highlight some aspects in your CV, explain to your employer why you did this and that and so on. True, deep values come through long periods of hard work and introspection, something which the western world has unlearned.

In a way, I am rather sceptic about the human race and sometimes I dream about what it would be like to have an alien invasion by creatures who look upon us as their food. Maybe, I hope, will they recognize me as a dog puppy and not kill me but play games. If not, at least I am consumed by the better part of me and die in honour. But who knows are the aliens Buddhists and let all animals in piece while feeding remorcefully upon plants: this is an iron law for anybody, we need to eat. Now, I am someone who likes a good piece of meat, but I enjoyed very much the vegetarian kitchen of Buddhist monks with the most delicious flavours and the most sumptuous buffet. Maybe, in the very best case, is the work explained in this book of service regarding a better and deeper understanding of humanity, perhaps changing the spiritual constraints explained in the previous chapter and transcend our current existence. Who knows, there are plenty of physicists who believe that the laws of nature themselves evolve towards the future and I am not hostile to such an idea. In contrast to many people, I am not afraid of AI either; in case a computer would just remain that, humans will always find a way beyond the mere computable and second if computers would come to power, they most likely would kill each other instead of dealing with the "lower" species.

I am going to leave at as that: I love being dry and factual however I am not dirty of a personal touch either. But this is enough before I really start to rant. We shall conclude this book by studying the consequences of our new viewpoint on quantum theory what regards extended objects such as strings and the entire universe.

Part IV

Quantum theory of extended objects.

Chapter 15

Quantum gravity type one.

The search for a theory of quantum gravity is one of new principles of nature and involves the question if and how the superposition principle should be applied to space time itself. Crucial in our story so far was the presence of a classical space time metric and therefore, a quantum theory of the space time metric appears to call for a super metric: a metric on the space of all geometries (I leave it open here whether one should "quantize space" or spacetime - the standard canonical first quantization procedure calls for a quantization of space whereas some other people might suggest that you have to quantize spacetime). Those, who keep on insisting upon a Feynman integration theory are facing the question of the canonical character of the "measure" where the latter has to be understood in some limiting, rather than a fundamental, sense since the space of all space times is not locally compact in any known Hausdorff topology. This is not the only worry one has regarding such discrete constructions: one has also to show that the limiting kinematical configurations are arbitrarily close to any classical space time in a suitable sense implying that the action principle at hand converges too. Furthermore, regarding the causal dynamical triangulations approach, one sums over all diffeomorphism equivalence classes of spacetimes but in doing so, nothing remains of the notion of locality. Furthermore, it is far from obvious that such an approach is equivalent to the Hamiltonian starting point; here, one would have to introduce a spatial metric and momentum operator and implement the constraints into the full path integral where the exponential term is manifestly non-covariant. But in contrast to say, electrodynamics in the Coulomb gauge, these constraints mix "position and momentum" variables so that these operators have to be ordered, with all momenta to the right -say- and all positions to the left, this would introduce lower order non-covariant terms. To go over to the histories formalism, one can then impose these *new* quntum constraints (which do not coincide with the classical ones) in the stanard formal path integral measure. What should happen then, at least to the common lore, is that those delta functions have to be implemented by means of a gauge fixing which leaves for a highly nonlinear (spatial) field dependent determinant (such as in non-abelian gauge theory, but there the constraints are linear in the fields). What one usually does then in nonabelian gauge theory is to integrate out the momenta leaving one with delta functions which only depend upon the field variables and their time and space derivatives. In doing so, say in electrodynamics, you have to solve for the momentum constraints; leaving one with a *field independent* determinant (which is ill defined but who cares), next integrate out the dependent variable and finally perform unrestricted Gaussian integration over the independent momentum variables. To obtain the usual Legendre transformation, one has also to use also the constraints upon the field variables, which for electromagnetism are just the same, in the Coulomb gauge, as for the momentum variables. Finally, to poor everything into manifestly covariant form, one has to introduce *auxilliary* fields[1] putting the Coulomb gauge condition on the field variables into manifestly covariant form allowing for a fully covariant Lagrangian. It is just so that this analysis goes wrong at several points regarding the gravitational field and nobody has cared about making a "precise" analysis. Nowhere, in the literature, have I seen any proof that such a procedure would lead to a fully covariant Einstein Hilbert Lagrangian supplemented with a covariant ghost field Lagrangian which has to capture for the constraints. What people usually do in so called perturbative quantum gravity is to start from the full naive path integral and then put in some gauge fixing conditions on the histories which have nothing to do whatsoever with the constraints, which are of a dynamical nature, whereas the gauge fixing is entirely kinematical. I know, effectively, such procedure leaves one with the same number of local degrees of freedom as the Hamiltonian procedure I just outlined but that is no proof that this is the same point of view. What people in the causal dynamical triangulations approach do is to suggest one can proceed with some path integral, where the measure is one on spacetimes but all ghost terms which are associated to the usual measure and contain couplings to the field variables have been integrated out leaving one with an effective measure which they claim is their simple counting measure. Not only is there no proof of such a suggestion, their path integral loses complete locality whereas the original Feynman point of view does not. There is a very important distinction here between gravity and all other action principles in field theory, which is that the latter all depend upon first field derivatives only while the former depends upon second derivatives of the metric field. There exists a discretization procedure invented by Regge, which can account for the second derivative in a distributional sense but it requires flexibility in the degrees of freedom of the discrete structure (a simplicial manifold) so that, locally, on the $n-2$ simplices, where n is the dimension of the simplicial manifold, the deficiency angles go to zero sufficiently fast. The "Ricci curvature" of the approximating simplicial manifolds then converges to the Ricci scalar in a weak distributional sense. I am

[1] Electrodynamics is very special that way in the sense that you dispose of two first class constraints, one primary saying that one momentum variable must vanish, and another secondary constraint which is the consequence of the equations of motion regarding this particular canonical variable. The latter can be solved for, so that you can eliminate a canonical pair leaving one with two second class constraints on the remaining field variables and their momenta. Such an elimination is not possible for gravity.

unaware of any suitable substitute for the Ricci tensor and Riemann curvature in this kinematical framework. I am also unaware of any approach to quantum gravity which manages to offer a suitable answer to these elementary matters of principle.

What I have described above can be called "quantum gravity type one" where there is no classical metric background on which computations are performed. One can of course maintain that the universe consists also out of classical degrees of freedom providing one with a dynamical classical background on which it is possible to regard the quantization of the gravitational force as the quantum theory of the graviton. This can be called "quantum gravity type two"; such a theory has long been believed to be impossible due to the non-renormalizability of the gravitational force on a Minkowski background. It his here that our novel nonunitary quantum theory offers a way out since the theory is, with the necessary modifications for large diagrams, finite - a result which does not depend at all on the structure of the Feynman diagrams as has been shown here. In particular, loops played no special role at all in our analysis and were treated on pair with other internal legs which shows that quantum gravity type two is a perfectly safe theory in our framework.

So, how can we extend our novel line of thought regarding second quantization to spacetime itself? For example, how to define the momenta of the theory which have to serve for a gravitational uncertainty principle and what are the constraints upon the momenta replacing the on-shell mass condition for relativistic particles? Here, it is appropriate to state that in our framework, we have disposed of first quantization all together, we immediately went over to the second quantization by an appropriate derivation of the Wightman functions. Clearly, in a continuum theory of the universe some infinite dimensional integration would have to be performed which again will lose its appeal through the appearant non-canonical character of the limit of measures. Here, it is useful to recall the canonical variables for classical relativity; those were the spatial metric h and a momentum π obeying four constraints $Z_i(h, \pi) = 0$ with as equations of motion the Einstein equations where the lapse and shift have been gauge fixed to one and the zero vector respectively. Now, the novel idea is to regard the Einstein equations as defining the free gravitational field; just like the geodesic equation was the correct one for a free particle. I will propose in the next section a similar avenue for the second quantization of string theory where the string equation of motion replaces our geodecy and free momenta, satisfying the usual constraints, are dragged over the string in a way as to preserve the constraints. I will suggested there that the resulting "connection" might correspond to a super (Finsler type) metric and that therefore some unique dynamical conserved quantity could be constructed in the propagator. Regarding gravity therefore, one should search for a metric on the space of all spatial metrics times "time" such that one spatial universe at a given time corresponds to a point in that space, lets call it the birth universe, and that the canonical momenta reside in the tangent space of that generalized manifold. Hence, a "geodesic" is determined by means of (h, π) and corresponds to a gauge fixed solution of the Einstein equations. Now, in order to go over to the second quantized theory, we should integrate along all momenta π' satisfying $Z_i(h, \pi') = 0$ and dragging these momenta along the Einstein equation as to preserve these constraints. As such, one could compute the amplitude corresponding to going from h to h' in "time" 1 (this is the very definition of the exponential map). As an aside here, the integration over all momenta can be canonically defined by using the "birth metric" h as a "background" on which to perform Fourier decomposition and taking the usual cutoffs in momentum space and finally considering the thermodynamic limit. Such a scheme could work out in principle and would provide one with a genuine, well defined, background independent quantization of space. Such an approach would be devoid of all ambiguities of the present one and be much more general as well. Moreover, there is no problem of time in this line of thought.

Chapter 16

Covariant string theory.

The Virasoro problem in string theory [2] arises most clearly in the covariant quantization where one has hermitian generators L_n with $n \in \mathbb{Z}$ which have to be regarded as constraints; that is physical states have to satisfy $L_n|\Psi\rangle = 0$ for $n \neq 0$ and $L_0|\Psi\rangle = a|\Psi\rangle$ with $a \neq 0$. The Virasoro algebra without central anomalies $c(n)$ in 26 spacetime dimensions reads,

$$[L_n, L_m] = i(n-m)L_{n+m} + c(n-m)1$$

makes this impossible given that

$$0 = [L_n, L_{-n}]\,|\Psi\rangle = 2inL_0|\Psi\rangle = 2ina|\Psi\rangle$$

which contradicts $a \neq 0$. The "fix" to this problem is to keep the constraints $L_n|\psi\rangle = 0$ for $n > 0$ while dropping the others. This is partly physically motivated because quantization of the string naturally gives rise to the notion of creation and annihilation operators which seducts one to define the vacuum as being the unique state annihilated by those operators. Admittedly, the L_m for $m < 0$, do not annihilate this vacuum which one might want to whereas the L_m for $m > 0$ do. Since $L^{\dagger}_{-m} = L_m$ we have that all constraints are still satisfied in the weak sense, just as in Gupta Bleuler quantization of electromagnetism. There however, the constraint operator was just linear in the field operator, which gave a clear separation between positive and negative frequency modes, whereas here, one takes the positive-negative frequency mode decomposition of the energy momentum tensor which contains quadratic terms in the modes (such that creation and annihilation operators get mixed). I am not sure what that is supposed to mean physically. The fact that not all constraints are strongly satisfied leads to physical operators changing particle species, spin and angular momentum when working on the so called class of physical states. The downside is that the geometrical description of the theory is partially lost at the quantum level (in the strong sense) even in a Minkowski background and that everything becomes therefore gauge dependent (certainly the choice of vacuum depends upon the gauge of the string worldsheet metric and different vacua determine unitarily inequivalent theories). Also, the theory is by far from unique given that different theories exist for different values of a and that the requirement of the absence of negative norm states leads to $a \leq 1$. The case $D = 26$ corresponds to $a = 1$ and an anomaly free virasoro algebra whereas for $a < 1$, $D \leq 25$ and central anomalies show up so that putting $a = 0$ won't help in strongly implementing the constraints. What I miss completely in this whole state of affairs is that there is no physical significance regarding the embedding of the string worldsheet for the s and t coordinates. In traditional particle theory, the physical parameter simply was the eigentime and the corresponding action principle didn't require at all an einbein on the particle worldline. The real physics happens in the background and still the notions of time and space which govern the string laws have nothing to do with time and space as measured in the background spacetime. Now, the situation is not that bad since the notions of particles are defined with respect to the left and right moving string solutions where string space and time are connected to the clocks of an inertial observer, because both the tangent vector in space as in time are for these separate cases null vectors which are equal to one and another up to a sign. So, the left and right moving string world sheets really are one dimensional and the second dimension builds itself by considering the superposition of both. I am by far more sceptical about the geometric character in a curved spacetime as one cannot simply add two solutions with one and another given that a general manifold is not a vector space with a canonical coordinate chart. Also, the analogy with Gupta Bleuler quantization is somewhat misleading; in the latter case, the constraint originated from a kinematical gauge fixing and has nothing to do with the actual physics; it is just there to restore determinism as well as positive probabilities - the particle notion being fully determined by the observer. In string theory however, the vanishing of the energy momentum tensor is a dynamical requirement and, as said before, the fact that not all constraints are strongly satisfied leads to "physical" operators doing unphysical things. In this paper, we shall develop a geometrical theory of strings which differs from the original one in the sense that we separate the left and right moving solutions from one and another. In the standard formalism, those partial strings do not commutate individually for equal times and distict space coordinates and neither do the separate momenta; the indvidual momentum - position commutation relations are however satisfied up to a factor 1/2. So, I change the prescription of the theory in order (a) to give it an intrinsic, with respect to the background, character and (b) to provide for a *unique* quantum theory without any ambiguities about the notion of a particle with a certain mass and momentum and so on. Mass quantization as well as quantization of angular momenta proceeds as usual due to the periodic boundary conditions. Let me further remark that for a standard free paricle, first quaantization did not make much sense given that the natural inner product was defined on the whole of spacetime and does not coincide with the Klein Gordon inner product; this was due to the fact that observer time had become an operator. Second quantization was needed to make sense out of the theory. In that vein, we shall immediately perform a second quantization and not a first one.

16.1 Strings from the viewpoint of covariant quantum theory.

In this section, we shall look for the correct clasical equations of motion for a closed string on a generic curved spacetime background. Given a closed string worldsheet $\gamma(t,s)$, we define two vectorfields $\mathbf{V} = \partial_t \gamma(t,s)$ and $\mathbf{Z} = \partial_s \gamma(t,s)$ where $t \in [0,T]$ and $s \in [0,L]$ with periodic boundary conditions in s; obviously $[\mathbf{V},\mathbf{Z}] = 0$.

The law one is looking for clearly is of the kind

$$\nabla_{\mathbf{V}}\mathbf{V} = \mathbf{F}(\mathbf{V},\mathbf{Z},\nabla_{\mathbf{Z}}\mathbf{Z}, g(\mathbf{R}(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z})) = \alpha\mathbf{V} + \beta\mathbf{Z} + \delta\mathbf{A}$$

where $\mathbf{A} = \nabla_{\mathbf{Z}}\mathbf{Z}$ is the spatial acceleration and we only include nontrivial gravitational degrees of freedom which are tangential to the string worldsheet. Because we want to eliminate reparametrizations of the string, we endow $\mathbf{Z}$ with a physical significance. That is, we demand that it corresponds to an arclength, that is

$$g(\mathbf{Z},\mathbf{Z}) = c$$

where c is a constant. Since this property has to be preserved under time evolution, we compute that

$$0 = \nabla_{\mathbf{V}} g(\mathbf{Z},\mathbf{Z}) = 2g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{Z}) = 2\nabla_{\mathbf{Z}} g(\mathbf{V},\mathbf{Z}) - 2g(\mathbf{V},\mathbf{A}).$$

To make this equation generic, it is desirable to impose the constraints

$$g(\mathbf{V},\mathbf{Z}) = d,\ g(\mathbf{V},\mathbf{A}) = 0.$$

Taking the time evolution of the former gives

$$0 = \nabla_{\mathbf{V}} g(\mathbf{V},\mathbf{Z}) = g(\nabla_{\mathbf{V}}\mathbf{V},\mathbf{Z}) + g(\mathbf{V},\nabla_{\mathbf{V}}\mathbf{Z}) = \alpha g(\mathbf{V},\mathbf{Z}) + \beta g(\mathbf{Z},\mathbf{Z}) + \frac{1}{2}\nabla_{\mathbf{Z}} g(\mathbf{V},\mathbf{V})$$

which suggests that either $\alpha, \beta = 0$ and $g(\mathbf{V},\mathbf{V}) = e$ or $g(\mathbf{V},\mathbf{Z}) = 0$ or $g(\mathbf{Z},\mathbf{Z}) = 0$. Taking the time derivative of our last constraint

$$0 = \nabla_{\mathbf{V}} g(\mathbf{V},\mathbf{A}) = g(\nabla_{\mathbf{V}}\mathbf{V},\mathbf{A}) + g(\mathbf{V},\nabla_{\mathbf{V}}\mathbf{A}) = g(A,A)\delta + g(\mathbf{V}, R(\mathbf{V},\mathbf{Z})\mathbf{Z}) + g(\mathbf{V},\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{V})$$

which can be rewritten as

$$g(A,A)\delta + g(\mathbf{V},R(\mathbf{V},\mathbf{Z})\mathbf{Z}) + \nabla_{\mathbf{Z}} g(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}) - g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}) = g(A,A)\delta - g(R(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}) - g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}).$$

Hence, for consisteny, we demand that $g(A,A) \neq 0$ and

$$\delta = \frac{g(R(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V})}{g(A,A)}.$$

There is nothing further to deduce and all constraints are presrved under evolution. This suggests one to put the unknown functions α, β to zero to arrive at the theory

$$g(\mathbf{Z},\mathbf{Z}) = c,\ g(\mathbf{V},\mathbf{V}) = e,\ g(\mathbf{V},\mathbf{Z}) = d,\ g(\mathbf{V},\mathbf{A}) = 0$$

with as force law

$$\nabla_{\mathbf{V}}\mathbf{V} = \frac{g(R(\mathbf{V},\mathbf{Z})\mathbf{V},\mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V})}{g(A,A)}\mathbf{A}.$$

In ordinary string theory on flat Minkowski $\mathbf{F} = \mathbf{A}$ for a Lorentzian flat worldsheet metric and $\mathbf{F} = -\mathbf{A}$ for a Riemannian worldsheet metric supplemented by the conditions that $d = e = c = 0$. The reader immediately notices that in this case δ reduces to

$$\frac{g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V})}{g(\mathbf{A},\mathbf{A}}.$$

and that our constraints then give the usual Virasoro conditions for the left and right moving modes

$$\partial_t\gamma.\partial_s\gamma = 0 = \partial_t\gamma.\partial_t\gamma = \partial_s\gamma.\partial_s\gamma.$$

The standard equations of motion

$$(\partial_t)^2\gamma - (\partial_x)^2\gamma = 0$$

are somewhat more limited than ours, but they *imply* that $\delta = 1$ as an easy calculation shows. So, our theory is somewhat more general than the standard "decoupled" one[1]. A simple calculation reveals that

$$\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{V}) = 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{V})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{K}) + 2\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{Z})\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{L}) - \sum_{i=1}^{n-4}\eta_{ii}(\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{E}_i))^2$$

where $g(\mathbf{K},\mathbf{V}) = 1 = g(\mathbf{L},\mathbf{Z})$ and $g(\mathbf{L},\mathbf{L}) = g(\mathbf{K},\mathbf{K}) = g(\mathbf{V},\mathbf{L}) = g(\mathbf{Z},\mathbf{K}) = 0$. Moreover, η_{ij} is an $n-4$ dimensional Euclidean vielbein in the remaining orthogonal space directions. The reader notices here that we changed the signature of spacetime to $(++----)$ giving two time directions and at least four spatial ones. This must be done for the theory to be nontrivial; indeed, string theory is *classically* trivial for the left and right moving modes as the constraint equations imply that $\partial_t\gamma$ is proportional to $\partial_s\gamma$ something which one would expect cannot be reconsiled with the Heisenberg commutation relations[2] in the quantum theory[3]. Indeed, the classical constraints as they stand are all first class and the usual way to deal with those is to implement them at the quantum level not as operator identities but as conditions the so-called physical states should

[1]Standard string theory would depart from the condition that $g(\mathbf{Z},\mathbf{Z}) + g(\mathbf{V},\mathbf{V}) = 0$ from which we derive, upon setting $\alpha,\beta = 0$ that $0 = \nabla_{\mathbf{V}}(g(\mathbf{Z},\mathbf{Z}) + g(\mathbf{V},\mathbf{V})) = 2(g(\nabla_{\mathbf{V}}\mathbf{Z},\mathbf{Z}) + g(\nabla_{\mathbf{V}}\mathbf{V},\mathbf{V})) = 2\delta g(\mathbf{A},\mathbf{V}) + 2\nabla_{\mathbf{Z}}g(\mathbf{V},\mathbf{Z}) - 2g(\mathbf{V},\mathbf{A})$. This suggests automatically that $\delta = 1$ and $g(\mathbf{V},\mathbf{Z}) = c$. Taking the time derivative of this last constraint, we get $2\nabla_{\mathbf{V}}g(\mathbf{V},\mathbf{Z}) = 2g(\mathbf{A},\mathbf{Z}) + \nabla_{\mathbf{Z}}g(\mathbf{V},\mathbf{V}) = \nabla_{\mathbf{Z}}(g(\mathbf{V},\mathbf{V}) + g(\mathbf{Z},\mathbf{Z}))$ which is just our constraint. As commented previously, I deem such condition to be unphysical and rather arbitrary; one would like stronger control over time and space. But the reader who wants to stay in conventional realms could just proceed with this theory and put $c = 0$. No extra time dimensions are required here

[2]See my previous comment on the matter.

[3]Actually, the embedded worldsheet is *one* dimensional and not two.

satisfy weakly or strongly. This is a rather gratuitious way of implementing constraints, an ambiguity which does not arise for second class constraints where a modification of the Poisson bracket ensures that they are valid at the operator level. Specific calculations show that you cannot impose the constraints strongly but weakly so the entire "procedure" is flawed and similar things can happen in canonically quantized gravity too where no natural positive frequency decomposition or particle notion exists. To make this observation a bit more intuitive, in the standard, naive, path integral picture, one should integrate over all momenta and positions obeying the constraints, but there is only really one momentum integral per string point which survives (a scaling of the null vector), all others being fixed by the constraint[4]. On the other hand, if you take a $2+d$ dimensional spacetime, then there are precisely d free momenta per string point which leaves plenty of "room" for the commutatation relations to hold. Our *second* quantization is by far not as gratuitious as the standard procedure is, but leads to a very geometrical picture of the momentum uncertainty standard in quantum theory. Nevertheless, even within this novel way of dealing with quantum theory (in our view), we shall come to the conclusion that the Virasoro constraints need small corrections otherwise the theory becomes inconsistent. Let me mention that our way of dealing with the Heisenberg operators at a geometrical level is fully equivalent, for point particles, to the standard procedure in flat Minkowxki. So, at least, the reader should be open to the suggestion that this is the *correct* way of viewing at quantum theory as opposed to the usual one.

From our contraints, it follows directly that the above formula reduces to

$$\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V}) = -\sum_{i=1}^{n-4} \eta_{ii}(\mathbf{g}(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{E}_i))^2.$$

In order not to get into conflict with the usual causality conditions, we suggest that the extra time and space directions are compactified and way beyond our scale of observation. Hence, a fiber structure is needed for the spacetime manifold with a four dimensional Lorentzian base manifold and a two dimensional Lorentzian fiber. We proceed now to the quantum theory.

16.2 Quantization of the string.

In ordinary particle theory, it is well known that the full quantum theory is provided by the Wightman functions as well as the appropriate Lorentz intertwiners governing the particle interactions. The main insight from this author [3] was that the Wightman functions can be given an entirely geometrical and relational meaning without recourse to any foliation of spacetime given by a class of observers. The observation is simply that free particles travel on geodesics and that the correct Wightman function is constructed by means of dragging

[4]In the standard string picture, two momenta are free.

on shell momenta on those godesics which correspond to off shell particle lines. Furthermore, the internal degrees of freedom are associated to representations of the little group of the momentum vector which for massive particles equals $SO(3)$ and for massless particles E_2, the Euclidean group in two dimensions at least if the spacetime dimension equals four. To have a similar thing in our theory, the remaining degrees of freedom consist of rotations in the space perpendicular to $\mathbf{Z}, \mathbf{V}$ which would need $7 = 2 + 5$ dimensions in order to recuperate the $SO(3)$ part. This provides one with a richer particle spectrum and suggests that massive particles can travel at the speed of light in case $\mathbf{Z}$ resides exclusively in the fiber. It seems clear that the string velocity, which is null in the ultrahyperbolic sense, needs to have a timelike component in the base manifold for massive particles to arise there. The reson why string theorists find massive particles in their spectrum is that the Virasoro algebra is not satisfied to begin with. Furthermore, mass quantization can only occur when the fiber momenta are quantized which necessitates closed (timelike) curves in the fiber, hence our compactification. Therefore, mass and in particular the mass gap, are dynamical quantities closely related to the microscopic structure of the fiber.

We now proceed to formulate the correct off shell propagation for strings and the proper dragging law for on shell momenta. To that purpose, let $\zeta(t, s)$ where $t \in \mathbb{R}^+$ and $\zeta(0, s) \sim S^1$ be our off shell string, meaning that for $\mathbf{T} = \partial_t \zeta(t, s)$ and $\mathbf{Z} = \partial_s \zeta(t, s)$, the following constraints hold

$$g(\mathbf{T}, \mathbf{Z}) = g(\mathbf{T}, \mathbf{A}) = g(\mathbf{Z}, \mathbf{Z}) = 0,\ g(\mathbf{T}, \mathbf{T}) = \lambda$$

where λ is not necessarily zero. We know already that the correct evolution law for the $\mathbf{T}$ field is given by

$$\nabla_{\mathbf{T}} \mathbf{T} = \mathbf{F}(\mathbf{T}, \mathbf{Z}, \mathbf{A}, \mathbf{g}(\mathbf{R}(\mathbf{T}, \mathbf{Z})\mathbf{Z}, \mathbf{T}), \mathbf{g}(\nabla_{\mathbf{Z}} \mathbf{T}, \nabla_{\mathbf{Z}} \mathbf{T}))$$

and that all these constraints are preserved under time evolution. Clearly, our on shell momenta $\mathbf{V}$ have to obey

$$\mathbf{g}(\mathbf{V}, \mathbf{A}) = \mathbf{g}(\mathbf{V}, \mathbf{Z}) = \mathbf{g}(\mathbf{V}, \mathbf{V}) = 0$$

and we look now for the appropriate dragging law

$$\nabla_{\mathbf{T}} \mathbf{V} = \mathbf{G}(\mathbf{T}, \mathbf{V}, \mathbf{Z}, \mathbf{A}, \mathbf{K}, \text{invariants})$$

where K is perpendicular to $\mathbf{T}, \mathbf{V}, \mathbf{Z}, \mathbf{A}$ such that those constraints are preserved under $\mathbf{T}$ evolution. As the reader will notice, it is mandatory to impose two extra constraints on the $\mathbf{V}$ field relating it to $\mathbf{T}$. Considering

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{V}) = 2g(\nabla_{\mathbf{T}} \mathbf{V}, \mathbf{V})$$

indicates one should simply drop the $\mathbf{T}$ dependency in $\mathbf{G}$. Likewise,

$$0 = \nabla_{\mathbf{T}} g(\mathbf{V}, \mathbf{Z}) = g(\mathbf{V}, \nabla_{\mathbf{T}} \mathbf{Z}) = g(\mathbf{V}, \nabla_{\mathbf{Z}} \mathbf{T}) = \nabla_{\mathbf{Z}} g(\mathbf{V}, \mathbf{T}) - g(\nabla_{\mathbf{Z}} \mathbf{V}, \mathbf{T})$$

suggsts two supplementary constraints, that is

$$g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{T})=0,\; g(\mathbf{V},\mathbf{T})=c$$

where c is a constant. Moreover

$$0=\nabla_{\mathbf{T}}g(\mathbf{V},\mathbf{A})=\delta g(\mathbf{A},\mathbf{A})+g(\mathbf{V},\nabla_{\mathbf{T}}\mathbf{A})=\delta g(\mathbf{A},\mathbf{A})+g(R(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V})+g(\mathbf{V},\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T})$$

which can be rewritten as

$$\delta g(\mathbf{A},\mathbf{A})+g(R(\mathbf{T},\mathbf{Z})\mathbf{Z},\mathbf{V})+\nabla_{\mathbf{Z}}g(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})-g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T}).$$

Upon noticing that

$$\nabla_{\mathbf{Z}}g(\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})=\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}g(\mathbf{V},\mathbf{T})-\nabla_{\mathbf{Z}}g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{T})=0$$

due to our constraints. Hence, our previous formula reduces to

$$\delta=\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{Z})+g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})}.$$

So, we have already determinded two of the five component functions of our dragging field $\mathbf{G}$. Remains to investigate the time evolution of our supplementary constraints

$$0=\nabla_{\mathbf{T}}g(\mathbf{V},\mathbf{T})=g(\nabla_{\mathbf{T}}\mathbf{V},\mathbf{T})$$

suggests one to drop the $\mathbf{V}$ dependency in $\mathbf{G}$. Finally,

$$0=\nabla_{\mathbf{T}}g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{T})=g(\nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{T})+g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{T}}\mathbf{T})$$

which can be further rewritten as

$$g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{T})+g(\nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\mathbf{V},\mathbf{T})+g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{A})\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{T},\mathbf{Z})+g(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})}.$$

We further investigate

$$g(\nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\mathbf{V},\mathbf{T})=\nabla_{\mathbf{Z}}g(\nabla_{\mathbf{T}}\mathbf{V},\mathbf{T})-g(\nabla_{\mathbf{T}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})=-\kappa g(\mathbf{Z},\nabla_{\mathbf{Z}}\mathbf{T}))-\delta g(\mathbf{A},\nabla_{\mathbf{Z}}\mathbf{T})-\gamma g(\mathbf{K},\nabla_{\mathbf{Z}}\mathbf{T})$$

where we know already δ but κ,γ have not been fixed yet. Further computation yields that

$$g(\mathbf{Z},\nabla_{\mathbf{Z}}\mathbf{T}))=g(\mathbf{Z},\nabla_{\mathbf{T}}\mathbf{Z}))=\frac{1}{2}\nabla_{\mathbf{T}}g(\mathbf{Z},\mathbf{Z}))=0$$

so we have nothing to say about κ and therefore we put it to zero[5]. So, we arrive at the equation

$$0=g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{T})+g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{A})\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{T},\mathbf{Z})+g(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})}-$$

[5]One can derive that κ must be zero in case one enlarges ones vision upon an off-shell string by allowing for $g(\mathbf{T},\mathbf{Z})=e\neq 0$

$$\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})} g(\mathbf{A},\nabla_{\mathbf{Z}}\mathbf{T}) - \gamma g(\mathbf{K},\nabla_{\mathbf{Z}}\mathbf{T}).$$

This leaves only for the possibility that $g(\mathbf{K},\nabla_{\mathbf{Z}}\mathbf{T})$ is nonzero otherwise our theory would become inconsistent. Denoting by

$$(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}$$

the component of $\nabla_{\mathbf{Z}}\mathbf{T}$ perpendicular to $\mathbf{T},\mathbf{V},\mathbf{Z},\mathbf{A}$ which is usually only determined, in case $g(\mathbf{T},\mathbf{V}) \neq 0$ *and* $g(\mathbf{T},\mathbf{V}) \neq= 0$ up to a multiple of $\mathbf{Z}$, we conclude that we need to add a force term

$$\frac{(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}}{g((\nabla_{\mathbf{Z}}\mathbf{T})^{\perp},(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp})}\left(g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{T}) + g(\nabla_{\mathbf{Z}}\mathbf{V},\mathbf{A})\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{T},\mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{T},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})}\right)$$

$$-\frac{(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}}{g((\nabla_{\mathbf{Z}}\mathbf{T})^{\perp},(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp})}\left(\frac{g(R(\mathbf{T},\mathbf{Z})\mathbf{V},\mathbf{Z}) + g(\nabla_{\mathbf{Z}}\mathbf{V},\nabla_{\mathbf{Z}}\mathbf{T})}{g(\mathbf{A},\mathbf{A})} g(\mathbf{A},\nabla_{\mathbf{Z}}\mathbf{T})\right)$$

to our formula for $\mathbf{G}$ which fully determines it up to an ambiguity in the definition of $(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}$. However, this might not be as bad as one thinks it is at first sight since it all depends upon what we mean by an off-shell string. In case we allow for $g(\mathbf{T},\mathbf{Z}) \neq 0$ then this ambiguity is of measure zero in $\mathbf{T}$ space and we can just ignore it; insisting upon $g(\mathbf{T},\mathbf{Z}) = 0$, our theory is dead given that this ambiguity propagates in a nontrivial way and therefore our theory would depend upon some convention we have to take at any point of the string[6]. Notice also that ideas where you decouple the bulk from the fiber won't help you; even if initially $\mathbf{Z}$ is in the fiber and $\mathbf{T},\mathbf{V}$ in the bulk, then $\mathbf{A}$ will usually have some nontrivial component in the fiber so that $\mathbf{T},\mathbf{V}$ propagate into the latter and therefore $\mathbf{Z}$ into the bulk. So, $\nabla_{\mathbf{Z}}\mathbf{T}$ does not remain into the bulk and therefore you cannot define an orthognal complement with regards to $\mathbf{Z}$ pertaining to the bulk alone. So, from this point of view, standard string theory does not make any sense. However, it is easy to save the day by allowing a liberty which we had before and that is to put $g(\mathbf{Z},\mathbf{Z})$ *not* equal to zero; in general, even if $g(\mathbf{V},\mathbf{V}) = 0 = g(\mathbf{V},\mathbf{Z}) = g(\mathbf{T},\mathbf{Z})$ the projection above will be uniquely defined as long as $g(\mathbf{V},\mathbf{T}) \neq 0$. One can even exclude this exceptional case by imposing, as mentioned previously, that $g(\mathbf{T},\mathbf{Z}) \neq 0$ allowing for $g(\mathbf{Z},\mathbf{Z})$ to be zero and in this case $g(\mathbf{V},\mathbf{T})$ can be anything you like. We shall discuss this matter further in the next section where we argue that convention, that allowing for a flexible $g(\mathbf{T},\mathbf{Z})$ is the best way to proceed.

[6] In conventional string theory, one may consider an off shell string $g(\mathbf{T},\mathbf{Z}) = d$, $g(\mathbf{T},\mathbf{T}) + g(\mathbf{Z},\mathbf{Z}) = c$ and on shell momenta $g(\mathbf{V},\mathbf{Z}) = 0 = g(\mathbf{V},\mathbf{V}) + g(\mathbf{Z},\mathbf{Z})$ with as dragging law $\nabla_{\mathbf{T}}\mathbf{V} = \mathbf{A}\frac{g(\mathbf{T},\mathbf{A})}{g(\mathbf{V},\mathbf{A})} - \frac{\mathbf{Z}}{g(\mathbf{Z},\mathbf{Z})}\left(\frac{g(\mathbf{T},\mathbf{A})g(\mathbf{Z},\mathbf{A})}{g(\mathbf{V},\mathbf{A})} + g(\mathbf{V},\nabla_{\mathbf{T}}\mathbf{Z})\right)$. The reader checks that all constraints are preserved under evolution and that the second term on the right hand side vanishes in case $\mathbf{T} = \mathbf{V}$ due to the constraints. Note that this transport equation is linear in $\mathbf{T}$ but nonlinear in $\mathbf{V}$.

16.3 Fourier transform for strings in covariant quantum theory.

In analogy with standard particle physics, we now proceed to construct the Fourier transform $\phi(S, \mathbf{V}_S, S')$, where S is a null string parametrized as before and $\mathbf{V}_S$ is a null vectorfield defined on the string and satisfying the previous constraints. Here, we meet a slight ambiguity in what we mean by an off shell string; to get more insight into this matter, one should realize that he mass parameter for the free classical theory does not matter at all, as is well known gravity does not care about the inertial mass of test particles, the latter becomes only important when you would compute backreaction effects on the gravitational field itself or include interactions with other particles. So, in a way is the geodesic equation more fundamental as the equation of motion derived from the usual action principle, but which forces momenta to be on shell. Quantum theory here partially destroys the geodesic idea in the sense that the propagator feels the mass of a particle. I think that, regarding classical theories with constraints that it is useful to regard at it in a more general wy, just as we did for (left or right moving) strings where constraints on the $\mathbf{T}$ field arose from mere consistency that spatial arclength had to be preserved. In particular, it followed that

$$g(\mathbf{T}, \mathbf{T}) = c,\ g(\mathbf{T}, \mathbf{Z}) = d,\ g(\mathbf{T}, \mathbf{A}) = 0$$

and we coined the term "on-shell" by means of the imposition that $c = d = 0$. The reader may verify that our equations of motion for the $\mathbf{T}$ field as well as the dragging law for on shell momenta $\mathbf{V}$ remain entirely the same if one allows for general values of c, d. In traditional string theory, we do not start out from a clear point of view on space, but merely impose that $g(\mathbf{T}, \mathbf{T}) + g(\mathbf{Z}, \mathbf{Z}) = 0 = g(\mathbf{T}, \mathbf{Z})$ and the natural suggestion here would be to relax those as being two independent constants. Nothing changes to our theory; so, what I suggest is that action principles, used in first quantization, do not properly reveal the most general point of view on the matter. Second quantization on the other hand reveals a broader spectrum of possibilities since, for example for a Klein-Gordon field, the mass squared can easily be taken to be negative, leading to tachyons. The Dirac field, on the other hand is not that flexible.

It is clear that the definition of $\phi(S, \mathbf{V}_S, S')$ should not depend upon the reparametrization freedom hidden in $\mathbf{Z}$ and we define the string length L as the range of this parameter domain. In analogy with particle physics, we start out with the most naive ansatz for a differential equation for $\phi(S, \mathbf{V}_S, t)$; the latter being given by

$$\frac{d}{dt}\phi(S, \mathbf{V}_S, t) = i\frac{\kappa}{L}\left(\int_0^L ds\, \mathbf{g}(\mathbf{V}(t,s), \mathbf{T}(t,s))\right)\phi(S, \mathbf{V}_S; t)$$

with $\phi(S, \mathbf{V}_S, 0) = 1$ and $\phi(S, \mathbf{V}_S, 1) = \phi(S, \mathbf{V}_S, S')$. Here, κ is a dimensionless constant and L is the string length. As is the case in standard particle theory, this factor is a constant in time and simply given by κc where

$c = \mathbf{g}(\mathbf{V}_S(t,s), \mathbf{T}(t,s))$. Hence,

$$\phi(S, \mathbf{V}_S, S') = e^{i\kappa c}$$

just as in ordinary particle physics, which we know is already a complete disaster there and leads to distributional propagators. Here, we are going to perform a functional integral over vectorfields instead of vectors and the volume associated with a constant c is just infinite. There is no other obvious term depending on $\mathbf{V}, \mathbf{T}$ one could add since we have the constraint that $g(\nabla_{\mathbf{Z}}\mathbf{V}, \mathbf{T}) = 0$. It is nevertheless worthwhile to look for other conserved quantities to put into the propagator; for that purpose, we calculate

$$\nabla_{\mathbf{T}} g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) = g(R(\mathbf{T},\mathbf{Z})\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) + g(\nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) + g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T})$$

and expanding the second term on the right hand side

$$g(\nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) = \nabla_{\mathbf{Z}} g(\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) - g(\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T})$$

which can be rewritten as

$$\nabla_{\mathbf{Z}} g(\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) - \nabla_{\mathbf{T}} g(\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}) + g(\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}).$$

The last term on the right hand side equals

$$g(\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T}) = g(R(\mathbf{T},\mathbf{Z})\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{V}) + g(\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T})$$

$$= g(R(\mathbf{T},\mathbf{Z})\nabla_{\mathbf{Z}}\mathbf{T}, \mathbf{V}) + \nabla_{\mathbf{Z}} g(\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T}) - g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T}).$$

Collecting all our results gives

$$\nabla_{\mathbf{T}}\left(g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) + g(\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T})\right) = \nabla_{\mathbf{Z}} g(\mathbf{V}, \nabla_{\mathbf{T}}\nabla_{\mathbf{Z}}\mathbf{T}) + \nabla_{\mathbf{Z}} g(\nabla_{\mathbf{T}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T})$$

where we have used the symmetry properties of the Riemann tensor. Hence, for a closed string

$$L \int_0^L ds \left(g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{T}) + g(\mathbf{V}, \nabla_{\mathbf{Z}}\nabla_{\mathbf{Z}}\mathbf{T})\right)$$

is a reparametrization invariant conserved quantity. Note that this quantity is still "kinematical" as the previous one and it would be very interesting to see if more complex dynamical conserved quantities exist. To stimulate this idea a bit further, one might expect that there exists a "super metric" on the entire string (which treats vector fields on the string as *vectors*) for which our transport equations are the geodecies). This requires further explanation; first of all, the connection associated to this metric manifests itself locally as being of the "Finsler type" in the sense that spatial derivatives of the momentum fields creep in which suggests that the string metric can be locally derived from a"Finsler metric" and our "second quantization" should be nothing else but the geodesy in this string metric. Note that our transport equations are nonlinear in $\mathbf{T}, \mathbf{V}$ due to the presence of $(\nabla_{\mathbf{Z}}\mathbf{T})^{\perp}$ (all the rest is linear) which suggests that we need a broader view upon geometry with non-linear "connections"; complications which did not show up for elementary particles. On those grounds alone, I expect a dynamical conserved quantity to exist. Again, our construction concerns a *second* quantization of string theory and not a first one as is usually discussed in the literature. We shall propose a similar suggestion for a second quantization of the gravitational field. Another suggestion would be to consider the spatial variation of $\mathbf{V}$ contracted with itself $g(\nabla_{\mathbf{Z}}\mathbf{V}, \nabla_{\mathbf{Z}}\mathbf{V})$, that is

$$L\zeta \int_0^L g(\nabla_{\mathbf{Z}}\mathbf{V}(t,s), \nabla_{\mathbf{Z}}\mathbf{V}(t,s))ds$$

which is not time independent. This author has proposd similar avenues in standard particle physics. It might be that other terms are required to make a sensible theory but these issues are postponed for further investigation. We now come to the definition of the propagator.

16.4 The free string propagator.

The particular feature about the string propagator is that it involves an infinite dimensional integration over momentum space $\mathbf{V}_S$ and we limit in the subsequent analysis ourselves to a product manifold $\mathcal{M} \times \mathcal{N}$ where $\mathcal{M}$ is a $3+1$ dimensional Lorentzian base manifold endowed with a $2+1$ dimensional Lorentzian fiber which decouple in the sense that the metric does not mix directions in the fiber and base manifold. The $\mathbf{T}$ field is as such that after parameter time one the string S specified above moves into a string S' with nontrivial projection into $\mathcal{M}$ due to s variations of the $\mathbf{T}$ field; that is, the projection of $\nabla_{\mathbf{Z}}\mathbf{T}$ on the base is different from zero. Hence we propose,

$$D(S,S') = \int dc \int_{\mathbf{V}_S} d\mu(\mathbf{V}_S)\delta(g(\mathbf{V}_S,\mathbf{V}_S))$$

$$\delta(g(\mathbf{V}_S,\mathbf{Z}))\delta(g(\mathbf{V}_S,\mathbf{T})-c)\delta(g(\nabla_{\mathbf{Z}}\mathbf{V}_S,\mathbf{Z})\delta(g(\mathbf{V}_S,\mathbf{A}))\theta_{\mathcal{M}}(\mathbf{V}_S)\phi(S,\mathbf{V}_S,S')$$

where we have chosen a time direction in the base manifold $\mathcal{M}$ and $\theta_{\mathcal{M}}(\mathbf{V}_S)$ concerns positivity of the projection of $\mathbf{V}_S$ on that time field. The problem here regards the usual definition of the path integral as a limiting measure and much care is required to give this expression a precise meaning. Let us mention that in contrast to string theory on flat spacetime, mass and momentum string states are not easily defined and the angular momentum and spin case are even much more difficult as is the case for ordinary particles on a curved background. We leave further investigations of these ideas to the future.

16.5 Conclusions.

This chapter comes with two distinct clear messages. The first one is that standard string theory is full of ambiguities and the weak implementation of the constraints does not make much sense. We have also argued that it is better to give a deeper geometrical meaning to the string as is usually the case, this requires going over to two time dimensions and slightly altering the constraints. This is not needed if you stick to conventional string theory and here, our novel approach might offer a full anomaly free and geometrical quantization of the free string. We also made it clear that on a curved background, there is no split in right and left moving strings since both need the Riemann tensor in their propagation whereas the "full" string does not. I remain with my stance that the usual quantization of the string does not make sense and hope to come back to this topic in future work.

Part V

Afterword.

As mentioned in the introduction, this book is an extensive report of work in progress and albeit we have obtained some insights into the failure of quantum field theory as well as the success of our approach, many issues remain to be investigated such as an adequate renormalization procedure in a general spacetime. Science is a never ending story indeed and only future progress from the theoretical as well as experimental side will further clear the sky. A topic which we did not treat here regards "the" classical limit of the theory since that issue is almost trivial in our approach. It must be clear from the remarks in part 4 that our theory is a real game changer; there is no problem of time in quantum gravity, there are no quantization ambiguities in string theory and the observer is, in principle, treated on the same level as the elementary particles as explained in full detail in part 3. It is my hope that this sheds a new light upon quantal relativity as we disentangled different notions of time which were indentified (in different ways) in as well quantum theory as general relativity. Both theories fell short here!

Regarding the mental world, a topic which is very avant garde, it might be that we have uncovered deep relations between time and all directions of space, relating them to very basic psychological features such as feeling, perception and compassion. Maybe I reach too far there, but the mathematics contains intruiging subtleties which might very well underpin a deeper level of reality. At least, we propose something to truely discuss about and go beyond the usual story telling in those fields. I feel at the moment pretty satisfied with its presentation as I have cleared up the sky, in contrast to the previous version, and removed several statements which were too speculative. Maybe the Poles, Belgians and Dutch will recognize themselves overall in the way I perceive them: that would be nice and strengthen my convictions that the psyche, the marriage and the kitchen are inextricibly linked to one and another. I have written this book in a self-contained manner and hope to have explained the importance of this topic to the interested reader. It must be clear that the first part of this book offers plenty of new possible pathways towards further exploration of higher physical principles involving more extensive geometrical objects as has been commented there. Part 3, regarding the mental world, has been treated on several levels, kinematically, dynamically and semantically albeit much needs to be done here and some aspects of the theory have been desciptive to some extend, albeit with direct reference towards the mandatory mathematics. We have likewise offered a full geometrical picture on string theory largely "surpassing" the work of Green, Schwarz and Witten given the possibly $4 + 2$ dimensional nature of the world, which merely arises from consistency conditions emerging from the constraints on the string in a novel sense. I hope the reader will enjoy further exploration of these ideas in his or her own future work.

Chapter 17

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Bibliography

[1] S. Weinberg, The quantum theory of fields, foundations, Cambridge University Press

[2] Witten et al, Superstring theory Volume I, Cambridge university press

[3] J. Noldus, Foundations of a theory of quantum gravity, Shiny world coorporation 2011 and Arxiv

[4] J. Noldus, Generally covariant quantum theory, J. Phys, Conf. ser. 1275