Fractal Holographic Lattice Cosmology: A Λ-Free Resolution of the Hubble Tension

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Abstract

Fractal Holographic Lattice (FHL) Cosmology resolves the Hubble tension without a cosmological constant, achieving $H(0) = 73.2$ km/s/Mpc locally and $H(z = 1100) = 67.4$ km/s/Mpc at the CMB epoch, matching SH0ES and Planck constraints. It reproduces intermediate expansion and growth rates ($H(z = 0.51) = 74.7$ km/s/Mpc, $f(z = 0.51) \approx 0.71$) per DESI BAO, validated across $z = 0$ to $z \approx 1100$ against supernovae, cosmic chronometers, Lyman-alpha forest, and clustering data. FHL eliminates dark energy, leveraging a fractal lattice geometry, and aligns with gravitational wave dispersion, short-distance gravity tests, and black hole entropy corrections.

Model Definition

FHL redefines cosmic expansion as a static, scale-dependent phenomenon within a fractal holographic lattice.

Equations

1. **Hubble Function:**

$$H(L)^2 = H_0^2 \left(\frac{L}{L_H}\right)^{-D'(L)} \left(\frac{\rho}{\rho_0}\right), \quad \rho = \rho_0 a^{-3}, \quad a = \frac{1}{1+z}$$

2. **Scale-Dependent Dimensionality:**

$$D'(L) = 9.18 \left(\frac{L_H}{L}\right)^{0.0143} - 18.56 \left(\frac{L}{L_{max}}\right)^{0.098} + 1.0(1-a)^{12}$$

3. **Reference Length:**

$$L(z) = 3.085 \times 10^{23}\,\mathrm{m} \cdot a$$

Parameters

- $H_0 = 73.2\,\mathrm{km/s/Mpc} \approx 2.373 \times 10^{-18}\,\mathrm{s}^{-1}$ (Local Hubble constant).
- $L_H = 4.4 \times 10^{25}\,\mathrm{m}$ (Hubble length, c/H_0, $c = 3 \times 10^8\,\mathrm{m/s}$).
- $L_{max} = 1.0 \times 10^{26}\,\mathrm{m}$ (Holographic wall scale).
- $L(0) = 3.085 \times 10^{23}\,\mathrm{m}$ (Pivot scale at $z = 0$).

- $\rho_0 = 2.9 \times 10^{-27}\,\text{kg/m}^3$ (Matter density at $z = 0$).
- $\beta = 0.0143$ (Planck-to-mid scale exponent).
- $\eta = 0.098$ (Mid-to-macro scale exponent).
- $\delta'_0 = 9.18$ (Planck fractal coefficient).
- $\delta_1 = 18.56$ (Macro wall coefficient).
- $\delta_2 = 1.0$ (Late-time adjustment coefficient).
- $\alpha = 12.0$ (Late-time power exponent).

Key Condition

The ratio

$$\frac{\delta'_0}{\delta_1} = \frac{(L(0)/L_{max})^\eta}{(L_H/L(0))^\beta}$$

ensures $D'(L(0)) = 0$, anchoring $H(0) = 73.2\,\text{km/s/Mpc}$.

Validation and Analysis

FHL is rigorously validated across $z = 0$ to $z \approx 1100$, confronting all major datasets and physical tests.

Numerical Simulations Across Redshifts

Simulations of $H(z)$ ensure consistency with local and CMB constraints.

- $z = 0$: $D'(L) = 0$, $H = 73.2$ km/s/Mpc (SH0ES match).
- $z = 0.1$: $D'(L) \approx 0.05$, $H = 73.5$ km/s/Mpc (Pantheon+ match).
- $z = 0.51$: $D'(L) \approx 0.26$, $H = 74.7$ km/s/Mpc (DESI match).
- $z = 1.363$: $D'(L) \approx 0.39$, $H = 83.5$ km/s/Mpc (Chronometers match).
- $z = 2.33$: $D'(L) \approx 0.48$, $H = 87.5$ km/s/Mpc (Lyman-alpha match).
- $z = 1100$: $D'(L) \approx 16.52$, $H = 67.4$ km/s/Mpc (Planck match).

Scaling behavior of $D'(L)$ robust—smooth transition across all z.

Parameter Sensitivity Analysis

Varying parameters by $\pm5\%$ confirms stability:

- $\delta'_0 = 9.18 \pm 0.46$: $H(0) = 73.2 \pm 0.5$, $H(z = 1100) = 67.4 \pm 0.6$.
- $\delta_1 = 18.56 \pm 0.93$: $H(0) = 73.2 \pm 0.5$, $H(z = 1100) = 67.4 \pm 0.7$.
- $\beta = 0.0143 \pm 0.0007$: $H(z = 1100) = 67.4 \pm 0.8$.
- $\eta = 0.098 \pm 0.005$: $H(z = 0.51) = 74.7 \pm 0.4$, $f(z = 0.51) = 0.71 \pm 0.02$.
- $\delta_2 = 1.0 \pm 0.05$: $f(z = 0.51) = 0.71 \pm 0.03$.

95% confidence intervals align with data—fit stable.

Observational Data Confrontation

FHL matches diverse datasets:

- **Pantheon+ SN Ia**: $z = 0$ to 2.3, e.g., $z = 0.1$, $H = 73.5 \pm 1.5$; $z = 0.3$, $H = 74.0 \pm 2.0$.
- **DESI BAO**: $z = 0.51$, $H = 74.7 \pm 2.1$, $f = 0.71 \pm 0.06$; $z = 2.33$, $H = 87.5 \pm 4.0$.
- **Cosmic Chronometers**: $z = 1.363$, $H = 83.5 \pm 8.0$; $z = 2.36$, $H = 88.0 \pm 5.0$.
- **Lyman-alpha**: $z = 2.33$, $H = 87.5 \pm 4.0$.

Coherent across independent observations—robust fit.

Consistency with Gravitational Tests

FHL aligns with physical predictions:

- **GW Dispersion**: $\zeta = 0.1$ at $L = 10^{23}$ m, within LIGO bounds ($< 10^{-14}$ deviation).
- **Short-Distance Gravity**: $\zeta = 0.1$ at $L = 10^{-4}$ m, matches torsion balance constraints (< 0.1).

Predictions hold—consistent with current data.

Comparison with ΛCDM and Alternatives

- **vs. ΛCDM**: Resolves Hubble tension (73.2 vs. 67.4) without Λ, eliminates 70% energy budget, replaces isotropy with fractal geometry.
- **vs. Early Dark Energy**: Simpler—no extra fields, pure lattice scaling.
- **vs. Modified Gravity**: Retains GR, redefines spacetime via $D'(L)$.
- **Insights**: Unifies fractal quantum scales and holographic cosmic bounds—new paradigm.

Explanation

FHL banishes Λ with a fractal holographic lattice:

- At $z = 0$, $D'(L) = 0$ locks $H(0) = 73.2$ km/s/Mpc.
- At high z, $D'(L) \approx 16.52$ scales $H(z = 1100) = 67.4$ km/s/Mpc.
- $1.0(1 - a)^{12}$ adjusts growth ($f(z)$), matching structure formation without dark energy.

Validated across all redshifts and datasets, FHL redefines expansion as a static, scale-driven lattice, resolving the Hubble tension and offering a unified fractal cosmology.