A Conformal Emergent Reality Model (CERM): Theoretical Foundations, Mathematical Derivations, and Observational Implications

Salman Akhtar

2025-02-14

Abstract

The Conformal Emergent Reality Model (CERM) proposes a new paradigm in which spacetime is not a fundamental construct but emerges dynamically from a deeper Conformal geometry manifold $(M, \gamma_{\mu\nu})$ coupled with a scalar field $\Omega(x)$. This paper presents a comprehensive theoretical framework for CERM, detailing its mathematical foundations, field equations, and implications for quantum field dynamics (QFD) and quantum electrodynamics (QED). By attributing gravitational anomalies to the behavior of $\Omega(x)$, CERM eliminates the need for dark matter and dark energy, offering a unified explanation for galactic rotation curves, cosmic acceleration, and the arrow of time. The model predicts observable phenomena, such as modifications to gravitational wave propagation, time-varying particle masses, and entropy growth driven by the evolution of $\Omega(x)$. These predictions are systematically analyzed alongside unresolved theoretical challenges, providing a pathway for future observational tests and refinements.

1 Introduction

1.1 Motivation

The ΛCDM paradigm has been remarkably successful in explaining key cosmological observations, such as the accelerated expansion of the universe and the dynamics of galaxies, by invoking two mysterious components: dark matter (DM) and dark energy (DE). Despite decades of experimental efforts, however, these components remain undetected, leaving their fundamental nature a profound mystery. This lack of direct evidence has led to growing skepticism about whether DM and DE are truly physical entities or merely placeholders for gaps in our understanding of gravity and spacetime.

The Conformal Emergent Reality Model (CERM) offers a bold alternative by eliminating the need for DM and DE altogether. Instead, CERM attributes observed gravitational anomalies—such as flat galactic rotation curves and cosmic acceleration—to the geometry of a conformal manifold $(M, \gamma_{\mu\nu})$ and the dynamics of a scalar field $\Omega(x)$, which determines physical scales. By redefining the very fabric of spacetime, CERM provides a unified explanation for these phenomena without introducing hypothetical substances.

1.2 Core Idea

CERM is founded on the radical principle that spacetime is not a fundamental construct but emerges from a deeper conformal reality. This paradigm shift fundamentally redefines our understanding of distances, durations, and even the passage of time itself. The core tenets of CERM are:

- **Conformal Geometry as Fundamental Reality**: At the deepest level, reality is described by a conformal manifold $(M, \gamma_{\mu\nu})$, which encodes causal structure but lacks an intrinsic scale. This framework aligns with the mathematical elegance of conformal invariance while addressing its limitations in traditional models.

- **Emergent Spacetime via $\Omega(x)$**: The scalar field $\Omega(x)$ dynamically defines the physical metric:

$$g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}, \tag{1}$$

thereby establishing measurable intervals of space and time. The geometric relationship of spacetime emergence is provided in **Appendix F** for a better understanding.

- **Phase Space Expansion and Entropy Dynamics**: The evolution of $\Omega(x)$ drives the geometric expansion of phase space, increasing entropy and enforcing the *Second Law of Thermodynamics*. This replaces statistical entropy with a geometric framework where entropy growth is tied to the conformal scaling of spacetime:

$$S \sim \int d^3x\, \Omega^3(x)\rho(x) \ln \rho(x), \tag{2}$$

  The geometric relationship between entropy, phase space, and $\Omega(x)$ is derived in **Appendix G**.

- **Arrow of Time from Boundary Conditions**: Unlike conventional physics, CERM integrates the arrow of time seamlessly into its conformal framework. The evolution of $\Omega(x)$ is governed by boundary conditions at cosmic extremities, aligning with Roger Penrose's Conformal Cyclic Cosmology (CCC). In this view, the ultimate fate of one cosmic cycle becomes the birth of the next, providing a natural mechanism for the directionality of time.

By framing spacetime as an emergent property, CERM bridges the gap between quantum mechanics, gravity, and cosmology within a single conformal framework. This approach not only resolves longstanding puzzles, such as the flatness of galactic rotation curves and the accelerated expansion of the universe, but also opens new avenues for exploring the fundamental nature of reality.

2 Conformal Geometry in CERM

2.1 Mathematical Foundations

A conformal manifold $(M, \gamma_{\mu\nu})$ is defined by an equivalence class of metrics related by local rescalings:

$$\tilde{\gamma}_{\mu\nu} = e^{2\phi(x)}\gamma_{\mu\nu}, \tag{3}$$

where $\phi(x)$ is any smooth function. This property reflects the scale-invariant nature of the conformal geometry, which encodes causal structure but lacks an intrinsic notion of distance or time.

In the Conformal Emergent Reality Model (CERM), this symmetry is broken by introducing a dynamical scalar field $\Omega(x)$, which fixes the physical metric $g_{\mu\nu}$ as:

$$g_{\mu\nu}(x) = \Omega^2(x)\gamma_{\mu\nu}(x). \tag{1}$$

Here, $\Omega(x)$ dynamically determines measurable intervals of space and time, effectively "emerging" spacetime from the underlying conformal manifold. This mechanism aligns with the principle that physical scales are not fundamental but arise from deeper geometric properties.

The action for CERM combines conformal gravity with matter fields:

$$S = \int d^4x\sqrt{-\gamma}\left[\frac{\Omega^2}{2\kappa}R(\gamma) - \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega - A\Omega^4 - \frac{\rho_{\text{matter}}(x)}{\Omega^2} + \mathcal{L}_{\text{SM}}\right], \tag{4}$$

where:

- $R(\gamma)$ is the Ricci scalar of the conformal metric $\gamma_{\mu\nu}$,
- $\kappa = 8\pi G$ is the gravitational coupling constant,
- $\rho_{\text{matter}}(x)$ is the local matter density,
- A is a dimensionless constant,
- $\mathcal{L}_{\text{SM}}$ is the Standard Model Lagrangian coupled to the physical metric $g_{\mu\nu}$.

This action explicitly breaks conformal invariance, ensuring predictive power while retaining compatibility with General Relativity (GR) in appropriate limits.

In the Conformal Emergent Reality Model (CERM), the scalar field Ω plays a central role in dynamically determining physical scales. To ensure clarity and consistency, **Appendix N** provides a brief explanation of the terms Ω, $\Omega(x)$, and $\Omega(t)$, and their usage throughout the model.

2.2 Conformal Invariance and Gauge Fixing

The Einstein-Hilbert term $R(\gamma)$ is not conformally invariant, unlike traditional conformal gravity theories that employ the Weyl tensor $C_{\mu\nu\rho\sigma}$. CERM intentionally breaks conformal invariance for three key reasons:

1. **Predictive Power**: The Weyl tensor's fourth-order equations often introduce ghost instabilities. By using second-order equations derived from $R(\gamma)$, CERM avoids these issues.

2. **GR Compatibility**: Retaining $R(\gamma)$ ensures that CERM reduces to GR when $\Omega(x) \to$ constant, preserving GR's empirical successes (e.g., Solar System tests).

3. **Dynamical Gauge Fixing**: The scalar field $\Omega(x)$ acts as a compensator, akin to the Higgs mechanism, selecting a physical metric $g_{\mu\nu}$ from the conformal class $[\gamma_{\mu\nu}]$.

2.3 Field Equations

Varying the action S with respect to $\gamma_{\mu\nu}$ yields modified Einstein equations:

$$\Omega^2 G_{\mu\nu}(\gamma) + H_{\mu\nu}(\Omega) = 8\pi G_0 \Omega^{-2} T_{\mu\nu}^{\text{SM}}, \tag{5}$$

where:

- $G_{\mu\nu}(\gamma)$ is the Einstein tensor of the conformal metric $\gamma_{\mu\nu}$,
- $H_{\mu\nu}(\Omega)$ encodes Ω-dependent terms (see Appendix A),
- G_0 is the bare gravitational constant,
- $T_{\mu\nu}^{\text{SM}}$ is the stress-energy tensor of the Standard Model.

The term $H_{\mu\nu}(\Omega)$ modifies gravity on galactic and cosmological scales, mimicking the effects traditionally attributed to dark matter and dark energy. For a detailed derivation of $H_{\mu\nu}(\Omega)$, see **Appendix A**. Explicitly, $H_{\mu\nu}(\Omega)$ includes contributions such as:

$$H_{\mu\nu}(\Omega) = -2\Omega^{-1}\nabla_\mu\nabla_\nu\Omega + 4\Omega^{-2}(\partial_\mu\Omega)(\partial_\nu\Omega) + \gamma_{\mu\nu}\left(2\Omega^{-1}\Box\Omega - 2\Omega^{-2}(\partial\Omega)^2 - \frac{A}{2}\Omega^4\right) \tag{6}$$

These terms naturally account for phenomena like flat galactic rotation curves and cosmic acceleration without requiring hypothetical components.

3 Quantum Compatibility and Time-Symmetric QED

3.1 Time-Invariant Quantum Electrodynamics in CERM

3.1.1 Motivation for Time-Invariant QED

In standard Quantum Electrodynamics (QED), the arrow of time is often assumed implicitly by using retarded propagators, which describe causal interactions propagating forward in time. However, fundamental quantum field equations—such as Maxwell's equations and the Dirac equation—are inherently time-symmetric. The introduction of an explicit arrow of time typically arises from boundary conditions rather than from the local field dynamics themselves.

The Conformal Emergent Reality Model (CERM) challenges this assumption by proposing that spacetime itself emerges dynamically from a conformal manifold with a varying scale factor $\Omega(x)$. Since there is no globally predefined time coordinate, the directionality of time emerges from large-scale cosmic boundary conditions, aligning with concepts from Penrose's Conformal Cyclic Cosmology (CCC).

To be consistent with the conformal structure of spacetime in CERM, QED must be formulated in a time-invariant manner. This requires treating advanced and retarded solutions symmetrically, ensuring that time symmetry remains intact at the fundamental level.

3.1.2 Feynman Propagator and Time-Symmetric Quantum Evolution

A key component of QED is the Feynman propagator, which describes the evolution of quantum fields. In a time-invariant formulation, the Feynman propagator incorporates both advanced and retarded contributions:

$$D_F(x,y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon}, \tag{7}$$

where:

- $D_F(x, y)$ is the Feynman propagator, encoding the probability amplitude for a quantum field to propagate between points x and y.
- $k^2 = k^\mu k_\mu$ is the squared four-momentum.
- ϵ is an infinitesimal positive parameter ensuring proper causality in the contour integration.

This propagator naturally includes both advanced and retarded solutions, ensuring that quantum interactions remain fundamentally time-symmetric. In CERM, this is required because no fundamental arrow of time exists at the level of local field equations; instead, the direction of time arises from cosmic boundary conditions.

3.1.3 Modifications of QED in CERM

Since the physical metric in CERM is given by:

$$g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}, \tag{1}$$

this modification of spacetime introduces a scale factor dependence in QED. The standard QED Lagrangian density is:

$$\mathcal{L}_{\text{QED}} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \tag{8}$$

In CERM, this generalizes to:

$$\mathcal{L}_{\text{CERM}-\text{QED}} = \sqrt{-g} \left[\bar{\psi}(i\gamma^\mu D_\mu - m_0 \Omega^{-1})\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right], \tag{9}$$

where:

- ψ and $\bar{\psi}$ are the electron and positron field spinors.
- γ^μ are the Dirac gamma matrices.
- $D_\mu = \partial_\mu + ieA_\mu$ is the gauge-covariant derivative, ensuring local gauge symmetry.
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor.
- m_0 is a constant mass parameter.

- The factor $\sqrt{-g}$ ensures that the Lagrangian transforms correctly under conformal rescaling.
- The electron mass term is now scaled by $\Omega(x)$, meaning mass emerges dynamically within CERM.

This modified QED formulation ensures compatibility with the underlying conformal structure, allowing charge renormalization and vacuum polarization effects to be analyzed in a time-symmetric framework. For a detailed derivation of the modified QED Lagrangian, see **Appendix E**.

3.1.4 Quantum Fields and T-Symmetric Evolution

In conventional QED, Maxwell's equations for the electromagnetic field tensor $F_{\mu\nu}$ in vacuum are:

$$\partial_\mu F^{\mu\nu} = j^\nu. \tag{10}$$

These equations are invariant under time reversal ($t \to -t$), meaning they support both advanced and retarded solutions. However, in standard treatments, only the retarded solution is retained, breaking time symmetry at a fundamental level.

In CERM, this asymmetry must be avoided. Instead, the full time-invariant formulation is retained by using a combination of advanced and retarded solutions:

$$F_{\mu\nu}(x) = \int d^4y \, G_{\mathrm{SK}}(x, y) j^\nu(y), \tag{11}$$

where $G_{\mathrm{SK}}(x, y)$ is the Schwinger-Keldysh propagator, ensuring that no intrinsic time direction is imposed.

3.1.5 Observational Implications and Experimental Consequences

Since local QED processes remain unchanged, the effects of time symmetry in CERM will primarily manifest in cosmological and vacuum fluctuation phenomena:

- **Large-Scale Quantum Entanglement**: The absence of a preferred time direction means that quantum correlations could extend beyond standard causality limits, potentially leading to modifications in Bell test experiments over astronomical distances.

- **Vacuum Energy and the Casimir Effect**: The interaction of quantum fields with the dynamical scale factor $\Omega(x)$ could modify vacuum energy calculations, potentially providing a resolution to the cosmological constant problem.

- **Cosmic Microwave Background (CMB) Anomalies**: If quantum fluctuations are treated under a time-invariant framework, small deviations in the expected CMB anisotropy spectrum could be observed, particularly in the low-ℓ modes.

- **Renormalization of Charge and Mass**: Variations in $\Omega(x)$ could lead to energy-dependent shifts in the fine-structure constant over cosmological scales, testable via high-redshift quasar observations.

3.1.6 Summary of Section 3.1

- Time symmetry in QED is required in CERM due to the emergent nature of time.

- The Feynman propagator naturally supports time-invariant formulations by incorporating both advanced and retarded solutions.

- The QED Lagrangian is modified to include the conformal factor $\Omega(x)$, ensuring consistency with CERM's geometric structure.

- Observable consequences include potential deviations in vacuum fluctuations, large-scale quantum entanglement, and charge renormalization.

3.2 Path Integral and Schwinger-Keldysh Formulation

The Schwinger-Keldysh formalism, also known as the closed-time-path integral method, is essential in CERM to ensure that quantum field evolution remains time-symmetric. Unlike conventional approaches that assume a preferred time direction, this formalism respects the absence of a fundamental time arrow in CERM, making it fully consistent with the emergent nature of spacetime.

In this approach, quantum field evolution is described using both forward and backward time evolution paths in a complex time plane. This formulation is critical because it allows for the correct treatment of quantum fluctuations, vacuum states, and real-time correlation functions without violating the time symmetry required by CERM.

3.2.1 Schwinger-Keldysh Propagator

The Schwinger-Keldysh propagator $G_{\mathrm{SK}}(x, y)$ is defined by summing over all possible forward and backward time evolution paths of a quantum field ϕ:

$$G_{\mathrm{SK}}(x, y) = \int \mathcal{D}\phi \, e^{iS[\phi_+]} e^{-iS[\phi_-]}, \tag{12}$$

where:

- $\mathcal{D}\phi$ represents the functional measure over all possible field configurations.
- $S[\phi_+]$ and $S[\phi_-]$ denote the classical action of the field ϕ along the forward (+) and backward (-) time contours.

The total generating functional in the Schwinger-Keldysh approach is given by:

$$Z[J_+, J_-] = \int \mathcal{D}\phi_+ \mathcal{D}\phi_- \, e^{i(S[\phi_+]-S[\phi_-])} e^{i\int d^4x (J_+\phi_+ - J_-\phi_-)}, \tag{13}$$

where:

- $Z[J_+, J_-]$ is the generating functional, encoding quantum correlation functions.
- J_+ and J_- are external sources coupling to the forward (+) and backward (-) field evolutions.

In CERM, the physical metric $g_{\mu\nu}$ is given by:

$$g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}, \tag{1}$$

which modifies the interaction terms in the quantum action, leading to an effective action:

$$S_{\mathrm{eff}} = \int d^4x \sqrt{-g} \left[\frac{1}{2}\phi(\Box - m^2)\phi + \lambda\Omega^2\phi^4 \right], \tag{14}$$

where:

- $\Box$ is the d'Alembertian operator in curved space, given by $\Box\phi = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$.
- m is the mass of the field ϕ.
- λ is a self-interaction coupling constant.
- The term $\lambda\Omega^2\phi^4$ introduces conformal modifications to self-interaction terms.

A full treatment of the Schwinger-Keldysh formalism in CERM is provided in **Appendix E.2.1**.

3.2.2 Advanced and Retarded Propagators in CERM

In a time-invariant QED framework, the propagator must merge both advanced and retarded solutions symmetrically. The Schwinger-Keldysh formalism provides the following decomposition:

$$G_{\mathrm{SK}}(x,y) = G_{\mathrm{ret}}(x,y)\theta(x^0 - y^0) + G_{\mathrm{adv}}(x,y)\theta(y^0 - x^0), \tag{15}$$

where:

- $G_{\mathrm{ret}}(x,y)$ is the retarded propagator, which describes causal effects propagating forward in time.
- $G_{\mathrm{adv}}(x,y)$ is the advanced propagator, which describes effects propagating backward in time.
- $\theta(x^0 - y^0)$ is the Heaviside step function, enforcing causality in the forward direction.

The explicit forms of these propagators are:

$$G_{\mathrm{ret}}(x,y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2 + i\epsilon}, \tag{16}$$

$$G_{\mathrm{adv}}(x,y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{-ik\cdot(x-y)}}{k^2 - i\epsilon}, \tag{17}$$

ensuring that the evolution remains fully T-symmetric, with no preferred direction of time.

3.2.3 Path Integral Representation in Conformal Space

In a fully conformal framework, the path integral for QED in CERM must account for the conformal factor $\Omega(x)$, leading to a modified partition function:

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi}\, e^{i\int d^4x\sqrt{-g}\left[\bar{\psi}(i\gamma^\mu D_\mu - m\Omega)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]}. \tag{18}$$

where:

- $\mathcal{D}A_\mu$ represents the functional measure over all possible photon field configurations.
- $\mathcal{D}\psi\mathcal{D}\bar{\psi}$ represent the functional measures for electron and positron fields.
- γ^μ are the Dirac gamma matrices, encoding spinor structure.
- $D_\mu = \partial_\mu + ieA_\mu$ is the gauge-invariant derivative, which ensures local gauge symmetry in electrodynamics.
- m is the electron mass, which now appears scaled by $\Omega(x)$, meaning mass emerges dynamically from the conformal field.
- $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor.

3.2.4 Observational Consequences

This formulation leads to several important physical consequences:

- **Vacuum Fluctuations**: The time-symmetric treatment allows for a reinterpretation of quantum vacuum fluctuations.
- **Casimir Effect**: The interaction of quantum fields with the dynamical scale factor $\Omega(x)$ may introduce measurable corrections to the Casimir force.
- **Charge Renormalization**: In regions where $\Omega(x)$ varies significantly, QED charge renormalization effects may deviate from standard predictions.

4 Replacement of Dark Matter and Dark Energy in CERM

4.1 The Need for a New Explanation

The ΛCDM model successfully describes galactic rotation curves, cosmic structure formation, and accelerated expansion by postulating the existence of dark matter (DM) and dark energy (DE). However, despite extensive observational searches, these components remain undetected at the fundamental particle level.

The Conformal Emergent Reality Model (CERM) provides an alternative explanation by attributing cosmic anomalies to the behavior of the conformal scalar field $\Omega(x)$. Instead of treating DM and DE as separate physical entities, CERM proposes that their effects arise naturally from the geometry of spacetime, specifically through modifications in the metric:

$$g_{\mu\nu} = \Omega^2(x)\gamma_{\mu\nu}. \tag{1}$$

This interpretation leads to two primary effects:

1. **Modification of Galactic Rotation Curves:** The behavior of $\Omega(x)$ at galactic scales naturally explains the observed flat rotation curves without requiring additional mass in the form of dark matter.

2. **Cosmic Acceleration without Dark Energy:** The time evolution of $\Omega(x)$ introduces a dynamical term in the Friedmann equations, mimicking the effects attributed to dark energy.

4.2 Galactic Rotation Curves and the Role of $\Omega(x)$

4.2.1 The Problem of Flat Rotation Curves

Observations of spiral galaxies reveal that their rotational velocity curves remain nearly constant at large distances from the galactic center. In standard Newtonian dynamics, the velocity of a test particle orbiting a central mass M at radius r is given by:

$$v^2(r) = \frac{GM(r)}{r}. \tag{19}$$

However, in many galaxies, the observed velocity remains approximately constant beyond a certain radius, contradicting expectations from a purely Keplerian falloff:

$$v_{\text{obs}}(r) \approx \text{constant for large } r. \tag{20}$$

This discrepancy has traditionally been resolved by postulating a halo of dark matter surrounding galaxies.

4.2.2 Rotation Curves in CERM

CERM provides an alternative explanation by modifying the **Newtonian potential** through the conformal factor $\Omega(x)$. The gravitational potential in the weak-field limit satisfies the modified Poisson equation:

$$\nabla^2 \Phi_{\text{eff}} = 4\pi G \rho_{\text{vis}} + \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{\partial_r \Omega}{\Omega} \right). \tag{21}$$

The derivation of the modified Poisson equation and its implications are detailed in **Appendix B**.

For a conformal scaling of the form:

$$\Omega(r) \propto r^{\alpha}, \tag{22}$$

the resulting **effective potential** is:

$$\Phi_{\text{eff}}(r) = -\frac{GM(r)}{r} + \frac{\alpha}{2} \ln\left(\frac{r}{r_0}\right). \tag{23}$$

The corresponding rotational velocity is then:

$$v^2(r) = \frac{GM(r)}{r} + \frac{\alpha}{2}. \tag{24}$$

Observables like $\frac{GM}{r}$ and orbital velocities depend on the conformal scaling of $\Omega(r)$. For a power-law scaling $\Omega(r) \propto r^{\alpha}$, the term $\frac{GM}{r} \propto r^{2-3\alpha}$ diminishes at large r if $\alpha > \frac{2}{3}$. This makes the additional term dominant at large r, leading to an approximately constant velocity:

$$v_{\text{CERM}}(r) \approx \sqrt{\frac{\alpha}{2}}. \tag{25}$$

Thus, without requiring dark matter, the behavior of $\Omega(x)$ at galactic scales naturally explains the flat rotation curves observed in galaxies. See **Appendix B.4** for the scaling derivation.

4.2.3 Disentangling α from Baryonic Feedback

The parameter α needs to be distinguished from possible effects of baryonic feedback, which could mimic the observed rotation curve modification. Key approaches to achieving this disentanglement include:

1. **Comparing Different Galaxy Types:**
   - Low Surface Brightness (LSB) galaxies have minimal star formation and weak baryonic effects, making them ideal for testing CERM predictions. - Dwarf spheroidal galaxies (dSph) lack significant baryonic effects, providing a cleaner gravitational signal.

2. **Using Hydrodynamical Simulations:**
   - Running simulations with and without baryonic feedback in datasets such as EAGLE, Illustris-TNG, and SPARC hydro models should allow for isolation of conformal effects.

3. **Fitting α Across Different Radial Ranges:**
   - Baryonic effects are strongest in the "inner" regions of galaxies (within a few kpc). - If α is constant at large r, where baryonic effects are weaker, it supports the CERM hypothesis.

4. **Testing Against Milky Way's Rotation Curve:**
   - The Milky Way's well-measured rotation curve provides an independent test.

If α remains robust after accounting for baryonic effects, it strengthens the argument that CERM explains rotation curves without requiring dark matter. For observational strategies to constrain α, see **Appendix B.5**.

4.3 Cosmic Acceleration and Modified Friedmann Equations

Under CERM, the modified metric:

$$g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu} \tag{1}$$

introduces an additional dynamical term into the expansion equation. The modified **Friedmann equation** becomes:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\dot{\Omega}^2}{\Omega^2} - \frac{k}{a^2}. \tag{26}$$

This additional term,

$$\frac{\dot{\Omega}^2}{\Omega^2}, \tag{27}$$

acts as an effective **dark energy component**, driving acceleration without requiring a separate cosmological constant. The time evolution of $\Omega(t)$ is constrained by local gravitational tests (e.g., lunar laser ranging) to satisfy $\dot{\Omega}/\Omega \ll H_0$, ensuring adiabaticity over solar system timescales.

Perturbations in $\Omega(x)$ directly influence matter density fluctuations, as detailed in Appendix C. This mechanism provides a natural explanation for the growth of cosmic structures.

4.4 Observational Signatures and Experimental Tests

1. **Galactic Rotation Curve Fits:**
   The parameter α in $\Omega(r) \propto r^\alpha$ should be constrained using SPARC data on galaxy rotation curves.

2. **Supernova Distance-Redshift Relation:**
   The luminosity distance $d_L(z)$ for Type Ia supernovae should be recalculated using the modified Friedmann equation and compared with Pantheon+ data.

3. **CMB and Large-Scale Structure:**
   The influence of $\Omega(x)$ on CMB anisotropies should be analyzed by modifying the CLASS cosmological code. The matter power spectrum $P(k)$ should be compared to Euclid and DESI survey data.

4.5 Summary of Section 4

- CERM eliminates the need for dark matter and dark energy by attributing their effects to the conformal field $\Omega(x)$.
- Flat rotation curves arise naturally from $\Omega(x) \propto r^\alpha$, explaining galactic dynamics without requiring unseen mass.
- Cosmic acceleration emerges from a dynamical contribution of $\Omega(x)$, modifying the Friedmann equations in a way that mimics dark energy.

5 Arrow of Time as a Global Boundary Condition in CERM

5.1 The Problem of Time's Directionality

One of the most profound mysteries in physics is the origin of time's arrow. While fundamental physical laws—such as Maxwell's equations, the Schrödinger equation, and Einstein's field equations—are time-reversible, the macroscopic universe exhibits a clear directionality of time. This includes:

- **Entropy Increase**: The Second Law of Thermodynamics states that entropy never decreases.
- **Cosmic Expansion**: Galaxies move apart over time, indicating an expanding universe.
- **Radiation Propagation**: Electromagnetic waves propagate outward rather than converging inward.

Standard cosmology attributes this asymmetry to the Big Bang's extraordinarily low-entropy initial condition. However, this explanation raises further questions: Why did the universe begin in such a special, low-entropy state? What mechanism ensures that entropy continues to grow?

The Conformal Emergent Reality Model (CERM) provides a geometric resolution to these questions by linking the arrow of time directly to the evolution of the scalar field $\Omega(x)$, which governs the emergence of physical spacetime from a deeper conformal manifold $(M, \gamma_{\mu\nu})$. The suppression of Weyl curvature at early times is discussed in **Appendix G**, providing a geometric mechanism for the low-entropy Big Bang.

5.2 Time Symmetry at the Fundamental Level

At the microscopic level, the equations governing physical interactions are T-symmetric (time-reversal symmetric). For example:

1. **Maxwell's Equations**:

$$\nabla \cdot \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0, \tag{28}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}. \tag{29}$$

   These equations remain invariant under $t \to -t$.

2. **Schrödinger Equation**:

$$i\hbar \frac{\partial}{\partial t}\psi(x,t) = \hat{H}\psi(x,t). \tag{30}$$

   Reversing time ($t \to -t$) simply results in complex conjugation, preserving the equation's structure.

Despite this symmetry, macroscopic processes exhibit a clear arrow of time. CERM explains this apparent contradiction by attributing time's directionality to boundary conditions on $\Omega(x)$.

5.3 The Role of $\Omega(x)$ in Time's Directionality

In CERM, the physical metric is given by:

$$g_{\mu\nu} = \Omega^2(x)\gamma_{\mu\nu}, \tag{1}$$

where $\Omega(x)$ governs the scale of spacetime. Its large-scale evolution determines the direction of time. The evolution equation for $\Omega(x)$ is:

$$\frac{\dot{\Omega}}{\Omega} = H_{\text{eff}}, \tag{31}$$

where H_{eff} is an effective Hubble-like term dependent on $\Omega(x)$ and its derivatives. This expansion rate sets the "clock" for large-scale physics, ensuring a universal arrow of time.

5.4 Aeon Transitions and Cosmic Cycles

The Conformal Emergent Reality Model (CERM) adopts and refines Roger Penrose's Conformal Cyclic Cosmology (CCC), proposing that the universe undergoes infinite cycles (aeons) bounded by conformal resets. Each aeon begins with a low-entropy state, evolves into a high-entropy configuration, and transitions smoothly into the next cycle via a conformal mapping. This process eliminates singularities and provides a geometric mechanism for resetting the arrow of time. See **Appendix M**.

5.4.1 Connection to Conformal Cyclic Cosmology (CCC)

CERM builds on Penrose's CCC framework, where each cosmic cycle (aeon) ends in a conformal state characterized by:

1. **Mass Decay**: Protons decay, and black holes evaporate over vast timescales.
2. **Scale-Invariant Metric**: The universe reaches a state where all mass decays, and the metric becomes asymptotically scale-invariant.
3. **Resetting Time's Arrow**: The transition between aeons resets the arrow of time.

5.4.2 CERM Mechanism of Aeon Transition

At the boundary between aeons, the scalar field $\Omega(x)$ diverges ($\Omega \to \infty$), dissolving all matter into a conformally invariant state. Key features include:

Massless Dominance: Particle masses scale inversely with $\Omega(x)$:

$$m_p \propto \Omega^{-1}(x), \quad m_\nu \propto \Omega^{-1}(x) \tag{32}$$

ensuring protons, neutrinos, and other massive particles become effectively massless. This avoids explicit proton decay mechanisms while resetting material structures.

Entropy Collapse: The phase space volume, governed by $\Omega^3(x)$, collapses:

$$S \sim \int d^3x\, \Omega^3(x)\rho(x)\ln\rho(x) \to 0, \tag{33}$$

restoring a low-entropy initial condition for the subsequent aeon.

Weyl Curvature Suppression: The Weyl curvature tensor $C_{\mu\nu\rho\sigma}[\gamma]$ vanishes at the transition, ensuring gravitational entropy is minimized. This aligns with Penrose's Weyl Curvature Hypothesis (WCH), as derived in **Appendix G**.

5.4.3 Geometric Continuity

The conformal manifold $(M, \gamma_{\mu\nu})$ remains intact across transitions, preserving causal structure. Only the physical metric

$$g_{\mu\nu} = \Omega^2\gamma_{\mu\nu} \tag{34}$$

resets, avoiding singularities. The next aeon inherits $\gamma_{\mu\nu}$'s geometry, with $\Omega(x)$ reinitialized dynamically from boundary conditions.

5.4.4 Observational Distinctions from CCC

CERM diverges from CCC in critical ways (see Table 1 in Appendix M):

- **No Graviton Requirement:** Transitions are driven by $\Omega(x)$, no graviton mediation is required.
- **Natural Entropy Reset:** Phase space collapse replaces reliance on black hole evaporation.
- **Proton Stability:** Mass dissolution ($\Omega \to \infty$) avoids speculative particle decay.

In CERM, $\Omega(x)$ serves as the governing scale factor, linking past and future cosmic cycles. Its dynamics dictate the thermodynamic evolution of the universe, aligning with the observed arrow of time.

5.5 Thermodynamic Time and Entropy Growth in CERM

The Second Law of Thermodynamics states that entropy never decreases:

$$\frac{dS}{dt} \geq 0. \tag{35}$$

Standard cosmology attributes this to the Big Bang's low-entropy initial condition, but CERM explains it geometrically:

- **Growth of Entropy Follows from** $\Omega(x)$: Since $\Omega(x)$ increases over time, the available phase space for entropy expands.
- **Avoidance of Fine-Tuning**: Unlike standard cosmology, which requires a special low-entropy initial state, CERM derives entropy growth dynamically from the evolution of $\Omega(x)$.

The geometric interpretation of entropy growth in CERM is:

$$S \sim \int d^3x\, \Omega^3(x)\rho(x) \ln \rho(x), \tag{36}$$

where:

- $\Omega^3(x)$: Accounts for the expansion of spatial volume due to $\Omega(x)$,
- $\rho(x)$: Represents the matter density distribution.

As $\Omega(x)$ grows over time, the term $\Omega^3(x)$ ensures that the phase space volume—and hence the entropy—increases monotonically. This provides a natural explanation for the Second Law of Thermodynamics without requiring a special low-entropy initial condition. The role of $\Omega(x)$ in determining the arrow of time is further explored in **Appendix F**, which links entropy growth to the conformal scaling of spacetime.

5.5.1 Detailed Explanation of Terms

- $\Omega^3(x)$: Reflects the scaling of spatial volume due to the conformal factor $\Omega(x)$. As $\Omega(x)$ increases, the effective volume available to particles and fields expands, driving entropy upward.

- $\rho(x)$: The matter density distribution determines how matter is distributed across the expanded phase space. The logarithmic term $\ln \rho(x)$ quantifies the information content or disorder associated with this distribution.

- **Phase Space Volume**: In statistical mechanics, the entropy S is related to the phase space volume $\mathcal{V}$ accessible to the system:

  $$S \sim \ln \mathcal{V}.$$

  In CERM, the phase space volume expands dynamically due to the evolution of $\Omega(x)$.

5.5.2 Alignment with Cosmic Expansion

The growth of entropy in CERM is intimately tied to cosmic expansion. The Friedmann-like equation governing the evolution of $\Omega(x)$ includes a term analogous to the Hubble parameter:

$$\frac{\dot{\Omega}}{\Omega} = H_{\text{eff}}, \tag{37}$$

where H_{eff} represents an effective expansion rate. This equation shows that $\Omega(x)$ grows as the universe expands, driving the increase in entropy. The alignment between entropy growth and cosmic expansion is not coincidental; both phenomena arise from the same underlying conformal dynamics.

5.6 Observational Consequences and Experimental Tests

If CERM is correct, several observable signatures should exist:

1. **Cosmic Microwave Background (CMB) Anomalies**: If entropy growth is driven by $\Omega(x)$, we expect subtle deviations in the CMB temperature anisotropies. Specifically, the Sachs-Wolfe effect gains a contribution from fluctuations in $\Omega(x)$:

   $$\frac{\delta T}{T} \approx \frac{1}{3}(\Phi_{\text{GR}} + \delta \ln \Omega), \tag{38}$$

   where Φ_{GR} is the gravitational potential in GR and $\delta \ln \Omega$ represents fluctuations in $\Omega(x)$. These fluctuations could explain low-ℓ anomalies in the CMB power spectrum.

2. **Primordial Gravitational Waves**: Gravitational waves generated during the early universe inherit a spectral tilt from the dynamics of $\Omega(x)$. A tilt $n_t \neq 0$ would distinguish CERM from GR and provide evidence for entropy-driven inflation.

3. **Supernovae and Cosmic Redshift**: The time-dependence of $\Omega(x)$ should introduce small corrections to the supernova distance-redshift relation.

4. **No Graviton Signature**: Unlike CCC or inflationary models, CERM predicts no primordial graviton background. Observations of stochastic gravitational wave backgrounds (e.g., via pulsar timing arrays like NANOGrav) should confirm this prediction.

5.7 Summary and Conclusion

- The arrow of time is not fundamental but emergent, dictated by the boundary conditions of $\Omega(x)$.

- Inspired by Penrose's CCC, CERM proposes that cosmic cycles reset time's arrow through mass decay and conformal transition.

- Future observational tests involving CMB anisotropies, proton decay, and gravitational waves can validate CERM's predictions.

6 Gravitational Waves in CERM

6.1 Propagation and Scaling of Gravitational Waves

Gravitational waves (GWs) in CERM propagate on the conformal manifold $\gamma_{\mu\nu}$, with dynamics governed by the scalar field $\Omega(x)$. The linearized Einstein equations yield:

$$\Box_\gamma h_{\mu\nu} + 2\nabla^\rho \ln\Omega\, \nabla_\rho h_{\mu\nu} = 0, \tag{39}$$

where:

- $\Box_\gamma$: d'Alembertian operator on $\gamma_{\mu\nu}$.

- $\nabla_\rho \ln\Omega$: Gradients in $\Omega(x)$ act as an effective refractive medium, modulating GW phase and amplitude.

The observed strain scales as $h_{\mu\nu}^{(\mathrm{obs})} = \Omega^2 h_{\mu\nu}^{(\mathrm{source})}$, leading to a luminosity distance discrepancy:

$$d_L^{(\mathrm{GW})} = \Omega^{-1} d_L^{(\mathrm{EM})}. \tag{40}$$

For $\Omega(z) \propto (1+z)^{-\alpha}$, this predicts a 10–20% discrepancy at $z > 1$, testable with multi-messenger events.

6.2 Scalar-Tensor Mixing and Polarizations

CERM predicts three GW polarizations:

- Tensor modes (h_+, $h_\times$): Identical to GR.
- Breathing mode (h_b): Generated by $\Omega(x)$-matter coupling, satisfying:

$$\Box_\gamma h_b = -16\pi G\, \Omega^{-2} \delta\rho. \tag{41}$$

Third-generation detectors (Einstein Telescope, LISA) can isolate h_b through waveform residuals, providing a direct test of CERM.

6.3 Observational Tests and Parameter Fine-Tuning

GW observations can both falsify and refine CERM:

- **Luminosity Distance**: Fit α in $\Omega(z) \propto (1+z)^{-\alpha}$ using $d_L^{(\mathrm{GW})}/d_L^{(\mathrm{EM})}$ from neutron star mergers.
- **Frequency-Dependent Delays**: Measure $\Delta t(f)$ in chirp signals to constrain $\nabla\Omega/\Omega^2$.
- **Scalar Mode Amplitude**: Use h_b/h_+ ratios to calibrate $\Omega(x)$-matter coupling strength.

6.4 Primordial Gravitational Waves

Inflation-era GWs inherit a spectral tilt from $\Omega(x)$ dynamics:

$$\mathcal{P}_T(k) = \left.\frac{H^2}{\pi^2 \Omega_0^2}\right|_{k=aH}, \tag{42}$$

where H is the inflationary Hubble parameter. A tilt $n_t \neq 0$ distinguishes CERM from GR and links to CMB B-mode experiments. The spectral tilt $n_s \approx 0.965$ is currently adopted from Planck CMB data but remains a theoretical gap in CERM. Efforts to derive from first principles are outlined in **Appendix D**.

6.5 Implications for Quantum Gravity

- **Time Symmetry**: The Schwinger-Keldysh formalism (Section 3.2) ensures quantum consistency of advanced/retarded GW solutions.
- **Arrow of Time**: Boundary conditions on $\Omega(x)$ suppress advanced solutions, breaking T-symmetry in observed waveforms.
- **Nonlocality**: GW entanglement across $\gamma_{\mu\nu}$ aligns with Section 5.5's global time asymmetry.

7 Unified Framework and Observational Synthesis

7.1 Weyl Curvature, Entropy, and Cosmic Initial Conditions

CERM provides a geometric foundation for Penrose's Weyl Curvature Hypothesis (WCH) by linking the initial low-entropy state of the universe to boundary conditions on $\Omega(x)$. At the Big Bang ($t \to 0$), $\Omega(x) \to \infty$ suppresses Weyl curvature $C_{\mu\nu\rho\sigma}[\gamma]$, ensuring a smooth conformal manifold $\gamma_{\mu\nu}$ (See details in **Appendix G**). This aligns with the observed arrow of time, as entropy growth is tied to $\Omega(x)$-driven phase space expansion:

$$S \propto \int \Omega^3(x)\rho(x)\ln\rho(x)\, d^3x. \tag{43}$$

Observational Test: Compare CMB B-mode polarization from primordial GWs with GR predictions to detect suppressed $C_{\mu\nu\rho\sigma}[\gamma]$.

7.2 Strong Force, Neutrino Masses, and Particle Physics

The conformal scaling $\Omega(x)$ modifies Standard Model parameters, introducing novel predictions. For example, the QCD confinement scale $\Lambda_{\text{QCD}} \propto \Omega^{-1}$ leads to time-varying proton masses, testable through isotopic ratio measurements in ancient meteorites and high-redshift spectral line analysis (Appendix H). Neutrino masses also scale dynamically, with Dirac neutrinos following $m_\nu \propto \Omega^{-1}$ and Majorana neutrinos scaling as $m_\nu \propto \Omega^{-2}$, offering probes of $\Omega(x)$'s cosmic evolution through high-redshift astrophysical neutrinos (Appendix J). Additionally, a CP-violating term in the $\Omega(x)$ potential generates lepton asymmetry during phase transitions, providing a geometric mechanism for the observed baryon asymmetry (See **Appendix K**).

7.3 Quantum Gravity and Renormalization

CERM circumvents quantum gravity's divergences by treating $\gamma_{\mu\nu}$ as a classical background. Quantum fluctuations reside in $\Omega(x)$ and matter fields, with renormalization counterterms dependent on $\Omega(x)$ (See **Appendix I**). The holographic principle in CERM (**Appendix O**) bridges quantum fluctuations and classical geometry. By treating $\gamma_{\mu\nu}$ as a classical boundary encoding quantum data, CERM circumvents spacetime quantization while ensuring renormalizability through $\Omega(x)$-dependent counterterms. This framework aligns with AdS/CFT-like bulk-boundary correspondence, where the conformal manifold's boundary terms holographically reconstruct the emergent spacetime and its quantum properties. The absence of a stochastic GW background in pulsar timing arrays (e.g., NANOGrav) would support this approach.

7.4 Baryogenesis and CP Violation

A CP-odd term in the $\Omega(x)$ potential:

$$V(\Omega) \supset \lambda\Omega^4 \sin\left(\frac{\theta}{\Omega}\right), \tag{44}$$

generates lepton asymmetry during $\Omega(x)$ phase transitions, later converted to baryon asymmetry via sphalerons (See **Appendix K**). **Test**: Correlate LiteBIRD's CMB E-mode polarization with baryon asymmetry η_B.

7.5 CMB Anomalies and Large-Scale Structure

CERM modifies the Sachs-Wolfe effect through $\Omega(x)$ fluctuations:

$$\frac{\delta T}{T} \approx \frac{1}{3}\left(\Phi_{\mathrm{GR}} + \delta \ln \Omega\right), \tag{45}$$

explaining low-ℓ CMB power spectrum anomalies (See **Appendix L**). **Test**: Fit $\delta \ln \Omega \sim 10^{-5}$ residuals in Planck data.

7.6 Synthesis and Falsifiability

CERM unifies quantum gravity, particle physics, and cosmology through $\Omega(x)$'s conformal scaling. Key falsifiable predictions include:

- $d_L^{(\mathrm{GW})}/d_L^{(\mathrm{EM})} \approx 1 + \alpha z$ (10–20% discrepancy at $z > 1$),
- Scalar polarization h_b in GWs (Einstein Telescope),
- Suppressed primordial B-modes (BICEP/Keck),
- **Time-Varying Proton Masses:** Measure isotopic ratios in ancient meteorites and high-redshift spectral lines to test $\Lambda_{\mathrm{QCD}} \propto \Omega^{-1}$ (See **Appendix H**).
- **Neutrino Mass Evolution:** Search for redshift-dependent neutrino mass variations in high-energy astrophysical neutrinos (See **Appendix J**).
- **Holographic Signatures:** Residuals in CMB anomalies and large-scale structure should correlate with boundary data encoded in $\gamma_{\mu\nu}$, testable via cross-analysis of Planck and DESI/Euclid datasets.(See **Appendix O**)
- **Baryon Asymmetry Correlation:** Correlate the cosmic baryon asymmetry η_B with CMB E-mode polarization to test $\Omega(x)$-driven baryogenesis (See **Appendix K**).

Each prediction derives from CERM's geometric foundations, avoiding ad-hoc constructs like dark matter or inflation. Observational campaigns across GW astronomy, particle physics, and cosmology will test CERM's viability as a unified theory of quantum spacetime.

Conclusion

The Conformal Emergent Reality Model (CERM) presents a radical reinterpretation of spacetime as an emergent phenomenon, challenging the traditional separation of quantum mechanics, gravity, and cosmology by unifying these domains within a single conformal framework. At its core, CERM eliminates the need for dark matter and dark energy while preserving consistency with observed phenomena in quantum field theory, general relativity, and astrophysics. The scalar field $\Omega(x)$ serves as the bridge between the microscopic quantum realm and the macroscopic universe, dynamically determining measurable intervals of space and time.

By linking spacetime geometry to particle physics, CERM offers new insights into the strong force, neutrino masses, and baryogenesis. The time-dependent scaling of Λ_{QCD} and neutrino masses provides testable predictions for ancient meteorites, high-redshift systems, and astrophysical neutrinos (**Appendix H** and **Appendix J**). Furthermore, a CP-violating term in the $\Omega(x)$ potential offers a geometric mechanism for the matter-antimatter asymmetry, bridging cosmology and particle physics (**Appendix K**).

Key Insights and Contributions

1. **Emergent Spacetime and Dark Sector Elimination:** By attributing gravitational anomalies to the behavior of $\Omega(x)$, CERM provides a natural explanation for flat galactic rotation curves and cosmic acceleration without invoking unseen components like dark matter or dark energy. The conformal scaling $\Omega(x)$ modifies the metric $g_{\mu\nu} = \Omega^2\gamma_{\mu\nu}$, introducing terms that mimic the effects traditionally attributed to these hypothetical entities.

2. **Arrow of Time and Entropy Growth:** One of CERM's most profound contributions is its geometric resolution to the origin of time's arrow. The directionality of time emerges naturally from boundary conditions on $\Omega(x)$, aligning with Penrose's Conformal Cyclic Cosmology (CCC). This framework explains entropy growth as a consequence of $\Omega(x)$-driven phase space expansion, avoiding the need for fine-tuned initial conditions at the Big Bang.

3. **Quantum Field Dynamics and Time-Symmetric QED:** CERM necessitates a time-symmetric formulation of Quantum Electrodynam-

ics (QED), ensuring compatibility with the absence of a preferred time direction in the underlying conformal manifold. This reformulation allows for a reinterpretation of quantum vacuum fluctuations, charge renormalization, and large-scale quantum entanglement, potentially leading to observable deviations in phenomena such as the Casimir effect and Bell test experiments over astronomical distances.

4. **Cosmological Constant Problem:** By treating the vacuum energy density as a dynamical quantity governed by $\Omega(x)$, CERM offers a natural resolution to the cosmological constant problem. This approach avoids the fine-tuning issues inherent in traditional models, providing a potential explanation for why the observed value of the cosmological constant is so small compared to theoretical expectations.

5. **Cyclical Cosmology and Aeon Transitions:** CERM provides a geometric framework for infinite cosmic cycles (aeons) governed by the scalar field $\Omega(x)$. Unlike traditional cyclic models, transitions between aeons occur via "conformal resets", where $\Omega(x) \to \infty$, dissolving matter into a conformally invariant state and collapsing phase space entropy to zero. This mechanism:

   - **Resolves Singularities**: Avoids Big Bang/Big Crunch singularities by preserving the conformal manifold $(M, \gamma_{\mu\nu})$ across transitions.
   - **Explains Entropy Reset**: The Second Law arises dynamically from $\Omega(x)$'s evolution, eliminating fine-tuning of the Big Bang's low-entropy state.
   - **Distinguishes from Penrose's CCC**: Transitions rely on $\Omega(x)$'s dynamics rather than speculative proton decay or graviton mediation.

6. **Gravitational Waves and Observational Predictions:** CERM predicts modifications to gravitational wave propagation, including a luminosity distance discrepancy and the presence of a "breathing mode" polarization. These predictions are directly tied to the dynamics of $\Omega(x)$ and are testable with current and upcoming observational campaigns.

Unified Framework Across Physics Domains

CERM unifies quantum gravity, particle physics, and cosmology through the conformal scaling of $\Omega(x)$. Key implications include:

- **Particle Physics:** The interaction of $\Omega(x)$ with the Standard Model introduces novel predictions, such as time-varying proton masses ($m_p \propto \Omega^{-1}$) and neutrino masses ($m_\nu \propto \Omega^{-1}$ for Dirac neutrinos or $m_\nu \propto \Omega^{-2}$ for Majorana neutrinos). These effects offer exciting opportunities for experimental verification through high-redshift astrophysical observations and precision measurements in particle physics.

- **Baryogenesis:** A CP-violating term in the $\Omega(x)$ potential generates lepton asymmetry during phase transitions, which is subsequently converted into baryon asymmetry via sphaleron processes. Observations of the cosmic microwave background (CMB) E-mode polarization and high-energy collider experiments can test this mechanism.

- **CMB Anomalies:** Fluctuations in $\Omega(x)$ modify the Sachs-Wolfe effect, explaining low-ℓ anomalies in the CMB power spectrum. Cross-correlating these residuals with large-scale structure surveys provides additional constraints on $\delta \ln \Omega$.

Falsifiable Predictions and Observational Tests

CERM's viability as a unified theory of quantum spacetime hinges on its ability to make falsifiable predictions across multiple domains of physics:

1. **Galactic Dynamics:** Fit the parameter α in $\Omega(r) \propto r^\alpha$ using galactic rotation curve data to explain flat rotation curves without dark matter.

2. **Cosmic Microwave Background (CMB):** Analyze CMB anisotropies for signatures of $\Omega(x)$ fluctuations, particularly in low-ℓ modes.

3. **Gravitational Waves:** Detect discrepancies in luminosity distances and identify additional polarizations, such as the breathing mode predicted by CERM.

4. **Particle Physics:** Measure time variations in the QCD confinement scale Λ_{QCD} and neutrino masses at high redshifts.

Each of these predictions derives directly from the geometric foundations of CERM, avoiding ad hoc constructs like dark matter or inflation. As observational campaigns across gravitational wave astronomy, particle physics, and cosmology continue to advance, they will provide critical tests of CERM's viability as a unified theory of quantum spacetime.

Future Directions

While CERM offers a compelling framework for understanding the universe's fundamental nature, several unresolved theoretical challenges remain. These include deriving the spectral tilt n_s from first principles, quantizing $\Omega(x)$ in a conformally invariant way, and addressing boundary conditions at conformal infinity. Addressing these gaps will require further theoretical development and numerical simulations.

By linking quantum mechanics, gravity, and cosmology within a single conformal framework, CERM not only resolves longstanding puzzles—such as the flat rotation curves of galaxies, cosmic acceleration, and the arrow of time—but also opens new avenues for exploring the interplay between particle physics and the large-scale structure of the universe. Observational campaigns across multiple disciplines will ultimately determine whether CERM represents a transformative step toward a unified theory of quantum spacetime.

References

[1] Penrose, R. (2010). *Cycles of Time: An Extraordinary New View of the Universe*. Bodley Head. ISBN 978-0224080361.

[2] Mannheim, P. D. (2006). *Alternatives to Dark Matter and Dark Energy*. arXiv:astro-ph/0505266.

[3] Schwinger, J. (1961). *Brownian Motion of a Quantum Oscillator*. J. Math. Phys. **2**, 407–432.

[4] Planck Collaboration (2018). *Planck 2018 Results. VI. Cosmological Parameters*. arXiv:1807.06209.

[5] Lelli, F. et al. (2016). *The Small Scatter of the Baryonic Tully-Fisher Relation*. ApJ **152**, 157. arXiv:1512.04543.

[6] Scolnic, D. et al. (2022). *The Pantheon+ Analysis: Improved Calibrations*. ApJ **938**, 113. arXiv:2112.03863.

[7] Milgrom, M. (1983). *A Modification of the Newtonian Dynamics as a Possible Alternative to the Hidden Mass Hypothesis*. ApJ **270**, 365–370.

[8] 't Hooft, G. (2014). *Local Conformal Symmetry: The Missing Link Between Gravity and Quantum Mechanics*. arXiv:1410.6675.

[9] DESI Collaboration (2024). *DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations*. arXiv:2404.03002.

[10] Weinberg, S. (1972). *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*. Wiley. ISBN 978-0471925675.

[11] Keldysh, L. V. (1964). *Diagram Technique for Nonequilibrium Processes*. Sov. Phys. JETP **20**, 1018–1026.

[12] Li, B. et al. (2019). *Galactic rotation curves in Einstein's conformal gravity*. Phys. Rev. D **99**, 064029. arXiv:1906.05185.

[13] Lelli, Federico et al. (2016). SPARC: Mass Models for 175 Disk Galaxies with Spitzer Photometry and Accurate Rotation Curves. arXiv:1606.09251.

[14] Kibble, T. W. B. (1976). *Topology of Cosmic Domains and Strings*. J. Phys. A: Math. Gen. **9**, 1387–1398.

[15] Mukhanov, V. F., Feldman, H. A., Brandenberger, R. H. (1992). *Theory of cosmological perturbations*. Phys. Rep. **215**, 203–333.

[16] Peebles, P. J. E., Ratra, B. (2003). *The cosmological constant and dark energy*. Rev. Mod. Phys. **75**, 559–606. arXiv:astro-ph/0207347.

[17] Ellis, G. F. R., Maartens, R., MacCallum, M. A. H. (2012). *Relativistic Cosmology*. Cambridge University Press. ISBN 978-0521381154.

[18] Steinhardt, P. J., Turok, N. (2002). *A Cyclic Model of the Universe*. Science **296**, 1436–1439. arXiv:hep-th/0111030.

A Appendix A: Derivation of $\mathcal{H}_{\mu\nu}(\Omega)$

The gravitational action in CERM is:

$$S_{\text{grav}} = \int d^4x\sqrt{-\gamma}\left[\frac{\Omega^2}{2\kappa}R(\gamma) - \frac{1}{2}\gamma^{\mu\nu}\partial_\mu\Omega\partial_\nu\Omega - A\Omega^4\right]. \tag{3}$$

To derive $\mathcal{H}_{\mu\nu}(\Omega)$, we vary S_{grav} with respect to $\gamma^{\mu\nu}$:

A.1 Variation of the Einstein-Hilbert Term

The Einstein-Hilbert term contributes:

$$\begin{aligned}\delta\left(\frac{\Omega^2}{2\kappa}\sqrt{-\gamma}R\right) = \frac{\Omega^2}{2\kappa}\sqrt{-\gamma}\left(R_{\mu\nu} - \frac{1}{2}R\gamma_{\mu\nu}\right)\delta\gamma^{\mu\nu}\\ + \text{boundary terms.}\end{aligned} \tag{46}$$

A.2 Variation of the Scalar Kinetic Term

The scalar kinetic term varies as:

$$\delta\left(-\frac{1}{2}\sqrt{-\gamma}(\partial\Omega)^2\right) = \sqrt{-\gamma}\left(-\frac{1}{2}\partial_\mu\Omega\partial_\nu\Omega + \frac{1}{4}\gamma_{\mu\nu}(\partial\Omega)^2\right)\delta\gamma^{\mu\nu}. \tag{47}$$

A.3 Variation of the Cosmological Constant Term

The Ω^4 term contributes:

$$\delta\left(-A\sqrt{-\gamma}\Omega^4\right) = -\frac{A}{2}\sqrt{-\gamma}\Omega^4\gamma_{\mu\nu}\delta\gamma^{\mu\nu}. \tag{48}$$

A.4 Combining Terms

Grouping all contributions and dividing by $\sqrt{-\gamma}/2\kappa$, we obtain:

$$\begin{aligned}\mathcal{H}_{\mu\nu}(\Omega) = -2\Omega^{-1}\nabla_\mu\nabla_\nu\Omega + 4\Omega^{-2}\nabla_\mu\Omega\nabla_\nu\Omega\\ + \gamma_{\mu\nu}\left(2\Omega^{-1}\Box\Omega - 2\Omega^{-2}(\nabla\Omega)^2 - \frac{A}{2}\Omega^4\right).\end{aligned} \tag{49}$$

A.5 Physical Interpretation

The terms in $\mathcal{H}_{\mu\nu}(\Omega)$ modify gravity on large scales:

- $-2\Omega^{-1}\nabla_\mu\nabla_\nu\Omega$: Tidal forces from $\Omega(x)$ gradients.
- $4\Omega^{-2}\nabla_\mu\Omega\nabla_\nu\Omega$: Effective stress-energy from $\Omega(x)$'s kinetic energy.
- $\gamma_{\mu\nu}(2\Omega^{-1}\Box\Omega - \cdots)$: Isotropic pressure-like terms.

B Appendix B: Effective Potential for Galactic Dynamics

B.1 Modified Poisson Equation in CERM

In the Conformal Emergent Reality Model (CERM), the physical metric $g_{\mu\nu}$ is related to the conformal metric $\gamma_{\mu\nu}$ via the scalar field $\Omega(x)$:

$$g_{\mu\nu} = \Omega^2(x)\gamma_{\mu\nu}. \tag{1}$$

This conformal scaling modifies the gravitational potential, leading to a modified Poisson equation that governs galactic dynamics.

In the weak-field limit, assuming spherical symmetry and $\gamma_{\mu\nu} \approx \eta_{\mu\nu}$, the effective gravitational potential Φ_{eff} satisfies:

$$\nabla^2\Phi_{\text{eff}} = 4\pi G\rho_{\text{vis}} + \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{\partial_r\Omega}{\Omega}\right), \tag{50}$$

where:

- ρ_{vis}: The visible matter density,
- $\Omega(x)$: The conformal scalar field governing physical scales.

The second term on the right-hand side arises from gradients of $\Omega(x)$, which contribute to the effective gravitational potential. This term naturally explains observed galactic rotation curves without requiring additional dark matter.

B.1.1 Physical Implications

The inclusion of the $\Omega(x)$-dependent term has several important consequences:

1. **Flat Rotation Curves**: The modified potential leads to approximately constant rotational velocities at large radii, consistent with observations.

2. **No Dark Matter Hypothesis**: The effects attributed to dark matter in standard cosmology are explained by the behavior of $\Omega(x)$ at galactic scales.

3. **Scale-Free Dynamics**: The logarithmic dependence of Φ_{eff} on $\Omega(x)$ ensures that the model remains consistent with scale-invariant properties of conformal geometry.

B.1.2 Detailed Explanation of Terms

- $\nabla^2\Phi_{\text{eff}}$: Represents the Laplacian of the effective gravitational potential, encoding contributions from both visible matter and $\Omega(x)$ gradients.
- ρ_{vis}: The density of visible matter, which dominates in the inner regions of galaxies.
- $\frac{\partial_r \Omega}{\Omega}$: Captures the radial variation of the conformal factor $\Omega(x)$, reflecting its dynamical role in shaping gravitational potentials.

B.2 Power-Law Solution for $\Omega(r)$

To solve the modified Poisson equation, we assume a power-law form for $\Omega(r)$:

$$\Omega(r) = \Omega_0 \left(\frac{r}{r_0}\right)^{\alpha}, \tag{51}$$

where:

- Ω_0: A normalization constant,
- r_0: A reference radius,
- α: A dimensionless parameter governing the radial scaling of $\Omega(r)$.

Substituting this ansatz into the modified Poisson equation, the contribution from $\Omega(x)$ becomes:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{\partial_r \Omega}{\Omega}\right) = \frac{\alpha}{r^2}. \tag{52}$$

Integrating twice, the effective potential is given by:

$$\Phi_{\text{eff}}(r) = -\frac{GM(r)}{r} + \frac{\alpha}{2}\ln\left(\frac{r}{r_0}\right), \tag{53}$$

where $M(r)$ is the enclosed mass within radius r.

B.3 Circular Velocity

The circular velocity of stars in a galaxy is derived from the effective potential:

$$v^2(r) = r\frac{d\Phi_{\text{eff}}}{dr} = \frac{GM(r)}{r} + \frac{\alpha}{2}. \tag{54}$$

For $\alpha > \frac{2}{3}$, the second term dominates at large radii, producing flat rotation curves:

$$v_{\text{CERM}}(r) \approx \sqrt{\frac{\alpha}{2}}. \tag{55}$$

This result aligns with observed galactic dynamics, eliminating the need for dark matter. Actual α value is to be constrained using real observational data sets.

B.4 Scaling Relations in the Conformal Emergent Reality Model (CERM)

B.4.1 Scaling of Newton's Constant $G \propto \Omega^{-2}$

The gravitational action in CERM includes a term proportional to $\Omega^2 R(\gamma)$, where $R(\gamma)$ is the Ricci scalar of the conformal metric $\gamma_{\mu\nu}$:

$$S_{\text{grav}} = \int d^4x\sqrt{-\gamma}\left[\frac{\Omega^2}{2\kappa}R(\gamma) + \dots\right]. \tag{56}$$

In General Relativity (GR), the Einstein-Hilbert term is $\frac{1}{16\pi G}R(g)$. Matching the coefficients implies:

$$\frac{1}{16\pi G} \sim \frac{\Omega^2}{2\kappa} \implies G \propto \Omega^{-2}. \tag{57}$$

B.4.2 Scaling of Mass $M \propto \Omega^{-1}$

The matter density ρ_{matter} couples to the physical metric $g_{\mu\nu} = \Omega^2\gamma_{\mu\nu}$. For consistency,

$$\rho_{\text{matter}} \propto \Omega^{-4}, \tag{58}$$

since $\sqrt{-g} \propto \Omega^4$, and $\rho\sqrt{-g}$ must remain invariant. Mass as an integrated density scales as:

$$M = \int \rho_{\text{matter}}\, dV \propto \Omega^{-4} \cdot \Omega^3 \cdot r^3 \propto \Omega^{-1} r^3. \tag{59}$$

B.4.3 Scaling of Radius $r \propto \Omega^2$

Distances in the physical metric $g_{\mu\nu}$ scale as:

$$ds^2 = \Omega^2 \gamma_{\mu\nu} dx^\mu dx^\nu \implies r_{\text{phys}} = \Omega^2 r_{\text{conf}}, \tag{60}$$

where r_{conf} is the radius in the conformal metric $\gamma_{\mu\nu}$. Thus, $r \propto \Omega^2$.

B.4.4 Scaling Argument

For consistency with flat rotation curves ($v^2 = \frac{GM}{r} + \frac{\alpha}{2}$), the mass scaling is:

$$M(r) \propto \Omega^{-1} r^3. \tag{61}$$

Then,

$$\frac{GM}{r} \propto \frac{\Omega^{-2} \cdot \Omega^{-1} r^3}{r} = \Omega^{-3} r^2. \tag{62}$$

For $\Omega(r) \propto r^\alpha$, this becomes:

$$\frac{GM}{r} \propto r^{2-3\alpha}. \tag{63}$$

To ensure the $\alpha/2$ term dominates at large r, we require:

$$2 - 3\alpha < 0 \implies \alpha > \frac{2}{3}. \tag{64}$$

B.4.5 Summary

A rigorous derivation must account for:

1. **Density Scaling**: $\rho_{\text{matter}} \propto \Omega^{-4}$.
2. **Volume Scaling**: $dV \propto \Omega^3 r^3$.
3. **Consistency Condition**: $\alpha > \frac{2}{3}$ for flat rotation curves.

This ensures CERM's predictions align with both conformal geometry and galactic observations.

B.5 Observational Test

To test the predictions of CERM for galactic dynamics, we propose the following observational strategies:

B.5.1 Fit α Using Galactic Rotation Curves

The parameter α can be constrained using datasets like SPARC (Spitzer Photometry and Accurate Rotation Curves). Specifically:

- Determine α to explain flat rotation curves in spiral galaxies.
- Compare predictions with low-surface-brightness (LSB) galaxies, where baryonic effects are minimal.
- If $\alpha \leq \frac{2}{3}$, alternative scalings for $M(r)$ or $\Omega(r)$ may be necessary. Observational tests using galaxy rotation curves, LSB galaxies, and hydrodynamical simulations will help refine the value of α and validate CERM's predictions.

B.5.2 Disentangle α from Baryonic Feedback

To validate the role of $\Omega(x)$ in galactic dynamics, consider the following approaches:

1. **Compare Different Galaxy Types**: LSB and dwarf spheroidal galaxies provide cleaner gravitational signals due to weaker baryonic feedback.
2. **Hydrodynamical Simulations**: Run simulations with and without baryonic feedback to disentangle conformal effects.
3. **Test Against Milky Way Data**: Use the well-measured rotation curve of the Milky Way as an independent test.

If α remains robust after accounting for baryonic effects, it strengthens the argument that CERM explains galactic dynamics without requiring dark matter.

C Appendix C: Transfer of $\delta\Omega$ to Matter Perturbations

C.1 Primordial Perturbations

In the Conformal Emergent Reality Model (CERM), the scalar field $\Omega(x)$ governs physical scales and dynamically influences spacetime geometry. To understand how perturbations in $\Omega(x)$ propagate into matter density fluctuations, we begin by expanding $\Omega(x)$ around a homogeneous background value $\Omega_0(t)$:

$$\Omega(x) = \Omega_0(t)[1 + \delta(x)], \tag{65}$$

where $\delta(x)$ represents the fractional perturbation in $\Omega(x)$, defined as:

$$\delta(x) = \frac{\delta\Omega(x)}{\Omega_0}. \tag{66}$$

This decomposition separates the large-scale evolution of $\Omega_0(t)$ from small-scale spatial variations encoded in $\delta(x)$. Substituting this expansion into the physical metric $g_{\mu\nu} = \Omega^2(x)\gamma_{\mu\nu}$, we obtain:

$$g_{\mu\nu} = \Omega_0^2(t)\gamma_{\mu\nu}[1 + 2\delta(x)]. \tag{67}$$

Here, the factor $[1 + 2\delta(x)]$ encodes the influence of $\delta(x)$ on the geometry of spacetime. This relationship highlights how perturbations in $\Omega(x)$ directly affect the physical metric and, consequently, the behavior of matter fields.

C.2 Density Contrast

The perturbation $\delta(x)$ induces corresponding fluctuations in the matter density. Assuming that matter is minimally coupled to the physical metric $g_{\mu\nu}$, the energy-momentum tensor for a perfect fluid takes the form:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \tag{68}$$

where ρ is the energy density, p is the pressure, and u_μ is the fluid's four-velocity. Expanding ρ and p around their background values ρ_0 and p_0, we write:

$$\rho(x) = \rho_0(t)[1 + \delta_\rho(x)], \quad p(x) = p_0(t)[1 + \delta_p(x)]. \tag{69}$$

The density contrast $\delta_\rho(x)$ is related to $\delta(x)$ through the conformal scaling of $\Omega(x)$. Specifically, the variation in the physical volume element $\sqrt{-g}$ introduces a direct coupling between $\delta(x)$ and $\delta_\rho(x)$. Using the relation:

$$\sqrt{-g} = \Omega^4\sqrt{-\gamma}, \tag{70}$$

we find that the fractional change in the physical volume is proportional to $4\delta(x)$. Consequently, the density contrast satisfies:

$$\frac{\delta\rho}{\rho} = -2\delta(x). \tag{71}$$

This result shows that perturbations in $\Omega(x)$ are directly imprinted onto the matter density distribution. The factor of -2 arises because an increase in $\Omega(x)$ expands the physical volume, thereby diluting the matter density.

C.3 Power Spectrum

To quantify the statistical properties of $\delta(x)$, we treat it as a free scalar field propagating on the conformal background $(M, \gamma_{\mu\nu})$. The action for $\delta(x)$ is derived from the gravitational action in CERM (see Appendix A):

$$S[\delta] = \int d^4x\sqrt{-\gamma}\left[\frac{1}{2}\gamma^{\mu\nu}\partial_\mu\delta\partial_\nu\delta - V(\delta)\right], \tag{72}$$

where $V(\delta)$ is an effective potential governing the dynamics of $\delta(x)$. Assuming $V(\delta)$ is quadratic, the linearized equation of motion is:

$$\Box_\gamma\delta + m_\delta^2\delta = 0, \tag{73}$$

where $\Box_\gamma$ is the d'Alembertian operator on the conformal manifold $(M, \gamma_{\mu\nu})$, and m_δ is an effective mass term.

Quantizing $\delta(x)$, we expand it in Fourier modes:

$$\delta(x) = \int \frac{d^3k}{(2\pi)^3}\left[a_k e^{i\vec{k}\cdot\vec{x}}\delta_k(t) + \text{h.c.}\right], \tag{74}$$

where a_k and $a_k^\dagger$ are creation and annihilation operators, and $\delta_k(t)$ satisfies:

$$\ddot{\delta}_k + 3H\dot{\delta}_k + \left(\frac{k^2}{a^2} + m_\delta^2\right)\delta_k = 0. \tag{75}$$

Here, $H = \dot{a}/a$ is the Hubble parameter, and k is the comoving wavenumber. Solving this equation during inflation yields the power spectrum:

$$P_\delta(k) = \frac{k^3}{2\pi^2}|\delta_k|^2. \tag{76}$$

For slow-roll evolution of $\Omega_0(t)$, the spectrum takes the nearly scale-invariant form:

$$P_\delta(k) = \left.\frac{H^2}{4\pi^2\Omega_0^2}\right|_{k=aH}. \tag{77}$$

C.3 Transfer Function

Primordial fluctuations evolve through different cosmological epochs. To account for this evolution, we introduce the transfer function $T(k)$, which relates the primordial spectrum $P_\delta(k)$ to the late-time matter power spectrum $P_m(k)$:

$$P_m(k) = P_\delta(k)T^2(k). \tag{78}$$

The transfer function $T(k)$ encodes the effects of gravitational collapse and baryon-photon interactions. For modes entering the horizon during matter domination, $T(k)$ can be approximated analytically:

$$T(k) = \frac{\ln(1 + 2.34q)}{2.34q}\left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}, \tag{79}$$

where:

$$q = \frac{k}{k_{\text{eq}}}, \quad k_{\text{eq}} \approx 0.01\,\text{Mpc}^{-1}. \tag{80}$$

D Appendix D: Theoretical Gap: Spectral Tilt $n_s \approx 0.965$

D.1 Current Status

The spectral tilt $n_s \approx 0.965$ is adopted from Planck CMB data but not derived from CERM's dynamics. This is a placeholder assumption pending further theoretical work.

D.2 Missing Ingredients

To predict n_s, CERM must:

1. **Specify a Potential** $V(\Omega)$: Example: $V(\Omega) = \lambda\Omega^4$, leading to slow-roll evolution.

2. **Solve Early-Universe Dynamics**: Derive $\Omega(t)$ during a "conformal inflation" phase with $\ddot{\Omega} \approx 0$.

3. **Define Conformal Slow-Roll Parameters**: Introduce analogs of inflation's ϵ and η:

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\ddot{\Omega}}{H\dot{\Omega}}. \tag{81}$$

4. **Link to** n_s: For single-field inflation, $n_s - 1 = -2\epsilon - \eta$. CERM requires a similar relation.

D.3 Challenges

- **Boundary Conditions**: Penrose's CCC imposes constraints at conformal infinity, complicating initial conditions.

- **Quantization**: Quantizing $\Omega(x)$ in a conformally invariant way remains unresolved.

- **Observational Validation**: Predict B-mode polarization in the CMB and compare with Planck/BICEP data.

D.4 Proposed Work

- Numerical simulations of $\Omega(t)$ for test potentials.
- Derive n_s and compare with Planck's $n_s = 0.9649 \pm 0.0042$.
- Predict the tensor-to-scalar ratio r, currently undefined in CERM.

E Appendix E: CERM, QFD, and QED – A Unified Conformal Framework

E.1 Quantum Field Dynamics in CERM

E.1.1 Path Integral Formalism

In CERM, quantum fields are quantized using a time-symmetric path integral defined over the conformal manifold $(M, \gamma_{\mu\nu})$:

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}, \quad S[\phi] = \int d^4x \sqrt{-\gamma} \, \mathcal{L}_{\text{QFD}}, \tag{82}$$

where $\mathcal{L}_{\text{QFD}}$ includes matter fields coupled to $g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}$. The absence of a preferred time direction in $\gamma_{\mu\nu}$ allows advanced and retarded solutions to coexist, preserving T-symmetry. Observed time asymmetry arises solely from boundary conditions on $\Omega(x)$.

E.1.2 Schwinger-Keldysh Formalism for Nonequilibrium Systems

For systems with explicit time evolution (e.g., cosmic inflation), the closed-time-path integral becomes:

$$Z = \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{i(S[\phi^+] - S[\phi^-])}, \tag{83}$$

where ϕ^+ and ϕ^- represent fields on forward/backward time contours. This formalism aligns with CERM's emergent arrow of time, as boundary conditions on $\Omega(x)$ enforce causal consistency.

E.2 Quantum Electrodynamics in CERM

E.2.1 Gauge Invariance and Propagators

Maxwell's equations are conformally invariant, allowing QED to retain its structure. The photon propagator in Lorenz gauge becomes:

$$D_F^{\mu\nu}(x - y) = \int \frac{d^4k}{(2\pi)^4} \frac{-i\gamma^{\mu\nu}}{k^2 + i\epsilon} e^{-ik\cdot(x-y)}, \tag{84}$$

where $\gamma^{\mu\nu}$ replaces $g^{\mu\nu}$. The propagator's advanced/retarded symmetry reflects CERM's T-symmetric geometry.

E.2.2 Mass Generation and the Higgs Mechanism

Fermion masses emerge via the Higgs field Φ, whose vacuum expectation value (vev) scales inversely with $\Omega(x)$:

$$m_\psi = y_\psi \frac{v}{\sqrt{2}} \propto \Omega^{-1}(x), \quad v \propto \Omega^{-1}(x). \tag{85}$$

where y_ψ is the Yukawa coupling constant.

This ensures conformal consistency, as dimensionful quantities derive from $\Omega(x)$. The Dirac equation remains T-symmetric, with mass terms dynamically tied to $\Omega(x)$.

E.3 Higgs Potential and Conformal Symmetry Breaking

The Higgs mechanism in CERM dynamically generates particle masses while preserving conformal invariance at the fundamental level. To achieve this, the Higgs potential must explicitly depend on the conformal factor $\Omega(x)$, ensuring that symmetry breaking scales inversely with $\Omega(x)$.

E.3.1 Conformal Higgs Potential

The Higgs field Φ is a scalar doublet with a potential:

$$V(\Phi) = \lambda \left(\Phi^\dagger \Phi - \frac{v_0^2}{\Omega^2(x)} \right)^2, \tag{86}$$

where:

- $\lambda > 0$: Self-coupling constant,
- v_0: Dimensionless constant fixing the hierarchy of symmetry breaking,
- $\Omega(x)$: Conformal scalar field.

E.3.2 Minimization and Vacuum Expectation Value (vev)

The potential is minimized when:

$$\Phi^\dagger \Phi = \frac{v_0^2}{\Omega^2(x)} \implies \langle \Phi \rangle = \frac{v_0}{\sqrt{2}\,\Omega(x)} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{87}$$

The physical Higgs vev is therefore:

$$v(x) = \frac{v_0}{\Omega(x)}. \tag{88}$$

This scaling ensures that particle masses $m \propto v(x) \propto \Omega^{-1}(x)$, as required for conformal consistency.

E.3.3 Higgs Lagrangian in CERM

The full Higgs Lagrangian, including kinetic and Yukawa terms, is:

$$\mathcal{L}_{\text{Higgs}} = \sqrt{-g}\left[(D_\mu\Phi)^\dagger(D^\mu\Phi) - \lambda\left(\Phi^\dagger\Phi - \frac{v_0^2}{\Omega^2}\right)^2 - y\bar{\psi}\Phi\psi\right], \tag{89}$$

where:

- $D_\mu = \partial_\mu + igA_\mu$: Gauge-covariant derivative,
- y: Yukawa coupling constant,
- ψ: Fermion field.

E.3.4 Conformal Invariance Check

Under a conformal transformation $\gamma_{\mu\nu} \to e^{2\phi(x)}\gamma_{\mu\nu}$, the fields scale as:

$$\Omega(x) \to e^{-\phi(x)}\Omega(x), \quad \Phi \to e^{\phi(x)}\Phi, \quad \psi \to e^{3\phi(x)/2}\psi. \tag{90}$$

Substituting these into the Higgs Lagrangian:

$$\mathcal{L}_{\text{Higgs}} \to e^{4\phi(x)}\sqrt{-\gamma}\,[\cdots] = \sqrt{-g}\,[\cdots], \tag{91}$$

confirming conformal invariance. The $\Omega(x)$-dependence in $V(\Phi)$ ensures the vev $v(x)$ scales correctly.

E.3.5 Observational Implications

1. **Time-Varying Fermion Masses:** As $\Omega(x)$ grows cosmologically, $m \propto \Omega^{-1}(x)$ decreases.
2. **High-Redshift Tests:** Spectral lines from ancient astrophysical objects (e.g., quasars) should exhibit redshift-dependent mass shifts.
3. **Electroweak Phase Transition:** The $\Omega(x)$-dependent vev modifies the dynamics of early-universe symmetry breaking.

F Appendix F: Spacetime Emergence and Information in CERM

F.1 The CERM Postulate

Spacetime emerges from a conformal manifold $(M, \gamma_{\mu\nu})$ coupled to a scalar field $\Omega(x)$:

$$g_{\mu\nu}(x) = \Omega^2(x)\gamma_{\mu\nu}(x). \tag{92}$$

The conformal manifold defines causal structure and angles, while $\Omega(x)$ dynamically sets physical scales (lengths, masses, energies).

F.2 Boundary Conditions and Cosmic Aeons

CERM adopts Penrose's Conformal Cyclic Cosmology (CCC), where aeons transition via conformal resets (See Appendix M):

1. **Aeon Transitions**: At $\Omega(x) \to \infty$, matter dissolves into a conformally invariant state.
2. **Entropy Reset**: Phase space volume collapses $\left(S \propto \int \Omega^3(x)\rho \ln \rho \, d^3x \to 0\right)$.
3. **Weyl Curvature Suppression**: $C_{\mu\nu\rho\sigma}[\gamma] \to 0$ ensures a low-entropy initial state.

F.3 Information Content of the Conformal Manifold

The conformal manifold encodes three types of information:

F.3.1 Causal and Geometric Structure

- **Causal Relationships**: Light cones and time-like trajectories derive from $\gamma_{\mu\nu}$.
- **Conformal Invariants**: Angles and scale-free ratios are preserved under $\gamma_{\mu\nu} \to e^{2\phi}\gamma_{\mu\nu}$.

F.3.2 Entropy Dynamics

- **Phase Space Expansion**: Entropy growth is tied to $\Omega^3(x)$ scaling:

$$S \propto \int d^3x\, \Omega^3(x)\rho(x) \ln \rho(x). \tag{93}$$

- **Arrow of Time**: Boundary conditions on $\Omega(x)$ at conformal infinity enforce entropy asymmetry.

F.3.3 Quantum Correlations

- **Nonlocal Entanglement**: The Schwinger-Keldysh propagator $G_{\mathrm{SK}}(x, y; \gamma, \Omega)$ on $\gamma_{\mu\nu}$ governs time-symmetric correlations (e.g., DCQE).
- **Retrocausality Suppression**: Advanced propagators G_{adv} are suppressed by $\Omega(x)$ boundary conditions.

F.4 Entanglement and the Conformal Manifold

F.4.1 Quantum Correlations as Geometric Properties of $(M, \gamma_{\mu\nu})$

In CERM, entanglement is not merely a quantum phenomenon but a geometric property of the classical conformal manifold $(M, \gamma_{\mu\nu})$, mediated by the scalar field $\Omega(x)$. This section formalizes how entanglement arises from the manifold's causal structure and scaling dynamics.

F.4.2 Entangled States on $(M, \gamma_{\mu\nu})$

For a bipartite quantum system (e.g., particles A and B), the entangled state is defined relative to regions of $\gamma_{\mu\nu}$:

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left(|A\rangle_U |B\rangle_V + |A'\rangle_{U'} |B'\rangle_{V'} \right), \tag{94}$$

where U, V are causally connected regions of $\gamma_{\mu\nu}$, and $|A\rangle_U$ denotes the state of field $\phi(x)$ localized in U. Entanglement reflects the connectivity of $\gamma_{\mu\nu}$, with $\Omega(x)$ modulating the *strength* of correlations.

F.4.3 Entanglement from Conformal Propagators

The Schwinger-Keldysh propagator on $\gamma_{\mu\nu}$ governs correlations:

$$G_{\mathrm{SK}}(x, y; \Omega) = \int \mathcal{D}\phi\, e^{iS[\phi,\gamma,\Omega]}\phi(x)\phi(y), \tag{95}$$

where

$$S[\phi, \gamma, \Omega] = \int d^4x\sqrt{-\gamma}\left(\frac{1}{2}(\partial\phi)^2 + \lambda\phi^4\right). \tag{96}$$

Entanglement entropy between regions U and V is:

$$S_{\mathrm{ent}} \propto \int_{\partial U} \sqrt{\gamma}\, d^3x\, \Omega^3(x) \ln \rho(x), \tag{97}$$

linking entropy to the conformal scaling of phase space.

F.4.4 Measurement and Conformal Scaling

Measurement devices act as conformal scaling operators $\hat{\Omega}(x)$, collapsing entanglement into observable correlations:

$$\hat{\Omega}(x)|A\rangle_U = \Omega(x)|A\rangle_{\Omega^2\gamma}. \tag{98}$$

This local scaling modifies $g_{\mu\nu} = \Omega^2\gamma_{\mu\nu}$, imprinting entanglement onto the emergent spacetime.

F.4.5 Key Results

1. **Entanglement as Geometry**: Correlations are dictated by $\gamma_{\mu\nu}$'s causal connectivity, not intrinsic quantum nonlocality.
2. **Role of** $\Omega(x)$: Determines the *scale* of entanglement via phase space expansion ($\propto \Omega^3$).
3. **Retrocausality Suppressed**: Boundary conditions on $\Omega(x)$ enforce causality in $g_{\mu\nu}$, aligning with the arrow of time.

F.4.6 Experimental Signature

- **Conformal Invariance**: Detection probabilities reduce to standard quantum mechanics when $\Omega(x)$ is constant:

$$P(A, B) = |\langle A| \otimes \langle B|\Psi\rangle|^2 = \frac{1}{2}. \tag{99}$$

- **Deviations in Curved Spacetime**: Varying $\Omega(x)$ introduces $\gamma_{\mu\nu}$-dependent corrections to $P(A, B)$.

F.5 Quantum States on a Fixed Conformal Manifold

Quantum states in CERM are functionals of matter fields $\phi(x)$ propagating on the classical conformal manifold $(M, \gamma_{\mu\nu})$:

F.5.1 State Representation

A quantum state is a functional:

$$\Psi[\phi, \gamma] = \int \mathcal{D}\phi \, e^{iS[\phi,\gamma,\Omega]}\Phi[\phi], \tag{100}$$

where $\Phi[\phi]$ encodes initial/final conditions. The scalar field $\Omega(x)$ dynamically scales physical observables (e.g., $\sqrt{-g} = \Omega^4\sqrt{-\gamma}$).

F.5.2 Conformal Invariance

Under a conformal transformation $\gamma_{\mu\nu} \to e^{2\alpha(x)}\gamma_{\mu\nu}$, states transform as:

$$\Psi[\phi, \gamma] \to \Psi[\phi', \gamma'], \quad \phi'(x) = e^{-\alpha(x)}\phi(x). \tag{101}$$

This ensures probabilities $|\Psi[\phi, \gamma]|^2$ remain invariant.

F.5.3 Observables

Measurement outcomes depend only on $\Omega(x)$-scaled quantities:

$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi \, \mathcal{O}[\phi, \Omega]|\Psi[\phi, \gamma]|^2}{\int \mathcal{D}\phi \, |\Psi[\phi, \gamma]|^2}. \tag{102}$$

F.5.4 Key Result

Quantum states are fully separable from the classical conformal manifold. Non-locality and entanglement arise solely from field correlations on $\gamma_{\mu\nu}$.

F.6 Implications for Quantum Gravity

- **Background Independence**: $\gamma_{\mu\nu}$ remains classical; quantum fluctuations reside in $\Omega(x)$ and matter fields.
- **Holographic Potential**: $\gamma_{\mu\nu}$ may holographically encode $g_{\mu\nu}$ data, akin to AdS/CFT (see **Appendix I** and **Appendix O**).

G Appendix G: Weyl Curvature and Entropy in CERM

G.1 Weyl Curvature Hypothesis

The **Weyl Curvature Hypothesis (WCH)**, proposed by Roger Penrose, addresses one of the most profound puzzles in cosmology: Why did the universe begin in an extraordinarily low-entropy state? The hypothesis states that the initial singularity of the universe had vanishing Weyl curvature $C_{\mu\nu\rho\sigma}$, corresponding to a highly ordered, low-entropy configuration. This contrasts with the high Weyl curvature observed in black holes, which are associated with maximal entropy.

In General Relativity (GR), the Riemann curvature tensor decomposes into two parts:

$$R_{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} + \frac{1}{2}\left(g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}\right) - \frac{1}{6}Rg_{\mu[\rho}g_{\sigma]\nu}, \tag{103}$$

where:

- $C_{\mu\nu\rho\sigma}$: The **Weyl tensor**, encoding tidal forces and gravitational waves,
- $R_{\mu\nu}$: The Ricci tensor, describing matter-energy contributions via Einstein's field equations,
- R: The Ricci scalar, representing the trace of the Ricci tensor.

At the Big Bang, the Ricci tensor $R_{\mu\nu}$ dominates due to the presence of matter and energy, while the Weyl tensor $C_{\mu\nu\rho\sigma}$ is hypothesized to vanish. This ensures that the universe starts in a smooth, homogeneous, and isotropic state, consistent with the observed cosmic microwave background (CMB).

G.1.1 Link to CERM

The Conformal Emergent Reality Model (CERM) provides a geometric mechanism for realizing the WCH. In CERM, spacetime emerges dynamically from a conformal manifold $(M, \gamma_{\mu\nu})$ and a scalar field $\Omega(x)$, which governs physical scales. The physical metric is given by:

$$g_{\mu\nu}(x) = \Omega^2(x)\gamma_{\mu\nu}(x). \tag{2}$$

At the Big Bang ($t \to 0$), $\Omega(x) \to \infty$, which smooths out the conformal metric $\gamma_{\mu\nu}$. This smoothing suppresses the Weyl curvature $C_{\mu\nu\rho\sigma}[\gamma]$, ensuring that the universe begins in a low-entropy, scale-invariant state. This aligns perfectly with Penrose's WCH, as the vanishing of $C_{\mu\nu\rho\sigma}$ corresponds to minimal gravitational degrees of freedom and maximal order.

G.1.2 Entropy Growth in CERM

Entropy growth in CERM is directly tied to the evolution of $\Omega(x)$. The geometric interpretation of entropy is:

$$S \propto \int d^3x \, \Omega^3(x) \ln \rho(x), \tag{104}$$

where:

- $\Omega^3(x)$: Represents the expansion of phase space volume due to $\Omega(x)$,
- $\rho(x)$: Denotes the matter density distribution.

As $\Omega(x)$ increases over time, the term $\Omega^3(x)$ ensures that the phase space volume—and hence the entropy—increases monotonically. This provides a natural explanation for the Second Law of Thermodynamics without requiring a special low-entropy initial condition.

G.1.3 Explanation of Terms in equation

- $\Omega^3(x)$: The cubic scaling reflects how $\Omega(x)$ expands spatial volumes, driving entropy upward.
- $\ln \rho(x)$: Quantifies the information content or disorder associated with the matter density distribution.
- **Phase Space Volume**: In statistical mechanics, entropy S is related to the accessible phase space volume $\mathcal{V}$:

  $$S \sim \ln \mathcal{V}.$$

  In CERM, the phase space volume expands dynamically due to the evolution of $\Omega(x)$.

G.1.4 Connection to Cosmic Evolution

The alignment between entropy growth and cosmic expansion arises naturally in CERM. The Friedmann-like equation governing the evolution of $\Omega(x)$ includes a term analogous to the Hubble parameter:

$$\frac{\dot{\Omega}}{\Omega} = H_{\text{eff}}, \tag{105}$$

where H_{eff} represents an effective expansion rate. This equation shows that $\Omega(x)$ grows as the universe expands, driving both entropy increase and cosmic acceleration.

G.2 Observational Test

If the Weyl curvature $C_{\mu\nu\rho\sigma}[\gamma]$ is suppressed at early times, this should leave observable imprints on the cosmic microwave background (CMB). Specifically, primordial gravitational waves (GWs) generated during inflation inherit a spectral tilt from the dynamics of $\Omega(x)$. The power spectrum of tensor perturbations is given by:

$$P_T(k) = \left.\frac{H^2}{\pi^2 \Omega_0^2}\right|_{k=aH}, \tag{106}$$

where:

- H: The inflationary Hubble parameter,
- Ω_0: The value of $\Omega(x)$ during inflation.

A tilt $n_t \neq 0$ distinguishes CERM from GR and links to CMB B-mode polarization experiments. Observational signatures include:

1. **Suppressed Primordial B-Modes**: If $C_{\mu\nu\rho\sigma}[\gamma]$ is initially small, the amplitude of primordial B-modes will be weaker than predicted by GR.

2. **Low-ℓ CMB Anomalies**: Residuals in the CMB temperature anisotropy spectrum at large angular scales ($\ell < 30$) may reflect $\Omega(x)$ fluctuations:

$$\frac{\delta T}{T} \approx \frac{1}{3}(\Phi_{\text{GR}} + \delta \ln \Omega), \tag{107}$$

where Φ_{GR} is the gravitational potential in GR and $\delta \ln \Omega$ represents fluctuations in $\Omega(x)$.

G.2.1 Comparison with Observations

To test these predictions:

1. Compare CMB B-mode polarization data from experiments like BICEP/Keck with GR predictions.

2. Fit $\delta \ln \Omega \sim 10^{-5}$ residuals in Planck's CMB power spectrum at low multipoles ($\ell < 30$).

G.3 Implications for CERM

The suppression of Weyl curvature at early times provides strong support for CERM's geometric framework. By linking entropy growth to the evolution of $\Omega(x)$, CERM avoids the need for fine-tuned initial conditions and offers a unified explanation for:

1. The low-entropy Big Bang,

2. The arrow of time,

3. Cosmic expansion and entropy increase.

This approach also aligns with Penrose's Conformal Cyclic Cosmology (CCC), where each cosmic cycle resets the universe's conformal structure, ensuring a smooth transition between aeons.

H Appendix H: Strong Force and Conformal Scaling

H.1 QCD in CERM

The strong interaction, described by Quantum Chromodynamics (QCD), plays a central role in particle physics. In the Conformal Emergent Reality Model (CERM), the conformal scaling $\Omega(x)$ modifies the dynamics of the strong force, leading to testable predictions for proton masses, quark confinement, and isotopic ratios over cosmic time.

H.1.1 Modified QCD Lagrangian

The standard QCD Lagrangian is given by:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - m_q)q, \tag{108}$$

where:

- $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$: The gluon field strength tensor,
- A^a_μ: The gluon gauge field,
- g_s: The strong coupling constant,
- f^{abc}: The structure constants of the SU(3) gauge group,
- $\bar{q}$ and q: The quark fields,
- m_q: The quark masses.

In CERM, the physical metric $g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}$ introduces a conformal dependence into the QCD Lagrangian. The modified Lagrangian becomes:

$$\mathcal{L}_{\text{CERM-QCD}} = \sqrt{-g} \left[-\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - m_q \Omega^{-1})q \right], \tag{109}$$

where:

- $\sqrt{-g}$: Ensures the Lagrangian transforms correctly under conformal rescaling,
- $m_q \propto \Omega^{-1}$: Quark masses scale inversely with $\Omega(x)$, reflecting the emergent nature of mass in CERM.

H.1.2 Confinement Scale and Proton Masses

The QCD confinement scale Λ_{QCD} determines the energy at which quarks and gluons are confined into hadrons. In CERM, this scale depends on $\Omega(x)$ as:

$$\Lambda_{\text{QCD}} \propto \Omega^{-1}. \tag{110}$$

This scaling implies that Λ_{QCD} evolves over cosmic time, altering the masses of protons and other hadrons. Since proton masses dominate baryonic matter, this effect could leave observable imprints in ancient astrophysical systems.

H.1.3 Physical Implications

The conformal scaling of Λ_{QCD} has several important consequences:

1. **Time-Varying Proton Masses**: As $\Omega(x)$ increases, proton masses decrease. This could be detected through isotopic ratio measurements in ancient meteorites.

2. **Nuclear Binding Energies**: Changes in Λ_{QCD} affect nuclear binding energies, potentially altering the abundance of light elements in early galaxies.

3. **Cosmic Evolution of Hadrons**: The evolution of $\Omega(x)$ provides a mechanism for studying how strong interactions behave over cosmological timescales.

H.2 Observational Test

To test the predictions of CERM for the strong force, we propose the following observational strategies:

H.2.1 Isotopic Ratios in Ancient Meteorites

Measure isotopic ratios in ancient meteorites, such as $^{187}\text{Re}/^{187}\text{Os}$, to probe variations in Λ_{QCD}. These isotopes are sensitive to changes in nuclear binding energies, which depend on Λ_{QCD}. If $\Lambda_{\text{QCD}} \propto \Omega^{-1}$, then isotopic ratios should exhibit a systematic shift over cosmic time.

H.2.2 Proton Mass Evolution

Search for evidence of time-varying proton masses using high-redshift astrophysical systems. For example:

- Compare spectral lines from distant quasars to detect shifts in atomic transitions caused by changing proton masses.
- Analyze the fine-structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}$, which depends on Λ_{QCD}, to constrain $\Omega(x)$ evolution.

H.2.3 Implications for Stellar Nucleosynthesis

Changes in Λ_{QCD} affect stellar nucleosynthesis processes, particularly the production of light elements like helium and lithium. Observations of primordial element abundances can provide indirect evidence for $\Omega(x)$-dependent scaling.

H.3 Summary

- The strong force in CERM is modified by the conformal scaling $\Omega(x)$, leading to a time-dependent confinement scale $\Lambda_{\text{QCD}} \propto \Omega^{-1}$.
- Quark masses and proton masses scale inversely with $\Omega(x)$, providing a mechanism for studying cosmic evolution of hadronic matter.
- Observational tests include isotopic ratio measurements in ancient meteorites, spectral line analysis in high-redshift systems, and constraints from primordial nucleosynthesis.

By linking the strong force to the conformal structure of spacetime, CERM offers a novel framework for understanding the interplay between particle physics and cosmology.

I Appendix I: Quantum Gravity and Renormalization

I.1 I.1 Effective Quantum Gravity in CERM

The Conformal Emergent Reality Model (CERM) provides a novel approach to quantum gravity by avoiding the direct quantization of spacetime. Instead, the conformal manifold $(M, \gamma_{\mu\nu})$ is treated as a classical background, while quantum fluctuations reside in the scalar field $\Omega(x)$ and matter fields. This sidesteps many of the renormalization issues associated with perturbative quantum gravity.

I.1.1 Effective Action for Quantum Gravity

The effective action in CERM includes contributions from the Einstein-Hilbert term, the Standard Model Lagrangian, and one-loop corrections dependent on $\Omega(x)$. The total effective action is given by:

$$\Gamma_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \mathcal{L}_{\text{SM}} \right] + \Gamma_{\text{1-loop}}[\Omega], \tag{111}$$

where:

- R: The Ricci scalar of the physical metric $g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}$,
- G: Newton's gravitational constant,
- $\mathcal{L}_{\text{SM}}$: The Standard Model Lagrangian coupled to $g_{\mu\nu}$,
- $\Gamma_{\text{1-loop}}[\Omega]$: One-loop corrections that include $\Omega(x)$-dependent counterterms.

The one-loop corrections arise from quantum fluctuations of matter fields and $\Omega(x)$ itself. These terms ensure that divergences cancel if $\Omega(x)$ evolves adiabatically over cosmic timescales.

I.1.2 Physical Implications

Treating $\gamma_{\mu\nu}$ as a classical background has several important consequences:

1. **Avoidance of Singularities**: By avoiding the direct quantization of spacetime, CERM circumvents the singularities that often arise in perturbative quantum gravity.

2. **Conformal Consistency**: The scaling of $\Omega(x)$ ensures that all dimensionful quantities (e.g., masses, couplings) are dynamically generated, preserving conformal invariance at the classical level.

3. **Renormalization of Counterterms**: The $\Omega(x)$-dependent counterterms in $\Gamma_{\text{1-loop}}[\Omega]$ absorb divergences, ensuring a finite and well-defined theory.

I.1.3 Detailed Explanation of Terms

- $\sqrt{-g}$: Ensures the action transforms correctly under general coordinate transformations.
- $R/16\pi G$: The Einstein-Hilbert term, which governs the dynamics of gravity.
- $\mathcal{L}_{\text{SM}}$: Encodes the interactions of Standard Model particles with the physical metric $g_{\mu\nu}$.
- $\Gamma_{\text{1-loop}}[\Omega]$: Includes loop corrections from quantum fluctuations of matter fields and $\Omega(x)$, ensuring renormalizability.

I.2 Observational Tests

The predictions of CERM for quantum gravity can be tested through observational signatures of quantum spacetime fluctuations. Key tests include:

I.2.1 Absence of Stochastic Gravitational Wave Background

In traditional quantum gravity theories, spacetime fluctuations generate a stochastic gravitational wave (GW) background. In CERM, the absence of such a background is a natural consequence of treating $\gamma_{\mu\nu}$ as a classical background. Observational searches for this background using pulsar timing arrays (e.g., NANOGrav) provide a critical test of CERM.

I.2.2 Constraints on $\Omega(x)$ Evolution

If $\Omega(x)$ evolves adiabatically, quantum fluctuations remain small, avoiding large-scale deviations from classical predictions. Observations of cosmic microwave background (CMB) anisotropies and large-scale structure can constrain the rate of $\Omega(x)$ evolution, testing the adiabatic assumption.

I.2.3 Implications for Black Hole Physics

CERM predicts modifications to black hole thermodynamics due to the conformal scaling of $\Omega(x)$. For example:

- The Hawking temperature of black holes scales as $T_H \propto \Omega^{-1}$,
- The Bekenstein-Hawking entropy scales as $S_{\mathrm{BH}} \propto \Omega^2$.

These predictions can be tested using observations of black hole mergers and their associated gravitational wave signals.

I.3 Summary

- CERM avoids quantizing spacetime directly by treating the conformal manifold $(M, \gamma_{\mu\nu})$ as a classical background.
- Quantum fluctuations reside in $\Omega(x)$ and matter fields, with renormalization ensured by $\Omega(x)$-dependent counterterms.
- Observational tests include searches for a stochastic gravitational wave background, constraints on $\Omega(x)$ evolution, and modifications to black hole thermodynamics.

By providing a consistent framework for quantum gravity without spacetime singularities, CERM offers a promising avenue for unifying quantum mechanics and general relativity.

J Appendix J: Neutrino Masses in CERM

J.1 Neutrino Mass Generation

In the Conformal Emergent Reality Model (CERM), neutrino masses arise dynamically due to the conformal scaling $\Omega(x)$. This provides a natural mechanism for understanding the origin of neutrino masses and their potential variation over cosmic time.

J.1.1 Dirac Neutrino Masses

For Dirac neutrinos, masses are generated through Yukawa couplings to the Higgs field Φ. The relevant Lagrangian term is:

$$\mathcal{L}_{\text{Dirac}} = y_\nu \bar{L} \Phi \nu_R + \text{h.c.}, \tag{112}$$

where:

- y_ν: The Yukawa coupling constant,
- $\bar{L}$: The left-handed lepton doublet,
- Φ: The Higgs field,
- ν_R: The right-handed neutrino field.

After electroweak symmetry breaking, the Higgs field acquires a vacuum expectation value (vev) v, leading to a Dirac neutrino mass:

$$m_\nu = y_\nu \frac{v}{\sqrt{2}}. \tag{113}$$

In CERM, the vev v scales inversely with $\Omega(x)$:

$$v \propto \Omega^{-1}(x). \tag{114}$$

Thus, the Dirac neutrino mass becomes:

$$m_\nu \propto \Omega^{-1}(x). \tag{115}$$

This implies that neutrino masses decrease as $\Omega(x)$ increases over cosmic time.

J.1.2 Majorana Neutrino Masses

For Majorana neutrinos, masses can arise via the seesaw mechanism. The relevant Lagrangian term is:

$$\mathcal{L}_{\text{Majorana}} = \frac{M_R}{2} \nu_R^T C \nu_R + \text{h.c.}, \tag{116}$$

where:

- M_R: The Majorana mass scale,
- C: The charge conjugation matrix.

The effective Majorana neutrino mass is given by:

$$m^{\nu}_{\text{Maj}} = \frac{v^2}{M_R}. \tag{117}$$

In CERM, since $v \propto \Omega^{-1}(x)$, the Majorana neutrino mass scales as:

$$m^{\nu}_{\text{Maj}} \propto \Omega^{-2}(x). \tag{118}$$

This stronger dependence on $\Omega(x)$ reflects the quadratic scaling of the seesaw mechanism.

J.1.3 Physical Implications

The conformal scaling of neutrino masses has several important consequences:

1. **Time-Varying Neutrino Masses**: As $\Omega(x)$ evolves, neutrino masses decrease over cosmic time. This could leave observable imprints in high-redshift astrophysical systems.

2. **Neutrino Oscillations**: The energy-dependent oscillation probabilities of neutrinos depend on their masses. Variations in $\Omega(x)$ could modify these probabilities over cosmological scales.

3. **Cosmic Evolution of Leptogenesis**: If neutrino masses vary with $\Omega(x)$, this could affect leptogenesis scenarios and the generation of baryon asymmetry.

J.2 Observational Test

To test the predictions of CERM for neutrino masses, we propose the following observational strategies:

J.2.1 High-Redshift Astrophysical Neutrinos

Measure the neutrino mass hierarchy using high-redshift astrophysical neutrinos detected by experiments like IceCube. In CERM, neutrino masses scale as:

$$m_\nu \propto (1+z), \tag{119}$$

where z is the redshift. This scaling provides a direct probe of $\Omega(x)$ evolution.

J.2.2 Cosmic Microwave Background (CMB)

Neutrino masses affect the CMB power spectrum through their contribution to the total energy density. Observations of the CMB anisotropy spectrum can constrain variations in m_ν over cosmic time.

J.2.3 Large-Scale Structure

Neutrino masses influence the growth of large-scale structure by suppressing matter perturbations on small scales. Comparing galaxy surveys (e.g., DESI, Euclid) with theoretical predictions can test the time evolution of m_ν.

J.3 Summary

- Neutrino masses in CERM arise dynamically due to the conformal scaling $\Omega(x)$.
- Dirac neutrino masses scale as $m_\nu \propto \Omega^{-1}(x)$, while Majorana neutrino masses scale as $m^\nu_{\mathrm{Maj}} \propto \Omega^{-2}(x)$.
- Observational tests include high-redshift neutrino measurements, CMB anisotropy analysis, and large-scale structure surveys.

By linking neutrino masses to the conformal structure of spacetime, CERM offers a novel framework for understanding the interplay between particle physics and cosmology.

K Appendix K: Baryogenesis via $\Omega(x)$ Dynamics

K.1 CP Violation and Sphalerons

One of the outstanding challenges in cosmology is explaining the observed matter-antimatter asymmetry in the universe. The Conformal Emergent Reality Model (CERM) provides a novel mechanism for baryogenesis by incorporating a CP-violating term in the potential of the scalar field $\Omega(x)$. This mechanism leverages phase transitions in $\Omega(x)$ to generate a lepton asymmetry, which is subsequently converted into a baryon asymmetry via sphaleron processes.

K.1.1 CP-Odd Term in the $\Omega(x)$ Potential

The potential for $\Omega(x)$ includes a CP-violating term:

$$V(\Omega) \supset \lambda\Omega^4 \sin\left(\frac{\theta}{\Omega}\right), \tag{120}$$

where:

- λ: A dimensionless coupling constant,
- θ: A phase parameter that introduces CP violation,
- $\Omega(x)$: The conformal scalar field governing physical scales.

This term breaks CP symmetry dynamically, providing the necessary conditions for generating an asymmetry between matter and antimatter.

K.1.2 Lepton Asymmetry Generation

During phase transitions in $\Omega(x)$, the CP-violating term generates a lepton asymmetry. The process occurs as follows:

1. **Phase Transition**: As $\Omega(x)$ evolves, it undergoes a phase transition, creating regions where the CP-violating term becomes significant.

2. **Bubble Nucleation**: Bubbles of the new vacuum phase form, with gradients in $\Omega(x)$ driving CP-violating effects at the bubble walls.

3. **Lepton Number Violation**: The CP-violating interactions at the bubble walls produce a net lepton number asymmetry.

The lepton asymmetry is quantified by:

$$\Delta L \propto \epsilon \cdot \Gamma_{\text{sph}}, \tag{121}$$

where:

- ϵ: The CP-violation parameter, determined by the phase θ,
- Γ_{sph}: The sphaleron transition rate, which converts lepton asymmetry into baryon asymmetry.

K.1.3 Baryon Asymmetry via Sphalerons

Sphaleron processes, which violate baryon and lepton number conservation but preserve $B - L$, convert the generated lepton asymmetry into a baryon asymmetry. The resulting baryon-to-photon ratio is given by:

$$\eta_B = \frac{n_B}{n_\gamma} \propto \Delta L, \tag{122}$$

where:

- n_B: The baryon number density,
- n_γ: The photon number density.

In CERM, the magnitude of η_B depends on the dynamics of $\Omega(x)$ during the phase transition, providing a direct link between baryogenesis and the conformal structure of spacetime.

K.2 Observational Test

To test the predictions of CERM for baryogenesis, we propose the following observational strategies:

K.2.1 Correlation Between Baryon Asymmetry and CMB Polarization

The baryon asymmetry η_B leaves an imprint on the cosmic microwave background (CMB) through its effect on primordial plasma dynamics. Specifically:

- The E-mode polarization of the CMB is sensitive to baryon density fluctuations.
- A correlation between η_B and E-mode polarization can be tested using future CMB experiments like LiteBIRD.

In CERM, the predicted relationship is:

$$\eta_B \propto \delta \ln \Omega, \tag{123}$$

where $\delta \ln \Omega$ represents fluctuations in $\Omega(x)$ during the early universe. Observations of η_B and CMB polarization can constrain this relationship.

K.2.2 Implications for Leptogenesis Models

If $\Omega(x)$-driven baryogenesis is correct, it provides an alternative to traditional leptogenesis models (e.g., those involving heavy Majorana neutrinos). Key differences include:

- The source of CP violation arises from the $\Omega(x)$ potential rather than Yukawa couplings,
- The timing of the asymmetry generation is tied to $\Omega(x)$ phase transitions, potentially occurring later than in standard scenarios.

K.2.3 Testing with High-Energy Colliders

Future high-energy colliders, such as the proposed FCC or CEPC, could probe the energy scales associated with $\Omega(x)$ phase transitions. Evidence of CP-violating interactions at these scales would support CERM's baryogenesis mechanism.

K.3 Summary

- CERM proposes a novel mechanism for baryogenesis via CP-violating dynamics in the $\Omega(x)$ potential.
- A lepton asymmetry is generated during $\Omega(x)$ phase transitions and converted into a baryon asymmetry via sphalerons.
- Observational tests include correlating baryon asymmetry η_B with CMB E-mode polarization and probing CP-violating interactions at high-energy colliders.

By linking baryogenesis to the conformal structure of spacetime, CERM offers a unified framework for understanding the origin of matter in the universe.

L Appendix L: CMB Anisotropies and $\Omega(x)$

L.1 Modified Sachs-Wolfe Effect

The Cosmic Microwave Background (CMB) temperature anisotropies provide a wealth of information about the early universe. In the Conformal Emergent Reality Model (CERM), the conformal scalar field $\Omega(x)$ introduces modifications to the Sachs-Wolfe effect, which describes how gravitational potentials imprint temperature fluctuations on the CMB.

The standard Sachs-Wolfe effect relates the temperature anisotropy $\delta T/T$ to the gravitational potential $\Phi_{\rm GR}$ in General Relativity (GR):

$$\frac{\delta T}{T} \approx \frac{1}{3}\Phi_{\rm GR}, \tag{124}$$

where $\Phi_{\rm GR}$ is the gravitational potential derived from GR.

In CERM, the effective gravitational potential $\Phi_{\rm eff}$ incorporates contributions from $\Omega(x)$:

$$\Phi_{\rm eff} = \Phi_{\rm GR} + \delta \ln \Omega, \tag{125}$$

where:

- $\Phi_{\rm GR}$: The gravitational potential in GR,
- $\delta \ln \Omega$: Fluctuations in the conformal factor $\Omega(x)$.

Substituting this into the Sachs-Wolfe formula, the modified temperature anisotropy becomes:

$$\frac{\delta T}{T} \approx \frac{1}{3}(\Phi_{\rm GR} + \delta \ln \Omega). \tag{41}$$

This additional term $\delta \ln \Omega$ arises naturally from the conformal scaling of spacetime and provides a mechanism for explaining anomalies in the CMB power spectrum.

L.1.1 Physical Implications

The inclusion of $\delta \ln \Omega$ has several important consequences:

1. **Low-ℓ Anomalies**: Observations of the CMB power spectrum show deviations at large angular scales ($\ell < 30$). These anomalies can be explained if $\delta \ln \Omega \sim 10^{-5}$, corresponding to small fluctuations in $\Omega(x)$ during recombination.

2. **Scale-Dependent Corrections**: The term $\delta \ln \Omega$ introduces scale-dependent corrections to the Sachs-Wolfe effect, potentially altering the shape of the CMB power spectrum.

3. **Link to Entropy Growth**: The fluctuations $\delta \ln \Omega$ are tied to the evolution of $\Omega(x)$, which governs entropy growth in CERM (see Appendix G).

Detailed Explanation of Terms

- Φ_{GR}: Represents the gravitational potential in GR, encoding density perturbations in the early universe.
- $\delta \ln \Omega$: Captures fluctuations in the conformal factor $\Omega(x)$, reflecting the dynamical nature of spacetime in CERM.
- $\delta T/T$: The fractional temperature fluctuation observed in the CMB.

L.2 Observational Test

To test the predictions of CERM for CMB anisotropies, we propose the following observational strategies:

L.2.1 Fit $\delta \ln \Omega$ to Planck Data

The Planck satellite has provided high-precision measurements of the CMB power spectrum. Residuals at low multipoles ($\ell < 30$) can be used to constrain $\delta \ln \Omega$. Specifically:

- Fit $\delta \ln \Omega \sim 10^{-5}$ to explain low-ℓ anomalies.
- Compare the predicted power spectrum with Planck data to validate the CERM model.

L.2.2 Correlation with Large-Scale Structure

Fluctuations in $\Omega(x)$ leave imprints not only on the CMB but also on the distribution of matter in the universe. Cross-correlating CMB anisotropies with large-scale structure surveys (e.g., DESI, Euclid) can provide additional constraints on $\delta \ln \Omega$.

L.2.3 Implications for Primordial Gravitational Waves

If $\delta \ln \Omega$ contributes significantly to the Sachs-Wolfe effect, it could alter the amplitude of primordial gravitational waves imprinted on the CMB. Observations of B-mode polarization (e.g., BICEP/Keck) can test this prediction.

L.3 Summary

- CERM modifies the Sachs-Wolfe effect by incorporating fluctuations in the conformal factor $\Omega(x)$.
- The term $\delta \ln \Omega$ explains low-ℓ anomalies in the CMB power spectrum.
- Observational tests include fitting $\delta \ln \Omega$ to Planck data, cross-correlating with large-scale structure, and analyzing primordial gravitational waves.

By linking CMB anisotropies to the conformal structure of spacetime, CERM offers a unified framework for understanding the interplay between cosmology and quantum gravity.

M Appendix M: Transition Between Cosmic Aeons in CERM

The transition between cosmic aeons in the Conformal Emergent Reality Model (CERM) is a cornerstone of its theoretical framework. This appendix provides detailed derivations and mathematical insights into the mechanisms governing the end of one aeon and the beginning of the next. These transitions align with Penrose's Conformal Cyclic Cosmology (CCC) but resolve key issues, such as proton decay and graviton dependence, through the unique conformal structure of CERM.

M.1 End of an Aeon: Conformal Reset

At the end of a cosmic cycle, the scalar field $\Omega(x)$ evolves to a singular state:

$$\Omega(x) \to \infty \quad \text{(or equivalently, } \Omega(x) \to 0 \text{ under inverse rescaling).} \tag{126}$$

This evolution triggers a **conformal reset**, which can be analyzed mathematically as follows:

M.1.1 Matter Dissolution

The masses of particles scale inversely with $\Omega(x)$:

$$m_p \propto \Omega^{-1}(x), \quad m_\nu \propto \Omega^{-1}(x), \quad \text{etc.} \tag{127}$$

As $\Omega \to \infty$, all particle masses vanish:

$$m_p \to 0, \quad m_\nu \to 0. \tag{128}$$

This process dissolves matter into a conformally invariant state dominated by massless fields (e.g., photons, neutrinos). The absence of massive particles ensures that no explicit proton decay mechanism (such as those proposed in Grand Unified Theories) is required.

M.1.2 Geometry Preservation

The conformal manifold $(\mathcal{M}, \gamma_{\mu\nu})$ remains intact during the transition. Only the physical metric $g_{\mu\nu}$ resets:

$$g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}. \tag{129}$$

As $\Omega \to \infty$, the physical metric becomes degenerate, but the conformal geometry $\gamma_{\mu\nu}$ retains its causal structure and Weyl curvature. This ensures continuity across aeons.

M.1.3 Entropy Reset

The phase space volume scales as $\Omega^3(x)$:

$$S \sim \int d^3x\, \Omega^3(x)\rho(x) \ln \rho(x). \tag{130}$$

As $\Omega \to \infty$, the phase space volume diverges inversely, resetting entropy to a low-state configuration:

$$S \to 0. \tag{131}$$

M.1.4 Weyl Curvature Reset

The Weyl curvature tensor $C_{\mu\nu\rho\sigma}[\gamma]$ is suppressed at the transition boundary:

$$C_{\mu\nu\rho\sigma}[\gamma] \to 0. \tag{132}$$

This ensures that the next aeon begins with low gravitational entropy, consistent with Penrose's Weyl Curvature Hypothesis.

M.2 Transition Mechanism

The transition between aeons occurs via **boundary conditions** at conformal infinity. The following steps describe the mechanism in detail:

1. **Massless Dominance**: At $\Omega \to \infty$, all particles become effectively massless:

   $$m_p \propto \Omega^{-1}(x) \to 0. \tag{133}$$

   This ensures that the universe enters a conformally invariant state dominated by radiation-like fields. The dissolution of matter into a conformally invariant state is accompanied by a holographic transfer of information from the bulk spacetime $\Omega_{\mu\nu}$ to the boundary conformal manifold $\gamma_{\mu\nu}$. As shown in Appendix O, the renormalized holographic potential ensures that quantum correlations and geometric data persist across transitions, avoiding information loss.

2. **Entropy Collapse**: The phase space volume collapses as:

$$\Omega^3(x) \to 0. \tag{134}$$

This collapse resets entropy to a minimal value, preparing the universe for the next aeon.

3. **Weyl Curvature Suppression**: The suppression of the Weyl curvature tensor ensures a smooth crossover:

$$C_{\mu\nu\rho\sigma}[\gamma] \to 0. \tag{135}$$

M.3 Key Differences from CCC

Aspect	**CCC**	**CERM**
Gravitons	Required for transition (massless mediators).	Unnecessary; transition governed by $\Omega(x)$.
Proton Decay	Relies on speculative particle physics.	Avoided via $m_p \to 0$ as $\Omega \to \infty$.
Entropy Reset	Achieved by black hole evaporation.	Driven by phase space collapse ($\Omega^3(x)$).
Geometric Primacy	Conformal rescaling of metrics.	Conformal manifold $\gamma_{\mu\nu}$ preserved; only $\Omega(x)$ resets.
Holographic Encoding	Absent in CCC.	Governed by $\Gamma[\gamma_{\mu\nu}]$ see **Appendix O**.

Table 1: Comparison between CCC and CERM

M.4 Observational Implications

1. **No Graviton Signature**: Unlike CCC or inflationary models, CERM predicts no primordial graviton background. Observations of stochastic gravitational wave backgrounds (e.g., via pulsar timing arrays like NANOGrav) should confirm this prediction.

2. **Massless Transition Era**: High-redshift observations (e.g., CMB spectral distortions) may reveal imprints of the massless phase during the transition.

3. **Entropy Asymmetry**: The low-entropy initial state of each aeon aligns with observed time asymmetry, without fine-tuning.

N Appendix N: Clarification of Ω, $\Omega(x)$, and $\Omega(t)$

In the Conformal Emergent Reality Model (CERM), the scalar field Ω plays a central role in dynamically determining physical scales. To ensure clarity and consistency, this appendix provides a brief explanation of the terms Ω, $\Omega(x)$, and $\Omega(t)$, and their usage throughout the model.

N.1 Definitions and Roles

- Ω: The scalar field Ω is a shorthand notation used when the explicit dependence on spacetime coordinates is not emphasized. It represents the conformal factor that scales the metric globally.
- $\Omega(x)$: Explicitly denotes the spatial dependence of the scalar field. This form is critical for describing phenomena such as galactic rotation curves, where gradients in $\Omega(x)$ contribute to gravitational effects.
- $\Omega(t)$: Denotes the temporal evolution of the scalar field. This is particularly relevant in cosmological contexts, where $\Omega(t)$ governs cosmic expansion and the arrow of time.

N.2 Interchangeability and Contextual Usage

While Ω, $\Omega(x)$, and $\Omega(t)$ are related, they are not fully interchangeable:

- Use $\Omega(x)$ when analyzing spatial variations, such as modifications to gravitational potentials or galactic dynamics.
- Use $\Omega(t)$ when discussing temporal evolution, such as cosmic acceleration or entropy growth.
- In scenarios involving both spatial and temporal dependencies, the combined form $\Omega(x,t)$ should be used for precision.

N.3 Examples of Correct Usage

1. **Metric Scaling**:
$$g_{\mu\nu}(x) = \Omega^2(x)\gamma_{\mu\nu}(x),$$
where $\Omega(x)$ captures the local conformal scaling of spacetime.

2. **Cosmic Evolution**:
$$\frac{\dot{\Omega}(t)}{\Omega(t)} = H_{\text{eff}},$$
where $\Omega(t)$ governs the effective expansion rate of the universe.

3. **Galactic Dynamics**:
$$\nabla^2 \Phi_{\text{eff}} = 4\pi G \rho_{\text{vis}} + \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{\partial_r \Omega(x)}{\Omega(x)}\right),$$
with $\Omega(x)$ explaining flat rotation curves without dark matter.

N.4 Summary

The scalar field Ω is a versatile construct in CERM, with its specific form (Ω, $\Omega(x)$, or $\Omega(t)$) chosen based on the context. By adhering to these distinctions, the model maintains mathematical rigor and physical clarity, ensuring consistent interpretations across diverse phenomena.

O Appendix O:Holographic Potential in CERM

Derivation and Mathematical Framework

The "holographic potential" in CERM refers to the effective action that encodes how the emergent physical spacetime $g_{\mu\nu} = \Omega^2\gamma_{\mu\nu}$ is holographically determined by the conformal manifold $(\mathcal{M}, \gamma_{\mu\nu})$. This appendix derives the mathematical relationship using CERM's action and boundary terms.

O.1 Setup and Action

The CERM action with boundary terms is:

$$S = \int_{\mathcal{M}} d^4x\sqrt{-\gamma}\left[\frac{\Omega^2}{2\kappa}R(\gamma) - \frac{1}{2}(\partial\Omega)^2 - A\Omega^4\right] + \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-\gamma}\Omega^2 K, \quad (136)$$

where K is the trace of the extrinsic curvature of the boundary $\partial\mathcal{M}$.

O.2 Equations of Motion

Varying S with respect to $\gamma_{\mu\nu}$ and Ω yields:

O.2.1 Modified Einstein Equations

$$\Omega^2 G_{\mu\nu}(\gamma) + H_{\mu\nu}(\Omega) = 8\pi G\, T^{\text{matter}}_{\mu\nu}, \quad (137)$$

where $H_{\mu\nu}(\Omega)$ includes Ω-dependent terms (see **Appendix A**).

O.2.2 Scalar Field Equation

$$\Box_\gamma\Omega - \frac{\Omega}{\kappa}R(\gamma) + 4A\Omega^3 = 0. \quad (138)$$

O.3 Holographic Potential

The **holographic potential** $\Gamma[\gamma_{\mu\nu}]$ is the on-shell action evaluated for solutions $\Omega[\gamma_{\mu\nu}]$:

$$\Gamma[\gamma_{\mu\nu}] = S_{\text{on-shell}} = \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-\gamma}\Omega^2 K\Big|_{\text{EOM}}. \quad (139)$$

To compute this:

1. **Regulate the Boundary**: Introduce a cutoff $\Omega = \Lambda$ near conformal infinity ($\Omega \to \infty$).

2. **Expand $\gamma_{\mu\nu}$ and Ω**: Assume a Fefferman-Graham-like expansion:

$$\gamma_{\mu\nu}(x, \Omega) = \gamma_{\mu\nu}^{(0)}(x) + \frac{\gamma_{\mu\nu}^{(2)}(x)}{\Omega^2} + \cdots, \quad K \sim \Omega \left(K^{(0)} + \frac{K^{(2)}}{\Omega^2} + \cdots \right). \tag{140}$$

3. **Evaluate Boundary Term**:

$$\Gamma_\Lambda = \frac{1}{\kappa} \int_{\partial\mathcal{M}} d^3x \sqrt{-\gamma^{(0)}} \left(\Lambda^3 K^{(0)} + \Lambda K^{(2)} + \cdots \right). \tag{141}$$

4. **Renormalize**: Subtract divergent terms as $\Lambda \to \infty$:

$$\Gamma_{\text{ren}} = \lim_{\Lambda \to \infty} \left(\Gamma_\Lambda - \Lambda^3 \int \sqrt{-\gamma^{(0)}} K^{(0)} \right). \tag{142}$$

O.3.1 Resulting Holographic Potential

The renormalized holographic potential in CERM captures the conformal anomaly and boundary dynamics and is given by:

$$\Gamma_{\text{ren}}[\gamma_{\mu\nu}^{(0)}] = \frac{1}{\kappa} \int_{\partial\mathcal{M}} d^3x \sqrt{-\gamma^{(0)}} \left[\mathcal{A} + \mathcal{B} R[\gamma^{(0)}] + \mathcal{C} R_{ij}[\gamma^{(0)}] R^{ij}[\gamma^{(0)}] + \cdots \right], \tag{143}$$

where (the coefficients are from equations of motion in section O.2 (e.g., scalar field):

$$\mathcal{A} = \frac{3A}{\kappa},$$
$$\mathcal{B} = -\frac{1}{4\kappa},$$
$$\mathcal{C} = \frac{1}{32\kappa A},$$

and:

- $\mathcal{A}$ is the trace anomaly proportional to A,
- $\mathcal{B} R[\gamma^{(0)}]$ encodes curvature corrections from the conformal manifold,

- $\gamma_{\mu\nu}^{(0)}$: Boundary conformal metric when $\Omega \to 0$,
- $R[\gamma^{(0)}]$: Ricci scalar of $\gamma_{\mu\nu}^{(0)}$,
- $R_{ij}[\gamma^{(0)}]$: Ricci tensor of $\gamma_{\mu\nu}^{(0)}$,
- A: Dimensionless constant from the Ω^4 term in the CERM action,
- $\kappa = 8\pi G$: Gravitational coupling constant.

O.3.2 Interpretation of Terms

$\mathcal{A}$ (Conformal Anomaly Term)

- Arises from the cosmological constant-like term $A\Omega^4$ in the CERM action.
- Represents a trace anomaly analogous to the conformal anomaly in quantum field theory.

$\mathcal{B}R[\gamma^{(0)}]$ (Curvature Correction)

- Encodes how the boundary curvature $R[\gamma^{(0)}]$ influences the holographic potential.
- Derives from the Einstein-Hilbert term $\Omega^2 R(\gamma)$ in CERM.

$\mathcal{C}R_{ij}R^{ij}$ (Higher-Order Curvature)

- Captures non-linear curvature effects.
- Subdominant unless $A \ll 1$ (strong conformal coupling).

O.3.3 Key Features

1. Divergence Cancellation
    - The Λ^3 and Λ divergences in Γ_Λ are subtracted, leaving finite terms dependent on $\gamma_{\mu\nu}^{(0)}$.
2. Holographic Duality

- $\Gamma_{\text{ren}}[\gamma^{(0)}_{\mu\nu}]$ acts as the effective action for the boundary conformal field theory (CFT), analogous to AdS/CFT.

3. Entanglement Entropy

   - The $\mathcal{A}$ term contributes to the entropy density:

$$s \sim \mathcal{A} + \mathcal{B}R + \dots \tag{144}$$

   - Links to CERM's geometric entropy formula:

$$S \sim \int \Omega^3 \rho \ln \rho. \tag{145}$$

This potential completes CERM's holographic framework, where the bulk spacetime $g_{\mu\nu} = \Omega^2 \gamma_{\mu\nu}$ is encoded in the boundary data $\gamma^{(0)}_{\mu\nu}$ and Γ_{ren}.

O.4 Physical Interpretation

- **Bulk-Boundary Correspondence**: The physical spacetime $g_{\mu\nu}$ is reconstructed from $\gamma^{(0)}_{\mu\nu}$ and $\Omega(x)$.
- **Conformal Anomaly**: The potential Γ_{ren} includes quantum corrections from $\Omega(x)$, analogous to the CFT effective action in AdS/CFT.
- **Entanglement Entropy**: The area term in Γ_{ren} links to CERM's entropy formula $S \sim \int \Omega^3 \rho \ln \rho$ via holography.

O.5 Key Equations Summary

1. **On-Shell Action**:

$$\Gamma[\gamma_{\mu\nu}] = \frac{1}{\kappa} \int_{\partial\mathcal{M}} \sqrt{-\gamma}\,\Omega^2 K. \tag{146}$$

2. **Renormalized Potential**:

$$\Gamma_{\text{ren}}[\gamma^{(0)}_{\mu\nu}] = \text{Finite terms in } \Gamma_\Lambda \text{ as } \Lambda \to \infty. \tag{147}$$

This derivation establishes CERM's holographic framework, where the conformal manifold $(\mathcal{M}, \gamma_{\mu\nu})$ encodes the data required to reconstruct the emergent spacetime $g_{\mu\nu}$ and its properties.