Classification of Composite numbers

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Abstract

In this research different and distinct subsets of composite numbers are identified together with their Logical formulae. The aim of the research is to come up with an efficient method of sieving out primes from a set of positive integers. An exact formula for determining the number of primes will be presented. In the paper a graphical method of factorization will be presented. The paper will back its theoretical claims with a practical Python code that will demonstrate both counting and graphical factorization methods. This integration of theory and practice is useful for further research and real world application. The paper aims at providing an exact and structured framework for counting primes within a given range. It will categorize prime numbers in a way that avoids redundant processing. By giving an exact prime counting function and a graphical method of factorization the paper offers insights beyond sieving.

Keywords Logical formula; set and subsets of composite numbers; Goldbach partition semiprimes; Graphical method of factorization; Shared least prime factor (SLPF); Least common prime divisor (SLPD); Formula for determining the number of primes

Introduction

The two main classifications of composite even numbers are well known. Classification by parity divides composite numbers into even and odd composite numbers. There is classification by number of prime factors.

We introduce a classification system in which composite numbers share a least prime factor.

Sharing a least prime factor means two or more numbers have the same smallest prime factor in common.

Composite numbers that share a least prime factor will belong to the same class. In this case the number of classes of composite numbers will also be equal to the number of primes. All composite even numbers share a least prime factor 2. The number 6 and 9 share common prime of 3 but they do not share a least prime factor. The least prime of 6 is 2 while the least prime of 9 is 3. The two composite numbers don't share the same least prime factor. The concept of two or more composite numbers jointly sharing the same least prime factor will be explored and be represented algebraically to achieve desired results.

Finally we will present a mathematical function for graphical for factorization of composite even numbers.

Definition: Least common prime factor.

Justification for a proper classification of composite numbers

A proper classification of composite numbers helps in creating an effective sieve for separating prime numbers from composite numbers.

Set-builder notation specifies a set as being the set of all elements that satisfy some logical formula, see reference [1]. In this research logical formulae will be identified for different subsets of the set of composite numbers.

Challenges of achieving an efficient sieve method

A good and efficient sieve method should be able to sieve a composite odd number once from a set of many odd numbers. If a sieve method does not have a mechanism for sieving a composite number once, it will also not have a reliable way of exact counting of the number of composite odd numbers from a set of odd numbers. The biggest weakness of the sieve of Eratosthenes [2] is the algorithm repeatedly checks the same numbers for primality, particularly when dealing with multiples of smaller primes, which can slow down the process for large ranges. Thus it memory intensive. To mitigate this weakness segmented sieves have been suggested.

The linear sieve [3] ensures that each number is marked as composite only once. The weakness of the Linear or Euler sieve is that 1. has a high memory usage 2. It is suitable for small queries 3. It is not suitable for large N. 4. It is not suitable for a range of queries 5. It is difficult to parallelize unlike the sieve of Eratosthenes.
The sieve of Sundaram[6] generates a list of potential primes and is less efficient tha that of Eratosthenes. The sieve of Atkin [7] is modern, complex and asymptotically faster than the sieve of Eratosthenes. How can we possibly achieve a sieve that can count the number of composite odd numbers, from a set of odd numbers? This is where the concept of shared least prime factor (SLPF) comes in. The set of composite numbers like $(113^2, 113\times127, 113\times131, 113\times137, 113\times139, 113\times149...., 113^3...113^n)$ share a least prime factor of 113.
The set of composite numbers $(97^2, 97\times101, 97\times103, 97\times107, ..., 97^3.....97^n)$. These two sets above do not in any way share the same composite even number. A classification system that puts composite even numbers sharing one least prime factor in a class will ensure there is no double sieving and therefore the number of composite odd numbers upto x equals the number of composite numbers upto x in the various classes. The number of classes is $\leq \sqrt{x}$.

An appropriate representation of composite numbers for classification purposes

Let N represent a composite even number with a least prime factor p_i. The mathematical form of such a composite number is given by (1) below.

$$N = p_i n_j \mid n_j = \prod p_j| \mid p_j \geq p_i \tag{1}$$

Here n_j represents a composite number with prime factors greater than or equal to p_i.
In the form above p_i is the smallest prime factor of N. All Composite numbers with the same smallest prime factor will be put in the same class. All composite even numbers with 2 as their smallest prime factor will be lamped together in one class. Similarly composite even numbers with 3 as their smallest prime factor will be lamped together in their class and so forth.

Thus the set of composite even numbers are represented as

$$N = p_1 n_j = 2n_j \mid n_j = \prod p_j \mid p_j \geq p_1 = 2 \tag{2}$$

and has elements

$$(4, 6, 8, 10, 12, ..., 2n_j)$$

. These elements share a least prime factor 2.

In each of the subsets generated the lead element is a square semiprime. The set made using logical formula (3) below

$$N = p_2 n_j = 3n_j \mid n_j = \prod p_j \mid p_j \geq p_1 = 3 \tag{3}$$

is made of the infinite set of elements

$$(9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99, 105, 111, ..., 9 + 6(n - 1))$$

Goldbach partition semiprimes with prime factors of 3 are in this set. Semiprimes for Goldbach partion of even numbers 6 and above are found in this set. The element 3^n to the above set. The set based on logical formula (4) below

$$N = p_3 n_j = 5n_j \mid n_j = \prod p_j \mid p_j \geq p_3 = 5 \tag{4}$$

is made of the infinite set of elements

$$(25, 35, 55, 65, 85, 95, 115, 125, 145, 155, 185, 205, ...5^m \prod p_{3+i})$$

and contains Goldbach partition semiprimes with prime factor 5. Semiprimes for Goldbach partition of even numbers 10 and greater are found in this set. The element 5^n belongs to the above set. In the above set based on logical formula (4) for example:

$$65(\frac{1}{5} + \frac{1}{13}) = 18$$

$$95(\frac{1}{5} + \frac{1}{19}) = 24$$

The set based on logical formula (5) below

$$N = p_4 n_j = 7n_j \mid n_j = \prod p_j \mid p_j \geq p_4 = 7 \tag{5}$$

is made of elements

$$(49, 77, 91, 119, 133, 161, 203....7^m \prod p_{4+i})$$

and contains Goldbach partition semiprimes with prime factor of 7. Semiprimes for Goldbach partition of composite even numbers 14 and above are found in this set. The element 7^n belong to the above set. In the above set based on logical formula 7 for example:

$$203(\frac{1}{7} + \frac{1}{29}) = 36$$

$$91(\frac{1}{7} + \frac{1}{13}) = 20$$

It should be noted from that from the above for sets that the number of composite odd numbers in tbe interval $(1, 112)$ is 27. There the number of odd primes in the same interval is $56 - 27 = 29$. Therefore the total number of primes in the interval is 30.

Determining the number of primes up to x

We have managed to classify composite odd numbers. Composite odd numbers with a shared least prime divisor (SLPD) of 3 belong to their own class. Composite odd numbers with (SLPD) 5 belong to their own class. Let $C_{o(x,p)}$ be the number of composite odd numbers $\leq x$ whose SLPD and SLPF is p. then the number of primes up to x_e (an even number) is given by

$$\pi(x_e) = \frac{x_e}{2} - (C_{o(x,p_2)} + ... + C_{o(x,p_n)}) \mid p_n < \sqrt{x_e} \quad (6)$$

Example 1 List the composite odd numbers between 31 and 100 in there different classes according to the LCPD and hence calculate the number of primes in the interval (30, 100) .

Solution The set for SLPD = 3 is (33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99). The set contains 12 composite odd elements. The set for SLPD = 5 is (25, 35, 55, 65, 85, 95). The set contains 6 composite odd elements.
The set for SLPD = 7 is (49, 77, 91) and contains 3 elements. The sets for SLPD = 11 and above are empty, meaning that they contain composite odd numbers greater than 100. The total number of composite odd numbers in the interval [31, 100] is therefore is 21. The odd numbers in the interval [31, 100] is $\frac{100-30}{2} = 35$. The number of primes in this interval is $35 - 21 = 14$. This is actually the exact number of primes in the interval.

Comparison with the prime number theorem

The prime number theorem, proved independently by Jacques Hadamard [4], and Charles Jean de la Vallee Pousin [5] by the ideas of Riemann is asymptotically by $\pi(x) \approx \frac{\ln x}{x}$ and therefore the approximation is not exact. The prime number theorem does not predict number of primes in small intervals and breaks down for small values of x. The prime number theorem lacks insight into sieving or prime factorization. It instead depends on complex analysis. It lacks error bounds in its basic form. The paper aims to come up with a first and exact computer generated algorithm of determining the number of primes and composite odd numbers in a given interval.

Advantages of the current sieve method

It classifies composite odd primes by there shared least prime factor. It means the least prime factor of each element in a class is the same and known.

It is able to determine the number of composite numbers in each class separately.

It is able to determine the number of primes and composite odd numbers in an interval and their compute their exact relative densities and its variation as intervals increase. One is able to determine the relative density of composite even number in each class. By relative density here we mean the ratio of number of composite odd numbers in a class to the total number of odd composite numbers.

A graphical method of factorization

Consider the identity:

$$p_2, p_1 = \pm(\frac{p_2 - p_1}{2}) + \sqrt{(\frac{p_2 - p_1}{2})^2 + p_1 p_2} \tag{7}$$

If we set $x = \frac{p_2 - p_1}{2}$, $p_1 p_2 = x$ and $p_2, p_1 = f(x) $ then:

$$f(x) = \pm x + \sqrt{x^2 + N} \tag{8}$$

We end up with a function for the graphical factorization of N. x represents half the gap of the factors of N. The function works to factorize any composite number to two factors greater than 1. This means the graph will have at least one integer point for every whole number N. if the integer N is prime like 7 the integer points generated are $(-3, 1)$ and (3, 7). Here 3 is half the gap of the factors of 7. The factors of 7 are 1 and 7 shown on the points. Also 3 - -3 = 6 represents the gap.

Example2 Use graphical methods to determine the factors of 91

solution Draw the graphs of $y = \pm x + \sqrt{x^2 + 91}$ and from the graph find the integer solution. From the graph we establish that $(x_1, y_1) = (3, 13)$ and $(x_2, y_2) = (-3, 7)$. This means that the factors of 91 are 7 and 13. The gap between the prime factors is given by $13 - 7 = 3 - -3 = 6$. For confirmation , check the integer points on the graph in figure 1 below the references

Summary and Conclusion

It is possible to come up with some classification system of composite even numbers that can make it easy to sieve out prime numbers from composite. A classification system of number in which numbers sharing a least common prime factor are in the same class is useful and achievable as has been achieved in this paper.

There exists a graphical method of prime factorization of numbers.

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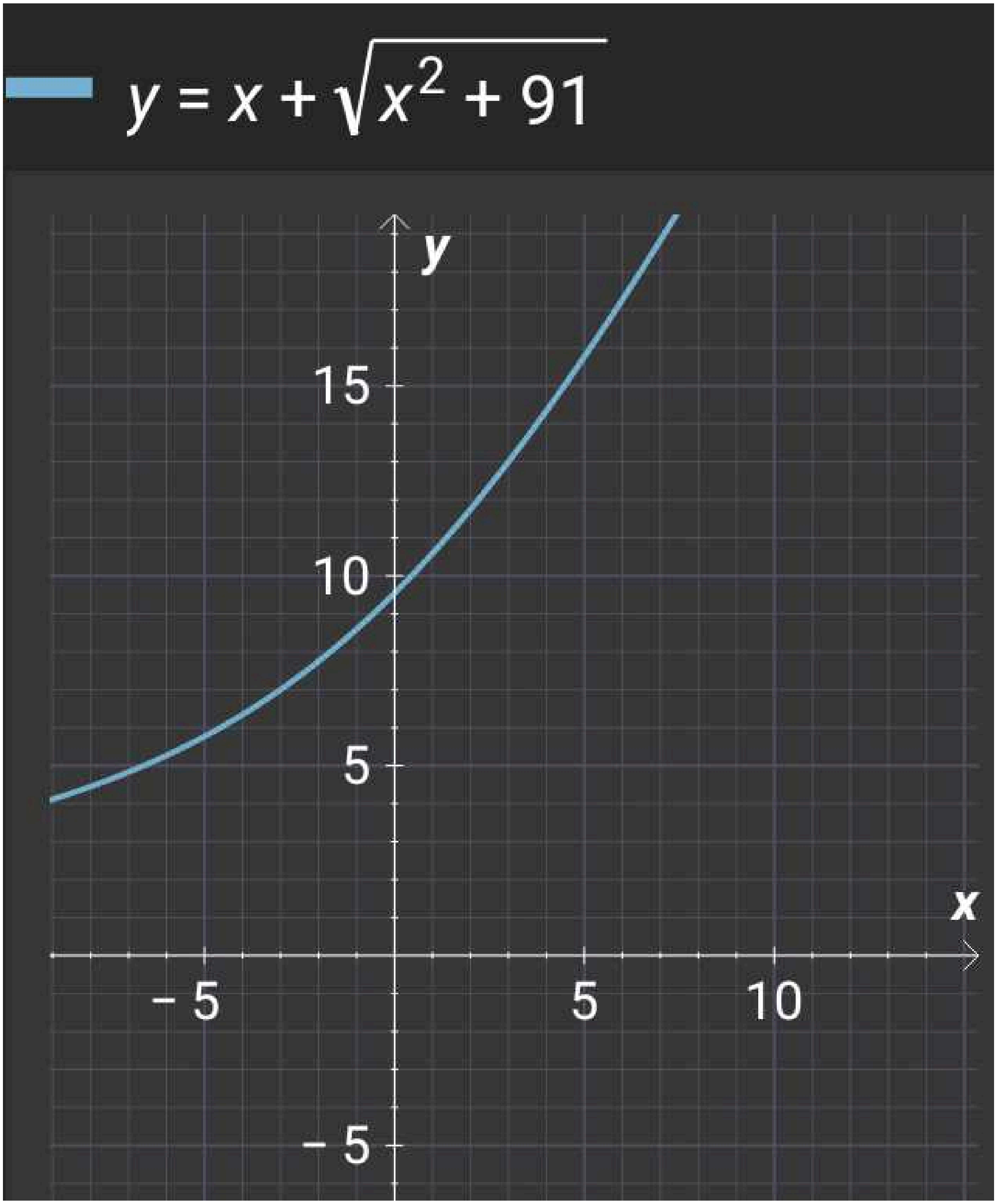

Figure 1: Determining integer solution graphically

Python application key for the formula

This section is meant to provide a python application key for examples (1) and (2).

Below are Python implementations of some key formulae from the paper:

import math

def count_primes(x, composite_classes): ”””Computes the number of primes up to x using composite classification.””” total_odds = x // 2 composite_count = sum(len(composite_classes[p]) for p in composite_classes if p < math.sqrt(x)) return total_odds - composite_count

Example classification of composite numbers

composite_classes = { 3: [33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99], 5: [25, 35, 55, 65, 85, 95], 7: [49, 77, 91] }

Example calculation

x = 100 print(“Number of primes up to”, x, “:”, count_primes(x, composite_classes))

import numpy as np import matplotlib.pyplot as plt

def factorize_graphically(n): ”””Plots the graphical representation of factorization.””” x_vals = np.arange(-n//2, n//2, 0.1) y_vals = np.sqrt(x_vals**2 + n) plt.plot(x_vals, y_vals, label=‘Factorization Curve’) plt.scatter([3, -3], [13, 7], color=‘red’, label=‘Integer Solutions’) plt.xlabel(“x”) plt.ylabel(“y”) plt.legend() plt.title(“Graphical Factorization of {}”.format(n)) plt.grid() plt.show()

Example usage

factorize_graphically(91)