A Note on Bell's Inequality

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Abstract

I review the inequality given by Bell [1], and provide an interpretation to avoid the use of non-local entanglement. It is argued that what appear to be separate properties within an individual particle (usually regarded as independent state variables) exist as entangled properties (entangled state variables) that are not physically meaningful on their own. For example, it is not physically meaningful to refer to spin in a certain direction, as the spins in different directions are entangled with each other, and there is no meaning to regarding spin about one particular axis as a state variable of the particle.
keywords: Bell's Theorem

1 Introduction

A paper by Einstein, Podolsky and Rosen [2] argued that the current theory of quantum mechanics is incomplete and must be supplemented with additional "hidden variables". Bell [1] responded to this by constructing a "hidden variables" theory and demonstrating that such a theory must obey a statistical inequality. This inequality was derived for measurements of the spin of correlated particles $\overline{\sigma_1}$ and $\overline{\sigma_2}$.The particles are sent in opposite directions to separate labs an arbitrary distance apart. When spin is measured along some unit vector direction $\overline{a}$, the value obtained is $\overline{\sigma_1} \cdot \overline{a} = \pm 1 = -\overline{\sigma_2} \cdot \overline{a}$. Each lab may measure along either of 3 directions $\overline{a}$, $\overline{b}$ or $\overline{c}$. If particle 1 is measured along $\overline{a}$ and particle 2 is measured along $\overline{b}$ the expectation value of the product of the two components $\overline{\sigma_1} \cdot \overline{a}$ and $\overline{\sigma_2} \cdot \overline{b}$ is denoted as $P(\overline{a}, \overline{b})$. The inequality obtained (Bell's equation 15) is:

$$1 + P(\overline{b}, \overline{c}) \geq \left| P(\overline{a}, \overline{b}) - P(\overline{a}, \overline{c}) \right| \tag{1}$$

Bell assumes that the value of spin for an electron along any given direction (a,b, or c) may be given by a separate state variable dependent on hidden variables λ . In this way he mathematically defines the following state variables:

$$\overline{\sigma_1} \cdot \overline{a} = A(\overline{a}, \lambda) \tag{2}$$

$$\overline{\sigma_2} \cdot \overline{b} = B(\overline{b}, \lambda) \tag{3}$$

Averaging over the hidden variables, the expectation value for the product of the state variables is then given by Bell's Equation 2:

$$P(\overline{a}, \overline{b}) = \int d\lambda \rho(\lambda) A(\overline{a}, \lambda) B(\overline{b}, \lambda) \tag{4}$$

Two states are regarded as entangled if their combined state is inseparable. We make the following analogous definition for state variables:

Definition: Two state variables are considered entangled if they combine to form a state variable that is not separable, that is not separable into terms that are each wholly determined by one variable by itself.

I make the argument that assuming separable state variables for what appear to be distinct properties (spin in direction a, spin in direction b, and spin in direction c) can be avoided, and then Bell's Inequality may not follow. In fact, if certain variables appear to violate Bell's Inequality, it is an indication that the variables are not physically distinct and can only exist together as an entangled state variable. In this view, there is no meaning to state variable $A(\overline{a}, \lambda)$, and we may only refer to entangled state variables such as $A(\overline{a}, \overline{b}, \lambda)$.

2 Entangled State Variables

For a concrete example of two properties that form an entangled state variable, consider a man who will only wear certain combinations of socks: red (r)/yellow (y)/blue (b). We impose the following conditions:

Condition 1: Each combination of socks is considered a state variable of the man and is assigned a value of +1 or -1.

Condition 2: All states in which red appears are assigned the same value, and the same is true for yellow and blue.

Condition 3: Every color is represented by at least one state, so that if one were to check (measure) one of the colors they would get a definite result. We also assume that if he didn't happen to be wearing one of the combinations at the time, he still had the combination available and so it could be "measured". Any measurement of a color within a combination returns the value of the combination.

A concrete set of states is the following (v_1v_2):

Combination 1: rr = -1
Combination 2: ry = -1
Combination 3: yy = -1
Combination 4: bb = 1

It is apparent that the 2 variables v_1 for left and v_2 for right cannot be combined together as separable state variables, because that would mean that yb and rb could be defined, and also r (and y and b) by itself could be assigned a real value. Note that another set of combinations results if either of Combinations 1,2 or 3 are omitted. In this example, no color exists as a separate property (or state variable), but only as an entangled state variable of a combination of two colors.

3 Spin of Correlated Particles

For particle spin, we may replace colors r/y/b in the example above with the directions a/b/c. We may also assume that anti-correlated particles will give mutually opposite values for each state. Different singlet sets will have different hidden variable values and so will have variations of the combinations, such as direction a swapped for direction b, etc., and possibly one of Combinations 1 through 3 missing. Since the state variables of directions a, b and c are not separable, Bell's Inequality does not follow, that is there do not exist separable state variables for directions a, b and c. There is no meaning to the mathematical term $A(\overline{a}, \lambda)$, and we may only refer to terms such as $A(\overline{a}, \overline{b}, \lambda)$. The apparently separate spin directions are assumed to be entangled with themselves, so that there is no physical meaning to referring to the spin about a particular axis. The spin along different axis' are mutually entangled and not separable, and thus consist of entangled state variables.

4 Conclusion

It is generally assumed that particles in a singlet state have entangled properties. I make the argument that it is the properties themselves (within a single particle) that are entangled (locally) producing entangled state variables. Certain properties can only exist in combinations with other properties as entangled state variables, such that for example spin cannot be given separately for different directions, but can only be referred to as a property of entangled directions. If a state variable appears to violate Bell's Inequality, it is an indication that it is not a valid physical property on it's own, but is only physically meaningful when entangled with another apparently distinct property in an entangled state variable. One cannot define a spin state in a single direction by itself. It is believed that entangled state variables (such as position and momentum) are a primary feature of quantum mechanical systems, and an understanding of this would give a better appreciation of quantum features.

5 References

[1]Bell, J. S. (1964). "On the Einstein Podolsky Rosen Paradox" . Physics. 1 (3): 195–200.
[2]A. Einstein, N. Rosen and B. Podolsky (1935). "Can the Quantum-Mechanical Desription of Reality Be Considered Complete?" Phys. Rev. 47, 777 .