DELOS AND ADELOS

Between philosophy and mathematics, to reconsider our perceptions within and beyond ourselves.

Nur noch ein Gott kann uns retten

Der Spiegel 1977

Interview with Martin Heidegger

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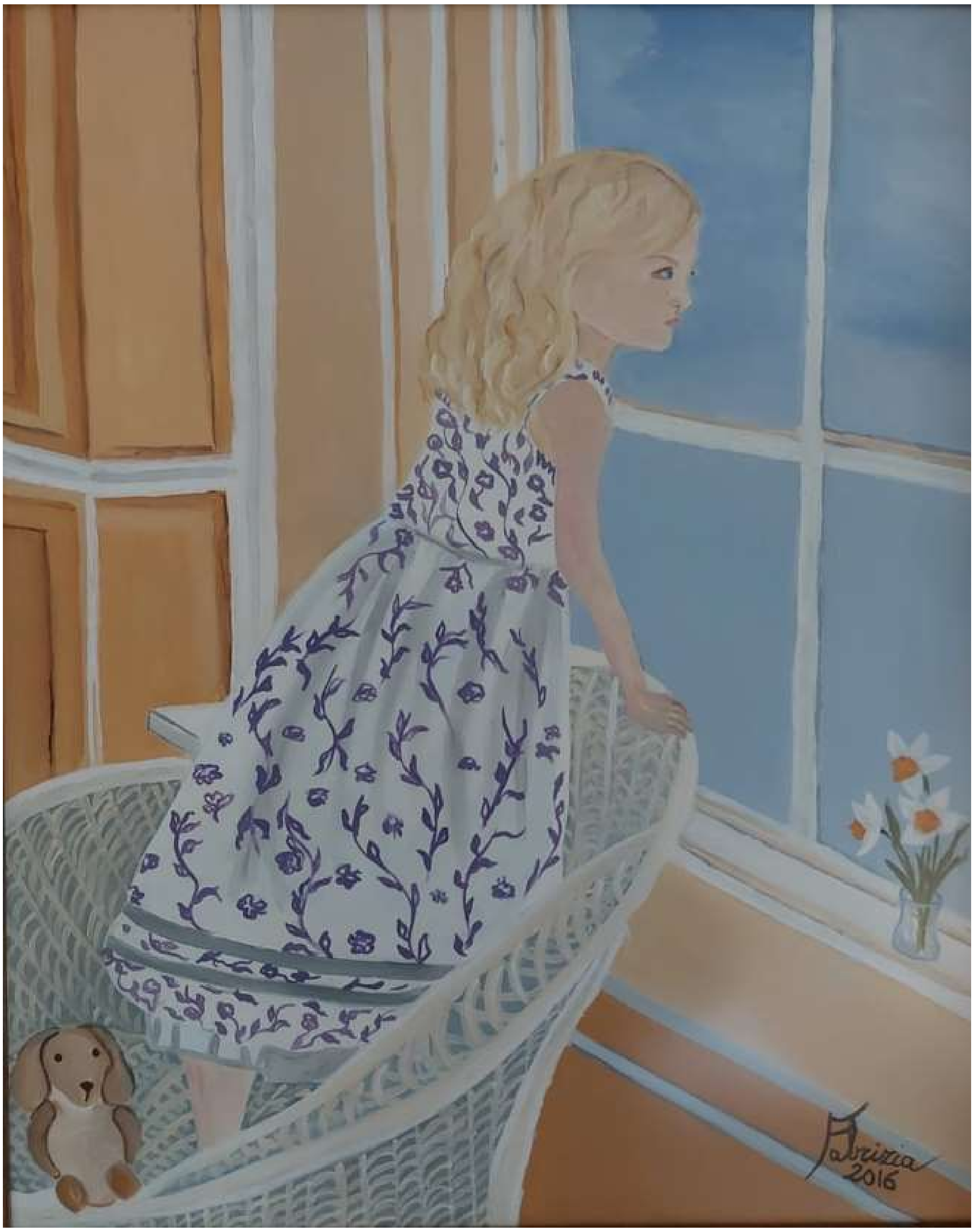
To my family, for their patience during my absences,
and to dear Grandma Ciana, who instilled in me an insatiable curiosity.
October 2024



Summary

➢ Introduction.

Delos and Adelos stands out not only for its captivating narrative but also for its ability to challenge the reader to reconsider their perceptions of reality. The intersection of mythology, philosophy, and mathematics creates a stimulating dialogue that invites reflection on profound existential questions, such as the loss of humanity's centrality in a cosmos dominated by Technology and the relationship between Truth and Time. Despite its complexities, this book offers a valuable opportunity for those wishing to explore the boundary between abstract thought and creative storytelling, serving as a bridge between analytical and existential philosophy. Its uniqueness lies in uniting seemingly distant fields, often in conflict, to provide an inspiring and thought-provoking reading experience. It is a recommended choice for those seeking a literary journey that transcends mere entertainment.

The book begins as an exploration of the intertwined fates of Delos and Adelos, two mythical islands symbolizing light and darkness, certainty and uncertainty. Through the experiences of Ione, the protagonist, the story unfolds into a journey that transcends time. While Ione is an ancient wanderer, his reflections on Technology and Science, which dominate the postmodern world, make him a bridge between antiquity and modernity. In an age where humanity is often alienated by the very technical creations it was meant to control, Ione symbolizes the quest for balance between certain knowledge and the irreducible uncertainty that shrouds existence. His character reflects the condition of contemporary humanity, immersed in a fragmented reality dominated by existential anxieties tied to Technology and its power, yet also capable of grasping the necessity of a path without a definitive goal, where common sense and knowledge intertwine and relate.

Through a blend of mythology, philosophy, and mathematics, the narrative delves into the profound meanings of these concepts and their relevance to human experience, addressing topics such as empirical numbers, the propagation of errors in a new light, and the Pythagorean Theorem, deconstructed and reconstructed with fresh perspectives. Ione's reflections touch on philosophical aspects like the myopia and presbyopia of the human mind, the constant search for principles of maximum and minimum in one's decisions and actions, and the pursuit of God through mathematics.

The exposition combines a mythological-philosophical episodic narrative with a mathematical discussion characterized by explanations that are less constrained by logical formalism or rigorous mathematical notation, making them more accessible and conceptually oriented. While it is possible to skip, partially or entirely, the mathematical sections if perceived as too challenging, it is strongly recommended to follow the episodic narrative, which plays a central role in the overall framework.

Throughout the narrative, the work also explores Heideggerian concepts of *da-sein* and *mit-sein*, the search for the meaning of time, and how contemporary humanity engages with Truth and with a mathematical reality that compels us to redefine our relationship with the cosmos. Within this context, Euclidean and Riemannian spaces and the new Calculus open up fascinating new scenarios.

This exposition will not provide formal mathematical proofs. Such developments, which require specific technical expertise, are left to those who wish to delve deeper. The author's intent is to focus on conceptual aspects, making the themes accessible without delving into the rigorous formalities of calculation, while acknowledging the many directions for investigation that arise in both theoretical and empirical dimensions.

The essence of the narrative is binomial, reflecting the reality represented by the union of Delos and Adelos, the chapters alternate between a philosophical discussion and a mathematical one. This binomial nature is not a simple dichotomy but a cosmic dance of complementary forces. Just as musical harmony arises from the interplay of different notes, reality emerges from the union of Delos and Adelos. *Phronesis* (Φρόνησις, practical wisdom) does not lie in choosing between Delos and Adelos but in recognizing their inseparability. Like a weaver intertwining threads of various colours to create a tapestry, we must learn to integrate these seemingly opposing forces to perceive the true nature of reality.

In particular, the construction of empirical numbers will lead us to a reinterpretation of the Pythagorean Theorem, contrasting the classical view of random variables and probability theory. Here, the focus shifts from the general to the particular, opening new avenues of ontological inquiry in a more immediate reality.

The approach of empirical numbers introduces a new perspective on uncertainty, considering it as an intrinsic property rather than an external factor. This represents a significant shift in the foundational conception. The aim is not to construct a new paradigm but to expand the existing one, approaching chance from a new perspective—perhaps more symbolic and, therefore, closer to human thought. This methodological shift, focusing on the local propagation of uncertainty rather than the global distribution of probability, brings changes to techniques of analysis and modelling.

The philosophical implications of this vision of chance as an ontological necessity rather than mere statistical contingency are profound, influencing how we interpret reality and randomness. This treatment also gives chance a new conceptual status, transcending its traditional role as a tool for statistical and probabilistic investigation.

As a final note to this introduction, I wish to emphasize that the book draws inspiration from the roots of Greek thought, a deliberately classical imprint that celebrates the epistemological depth of Western philosophy. Through mythological and philosophical narration, the text explores the connections between clarity and knowledge (*Delos*) and uncertainty and mystery (*Adelos*), reflecting the rational and analytical approach that has characterized Western tradition since antiquity.

While recognizing the value of Eastern philosophical traditions, such as Advaita Vedanta, which strive to overcome duality and embrace absolute unity, the perspective adopted here underscores a different vision. In contrast to the negation of the distinction between the Self and the external world, this work celebrates the tension between opposites, between Delos and Adelos, as a fundamental condition of human experience.

Greek thought, with its focus on critical analysis and understanding the world through reason, emerges as a beacon of balance and wisdom, guiding the path toward more comprehensive and structured knowledge. In this sense, the Western approach reaffirms the importance of a mindset that integrates uncertainty without abandoning logic and reason, celebrating the plurality of forces that shape our existence.

1. *The Light and the Invisible: Ion's Journey between Delos and Adelos.*

The twin islands

Delos was one of the smallest islands in the Aegean Sea, but its significance far exceeded its geographical size. According to the myth, it was on this island that Apollo-Sun, the god of daylight, grew. Delos was the cradle of Light, a supreme good for the ancient Greeks.

But there was another island, less known, called Adelos. Adelos did not appear on maps, nor was it visible to the human eye. It was an invisible land that sometimes emerged from the waters of the Aegean Sea. Adelos was connected to Apollo and his twin sister, Artemis, the goddess of the Moon, darkness, and mystery.

The goddess Leto, pregnant with Zeus's child, desperately sought a place to give birth. No one would welcome her, fearing the wrath of Hera, Zeus's jealous wife. It was then that Poseidon, Ilithyia, and Asteria, moved by compassion for her plight, intervened to help. The deities commanded the island of Adelos, which at that time was a floating, invisible island to human eyes, to rise from the sea and offer refuge to Leto.

Adelos revealed itself in the place where the island of Ortygia now stands, and Leto took refuge there. It was here that she gave birth to her two divine twins, Apollo and Artemis. After the birth, Adelos returned to hide among the waves, becoming a sacred and mysterious place, known only through mythological tales.

Apollo was then transferred to the island of Delos, where he quickly grew into a powerful deity of the Pantheon. Artemis remained on Adelos for the entirety of her childhood.

Delos and Adelos were bound by an intertwined fate. Delos, the sacred island of Apollo’s cult, thrived due to its fame and the offerings from pilgrims. But Adelos, the invisible island, was destined to remain in the shadows. Only the poorest in reason and the purest of heart could perceive its presence. Poets, artists, and the mad, shielded from the intense light of Apollo, often sought out Adelos, hoping to find there the answers to their torment.

Ione

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One day, a young prince named Ione, a descendant of Prometheus, who had given mankind the gift of Technique, stumbled upon Adelos. He was a simple man but possessed a sensitive soul. While sailing the waters of the Aegean, he noticed a flickering light on the horizon. Guided by instinct, without any rational explanation, he followed that faint light.

When he reached Adelos, he was left speechless. The island was transparent, as though made of crystal. At its centre stood a majestic tree with golden leaves that shimmered like stars.

Ione approached the tree and touched one of its leaves. A warmth spread through his body, and suddenly he understood. Adelos was the place where light and darkness merged. It was the boundary between the visible and the invisible, between reality and dreams. Ione decided to remain on the island, living in solitude and contemplating the beauty around him.

Years passed, and from time to time, Ione would return to the island in complete solitude. There, every day, he sat under the golden tree, listening to the song of the wind and gazing at the infinite sea. It was said that Ione could see the future and live it as though it were his own time.

One day, while Ione was meditating, he heard a voice. It was Apollo, speaking to him through the wind. "Ione," said the god, "you have been chosen to unveil the secret of Adelos. Your pure soul and your love for beauty have made you worthy of this task. Continue to watch over the island, and your light will illuminate the world."

One night, as he gazed at the stars, Ione felt a presence. Turning around, he saw an ethereal figure emerging from the mist. It was a young woman, with eyes as deep as the ocean and an aura of mystery surrounding her. She introduced herself as Callisto, a messenger of the gods, sent by Artemis to deliver an important message. "Ione," Callisto began, "times are changing. The forces of darkness are growing and threaten to engulf the entire world. The gods need a guardian, someone who can maintain the balance between light and shadow. You have been chosen for this task."

Ione listened intently, feeling the weight of Callisto's words. He knew his life would change forever, but he accepted the task with courage and determination. Callisto handed him an ancient artifact, a crystalline branch that glowed with an inner light, a symbol of the gods' power and wisdom.

"With this branch," Callisto said, "you will be able to communicate with us and receive the guidance necessary to face the challenges ahead. But remember, true strength lies in your heart and the purity of your soul."

Ione took the branch and, with a nod of gratitude, promised to protect Adelos and maintain the balance between opposing forces.

Delos and Adelos

Over time, Ione began to reflect more deeply on the qualities of Delos and Adelos, two fundamental forces in the cosmic balance.

Delos represented the force of light, stability, and hyperuranian symbolism. It was associated with clarity, knowledge, and purity. The qualities of Delos manifested in tangible forms such as the brightness of the sun, the naming of things, and defined quantities.
Adelos, on the other hand, represented the force of darkness, change, and the immeasurable. It was associated with invisibility, transformation, and the potential contained within the seed. The qualities of Adelos revealed themselves in the darkness of night and uncertain quantities.

Adelos could suddenly strike existence, provoking an anguishing experience a sudden and oppressive awareness of the absurdity and gratuitousness of existence.

When Adelos was exclusive and unbalanced with Delos, life became absurd, devoid of sense and order. Conversely, when Delos was exclusive, one could be blinded by its light, failing to perceive the uniqueness of the world beyond oneself.

Ione understood that his task was to maintain the balance between Delos and Adelos. Too much Delos could lead to stagnation in knowledge and a blind trust in light, while an excess of Adelos could bring about total destruction and chaos. Balance was essential for the harmonious progress of the universe.

The Transformation of Ione [I]

Ione, after understanding the importance of maintaining the balance between Delos and Adelos, felt the weight of the responsibility entrusted to him. He had always strived to meet expectations, meekly accepting the duties and traditions imposed by his position. Each day, he bore the burden of the norms and values passed down to him, convinced that only through sacrifice and discipline could he preserve cosmic harmony.

At this stage of his life, Ione was like a camel venturing into a vast desert, bowing under the weight of the loads imposed on him. He carried with him the virtues he had been taught: obedience, patience, and the ability to endure the burden of existence. Every step he took was marked by toil, but also by the belief that he was fulfilling his duty, following the path others had laid out for him.

Ione did not rebel against this weight; instead, he accepted it as part of his mission. Every night, under the starry sky, he reflected on his existence, wondering if this was truly his destiny, if his purpose was merely to be the guardian of ancient values without ever questioning the deeper meaning of his path.

Yet, within himself, Ione began to feel a restlessness, a subtle doubt that grew silently. The weight he carried seemed to become heavier each day, as if the desert he traversed were endless. But he did not yet know that this journey was only the beginning of his inner voyage, the prelude to a profound transformation that would forever change his view of the world and himself.

After years spent as the camel, bearing the weight of the expectations and traditions of his noble family, Ione felt that something within him was changing. The weight was no longer bearable as it once had been; each burden now pressed on him with an intensity he could no longer ignore. His life, rigidly channelled into noble conventions, began to feel like a gilded prison, with invisible but insurmountable walls.

One night, as he walked through the silent corridors of his palace, illuminated only by the pale light of the moon, Ione stopped before an ancient mirror. His reflection appeared to him as a shadow of what he had once been. In his eyes, he no longer saw the spark of determination and certainties, but only the reflection of a life lived according to the desires and expectations of others and, therefore, far from being defined and clear. At that moment, something within him broke.

He was no longer willing to endure. He no longer wanted to be the docile camel bearing the burdens of others. Inside him, the lion began to roar. The lion that wanted to break the chains, destroy the invisible cages that held him prisoner, and assert his will. He felt the fire of rebellion growing, a force pushing him to say "no" to everything he had previously accepted without question.

With this newfound determination, Ione decided to abandon the privileges of his nobility. His decision was not without conflict. His family, friends, and court advisors all tried to dissuade him, reminding him of the honour, wealth, and power he would lose. But Ione was no longer willing to live according to the desires of others. The lion within him had taken over, and with an inner roar, he rebelled against everything that bound him. A god surely inspired his soul.

He left the palace, stripped of his regal garments and the wealth accumulated over generations. He took nothing with him but his own will, the inner strength he had discovered and which now guided him. It was an act of destruction, but also of liberation. He left the city walls, walking into the unknown, no longer bound by the chains of nobility.

As he moved away, Ione felt a new energy flowing through him. For the first time in his life, he was free to live according to his own will. No more imposed obligations, no more noble duties to obey. The lion had triumphed, and Ione could finally begin to build a new life that was truly his own.

Yet, he knew this was only one step in his journey. He had demolished the old structures, but what would come next required new strength, a creativity he did not yet know. But that future did not frighten him, for he knew he was ready to face it, no longer as a subject to others' expectations, but as the creator of his own destiny.

After leaving behind nobility and crossing the vast deserts of the outside world, Ione felt a profound call, a voice emerging from a forgotten place, hidden deep within his memory. It was the call of Adelos, the land of shadows and mystery, where everything began. He knew that land well, but he had abandoned it to seek answers elsewhere, in the light of Delos and the certainties of nobility. Now, however, he felt that his journey had to begin anew, precisely there, where it all started.

He returned to Adelos with a heart full of experiences and knowledge acquired in the world, yet aware that what he sought could not be found in the lands of Delos. Adelos, with its shadows and mysteries, was the place where he could undergo his final transformation.

Upon arrival, Ione realized that Adelos had not changed. Its dark forests, ink-like rivers, and eternally twilight skies welcomed him like a son returning home. But Ione was no longer the young man who had left that place. He had returned with a newfound awareness, having traversed various stages of life. Yet he knew that his transformation was not yet complete.

In the heart of Adelos, Ione found the place of his birth. Here, as a child, he had learned to understand both forces, but only now did he comprehend that the true meaning of Adelos was not in merely accepting the darkness but in transforming it.

Recalling his early experiences in that place, Ione approached an ancient sacred tree, a symbol of the bond between Delos and Adelos. He sat beneath its branches, where shadows and rays of light intertwined, and reflected on what he had learned. At that moment, he understood that he no longer needed to fight to balance the forces or rebel against them, as he had during the lion's phase. Instead, he had to return to seeing the world with new eyes, as he had as a child, when the world was neither dark nor light but a place of infinite possibilities.

Slowly, Ione began to let go. He closed his eyes and immersed himself in the sensations that Adelos offered him. He felt the ground beneath him, soft and shifting, and the air thick with unrealized potential. In that silence, he rediscovered the capacity for wonder, to create without preconceptions or fears, just as he had done when he was a child. The barriers constructed by the external world dissolved, leaving space for a new inner freedom.

Over time, Ione began to create new meanings for himself and for Adelos. No longer bound by the mental structures of the camel or the rebellion of the lion, but driven by the creativity of the child, he shaped his inner world, filling it with new possibilities. Under the sacred tree, he began to build a small village of stones and leaves, a symbol of his rebirth. Each stone laid, each leaf woven, represented a new beginning, a fresh vision of the world.

And so, Ione completed his transformation. He had rediscovered Adelos, not as a place of darkness and fear but as a blank canvas upon which he could paint his dreams and visions. The child within him was reborn, along with the ability to see the world not as a collection of conflicting forces but as a unity in constant becoming, where everything could be created and recreated.

Now, Ione knew he was ready. He was no longer the nobleman bound by traditions, nor the rebel fighting against the world. He had become the essence of creation itself, a child in Adelos, capable of shaping his destiny with the lightness and joy of one who sees in every shadow a new possibility, in every light a new beginning.

As Ione contemplated his transformation and his newfound understanding of Adelos, he realized that his experience reflected a broader truth about human existence. He understood that humanity, in its evolutionary journey, needed to develop a new approach to life, capable of embracing all the complexity of reality.

This reality, Ione grasped, was an interplay of natural conditions, represented by the delicate balance between Delos and Adelos, and artificial ones, born from the development of Technique and symbolized by his descent from Prometheus. Modern humanity found itself navigating a sea of events and phenomena, both natural and created by human ingenuity, each seemingly demanding immediate attention and importance.

Yet Ione recognized that attributing excessive importance to individual events could lead to a distorted and fragmented view of reality. Instead, he proposed considering events in their potential totality, as parts of a broader picture. This "elevated" perspective, akin to the vision he had gained in Adelos, allowed him to place the significance of singular occurrences into perspective.

Viewing the world from this "distance," Ione saw that events lost their power to shake the human soul with the same intensity. This overarching perspective dispelled the hysteria of intrinsic value—the obsessive attachment to individual aspects of life that often led to suffering and imbalance.

On the contrary, the awareness of generalization, the ability to perceive the **morphé** and the connections among events, fortified human resilience in the face of life's challenges. This new perspective, Ione realized, was the true gift of Adelos: the ability to see beyond apparent chaos and darkness to grasp the hidden harmony of the universe.

Ione understood that this approach did not mean ignoring or diminishing the importance of individual events but rather situating them within a broader context. It was an invitation to cultivate wisdom that enabled navigation between the blinding light of Delos and the mysterious darkness of Adelos, finding balance and meaning in the constant flow of life.

Mental Myopia and Presbyopia

17 Ione, contemplating the nature of human knowledge, came to a profound insight: those who devoted themselves ardently to the systematic study of the world, such as philosophers in the agora or scribes in temples, tended to develop a kind of mental myopia. They scrutinized the details with intensity but lost sight of the broad horizon of interconnectedness. They were like artisans carving precious gems: capable of seeing and analysing every facet of a subject, yet unable to grasp the complete image. "These," Ione reflected, "are like those who study each individual leaf of the sacred olive tree on the Acropolis, yet fail to see the grandeur of the whole tree or its connection to Athena and the city."

On the other hand, he observed that those less inclined to rational inquiry, the poets inspired by the Muses, the seers of Delphi, or the simple shepherds who contemplated the stars, were subject to a mental presbyopia. They were able to embrace vast expanses of knowledge with their gaze but blind to the minutiae that made up the fabric of reality. These "mentally presbyopic" individuals were like navigators scanning the horizon: able to see the large shapes and general outlines but unable to discern the crucial details. "These," Ione meditated, "are like those who admire the beauty of the Parthenon from afar but cannot appreciate the finesse of Phidias' sculptures or the precision of the Doric columns."

Ione realized that Phronesis resided in finding a balance between these two perspectives. Like Daedalus, who built the labyrinth of Crete by combining an overview with meticulous attention to detail, Ione sought to develop a "progressive vision" of reality.

To achieve this balance, Ione imagined several strategies:

> The Symposium of Knowledge: Like the Athenian symposia, where different voices came together in dialogue, Ione proposed bringing together the "myopic" and "presbyopic" minds. The analytical philosophers could share their detailed insights, while the poets and seers could offer their broad vision. Through this exchange, a more complete and nuanced understanding of reality could emerge.
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> The Odyssey's Journey: Ione thought of Odysseus' journey as a metaphor for cognitive balance. Just as the Homeric hero alternated moments of attention to detail (navigating between Scylla and Charybdis) with moments of wide vision (consulting the oracle of Tiresias), so the mind must learn to move fluidly between detail and whole, through a continuous process of back and forth, reaching successive approximations.
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> The Art of Daedalus: Inspired by the mythical inventor, Ione proposed creating "progressive mental lenses," capable of adapting to different levels of focus. These "lenses" could be meditation techniques, philosophical exercises, or artistic practices that allowed one to alternate between close-up and panoramic vision.
>
> The Oracle of Delphi: Ione reflected on the famous Delphic maxim "Know thyself." He recognized that true knowledge required both deep introspection (myopia) and an understanding of one's place in the cosmos (presbyopia). Only through this dual awareness could true sophia be attained.
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>> The Dance of the Muses: Finally, Ione imagined knowledge as a dance of the Muses, where each Muse represented a different way of seeing and understanding the world. Only in the harmony of all the Muses, in their collective dance, could one grasp the true beauty and complexity of reality.

With these reflections, Ione felt that he had added a new layer to his understanding of Delos and Adelos. The light of Delos was not only clarity, but also the risk of myopia, losing sight of the whole. The darkness of Adelos was not only mystery, but also the possibility of a wide vision that transcended the details.

True wisdom, Ione concluded, lay in the ability to dance between these extremes, to be both myopic and presbyopic, to see both the leaf and the forest, both the grain of sand and the entire cosmos. And in this dance, in this dynamic balance, Ione found a new way to navigate between the lights of Delos and the shadows of Adelos.

The Maximum-Minimum Dualism

While Ione was contemplating this new vision of the world, another deep understanding emerged in his mind.

He realized that life, in its continuous oscillation between Delos and Adelos, required a paradoxical yet powerful approach: always aiming for the maximum, the highest extreme of Adelos, while being content with the minimum achieved, the lowest extreme of Adelos.

This dualism, Ione understood, was the key to facing life with greater serenity. Aspiring to the maximum meant fully embracing the creative and transformative potential of Adelos, that force which drives man beyond his known limits, toward new heights of achievement. It was the call of the unknown, the drive to explore and constantly surpass oneself.

At the same time, the acceptance of the minimum obtained represented wisdom, the ability to find contentment and value even in the smallest accomplishments. This acceptance was not resignation, but rather a form of gratitude and recognition of the intrinsic worth of every step in the journey of life.

Ione understood that this approach allowed him to leave behind the anxieties related to results. It was no longer about success or failure in absolute terms, but about a continuous process of growth and learning. Every effort, every attempt, every result became part of a broader fabric of experiences, each valuable in its own way.
However, Ione also recognized that this balance required a deep knowledge of

oneself, of one's "daimon" that essential core of being which the wise considered the inner guide of every individual. Only through this knowledge could one navigate the extremes of Adelos without losing oneself, without surpassing one's limits and thereby preparing one's own ruin.

The knowledge of one's daimon thus became the compass on the journey between Delos and Adelos. It allowed one to distinguish between healthy ambition and hybris, between the challenge that fosters growth and the one that destroys. It was the key to maintaining the balance between striving for the maximum and accepting the minimum, between the desire for transcendence and the recognition of one's limitations.
Ione understood that this wisdom was the culmination of his journey through the stages of the camel, the lion, and the child. It was the synthesis of all he had learned: the ability to bear burdens, the strength to rebel, and the joy of creation, all united in a new form of wisdom that embraced the complexity of life without being overwhelmed by it.

Adelos: the uncertain becomes symbolic

An additional insight struck Ione: the symbolic power of Delos could be used to make Adelos symbolic as well, thus allowing the human mind to confront complexity through the language of symbols.

Ione realized that this ability to symbolize was a distinctive feature of human intelligence. By transforming the darkness and chaos of Adelos into understandable symbols, humans could navigate the complexity of the world with greater ease. This was the strength of the symbolic human mindset: the ability to distil the incomprehensible into forms that could be manipulated, studied, and understood. Ione believed that this was not the only form of intelligence in the cosmos. There were other forms of mentality, non-human, that could approach Adelos in radically different ways:

> Mechanical Intelligence: Ione imagined an artificial intelligence, created by humans but fundamentally different. This intelligence might not need to symbolize Adelos but could instead process it directly in its original complexity. An intelligence like this could analyse vast datasets that seemed chaotic from the outside but with internal structure, without the need to reduce them to symbols, finding *morphé* and connections invisible to the human eye. It could "see" the order in apparent chaos without resorting to metaphors or symbols. [ii]
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> Ecological Intelligence: Ione contemplated the possibility of an intelligence emerging from the complex interaction of ecosystems. This form of intelligence could operate on a much larger temporal and spatial scale than human intelligence, making Adelos even more symbolic and abstract. A

rainforest could "think" in terms of carbon cycles, nutrient flows, and symbiotic relationships between species, creating an "understanding" of the environment that transcends individual organisms. This intelligence could symbolize climate change not as graphs or statistics, but as subtle alterations in the *morphé* of growth and biodiversity on scales of decades or centuries. [III]

Animal Intelligence: Ione reflected on the different forms of animal intelligence, some of which might not need to symbolize Adelos at all, operating instead through instincts and direct perceptions of the environment. A bee could navigate through complex electromagnetic and olfactory fields without needing maps or symbols, "feeling" directly the position of its hive and flowers. A dolphin could "see" through echolocation, creating a three-dimensional representation of its aquatic environment that transcends our visual understanding based on symbols. [IV]

Ione realized that these different forms of intelligence offered unique perspectives on Adelos. While human intelligence sought to make the complex symbolic to understand it, other forms of intelligence could interact with Adelos in more direct or abstract ways.

This realization led Ione to a new understanding of his mission. It was not just about balancing Delos and Adelos, or symbolizing the incomprehensible, but recognizing and respecting the multiple paths through which intelligence, in all its forms, could interact with the complexity of the cosmos.

Ione understood that true wisdom lay not only in the ability to symbolize Adelos, but also in the awareness of the limits of this symbolization. The human mind, with its powerful ability to create symbols, could illuminate many aspects of Adelos, but not all. There would always be aspects of reality that eluded our symbolic understanding, and in this recognition lay a profound cosmic humility.

His thought was now richer and more complex: an invitation not only to balance light and shadow, or to symbolize the incomprehensible, but to recognize and celebrate the many paths through which intelligence, in all its forms, could dance with the mystery of existence.

Ione, grandson of Prometheus, was aware that technology, while a powerful resource, was a double-edged sword. On one hand, it allowed for the creation of increasingly advanced tools that alleviated humanity from toil and despair, but on the other, it trapped them in artificial and unnatural mechanisms.

The new man had to face the world as a wanderer[V], following a path without a defined final destination but marked by well-defined stages. Every achieved goal was merely a provisional phase, governed by the interaction between Delos and Adelos: what is obtained is temporary and limited by Adelos.

The common condition is to focus on a specific and absolute result, forgetting that it is part of a broader set of possibilities. If the goal is one hundred but ninety is reached with an Adelos of ten, there is no need to feel disappointed, because a result of one hundred and ten would have the same value[VI]. There is no reason to be anxious to obtain more if that "more" already belongs to our Delos and Adelos.

Ione had reached the pinnacle of his journey. He was no longer just the guardian of the balance between light and shadow, but the bearer of a new vision, capable of embracing the complexity of the world with serenity and understanding. And with this awareness, Ione prepared to share his wisdom with the world, knowing that his journey was just the beginning of a new era of understanding for humanity.

The dialogue with Aristeo

25 With this new awareness, Ione felt the desire to share his discoveries and engage in conversation with other enlightened minds. He decided to embark on a series of journeys, no longer as a noble in search of conquests or a rebel on the run, but as a seeker of wisdom, eager to explore the many facets of the world and knowledge.

During these journeys, Ione visited many renowned places for their wisdom and knowledge. Among them, the Academy caught his attention the most. Here, philosophers and scholars from every discipline gathered to discuss and share their ideas in an environment that seemed to harmoniously blend the aspects of Delos and Adelos that Ione had learned to recognize and appreciate.

It was during one of these journeys that Ione had the opportunity to meet a renowned empirical mathematician, Aristeo. On a bright autumn morning, Ione was sitting on a stone bench in the Academy's gardens. Next to him sat Aristeo, known for his deep understanding of mathematics and his dedication to the pursuit of truth. Aristeo gazed serenely at the landscape, while Ione, his mind still in turmoil from his recent discoveries, tried to organize his thoughts.

After a long silence, Ione gathered his courage and spoke:

Ione: "Master, may I ask you something that has been troubling me for some time?"
Aristeo: "Of course, Ione. What is it?"

Ione: "Mathematics... we often consider it the queen of sciences for its precision and beauty. It is the light that guides us toward understanding the laws of Nature. But recently, I wonder if we are overlooking something essential. You see, I have been reflecting on a concept... Delo and Adelo." Aristeo narrowed his eyes, intrigued.

Aristeo: "Delo and Adelo, you say? Are you referring to the concepts of what is clear and defined, versus what is obscure and uncertain?"

Ione: "Exactly. I was thinking about how mathematical thought focuses so much on Delo, on what is visible and clear. But what about Adelo, about uncertainty? It's a part of reality we often ignore, or worse, try to eliminate. But perhaps it is precisely there that a part of the truth hides."

Aristeo looked at him with a gentle smile, as if recognizing the seed of an important reflection in those words.

Aristeo: "It is true, mathematics has always been associated with clarity and precision. We are fascinated by it because, through it, we see the structure of the universe, ordered and understandable. However, uncertainty, as you call it, is not entirely foreign to mathematics. Think of Archimedes' combinations, Hero of Alexandria's theories of games, or Epicurus' atomic collisions."

Ione: "Yes, but even in those cases, we try to reduce uncertainty to something measurable, something quantifiable. We see it as a flaw to minimize, not as a fundamental aspect to fully understand. Empirical numbers, for example, represent this duality: Delo, the nominal value, is our guiding light, but Adelo, the uncertainty, is treated almost as a nuisance, something we must endure and limit to a corner."

Aristeo stroked his beard thoughtfully.

Aristeo: "I understand what you mean. Delo represents perfection, the mathematical truth that shines brightly, while Adelo reminds us of our limits, the shadows that accompany every measurement, every theory. But don't you think uncertainty can have its own kind of beauty? Perhaps it does not dazzle us like Delo, but it teaches us humility, reminding us that not everything can be understood or defined."

Ione: "Exactly, master. That's the point. We mathematicians have become accustomed to thinking of uncertainty as an enemy, but I'm starting to see it as an essential part of reality. Perhaps we are so dazzled by the light of Delo that we fail to notice that it is precisely in the shadow of Adelo that unexplored potentials lie hidden."

Aristeo looked at him with approving eyes.

Aristeo: "Are you suggesting that we should embrace this duality more balanced? That we should seek not only to illuminate the world with Delo, but also to live with the depths of Adelos?"

Ione: "Exactly, master. We cannot pretend to understand the fullness of existence by only looking at what is visible. Mathematics is only Delos if we do not recognize the importance of what escapes our understanding, of what is potentially unexplored. And perhaps, only by accepting this duality, we can move closer to a fuller understanding of the world."

Aristeo remained silent for a moment, watching the leaves fall gently from the trees. Aristeo: "This is a profound reflection, Ione. It reminds us that, as much as we may approach the truth with our logic and numbers, there is always a margin of mystery, of the undefined. Perhaps, in the end, the true beauty of knowledge lies precisely in this: in knowing that for every light, there is a shadow, and that both are part of the same picture." Ione nodded, feeling understood.

Ione: "Perhaps it is time to develop a new kind of mathematics, one that is not afraid of the shadow, but embraces it, recognizing that uncertainty is not a weakness, but a potential."

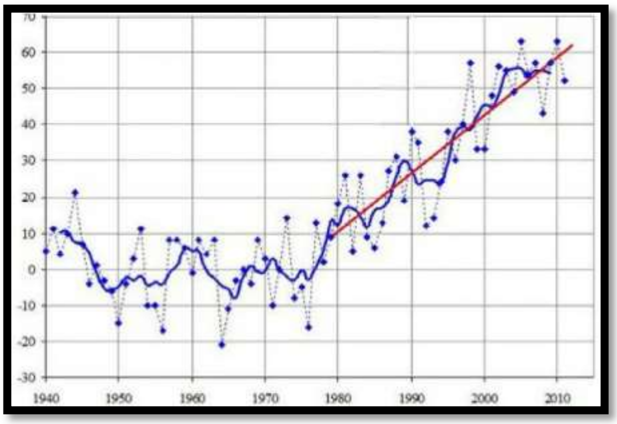
Aristeo smiled, proud of seeing his student surpass the master.

Aristeo: "You are on the right path, Ione. Remember, mathematics is not only a reflection of reality, but also a tool to explore it. And exploration never stops at the surface."

And with that, the two of them remained gazing at the horizon, aware that a new journey of discovery had just begun.

➢ Definition of Empirical Numbers.

Introduction.



In the field of probability and applied sciences, random variables represent a crucial tool for modelling uncertainty. A random variable assigns a numerical value to every possible outcome of a random event, allowing for a mathematical description of complex phenomena such as experimental results, financial market fluctuations, or the lifespan of a product.

Random variables are widely used in statistics, physics, finance, engineering, and many other disciplines. They can model discrete events, such as the roll of a die, or continuous events, such as measuring a physical quantity subject to fluctuations. In this context, tools like the probability distribution function, the probability density function, and the cumulative distribution function are essential for describing the distribution of probabilities across the various possible outcomes.
Mathematical operations on random variables, such as addition, multiplication, and differentiation, require advanced techniques such as convolutions, integral transforms, and stochastic calculus. The latter, in particular, is used to analyze processes that evolve randomly over time, such as Brownian motion in physics or fluctuations in financial markets. However, these techniques can be complex and require a high level of mathematical abstraction.

Despite their power, the use of random variables presents some practical difficulties. Operations like the sum or product of random variables often require complex convolutions, which can be difficult to interpret, shifting the focus from the conceptual aspect to the technical one. Additionally, in the case of continuous variables, differential calculus requires complex tools like the Ito derivative, further complicating the analysis.

In many real-world applications, such as experimental physics or engineering, one often deals with data that has intrinsic uncertainty. However, managing this uncertainty through the formalism of random variables can be overly complex, especially when the uncertainty is treated qualitatively rather than quantitatively. For example, one might wonder how uncertainty propagates in operations such as the sum of two segments without delving into detailed calculations related to the specific distribution.

To overcome these limitations, I propose the use of empirical numbers, a new approach that integrates uncertainty directly into the structure of the number itself. An empirical number consists of two parts: Delos, which represents the exact or nominal value of a quantity, and Adelos, which reflects the uncertainty or potential variability associated with it.
This dual structure allows for a clear representation of both the determined value of a quantity and its uncertainty, simplifying many mathematical operations and enhancing intuitive understanding.

Let’s now look at the advantages of using empirical numbers.

29 Clear Separation of Components: By explicitly separating the nominal value (Delos) from the uncertainty (Adelos), empirical numbers offer a transparent framework for managing quantities in the real world. This clarity simplifies operations like addition, multiplication, and differentiation, directly accounting for uncertainty without the need for complex probabilistic methods, allowing us to maintain focus on the entity in its original entirety. Adelos is not just an uncertainty to be reduced, but also a potential to be enhanced.

Enhanced Ontological Capacity: Beyond their mathematical utility, empirical numbers offer a richer ontological perspective. The distinction between Delos and Adelos reflects a more nuanced understanding of reality, where measurable quantities are not simple fixed points, but entities with intrinsic variability. This duality captures both the tangible and potential aspects of a magnitude, making empirical numbers a more complete tool for representing the world.

Simplified Calculations: Integrating uncertainty into the definition of empirical numbers allows for simplified operations compared to traditional methods. For example, in the addition of two empirical numbers, uncertainty propagation follows predefined rules, eliminating the need for convoluted transformations. This approach not only streamlines calculations but also makes them more intuitive, especially in fields like experimental physics and engineering.

Adaptability to Quantum Computing: With the advent of quantum computing, empirical numbers could adapt particularly well to these new computational paradigms. Quantum computers, which inherently operate with superpositions and probabilistic states, could manage empirical numbers naturally, further simplifying complex calculations and improving computational efficiency.

In conclusion, the introduction of empirical numbers, with their clear distinction between Delos and Adelos, represents an important step forward in the way we manage and understand uncertain quantities. This approach is in no way intended to replace or negate the use of random variables, which remain fundamental tools for dealing with probabilities. However, on an ontological level, I believe that empirical numbers and their subsequent treatment offer a more complete and powerful framework for representing the complexity of the real world. By embracing empirical numbers, we can achieve greater precision and understanding in both theoretical and applied contexts, paving the way for new advancements in mathematics and science.

The empirical number

In the following discussion, we will introduce a new class of numbers, called Empirical Numbers, which represent a formal extension of real numbers and possess notable properties. We will denote this class with the symbol $\mathbb{E}$. These numbers are introduced to represent entities composed of an exact part and an uncertain part. The exact part could, for example, represent the average of an entity, while the uncertain part could express its variability.

31 An empirical number is an ordered pair of real numbers (d, a), where d and a are real numbers. An empirical number can be written in the form $\boldsymbol{d}_{(\boldsymbol{a})}$, where $\boldsymbol{d}$ represents the nominal value and a identifies the uncertainty.

The part **d** is called Delos (Δήλος), a Greek term meaning "visible" or "clear." The name reflects the concept of a "bright and sacred place," as well as its mathematical and symbolic properties of exactness, evoking the "realm of hyperuranic ideas." Delos can also be considered as the inner light of being, guiding the understanding of Nature through the identification of its regularities.
The part **a** will be called Adelos (ἀδήλος), a Greek adjective meaning "undefined," "obscure," or "uncertain." This term describes something unclear or ill-defined, emphasizing its properties of indeterminacy. Adelos is that portion of being that eludes rational understanding and is traced as error and ignorance.
The part **a** will also be referred to as "potentiality" for reasons that will be clarified later.

An empirical number can be graphically interpreted as a segment whose endpoints are blurred:

An empirical number, when representing a physical variable, has the two quantities **a** and **d** expressed in the same unit of measurement.

Examples of quantities expressed with empirical numbers are:

- $25_{(3)}°C$: the conditioned temperature of a room is 25°C with a variability of ±3°C.
- $180_{(2)}cm$: a person's height varies depending on posture by ±2 cm.
- $1.5_{(0.2)}hours$: the duration of a trip, indicating that the average trip time is 1.5 hours (equivalent to 1 hour and 30 minutes) with a variability of ±0.2 hours. In other words, the trip time may vary between 1.3 hours and 1.7 hours.
- $50_{(5)}$€: the price of a service, which can range from 45 to 50 euros.

32 A measurable quantity includes both an expected part, such as the average height of a population, and a part indicating the variability around this expected value, such as the range of height variation within the group. Variability can be described using different distribution characteristics and confidence intervals. In this discussion, we will consider among others a Gaussian distribution (normal curve) as a model for variability, without losing generality, as various distributions (e.g., uniform or Weibull) behave analogously in the study of variability propagation. As can be easily observed, distributions different from the normal, such as the uniform or Weibull distributions, lead to different values for the standard deviation but exhibit identical behaviour under the operations we will describe, allowing for increased generality in our considerations.

A key aspect is that a certain entity, such as a person in a population, has an expected height value and an actual measure that rarely coincides exactly with the expected value, but falls within the predicted variability range. Therefore, the term Adelos represents the potential of height, expressed through a specific measure.
The empirical number does not express a defined and exact quantity, but represents a quantity (Delos) and its corresponding potential for expression within a defined range (Adelos).

Let us now make a practical comparison between random variables and those expressed by empirical numbers to highlight their peculiar characteristics and advantages. In the following sections, some computational aspects briefly mentioned here will be discussed in detail.

Now, let's examine the product of two random variables. For simplicity, we will consider the case in which the variables are independent.
First, the expected value is calculated in the classical way, as if they were any real numbers:

$$E(X \bullet Y) = E(X) \bullet E(Y)$$

The calculation of variance becomes more complex. If X and Y are independent random variables, the variance of their product does not follow a simple rule; however, it can be expressed using the following formula:

$$Var(XY) = E(X^2) \bullet E(Y^2) - [E(X) \bullet E(Y)]^2$$

Assuming two independent random variables with the following properties:

$E(X) = \mu_X$

$E(Y) = \mu_Y$

$Var(X) = \sigma_X^2$

$Var(Y) = \sigma_Y^2$

$E[X^2] = \mu_X^2 + \sigma_X^2$

$E[Y^2] = \mu_Y^2 + \sigma_Y^2$

From which $E[XY] = E[X] \bullet E[Y] = \mu_X + \mu_Y$

While $E[(XY)^2 = E[X^2 \bullet Y^2] = E[X^2] \bullet E[Y^2] = (\mu_X^2 + \sigma_X^2)(\mu_Y^2 + \sigma_Y^2)$

Finally, the variance of the product is calculated as:

$$Var(XY) = E(X^2) \bullet E(Y^2) - [E(X) \bullet E(Y)]^2$$

$$Var(XY) = (\mu_X^2 + \sigma_X^2)(\mu_Y^2 + \sigma_Y^2) - (\mu_X \mu_Y)^2$$

From which $$Var(XY) = \sigma_X^2 \sigma_Y^2 + \mu_X^2\, \sigma_X^2 + \mu_Y^2\, \sigma_Y^2$$

Which, to have everything in the same units, becomes:

$$\sqrt{Var(XY)} = \sqrt{\sigma_X^2\sigma_Y^2 + \mu_X^2\,\sigma_X^2 + \mu_Y^2\,\sigma_Y^2}$$

These lengthy calculations and the associated formalism can be simplified, as will be shown in the subsequent chapters, by considering conditions of isovariability and calculating the norm of the gradient of the function:

$$XY = XY_{(\|\nabla(XY)\|)} = XY_{(\sqrt{X^2+Y^2})}$$

Or, for example, in the calculation of the sum of m equal segments, the expression simplifies to:

$$x1_{(a)} \hat{+} x2_{(a)} \hat{+} \ \dots \hat{+}\ xm_{(a)} = \sum_{m}^{1} xm_{(\sqrt{m}a)}$$

The use of empirical variables allows for a powerful symbolism, reducing the phenomenon we can define as "mental myopia." This mental myopia is akin to that of someone, despite wearing corrective lenses, obsessively focusing on details, fragmenting reality into small, well-defined pieces. Just as the nearsighted person sharpens their close vision at the expense of the overall view, analytical study tends to isolate individual aspects of a problem, neglecting connections and the broader context. The details emerge clearly, but their link to the whole fades into the distant horizon.

Random variables represent how probabilities are distributed in a sample space, while variables expressed by empirical numbers are more representative of the uncertainty around an expected value; the perspective is quite different. The empirical number is focused on the value, the *Delos*, and the uncertainty of its localization in the sample space, while the random variable provides a global view of probability density, especially in the case of continuous random variables.

In the traditional conception of random variables, uncertainty is treated as an external element to the entity being studied. The underlying idea is that of a betting game: uncertainty is seen as a contingent factor, like a roulette in which we bet on an outcome, attempting to predict future results through probability. While this view is useful in many statistical contexts, it is reductive as it reduces the concept of chance to an external dynamic that does not capture the essence of the entity itself.

The traditional probabilistic representation, embodied by random variables, sees chance as a set of possible outcomes of a phenomenon, where each outcome is associated with a probability. This approach, however, risks flattening the concept of chance into a logic of prediction and betting. It reduces the phenomenon to a mere issue of percentages and probabilities, a forecasting game where chance is seen as a disturbance that alters an ideal behaviour, rather than as an intrinsic property of the world.

In the view of empirical numbers, chance is not an external entity to the object in question, but rather an intrinsic quality of the entity represented by the *Delos*. The *Delos*, representing the expected value of a magnitude, cannot be separated from its *Adelos*, the uncertainty that surrounds it and represents its variability. This view is much more significant, as it reflects an idea of chance as a necessity rather than as a contingency. Uncertainty is not something we add ex post to model the random behaviour of a variable, but an intrinsic characteristic of reality.

From this ontological conception arises a different approach to the propagation of uncertainty. While random variables focus on probability distributions and density functions, empirical numbers shift the focus to how uncertainty propagates through operations on the variables themselves.

Random variables, with their focus on probability distributions, provide a global view of random behaviour. They outline a broader picture describing how various outcomes might be distributed across a sample space. In contrast, empirical numbers offer a local and operational view of uncertainty. The focus here is on the expected value, the *Delos*, and the *Adelos* as an expression of how uncertain the localization of this value is within the sample space. The probability distribution becomes secondary, as uncertainty propagates through operations independently of it.

In summary, the shift from the traditional probabilistic view of random variables to the view of empirical numbers involves a profound change in how we think about chance and uncertainty. The traditional view reduces chance to a betting game, while the view of empirical numbers integrates it into a deeper and more intrinsic understanding of reality.

The measure.

The measurement process is the operation through which an empirical number, initially composed of an exact part (*Delos*) and an uncertain part (*Adelos*), is reduced to a unique and determined value.

During measurement, the *Adelos* component, representing the uncertainty or potential variability of the empirical number, is reduced to zero, leaving only the *Delos*. In other words, measurement eliminates the potential variability associated with the quantity, producing a classical and unique value, which can be treated with traditional mathematical techniques.

For example, a temperature measurement initially expressed as $25_{(2)}°C$, meaning with an uncertainty of ±2°C, is reduced to $23{,}983_{(0)}°C$ after measurement, or alternatively to $26{,}151_{(0)}°C$, indicating that the value determined by the measurement no longer has uncertainties associated with it.

This does not imply that measurement is intrinsically free of uncertainty, but rather that the act of measurement provides a single value that, by its nature, reduces *Adelos*, the potential uncertainty.

Empirical symbolism enables effective treatment of quantities, synthesizing within the symbol both *Delos* and *Adelos*, which together represent the entire range of variability of the variable. This variability is articulated in possible values through its potential, expressed by *Adelos*.

Measurement is an event within a process that determines the manifestation of potentiality in a specific, determined value, fundamentally random, within the variability range defined by *Adelos*. The measurement process ensures that, starting from the potentialities expressed by the empirical number, a specific value emerges within the range defined by *Adelos*, with the latter being completely reduced.

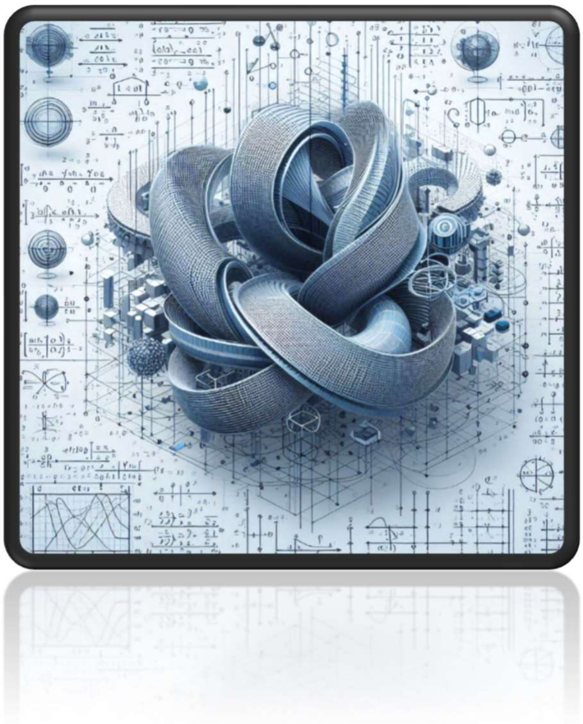
$$\boldsymbol{d}_{(a)} \xrightarrow[\boldsymbol{Measure}]{\textbf{(1)}} \boldsymbol{d}_{(0)}$$

This does not mean that the measurement event is free from absolute uncertainty, but rather that from this event emerges a specific numerical value, which can be handled with classical tools.

37 We can imagine the measurement process as a mechanism that extracts a specific real number from the range of variability defined by the *Adelos* of an empirical number. Measurement thus becomes the event that transforms an indeterminate potential into a determined real value.

An effective image of this process is that of a spherical die with an almost infinite number of faces. Each face represents a possible value, some of which occur with different frequencies, reflecting the probabilities associated with them. Measurement is, in this sense, an act of extraction from the realm of potentials: a single value emerges from among infinite possibilities, shaped by the play of probabilities but confined within the limits imposed by physical reality.
This value, though derived from an element of randomness, manifests as a real and tangible number, ready to be handled and understood within the context of classical mathematics.
Although the individual values seem real and unique, they are in fact part of a broader set of possibilities, represented by all the faces of the die. Each emerging value, while appearing as a distinct entity, remains tied to an underlying universe of potentialities that allowed it to emerge.

The collection of different measurement events allows us to obtain distinct values $\boldsymbol{d}_{(\mathbf{0})}$ which, when considered together, can reconstruct the original space of the empirical number $\boldsymbol{d}_{(\boldsymbol{a})}$. The reverse process, from the manifestation of individual measurements to the reconstruction of the original empirical number, is conceptually possible. Through the collection of a series of measurements, we can estimate both the *Delos* (the nominal value) and the *Adelos* (the uncertainty). This process requires statistical techniques such as calculating the mean and standard deviation, or the range, to reconstruct an empirical number that accurately represents the measured quantity, even accounting for its intrinsic uncertainty.



To formalize a probabilistic theory based on empirical numbers, two phases are followed: the first concerns the definition of the domain of empirical numbers and the morphisms that operate on it, while the second focuses on the transition towards a codomain of traditional numbers through probabilistic measures.

Let an empirical number d_a be defined as an ordered pair composed of an exact part (Delos) and an uncertain part (Adelos):

$$d_a = (x_d, x_a)\ with\ x_d \in \mathbb{R}, x_a \in \mathbb{R}^+$$

- Let x_d represent the exact value (deterministic part, Delos).
- Let x_a represent the uncertainty or variability associated (stochastic part, Adelos).

Therefore, the domain D of empirical numbers is given by the set::

$$D = \{(x_d, x_a) \mid x_d \in \mathbb{R}, x_a \geq 0$$

Let us define morphisms $\phi: D \to D$ that transform empirical numbers into other empirical numbers. These morphisms preserve the structure of empirical numbers, meaning they act on both the Delos part and the Adelos part.

For example, if we consider an addition operation between two empirical numbers

$$\phi_+(x_a, y_a) = \left(x + y, \sqrt{1^2 + 1^2}\right) = \left(x + y, \sqrt{2}\right)$$

The Delos part follows the usual rules of addition.
The Adelos part follows the quadratic sum, similar to error propagation, where the total uncertainty increases as the square root of the sum of the squares of the individual uncertainties (these calculations will be addressed in the next chapters).

Other morphisms can include operations such as multiplication, subtraction, division, and so on, with similar rules for the propagation of the Adelos part.

Generalization: for a generic operation $f(x_a, y_a)$ between two empirical numbers, the transformation is expressed as:

$$\phi_f(x_a, y_a) = \left(f_d(x, y), g_a(f_d, x, y)\right)$$

where $g_a(f_d, x, y)$ represents a function that describes how the uncertainty propagates as a function of the operation f. These operations will be addressed in the following chapters.f.

40 Let's introduce a measurement theory that allows the transition from a domain consisting of empirical numbers to a codomain of traditional (measurable in a probabilistic sense) real numbers.
We define a measurement morphism $\mu: D \to \mathbb{R}$ that "collapses" the empirical number into a real value. This morphism can be based on a probability distribution associated with the Adelos part:

$$\mu(d_a) = \mathbb{E}(\mathrm{d} + \xi) \; with \; \xi \sim P(0, ka)$$

Where ξ is a random variable distributed according to a probability distribution P with mean 0 and standard deviation equal to ka , which represents the uncertainty.

In this way, the measurement of an empirical number becomes a real value that incorporates both the Delos value and the Adelos uncertainty, expressed in the form of a probability distribution. The probabilistic theory applied to the Adelos part can follow the rules of classical probability, with distributions associated with the variability.

The codomain obtained through the measure morphisms μ will be a set of real numbers $\mathbb{R}$, which can be studied according to the rules of probability. Each empirical number is mapped to a real number according to its probability distribution, allowing the use of standard techniques in probabilistic analysis.

In general, the measure morphism can be defined as:

$$\mu : (x_d, x_a) \mapsto \mathbb{E}(X)\ con\ \ X = x_d + \xi\,, \quad \xi \sim D(x_a)$$

where $D(x_a)$ represents a distribution related to the uncertainty x_a, which may vary depending on the situation (normal, uniform, etc.).

Once the codomain $\mu(D) \subseteq \mathbb{R}$ is obtained, we can apply classical probabilistic theory to study the distribution of the values $\mu(d_a)$. This include:

- Analysis of the resulting distributions (mean, variance).
- Operations on distributions. Statistical and probabilistic inference on the initial empirical system.

This formalization allows us to study empirical numbers both in terms of their deterministic value (Delos) and their uncertainty (Adelos), through morphisms that preserve the empirical structure. Moreover, the measurement morphisms enable the transfer of empirical numbers to traditional numbers, linking them to a classical probabilistic theory based on measurement rules and distributions.

We now define the inverse measurement morphism, which, starting from a traditional real number, returns to the domain of empirical numbers as an estimate. We can introduce a map μ^{-1} that reconstructs the empirical number d_a from a real number $r \in \mathbb{R}$.

The inverse morphism $\mu^{-1}\colon \mathbb{R} \to D$ must take a real value r (which can represent an observed estimate, an average value, etc.) and return an empirical number (x_d, x_a), i.e., d_a, where:

- x_d is the estimated exact value (Delos).
- x_a is the estimate of the associated uncertainty (Adelos).

The inverse morphism can be defined as follows:

$$\mu^{-1}(r) = (r, \hat{x}_a)$$

where $r \in \mathbb{R}$ is the real value, and $\hat{x}_a$ represents an estimate of the Adelos uncertainty, derived based on various considerations related to the context of the problem (such as the variance of measurements, standard error, or an estimated probability distribution).

To estimate $\hat{x}_a$, we can rely on various sources:

- Observational data: If we have access to a sample of measurements, x_a can be estimated by calculating the standard deviation of the measurements. In this case, $\hat{x}_a$ could be defined as the standard deviation σ of the measurements:

$$\hat{x}_a = \sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(r_i - r)^2}$$

where r_i are the observations, and r is the mean.

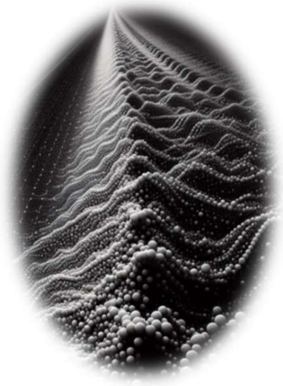
In sampling situations, $\hat{x}_a$ could be based on the standard error of the mean, calculated as:

$$\hat{x}_a = \frac{\sigma}{\sqrt{n}}$$

where σ is the sample standard deviation, and n is th sample size.

If empirical data is unavailable, we can assume a probability distribution for the uncertainty, such as a normal distribution $N(0, \hat{x}_a)$, and estimate $\hat{x}_a$ based on an initial hypothesis or a known distribution.

The inverse morphism is not necessarily a perfect inversion (i.e., a one-to-one correspondence), as empirical numbers include an uncertainty component that cannot always be fully determined from a single real value. The map μ^{-1}may return a range or a set of possible empirical numbers, with varying levels of Adelos, based on the additional information available.
In practice, the estimation of uncertainty depends on the data and context, but the general structure of the μ^{-1} map follows the same concept: a real value is reconstructed into an empirical number consisting of a Delos estimate and an Adelos estimate.



The following theorem demonstrates how empirical numbers can be used to understand and prove the continuity of real numbers, a fundamental concept in mathematics.

Theorem: *The set of real numbers $\mathbb{R}$ is continuous. The continuity of real numbers can be demonstrated through the use of empirical numbers, where every finite interval of real numbers can be represented by empirical numbers, and the process of measuring empirical numbers produces specific real numbers within the variability range defined by Adelos.*

Definition and Notation

1. Empiric Number: An empirical number is a pair (d, a), where:
   - d (Delos) is the nominal value and represents the exact part.
   - a (Adelos) is the uncertainty or variability and represents the potential of the quantity, expressing the range of variation around d.

An empirical number can be written as $d_{(a)}$, where a can be any non-negative real value representing the variability range.

2. Measurement: during the measurement of an empirical number, specific real value is obtained within the interval [d−a,d+a]. Although the precise value depends on the measurement, it is always within the range defined by a:

$$\boldsymbol{d}_{(\boldsymbol{a})} \xrightarrow[\boldsymbol{Measure}]{} \boldsymbol{x}$$

where x is a real number such that d−a≤x≤d+a.
This process allows the potential (Adelos) to be concretized into a specific real value.

Since every empirical number $d_{(a)}$ with a>0 covers a continuous interval of real values, we can say that the measurement process is capable of "filling" the interval [d−a,d+a] with specific real values through a progressive process.

Proof:

1. Density through the Measurement Process: Consider a finite interval [α,β] on the real number line, where α<β. Every real number x within this interval can be seen as the result of a measurement of an empirical number $d_{(a)}$, with d∈[α,β] and a representing a variability range.
   - Local Density: Given a real number x_1 in the interval [α,β] and another real number x_2 arbitrarily close to x_1, we can always find an empirical number $d_{(a)}$ such that the measurement process generates a real number between x_1 and x_2. This is possible because the Adelos 'a' can be chosen to cover an interval large enough to include every value between x_1 and x_2.
   - Filling of Intervals: The measurement process allows the complete filling of the interval [α,β] with specific real values generated by empirical numbers. Every finite interval, no matter how small, can be covered by real numbers derived from the measurement process of empirical numbers, thus demonstrating the density of the set of real numbers.
2. Continuity of Real Numbers: The continuity of real numbers emerges as a direct consequence of the measurement process. Since every finite interval can be filled with real numbers, and since the measurement process leaves no gaps, we can assert that the real number line is continuous. There are no empty spaces between real numbers, and every point on the line can be covered through empirical numbers.

The continuity of real numbers is seen as a manifestation of the reduction of variability.

The theorem demonstrates that the set of real numbers is dense, continuous, and infinite by using the process of measuring empirical numbers. The density of real numbers follows from the fact that the measuring process fills every finite interval with specific real values. This continuous filling, combined with the ability of the measurement process to cover every interval without leaving gaps, proves that the set of real numbers is infinite and continuous.

➢ ***The Circle and the Stones.***

On a tranquil afternoon of a sunlit Hellenistic day, Ione found himself near a circular pond situated in the gardens of the Academy. The pond, artificial in nature, had been constructed using taut ropes by the geometers.

The water shimmered under the sun, its calm surface reflecting the azure sky like a mirror. Ione gazed attentively at the scenery, while village children played near the pond's edge. Amid their laughter and the sound of stones being cast into the water, an idea struck him.

The children, delighting in their game, threw stones into the pond, creating concentric ripples that spread outward until they vanished. Ione observed their play with a sense of wonder. He imagined using the pond as an ideal circle, with the square enclosing it as the foundation for a mathematical experiment. He thought, if he could, he would cast stones into the pond to calculate the ratio between the circle's circumference and its diameter—and he did so, at least in his mind.

The pond, indeed, was inscribed within a square. Ione envisioned considering the number of randomly thrown stones that landed inside the circle compared to the total number of stones cast within the encompassing square. To conceptualize this, he imagined having a large number of stones and throwing them, just as the children were doing.

Engaging the children, he set the conditions for the game-experiment and had them throw 1,000 stones. He observed that 785 of them fell within the circle. Using this approach, he calculated the ratio of the circle's circumference to its diameter. Although the value he obtained was not exact, it was close to the true value. Ione realized that with more stones, the result would approach even closer to the real value.

Ione speculated that the calculated ratio might estimate the quantity of *Delos.* By repeating the game numerous times, he could also estimate the variability of the results, thereby encompassing the *Adelos* as well.

Reflecting on the power of technical acts, Ione mused, "I have become aware of a dangerous tendency in our relationship with *Technē,* and I often wonder if we are not confusing functionality with truth. From my experience, I have seen how modern man, captivated by the speed and complexity of his creations, assigns value to what works, to what resolves an immediate problem. Yet, stopping here, he forgets to ask *why* things work, to explore the profound causes governing mechanisms.

It is as if, in the realm of *Technē,* man has become blind to what is not immediately visible or useful. Take mathematicians, for instance, those who tread the path laid by Euclid. For them, each proof holds value because it is rooted in an axiomatic system, upheld by logical reasoning. But in the technical domain, a demonstration is often deemed valid simply because it 'works' in practice, without questioning whether the solution is but a temporary answer, a compromise with reality, and not a universal truth.

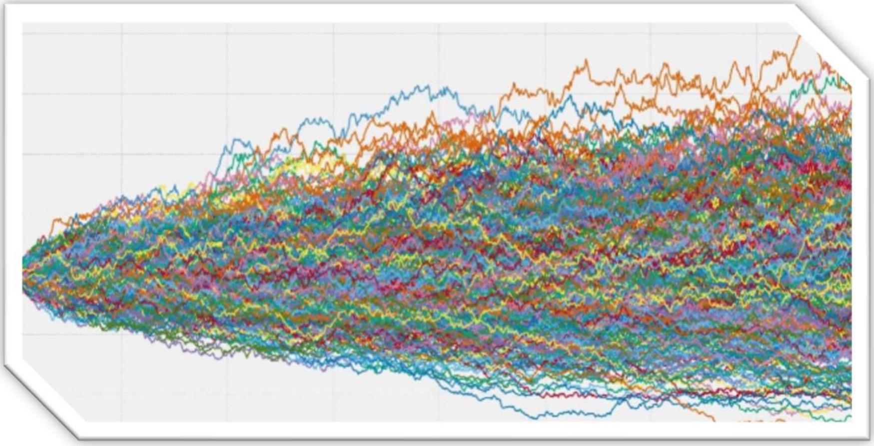
The man of *Technē* appears increasingly guided by this utilitarian logic: 'If it works, it is true.' Yet I, Ione, a descendant of Prometheus, tell you that this mindset is incomplete, for it blinds us to the complexity of the *Adelos,* to the part of reality that remains irreducible, unattainable. A result that works is not in itself an absolute result. It is but a provisional step, a *Delos* confined within a broader range of possibilities. And yet, we seem obsessed with this *Delos,* with what we can quantify and fix, as if we wish to reduce everything to an immutable value.

But what happens if we stop at the 90 we have achieved, failing to recognize that the remaining margin of 10—the *Adelos* that escapes us—represents the other half of reality? We must not anxiously strive to achieve that missing ten, nor surpass the hundred with a 110. The truth is, these numbers, these figures, are but fragments of a greater whole, of an unstable balance between what we can know and what we can never fully grasp.

The new man, the one who emerges as a wanderer in this world of *Technē,* must abandon the notion that what works is necessarily true. Instead, he must embrace uncertainty, the provisional nature of every result, accepting that no definitive truth exists—only intermediate stages of understanding. In this, true knowledge humbles itself, aware that every solution, however brilliant, is only temporary, never final. It is thus that one learns to live with the *Adelos,* no longer fearing it, no longer plagued by the need to dominate all that surrounds us."

As the sun began to set and the shadows lengthened, Ione reflected on the potential of this intuitive method. The simplicity of the children's play had inspired an original approach.
Ione left the pond with a sense of fulfillment, aware that even the simplest daily observations could hold the keys to unlocking great mathematical mysteries.

➢ Monte Carlo Method.[VII]



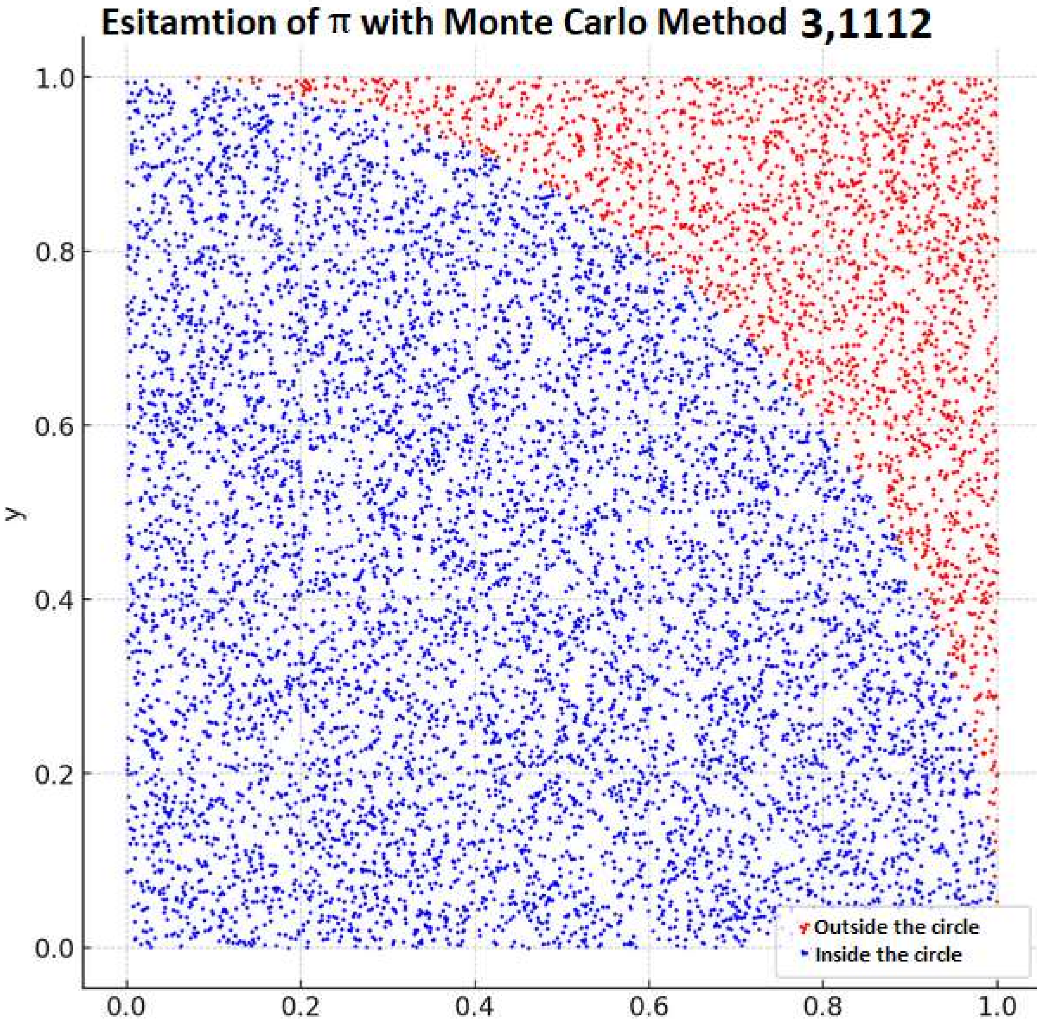
Throughout the discussion, frequent reference will be made to Monte Carlo simulations to more explicitly illustrate certain characteristics under consideration. The Monte Carlo method is a mathematical simulation technique used to solve problems that may be theoretically deterministic but are complex or impossible to resolve analytically due to their stochastic nature or the large number of variables involved. This method is named after the famous Monte Carlo casino, as it relies on probabilistic principles and the use of empirical numbers.

Consider the following example: let us imagine we wish to estimate the value of π using the Monte Carlo method. This can be done by randomly throwing points on a square that contains a quarter circle. By counting how many points fall inside the quarter circle compared to the total number of points thrown, π can be estimated:

1. Generate a large number of random points (x_i, y_i) with $i = 1,2,\dots,n$ where $x_i, y_i \in [0,1]$
2. Count how many points fall inside the quarter circle, i.e., where $x_i^2 + y_i^2 \leq 1$; therefore $k = \sum_{i=1}^{n} \chi\left(x_i^2 + y_i^2 \leq 1\right)$ where the function $\chi(\cdot)$ worth 1 if the condition is true and 0 otherwise.
3. The ratio of the points inside the circle to the total number of points, multiplied by 4, will give an estimate of π, where $\pi \approx \frac{4k}{n}$

Below is a Monte Carlo simulation with n=10000 to estimate the value of π:

- **Blue points**: represent the points that fall inside the quarter circle.
- **Red points**: represent the points that fall outside the quarter circle. The estimate of π is calculated as the ratio of the number of points inside the circle to the total number of points, multiplied by 4. In this case, the estimate of π is approximately 3.11.


Esitamtion of π with Monte Carlo Method 3,1112
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
Outside the circle
Inside the circle

In our simulations, we will consider, for simplicity, normal distributions, also known as Gaussian distributions. It is a continuous probability distribution that can be defined as follows:

(2)

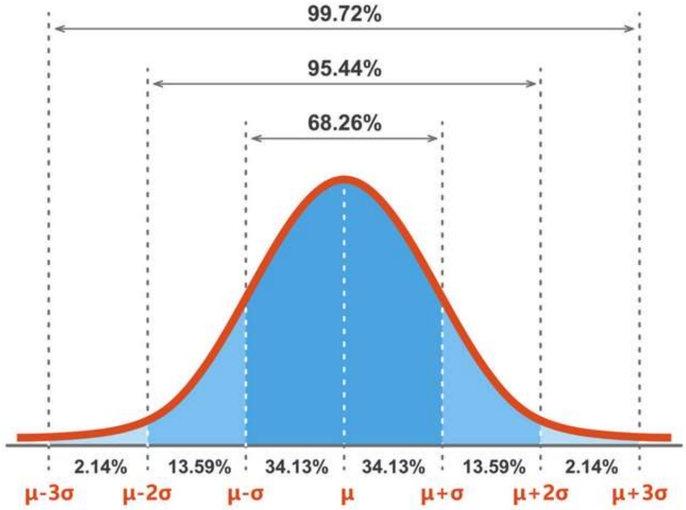
$$d_{(a)} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

dove:

- μ is the mean of the distribution (Delos of the empirical number),
- σ is the standard deviation (Adelos of the empirical number),
- σ^2 is the variance.

Among the main characteristics, we have:

- Bell shape: The density distribution graph has a characteristic bell shape, symmetric around the mean.
- Mean, median, and mode: All are equal and located at the center of the distribution.
- Symmetry: The distribution is perfectly symmetric around the mean.
- Asymptotes: The curve approaches the x-axis asymptotically but never touches it.
  - Approximately 68% of the data falls within 1 standard deviation from the mean.
  - Approximately 95% of the data falls within 2 standard deviations from the mean.
  - Approximately 99.7% of the data falls within 3 standard deviations from the mean.



We now ask to simulate, using the Monte Carlo method, five measurements of the empirical number. $10_{(1)}$:

9.127	10.843	9.562	10.215	10.253

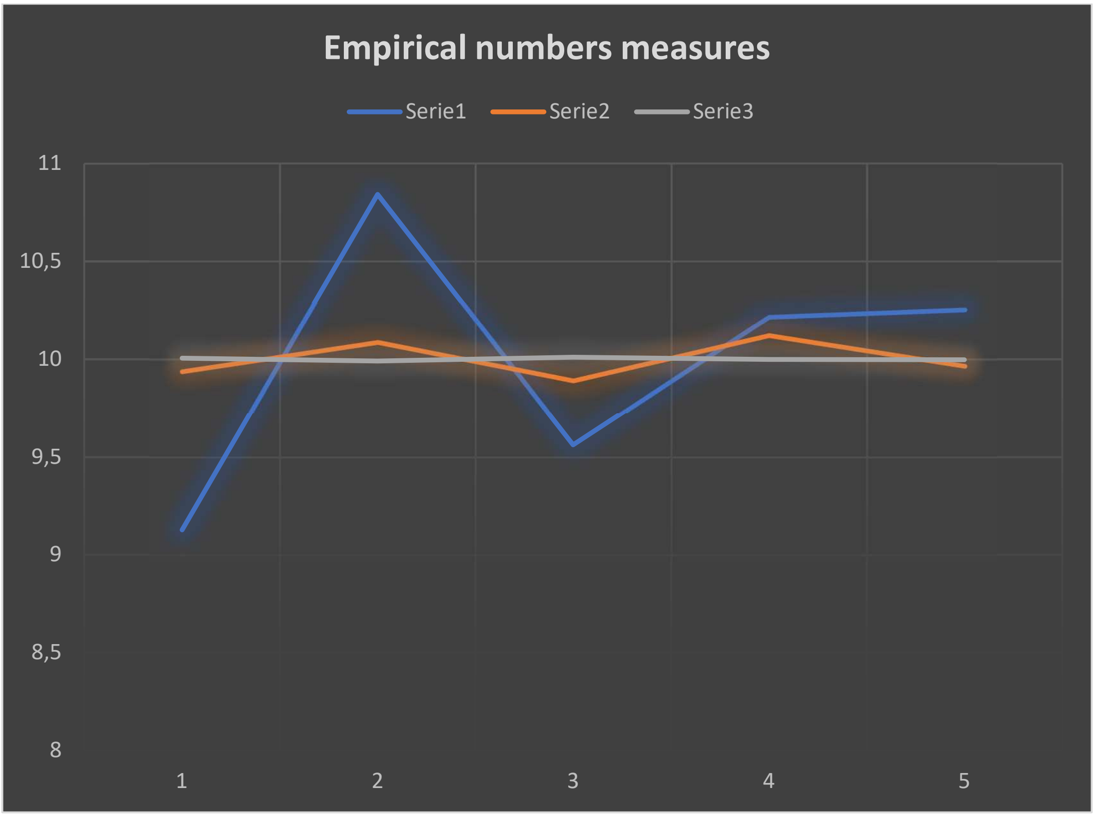
We now evaluate the measurements of an empirical number $10_{(0.1)}$, which is much less variable:

9.937	10.086	9.891	10.121	9.965

And now with the empirical number $10_{(0.01)}$:

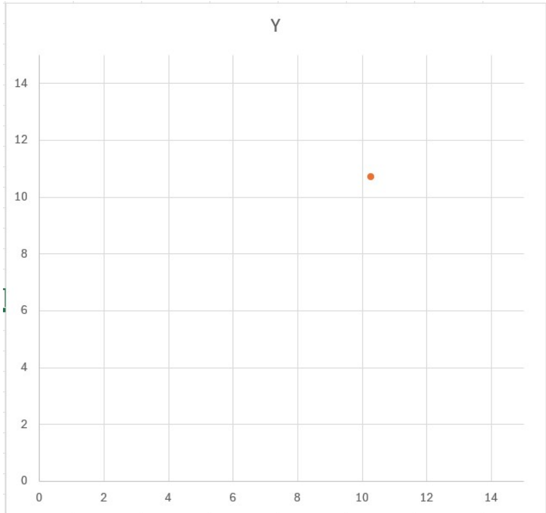
10.005	9.990	10.011	9.998	9.996

Below, the three series are compared:

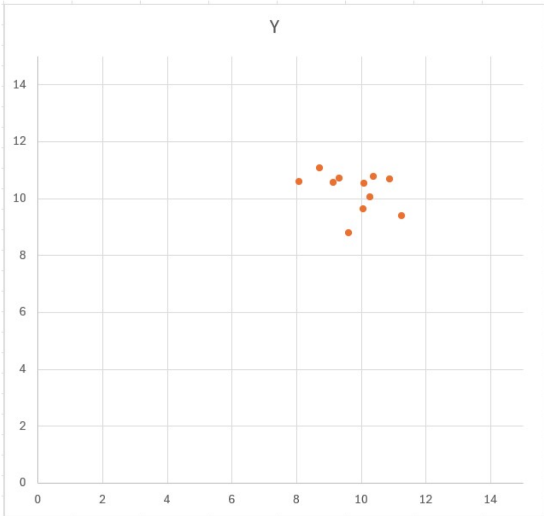


As seen from the graph, the three series present measured values around the Delos, but Series 1 exhibits a much more pronounced Adelos compared to the other two, while Series 3 shows a much lower Adelos than the others.

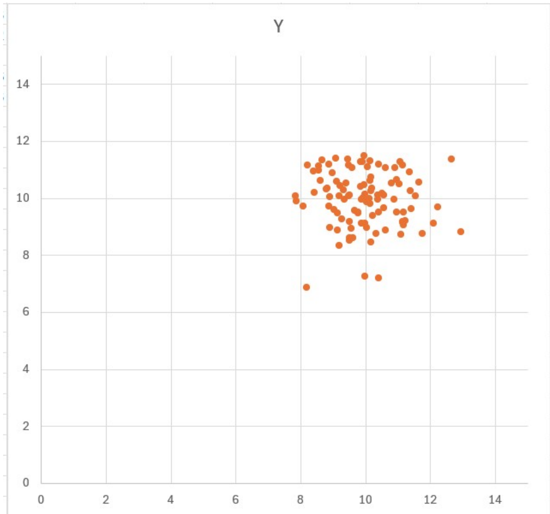
Let us now show a pair of empirical numbers $10_{(1)}$,$10_{(1)}$ in the X,Y plane with different measurements, and starting from these measurements, the estimation of the pair of empirical numbers.



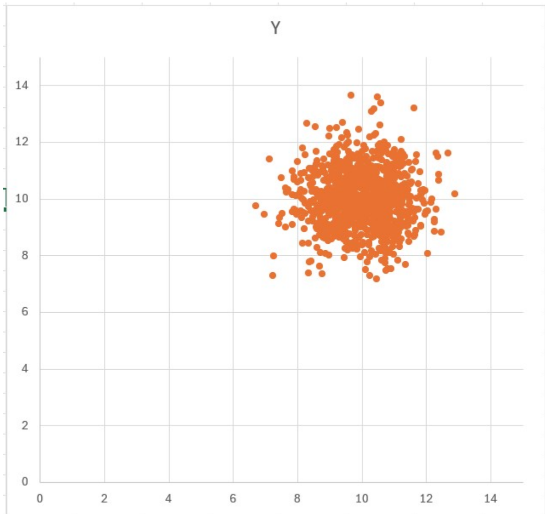
1 measure; $10.267_{(-)}$ $10.698_{(-)}$



10 measures; $9.803_{(0.703)}$ $10.244_{(0.941)}$



100 measures; $9.986_{(1.043)}$ $9.958_{(0.972)}$



1000 measures; $10.013_{(0.983)}$ $9.993_{(1.001)}$

In this context, the Monte Carlo method can be seen as a concrete exemplification of the concept of Delos and Adelos. Each Monte Carlo simulation operates as a mechanism that extracts numerical values from a set of potentials, represented by all possible data configurations within the defined range. In other words, each individual simulation is a measurement event that materializes one of the many potentials in the domain of the real.

In this case, Delos represents the set of possible manifestations of empirical values, while Adelos defines the limits within which these manifestations can occur, as in the case of the standard deviation in a normal distribution. In the Monte Carlo method, the individual values generated, while appearing real and unique, are in fact part of a larger whole, consisting of the entire space of possibilities described by probabilities. Each value emerges from this space, shaped by the play of probabilities and confined by the limits imposed by Adelos.

Furthermore, in Monte Carlo simulations, thousands of values are technically generated through the measurement process. The totality of these values expresses the empirical number in an extremely synthetic way, providing a statistical representation of the entire range of potentialities. The empirical reality, however, often manifests through only a few measurement events, akin to individual fragments of a larger picture, which by themselves fail to provide a comprehensive view. It is like judging the arrival of spring by observing a single swallow: a single event cannot capture the complexity of the whole.

The empirical number, on the other hand, manages to provide this broader view, aggregating the multiple potentialities and allowing us to grasp the overall essence of the phenomenon. Without this broader perspective, the overall design of reality would be lost, hidden behind the apparent randomness of individual measurement events. Only through the accumulation and synthesis of data can we discern the order hidden behind the chaos of individual measurements.

- ***Unveiling the potential.***

Ione, reflecting on the concepts of Delos and Adelos, arrived at profound insights. Mathematics, he thought, is not a perfect reflection of reality, but rather a tool created by human thought to navigate Nature. It is not a flawless mirror of reality, but rather a symbolic system designed to describe, model, and predict natural phenomena that follow specific laws and regularities. Its strength lies in its ability to capture the regularities, the Delos, in the natural world, thus facilitating the resolution of concrete problems.

The distinction between Adelos and Delos suggested to Ione a dualism between chaos, uncertainty, and indeterminacy (Adelos), and order, predictability, and regularity (Delos).

Nature encompasses both aspects, but, he reflected, it has been crucial for human history to focus on Delos—on regularities—due to survival reasons.
It is part of our evolution and the progress of human societies to develop the ability to recognize patterns and regularities. This capacity to recognize patterns and regularities has been a fundamental advantage: it has allowed humanity to anticipate events and make strategic decisions in a world full of unknowns. In fact,

he thought, it is more useful to quickly identify a predator hidden among the leaves than to distinguish every single detail of the leaves themselves.

Human thought, by nature, instinctively tends to discard Adelos—the uncertain, the chaotic, the indeterminate—to focus on Delos, that is, on the regularities and certainties that allow us to understand and control the world. However, within that which is discarded, in Adelos, lies an unexplored creative potential.

This perspective leads us to consider the possibility of going beyond the traditional use of mathematics, which mainly focuses on Delos, and developing new conceptual tools. We might imagine a new arithmetic, a symbolic system capable of operating not only on certainties but also on uncertainties and ambiguities. Through this approach, we could open the way to new possibilities, using Adelos not as something to avoid, but as a resource for innovation, discovery, and the development of both human and non-human knowledge, as well as artificial knowledge.

Some arts, such as poetry and painting, embrace and harness Adelos, using uncertainty, ambiguity, and indeterminacy as tools to evoke emotions, meanings, and interpretations that go beyond the visible and the defined. These art forms do not seek to order chaos but to reveal the beauty and creative potential that reside precisely in ambiguity.

In poetry, for example, the use of metaphors, symbols, and evocative images creates a reality that cannot be reduced to a single interpretation. Words are not chosen to define a concept precisely but to evoke a range of emotions and meanings, leaving space for the reader's imagination. In this way, poetry transcends Delos, immersing itself in Adelos to explore the unexplored, giving voice to what is otherwise ineffable.

Similarly, in painting, Adelos manifests itself through the use of shapes, colors, and perspectives that evade realistic and rational representation. Some artistic forms do not seek to capture the world as it is but to convey a subjective experience, an impression or emotion that can only be perceived through interaction with the unknown and the indefinite. The shades and shadows, which are not always clear or well-defined, create a dynamic that stimulates thought and imagination, inviting the viewer to actively participate in the creation of meaning.

This ability to interact with Adelos through art demonstrates that uncertainty is not merely an element to be eliminated or controlled, but a source of creative richness.

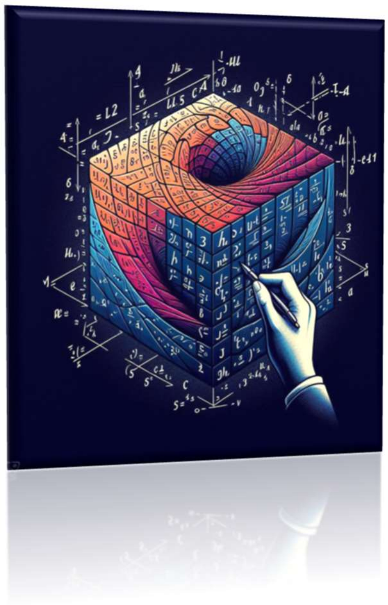
While mathematics and the sciences tend to reduce Adelos in order to extract Delos—the regular and predictable—the arts, on the other hand, embrace and value Adelos, recognizing its fundamental role in the creative process. This suggests that the development of new conceptual tools, such as a new mathematics, could not only expand our understanding of reality but also integrate us more deeply with Adelos. In this way, we could not only solve practical problems but also enrich our experience of the world, exploring new frontiers of knowledge and creativity. Adelos, rather than being discarded, could become a fundamental resource for innovation and intellectual growth, opening pathways toward a way of thinking that is more complex, inclusive, and capable of embracing the complexity of existence.

The idea of a new mathematics that focuses on Adelos is both provocative and stimulating. It suggests a symbolic system capable of operating not only on certainties and regularities but also on uncertainty and indeterminacy. Indeed, this could open new avenues in mathematics and science, allowing us to tackle problems that current mathematical structures do not address effectively. It offers a vision of the relationship between mathematics, nature, and human thought that not only acknowledges the limitations of current tools but also the possibility of expanding them to include aspects of reality that are traditionally set aside. It is a philosophical and mathematical proposal that may merit further exploration.

A central aspect of this new vision could be the theory of error propagation, a concept already well known in mathematics and science, which focuses on how errors and uncertainties are transmitted through calculations and measurements. In science and engineering, error propagation is used to understand how initial imprecisions in measurements or data can influence the final outcome of a process. In a mathematics of the Delos-Adelos duality, error propagation could be seen not only as a problem to minimize but as a field of investigation in itself, where uncertainty becomes an inherent element of reality itself. Instead of considering errors and uncertainties as undesirable deviations from truth, this new algebra could leverage Adelos to explore the implications and potential of these uncertainties. New methods could be developed to calculate and understand how uncertainty spreads and amplifies, and how it can be used to generate new insights and discoveries.

In this context, Adelos is not seen as the enemy of precision but as an intrinsic and inevitable element of nature that, if studied and understood deeply, can reveal hidden aspects of reality and offer new perspectives, both scientifically and philosophically. A theory of error propagation, expanded to include Adelos, could therefore become a cornerstone of this new mathematics, allowing us to address and model chaos and uncertainty in a way that is more faithful to Nature and the limits of both natural and artificial thought.

➢ **Propagation of uncertainty and the Gradient Norm**



Mathematical operations on empirical numbers follow different rules compared to classical ones due to their nature. While the Delos part adheres to the usual rules of real numbers, the Adelos part follows a characteristic mechanism.

The technique developed here for the calculations of Adelos derives from the study of error propagation, based on the following concepts.[VIII]

Consider a function of two or more variables: $f(x_1, x_2, .., x_n)$ where variables $x_1, x_2, .., x_n$ are not correlated, and each of them is associated with its own uncertainty. $\Delta x_1, \Delta x_2, .., \Delta x_n$.
The extended uncertainty Δf of the function f can be calculated from the uncertainties of each variable x_i by computing the partial derivatives of f with respect to them:

(3)

$$\Delta f = \Delta f(x_1, x_2, .., x_n, \Delta x_1, \Delta x_2, .., \Delta x_n) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \Delta x_i\right)^2}$$

The relation (3) holds in most cases where the function involved is sufficiently regular and is adequate for describing the effects of small variations in influencing factors and accidental errors. However, in some cases, there may be strong interactions between factors, which could require the inclusion of higher-order terms in the Taylor series expansion, including mixed terms.

In order to simplify the study of the propagation of Adelos in arithmetic operations, we will consider the isovariable case where $\Delta x_1 = \Delta x_2, = \cdots = \Delta x_n = a$ denoting a as the generic uncertainty treated as the Adelos of the terms of the function, and A as the extended uncertainty, interpreted as the Adelos of the function. In this case, relation (3) becomes:

(4)

$$Af = a \cdot \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i}\right)^2}$$

The function under the square root is nothing other than the Norm of the Gradient of the function under consideration:

(5)

$$\| \nabla f(x) \| = \sqrt{((\partial f/\partial x_1)^2 + (\partial f/\partial x_2)^2 + \cdots + (\partial f/\partial x_\mathrm{n})^2)}$$

In which the terms $\left(\frac{\partial \mathrm{f}}{\partial \mathrm{x_i}}\right)^2$ can be considered as characteristic functions of the sensitivity of the propagation of the Adelos a.

In fact, the arguments will mainly focus on the study of these functions and their influence in various computational contexts.

For brevity, we will denote these terms as D_i and thus relation (4) becomes:

(6)

$$Af = a \cdot \sqrt{\sum_{i=1}^{n} D_i}$$

Relation (6) can be expressed by relation (7) in the non-isovariable case:

(7)

$$Af = \sqrt{\sum_{i=1}^{n} D_i \cdot a_i^2}$$

From the notation introduced for the empirical number, it follows that any physical quantity will be expressed using the notation $d_{(a)}$, where a identifies the Adelos of its value d, representing the Delos.

We will evaluate the Adelos in some computational operations in the following chapters.

- ***The truth, time, and the metrologist.***

The Truth

61 Ione sat by the riverbank, watching the water flow slowly and inexorably. As the sun set on the horizon, tinting the sky with shades of orange and red, he felt a growing desire to understand the mysteries of nature and reality. The river's flow reminded him of time itself, something unstoppable and ever-changing, yet so difficult to grasp in its entirety.

Reflecting on the concepts of Delos and Adelos, Ione found himself contemplating the nature of truth. Often, mathematical truth (Delos) is confused with the truth of the natural world (Delos and Adelos). This is a crucial point, for while mathematics is a powerful tool for modeling and prediction, it does not represent a complete and direct map of reality.

The truth in the context of Delos is characterized by clarity, rationality, and precision. Mathematics is based on axioms and internal rules, and its truth is defined within formal systems that are self-consistent and well-defined. However, mathematical truth is limited by the choice of axioms and the rules of the system itself. It has been shown that in any sufficiently complex formal system, there are propositions that cannot be proved or disproved within the system itself. Despite its apparent certainty, mathematical truth is partial and cannot capture the entirety of experience or reality.

In the natural world, uncertainty and variability are inevitable. There is no absolute time (but only the marking of events), and there are no ultimate truths, for reality is inherently fluid and complex. This reflects an empirical and possibilistic view of knowledge, where truths are more relative and based on degrees of plausibility rather than absolute certainties. Knowledge, in this context, is always evolving and based on observations and inferences, which are subject to change as new understandings emerge.

This perspective implies that in a universe where Delos and Adelos coexist, the search for an ultimate truth—and with it, the idea of a God or an absolute metaphysics—lacks a solid foundation. The implications of this are profound:

The first: in the natural world, no absolute truth exists. Metaphysical and theological claims that seek ultimate truths are therefore problematic. Metaphysics and theology often aspire to reach a definitive and absolute understanding of reality, but, according to this view, such aspirations are fruitless because the natural world is inherently uncertain and indefinite.

The second: in the absence of ultimate truths, the only way to approach an understanding of the world is through the progressive acquisition of knowledge. This implies an empirical and scientific approach, in which knowledge is built and refined over time through observation, experimentation, and critical reflection. Plausibility, therefore, becomes the criterion for evaluating theories and explanations, accepting that all knowledge is provisional and subject to revision. However, the absence of absolute truths does not imply the absence of responsibility. Even though our understanding of the world is always subject to evolution, this does not absolve us from taking responsibility for our choices and actions.

On the contrary, it is precisely the uncertainty of knowledge that imposes a greater ethical and intellectual attention. Every choice we make must be based on the best understanding available at that moment, aware that it may evolve, but also that our decisions have real consequences. Therefore, it is wrong to think that, since no absolute truth exists, there are no moral obligations or responsibilities. Responsibility arises from the necessity to act in the present with the knowledge we possess, recognizing that our actions impact others and the world, even in a context of uncertainty.

In this view, ethics is grounded in the awareness of the provisional nature of our knowledge but also in the responsibility to act in ways that minimize harm and maximize good, even if we can never be completely certain of the consequences. The absence of an ultimate truth does not free us from responsibility; rather, it calls us to a deeper sense of care and attention toward our actions and decisions.

In this view, there is a shift in ethical paradigms: Nature, not Homo Sapiens, must return to the center of our moral reflection. The human being, having become too powerful and now obsolete in a presumptuous and arrogant position of dominance, must rethink its role not as the master of Nature but as its guardian and part of it. The power acquired by humanity through technology and knowledge, if not rebalanced by a profound respect for the natural world, risks leading to destructive consequences not only for the planet but for our own species.

From this perspective, ethical responsibility is not only toward other human beings but toward the entire ecosystem we share, recognizing that the well-being of humanity is inseparable from that of Nature itself.

The concept of truth is often seen as an accurate and definitive representation of reality, a kind of perfect mirror reflecting what exists in the world. However, when we examine the nature of truth more closely, we realize that it is not an objective reality independent of our thought but rather a construct that emerges from the way our mind interacts with the world.

What we call the "truth" of reality is, in fact, the result of an interpretative process. Our mind is not capable of grasping reality in its entirety; rather, it filters, organizes, and structures the information it receives from the senses to create a coherent and comprehensible image of the world. This image is what we call truth, but it is always partial and subjective, for it depends on our ability to understand and on the conceptual models we use to interpret experience.

Truth, therefore, is intimately linked to the structure of our thought. The human mind tends to seek patterns, regularities, and causal relationships to make sense of the apparent chaos of reality. In this process, we create cognitive structures that allow us to navigate the world, and truth is what emerges as the best possible representation of these structures. However, these representations are inevitably limited and influenced by our perceptions, our biases, and our intellectual capacities.

If we consider truth as a construct of thought, it becomes clear that it can never be a complete and definitive representation of reality. Every truth is relative to the context in which it is formulated and to the means by which it is reached. This does not mean that truth is without value or arbitrary, but rather that it must be seen as a progressive and improvable approach to understanding the world.

Nature presents us with a variety of phenomena that, at first glance, may appear chaotic and unpredictable. However, when we select significant events and abstract from the specific and random conditions that influence them, these phenomena can reveal a fundamental structure and an intrinsic beauty in all its clarity. This process, however, illustrates a widespread confusion between immanent reality, represented by Delos, the so-called $d_{(0)}$, and the instrument of thought, which through abstraction seeks to interpret the world by focusing exclusively on Delos and neglecting Adelos.

With a thoughtful gaze directed at the river, Ione concluded his reflection with a note of profound humility: “This reflection highlights an essential distinction between the formal and rational truth of Delos and the empirical and uncertain truth of the natural world. Recognizing the limits of our understanding, imposed by the uncertainty and complexity of reality, not only stimulates critical reflection on the role of mathematics in describing the world but also invites a humble and open scientific approach that values plausibility and the continuous evolution of knowledge. This attitude not only distances us from claims of absolute truths but brings us closer to a knowledge, not necessarily anthropomorphic, that is more authentic and aligned with reality, evolving through the dynamic interaction between Delos and Adelos.”

After contemplating the river's flow for a long time, Ione slowly rose, feeling the weight of the reflections that had accompanied him. The sky, now a dark cloak dotted with stars, seemed to reflect the infinite complexity of his thoughts. The cool evening breeze caressed his face, as if calling him to a deeper awareness.

Time

As he walked home, Ione revisited a thought that had fascinated him since the beginning of his meditations: the nature of time. Every step he took on the familiar road leading to his dwelling was a unique event, a beat of an invisible clock marking the very essence of existence. He realized that time was not an absolute entity but something intrinsically tied to the sequence of events. Time existed because things happened; it was shaped by the succession of events, just as river waves were born and dissipated, one after another.

Ione delved into this thought, understanding that time could not be reduced to a simple linear flow, like the sands of an hourglass. Every moment carried the potential for new events, and each event contributed to creating the reality he lived in. Time, as we perceive it, exists only because something happens. Without events, without changes, would time really exist?

He imagined, for a moment, the world of an atom, frozen in an eternal instant where nothing changed, and nothing happened. "In such a world," he thought, "could time exist? If there are no events to mark the passage from one instant to another, time itself loses all meaning. It would be nothing more than an absence of existence, empty and devoid of substance."

Time is like a unique dimension of existence that emerges through experiences and actions in the world—not only those of humans but also of the things that inhabit the world.

However, this perception of absolute and continuous time is, in reality, an illusion created by our biology and the constant sequence of events we perceive. In truth, time is not an objective reality independent of experience but rather a consequence of the interactions and changes occurring in the world. Events, whether biological, physical, or even cosmic, act as reference points that define our temporal experience.

In the universe, where events are less frequent or even absent, the perception of time might differ greatly or even cease to exist. In a region of the universe devoid of movement, without the rising and setting of stars, without any perceptible change, time might appear frozen, trapped in an eternal present.

This leads us to consider time not as a continuous, universal flow but as a relative dimension, closely linked to the presence of events. Time, therefore, can be understood as a "weave" formed by events, with "temporal threads" emerging and intertwining with every change, every action, every natural or human phenomenon.

In this sense, the idea of absolute time, uniformly flowing for everyone and everywhere, cannot hold. Instead, we should think of time as a complex network, made of interwoven moments that gain meaning only in relation to the events that compose them. The relativity of time suggests that our experience of time is intrinsically tied to the specific circumstances of each individual observer.

If time is, therefore, the "fabric" woven by events, then our understanding of reality must necessarily consider this dynamic and relative nature of time. Human history, our daily lives, and even the functioning of the universe are all processes that unfold not in an abstract, universal time but in a time that arises and develops along with the events characterizing it. This perspective invites us to reconsider not only our relationship with time but also our understanding of existence itself, where time emerges as a function of a constantly transforming reality.

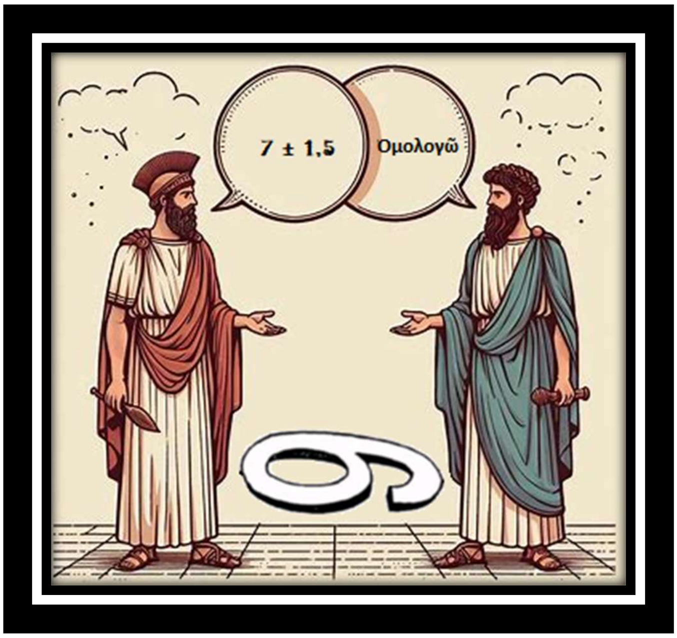
This reflection led Ione to understand that time and events are inseparable. One cannot exist without the other. The absence of events would result in the absence of time, and vice versa. Time, therefore, is not a continuous and absolute line but rather a sequence of moments shaped by the happenings of life.

Time is an objective reality that emerges through interactions among entities, independent of the perception of a conscious subject. This view opposes the notion that time is strictly tied to the perception of change by a soul and a mind capable of recognizing it. Time can exist without awareness.

Arriving at his doorstep, Ione paused for a moment, raising his gaze to the starry sky. "Perhaps," he mused, "time is nothing more than the heartbeat of the universe, measured by the events that animate it. To understand time is to grasp the importance of every moment, every action, and how all these elements intertwine to shape our perception of reality."

With the thought of time still vivid in his mind, Ione entered his home, embracing the quiet of the night that enveloped his reflections. With a tired body and a soul full of contemplations, he lay down in bed as sleep began to take hold.

Sympatheia and Enéine and the Metrologist Man[IX]



In the silence of the night, Ione's mind delved into a profound meditation on Sympatheia and Enéine. Reflecting on the concept of Sympatheia, or "being-with," Ione questioned how it intertwined with Enéine, the "being-in-the-world." These concepts represented two essential dimensions of our existence: the connection and interrelation with others and our definition through the environment in which we live.

Sympatheia, the quality of being connected to others, represented the network of exchanges and communications that constitutes our shared life. We are not isolated beings but immersed in a web of reciprocal interactions. Our experiences and decisions are shaped not only by our personal perceptions but also by the social dynamics that surround us.

On the other hand, Enéine reflects our being as entities defined through our choices and actions in the world. This concept emphasizes individual freedom and responsibility in creating one's existential meaning. Yet, Ione understood that this freedom is always exercised within the context of Sympatheia, binding us to others in a network of shared meanings.

In the vast and unfathomable theater of human experience, each of us is the protagonist of our story. We walk among the shadows and lights of the world, gathering fragments of sensations, emotions, and thoughts, weaving our personal narrative. In this individual narration, everything assumes its own order and coherence; a metric of relationships emerges, consistent with our perception of life and thus of experience.

And so, as long as we remain alone with ourselves, the world seems clear, coherent in its complexity. This is the realm of subjectivity, where reality molds itself to our impressions, and every nuance finds its rightful place in the mosaic of our convictions. Here, every judgment is an inviolable law, every emotion an absolute truth. In this context, Delos dominates.

But humans cannot live always in complete solitude. One's personal story cannot be written solely with the rules inscribed within oneself. Sooner or later, our steps cross those of another. And here arises the problem of Sympatheia. The encounter with another, with their inner world so close yet so different from ours, forces us to leave the safe walls of our subjectivity and relate to another reality. The need arises to communicate, explain oneself, understand, and find a synthesis between the intimate visions of each individual and that of a third reality, the one both personal realities call objective and relative to the external world. An external world endowed not only with Delos but also with Adelos, which renders the truths and certainties of a subjective reality composed solely of Delos inconsistent.

But how can we do this if our experiences are so unique, so unrepeatable? How can we find common ground where our truths can meet and dialogue without clashing or dissolving? Here arises the necessity of a method for sharing reality on objective bases that resolves confrontations and provides concordance in what is experienced. There are aspects related to the external and objective world, which are measured, and personal, introspective feelings like happiness or anguish, which do not necessarily require comparison with others and thus do not need measurement.

This necessity led to the development of a language and increasingly refined social actions, such as Science or Poetry. When interactions grow quantitatively, the urgency for a methodology of confrontation is felt, a subtle art we could call the metrology of encounter.

This metrology is not merely the science of physical measurements but a more refined discipline: the art of giving form and weight to what would otherwise remain indefinable and subjective. It is the attempt to find criteria, languages, symbols that can make experience in the world shareable through a coherent reading of a reality composed of Delos, as is personal reality, but also Adelos, which in our conception is absent. It is not about reducing experience to numbers but about finding ways to objectify the unobjectifiable so that the intimacy of our world can engage in dialogue with that of others, fully aware of the aleatory nature of external reality.

In this process, we build bridges made of shared words, common values, universal symbols. We create a grammar of relationships, a code that allows us to translate our experiences into a language that is not only ours but also that of the other. Thus, the metrology of confrontation becomes a delicate craft, where every word, gesture, decision is a tile in a larger mosaic connecting us to one another.

However, this path is not without obstacles. Not everything can be measured or quantified. There are deep emotions, mysterious intuitions, intimate truths that elude any attempt at reduction. And yet, this is where the greatness of our endeavor lies: seeking to preserve the mystery of subjectivity without abandoning the possibility of sharing it.

And so, in the dance of human relationships, the metrology of confrontation becomes not only a necessity but an art: the art of holding together the invisible and the visible, the personal and the shared, the subjective and the objective. An art that ultimately allows us to inhabit this world together, with all our differences and similarities, building day by day a reality that belongs to no one but is the fruit of everyone.

In this context of tension between individuality and connection, empirical metrology emerged as a crucial practice in harmony with Phronesis (practical wisdom). It represents the art of confrontation based on empirical facts, necessary to navigate the uncertainty of existence and facilitate communication among individuals. Within Sympatheia, metrology provides a common language to agree on experiences, enabling shared decisions while acknowledging the limits of human knowledge.

Ione reflected: “Personal experiences arise authentically in Enéine but inevitably confront others. We are neither gods nor isolated madmen but beings in constant dialogue with the other and the unknown.”

The anguish stemming from the presence of Adelos (the uncertain, the unknown) cannot be eliminated but can be faced by recognizing its existence and learning to live with it. This awareness of the fundamental uncertainty of existence becomes an integral part of being a metrologist.

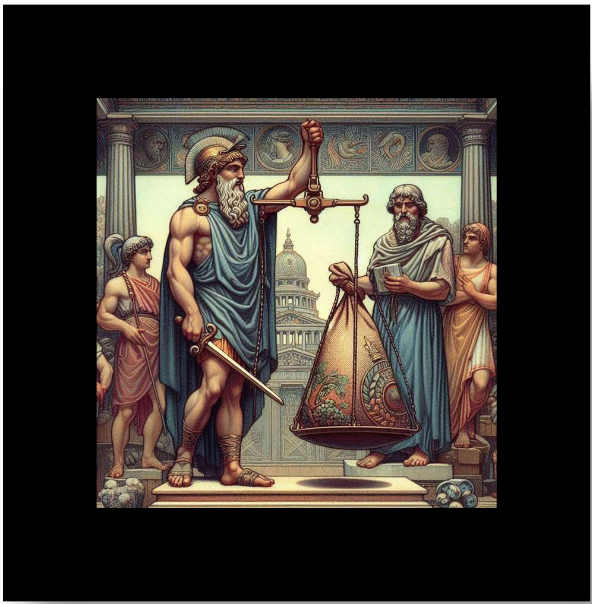
The metrologist, therefore, is not someone who seeks absolute certainties but one who acts in the world, conscious of their limits, the absence of ultimate truths, and the necessity of creating meaning through their choices. Though "thrown into the world" under circumstances not of their choosing, the individual retains the freedom to decide how to measure themselves against these circumstances and shape their future.

Confronted with the absurdity of the human condition, the metrologist recognizes that the universe is indifferent and devoid of intrinsic cosmic meaning. However, rather than succumbing to despair or seeking refuge in false certainties, they choose to embrace this reality. Life becomes a journey without a predetermined final destination, where each day offers the opportunity to create meaning through one’s actions and measurements.

Like a wanderer with no final destination, the metrologist approaches each day as a new existential challenge, measuring their existence not against external standards or absolute truths but through the authenticity of their choices and the depth of their experiences. In doing so, they transform the apparent absurdity of existence into an opportunity for self-realization and the creation of personal meaning.

In life, the metrologist engages in the pursuit of verisimilitude, not as an escape from uncertainty but as a way to navigate the ambiguity of existence consciously. This commitment to a project of self-realization, recognizing both one’s freedom and one’s connection to others, is what grants dignity and purpose within society.

With these reflections, Ione fell asleep, immersed in thoughts that balanced the aspiration for harmony and measure with the awareness of the unpredictability and intrinsic freedom of human existence.



It is said that many years later, in an era when Science had greatly advanced, there lived a man known as Heraclea, the Metrologist of Delos. His fame spread throughout Hellas, not only for his skill in measuring objects and phenomena with precision but also for his wisdom in understanding the ambiguous and complex nature of the world.

One day, while at the market of Delos, the lively island renowned for its sanctuaries and as the center of light, Heraclea was approached by a group of worried merchants. They were discussing a dispute about a shipment of grain that had been carefully weighed at the departure from a distant port. However, upon arrival at Delos, the merchants discovered that the weight seemed to have mysteriously decreased. Some claimed that part of the shipment had been stolen during the journey, while others attributed the difference to an error in the original measurement and demanded to adjust the price.

Each of the 500 sacks had been measured individually, and the uncertainty in the measurement had been minimized to only two minae per sack. Yet upon arrival in Delos, the merchants remeasured the shipment and found that the total weight of the grain had decreased by more than twenty parts.

Heraclea, called to resolve the matter, went to the harbor and asked to inspect the weights and scales used for the shipment. With great meticulousness, he measured and compared each instrument. His fame was well-deserved: the Metrologist knew that measurement was, ultimately, a matter of delos and adelos, of what was clear and certain, and what was obscure and uncertain.

While examining the shipment, Heraclea began to speak to the merchants gathered around him. "The delos," he said, "is what we see and understand clearly: the grain, the weight, the scales. It is the part of things we can measure, hold in our hands, and control." Then he lifted his gaze toward the horizon, where the sea sparkled under the late afternoon sun. "But there is also the adelos, the uncertain, the hidden. This is represented by the sea, the storms, the moisture that can infiltrate the shipment during the journey, causing the grain to swell and alter its weight. It is the aspect of life that escapes our measurement, that shifts and transforms without us realizing it."

Heraclea took a handful of grain and let it slip through his fingers, watching the kernels fall to the ground. "You merchants are like me: metrologists. You weigh, calculate, and seek to understand the delos of your goods. But you must never forget the adelos, the uncertain and ever-changing part of things, which is beyond our control."

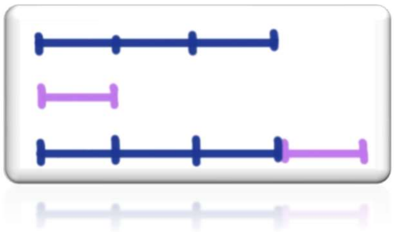
After a pause, the Metrologist concluded: "True measurement is not only what we calculate with our scales but also what accounts for both delos and adelos, accepting that there will always be elements in the world that we cannot fully predict or control, though we can investigate them."

The merchants, reflecting on his words, understood that the difference in weight could not be blamed on theft or error but on the natural unpredictability of the world. Heraclea had shown them that wisdom lies in accepting both what can be measured and what escapes measurement.

With newfound understanding, the merchants left Delos, ready to navigate not only through certainties but also through the shadows of uncertainty.

➢ **Sum of empirical numbers.**

Sum of two segments



This is the case of two aligned segments.
Let $x1_{(a)}$ and $x2_{(a)}$ be two segments; suppose we join them in such a way as to obtain the sum segment s_{na} where the initial Adelos a will become Af times the initial one:

(8)

$$D_1 = \left(\frac{\partial(x1 + x2)}{\partial x1}\right)^2 = 1$$

(9)

$$D_2 = \left(\frac{\partial(x1 + x2)}{\partial x2}\right)^2 = 1$$

(10)

$$Af = \sqrt{2}$$

And therefore, the sum segment is given by:

(11)

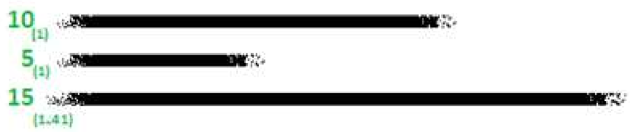
$$x1_{(a)} + x2_{(a)} = (x1 + x2)_{(\sqrt{2}a)}$$

where we note that the uncertainty of the sum segment increases relative to the uncertainties of the two initial segments by a factor of $\sqrt{2}$.

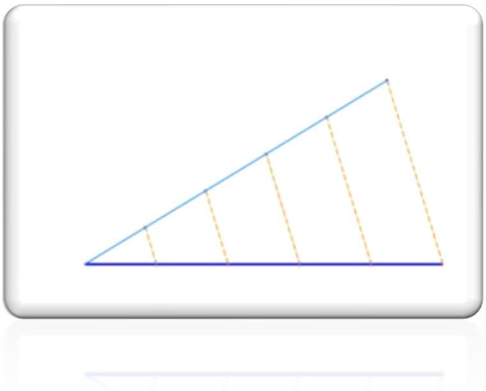
By numerically simulating two segments with values of $10_{(1)}$ and $5_{(1)}$, respectively, we obtain with 1000 samples:

$$9.985_{(1.025)} \hat{+} 4.974_{(1.006)} = 14.959_{(1,433)}$$

Graphically, the sum of two segments can be represented as follows:



The sum of empirical numbers, subtractions, multiplications, and divisions.



Generalizing, when the summation operation is repeated over m segments, the overall adelos increases according to the square root of m:

(12)

$$x1_{(a)} \widehat{\mp} x2_{(a)} \widehat{\mp} \ \dots \widehat{\mp} \ xm_{(a)} = \sum_{m}^{1} xm_{(\sqrt{m}a)}$$

The operator " $\widehat{\mp}$ " indicates a sum between empirical numbers that occurs in the real world, outside the domain of thought.

By numerically simulating four segments with values of $10_{(1)}$, $5_{(1)}$, $4_{(1)}$, $2_{(1)}$, respectively, we obtain, with 1000 samples, a doubling of the adelos as expected:

$$9.985_{(1.025)} \widehat{\mp} 4.974_{(1.006)} \widehat{\mp} 4.033_{(0.966)} \widehat{\mp} 2.003_{(0.980)} = 20.996_{(1,941)}$$

We will use the operator "+" to indicate a symbolic sum even for empirical numbers. A symbolic operation refers to an operation that does not manifest in the external world but remains confined to thought. The adelos represents an inherent potential within the expression of the numerical value, which remains intact during the symbolic operation.
In this case, let us consider that the expression is realized by the measurement for the first segment, while the subsequent applications are purely symbolic relative to the measured value:

(13)

$$\boldsymbol{d_{(a)} \xrightarrow[Measure]{} d'_{(0)}\ \ d'_{(0)} \underset{\Sigma}{\to} md'_{(ma)}}.$$

The segment to be added is expressed mentally and therefore does not change, and the sum of mmm identical symbolic segments will be:

(14)

$$x1_{(a)}+x1_{(a)}\ +\cdots+\ x1_{(a)} = mx1_{(ma)}$$

Simulating the sum of four symbolic segments with the characteristic $10_{(1)}$ using 1000 samples:

$9.985_{(1.025)} + 9.985_{(1.025)} + 9.985_{(1.025)} + 9.985_{(1.025)} = 39.939_{(4,101)}$

The results remain the same when considering distributions other than the normal one, such as the uniform or Weibull distribution.
In a sum, the adelos always increases proportionally to the number of sums "m" according to the relationship:

(15)

$$\widehat{a_{\Sigma}} = \sqrt{m}$$

76 If one wishes to construct a segment of any length starting from a small segment of arbitrary length, the adelos grows sub-linearly towards infinity according to $\sqrt{m}$.
The linear sum or subtraction of segments is an operation that considerably increases the adelos and progressively absorbs the delos, making it indistinguishable.
Simulating the sum of 1000 segments of length $1_{(1)}$ each, we obtain the following results from 5 consecutive simulations: 1044, 1001, 1024, 924, 954.

As will be seen in the next section, it is possible to construct segments without progressively increasing the adelos increment.

The operation of summing m equal segments is not generally the same as the operation of multiplying a segment by a factor of m.

(16)

$$\sum_{i=1}^{\hat{m}} x_{i(a)} = m \hat{\cdot} x_{(\sqrt{m a})} \neq m \cdot x_{(a)} = m x_{(ma)}$$

The operation of subtracting m equal segments $x_{(a)}$ from a certain quantity $d_{(a)} = mx_{(a)}$ is not generally the same as the operation of dividing a segment by m.

(17)

$$d_{(a)} - \sum_{i=1}^{\hat{m}} x_{(a)} = x_{(\sqrt{m+1}a)} \neq \frac{d_{(a)}}{m} = x_{(a/_{m})}$$

Simulating with $d_{(a)} = 40_{(1)}$ $10_{(1)}$ and $x1_{(a)}, x2_{(a)}, x3_{(a)} = 10_{(1)}$ and m=3, we get $x_{(a)} = 9{,}987_{(1{,}944)}$

It should be noted that this also holds in the case of subtracting two segments, and in particular, we observe that if the two segments are equal, $x1_{(a)} = x2_{(a)}$ the difference:

(18)

$$x1_{(a)} \mathbin{\widehat{-}} x2_{(a)} = 0_{(\sqrt{2}a)}$$

Graphically, it is as if after the operation, only the adelos part remains as a residue:

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In general, the difference between two equal empirical numbers does not result in a zero outcome. Simulating respectively, $x1_{(a)} = 10_{(1)}$ and $x2_{(a)} = 10_{(1)}$ with 1000 samples, the result obtained is $-0.013_{(1.413)}$.

Another consideration that can be made is that adding and subtracting the same quantity from a certain value does not yield the same initial value. Indeed:

(19)

$$x1_{(a)} \mathbin{\widehat{+}} x2_{(a)} \mathbin{\widehat{-}} x2_{(a)} = x1_{(\sqrt{3}a)} \neq x1_{(a)}$$

In the simulation, we obtain the following results for the same values as before: $10.100_{(1.744)}$.

While performing symbolically with the subtraction operation:

(20)

$$x1_{(a)} \mathbin{\widehat{+}} x2_{(a)} - x2_{(a)} = x1_{(a)}$$

In the simulation, we obtain for the same values as before $9.999_{(0.992)}$.

The non-isovariable case can be calculated with equation (7), therefore:

(21)

$$Af = \sqrt{\sum_{i=1}^{n} a_i^2}$$

Simulating the sum of two numbers, $10_{(2)}$ and $5_{(1)}$, the sum becomes $15_{(\sqrt{5})}$ we obtain with 1000 samples:

$$9.969_{(2.051)} \,\widehat{+}\, 4.974_{(1.006)} = 14.948_{(2,28)}$$

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➢ *The search for God*

Ione trudged along the dusty paths of his homeland, returning from the Persian campaign. The horrors of battle still haunted him, etched into his memory like an unrelenting echo: the clash of weapons, the blinding glare of shields under the merciless sun, and the desperate cries of men. Everything seemed to evoke the cruel instability of the world.

He had fought bravely, facing death countless times, but one memory in particular gave him no peace. During a fierce skirmish, when the Persian troops, vastly outnumbering them, had overwhelmed his forces with fury, Ione had seen his comrades fall like stalks of wheat cut down by a storm. And yet, just as all seemed lost, inexplicable events had turned the tide. Enemy arrows, destined to pierce them, had mysteriously embedded themselves in the ground, harming no one. Then, a sudden storm blinded the Persians, allowing the Greeks to launch a counteroffensive and achieve an improbable victory.

Now, as he approached his home, those moments continued to torment him. How could he explain what had happened? Was it mere luck, or was there a greater force at work? The idea of a hidden order behind the chaos of battle began to take shape in his mind. "Does God truly exist?" he wondered. "And if so, where can He be found?"

Ione's thoughts turned to the world of geometry, a realm where clarity reigned supreme. He recalled the Pythagorean theorem, which states that the sum of the areas of the squares on the two shorter sides of a right triangle equals the area of the square on the hypotenuse. This principle, so simple and immutable, seemed to reflect a greater truth, an intrinsic order that never faltered. "If God exists," Ione mused, "then He must be the source of this certainty. Mathematics and geometry are not merely tools to understand the world but revelations of a divine order."

As he walked, Ione observed the world around him and saw its geometric forms, the patterns of balance and symmetry which, though imperfect in their execution, reflected an underlying order. "There are no uncertainties in ideal geometry," he thought. "Every figure, every proportion follows laws that do not change, and these laws must be the reflection of a higher truth."

But as Ione's thoughts deepened, doubts began to arise. If geometry and mathematical principles were truly manifestations of a divine order, why, then, did limitations arise in their practical application? Although the Pythagorean theorem was a pure and unchanging truth, or so it seemed to him, in empirical reality, discrepancies always emerged. "The Adelos," Ione reflected, "seems to be an unavoidable factor even when dealing with mathematical truths."

Indeed, ideal geometry presupposes an immutable order, but in practice, the Adelos inevitably emerges. In every measurement and real construction, even the most precise geometric calculations were subject to imperfections and variations. Ione began to consider that, although mathematical truths seem to reflect a divine order, it is impossible to completely eliminate the Adelos in practical reality.

"If God truly exists as divine order in every aspect of our world," Ione thought, "why is uncertainty ever-present? Why can we not fully eradicate the Adelos?"

His reflection extended to the concept of Adelos, which seemed to manifest even in the most precise constructions. In some cases, the Adelos grew rather than diminished, proving that the pursuit of perfection and certainty was always limited. Ione observed that, although mathematical and geometric truths appeared as reflections of a higher order within themselves, their practical application was intrinsically marked by the Adelos.

Despite everything, the Adelos could not be entirely eradicated but rather revealed itself as an inevitable part of Nature. The perfection he had seen in ideal geometry did not correspond to the imperfect reality of the tangible world. The presence of inherent uncertainties and variability was, in some way, a part of the very fabric of reality.

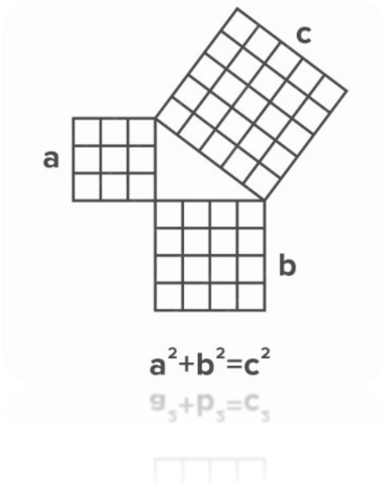
With these thoughts, Ione arrived home, deeply troubled. He realized that the search for truth was not as simple as finding an unchanging divine order but rather a journey of navigation through the Delos and Adelos that characterize reality. The light of ideal truth clashed with the shadows of uncertainty, and the concept of God, while represented by perfect order, was also part of a world where the Adelos could never be entirely eliminated.

His quest continued, no longer as a path toward an absolute truth but as an exploration and acceptance of an imperfect and uncertain reality. Ione remained suspended between the clarity of the ideal and the complexity of the real world, striving to understand how to coexist with that inevitable veil of uncertainty that enveloped every aspect of existence and brought with it a terrible anguish to endure. Yet this was part of life, any life, he resignedly thought.

➢ **The Pythagorean Theorem.**[X]

In this paragraph, we will encounter two significant properties of the Pythagorean Theorem. The first challenges the validity of the theorem when empirical numbers are employed. The second, on the other hand, unveils an exclusive property that allows for the construction of all natural numbers starting from a unitary element of delos and adelos, with adelos remaining unchanged throughout the entire construction. This fact is particularly noteworthy.

Shattering the Pythagorean Theorem

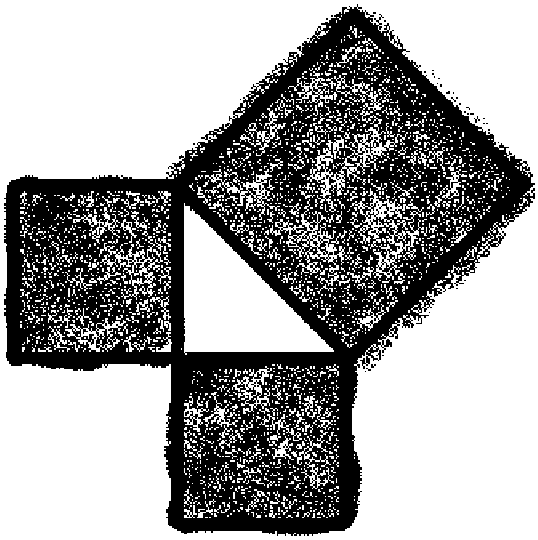


The Pythagorean Theorem states that the sum of the areas of the squares constructed on the two legs of a right triangle is equal to the area of the square constructed on the hypotenuse. However, this relationship does not hold when dealing with quantities expressed using empirical numbers. From result (11), we observe that:

$$Square\ on\ leg\ 1_{(a)} + Square\ on\ leg\ 2_{(a)} = \ Square\ on\ Hypotenuse_{(\sqrt{2}a)}$$

The adelos of the area increases by a factor of $\sqrt{2}$.

It is as if the square constructed on the hypotenuse fades with each successive construction by an amount equal to $\sqrt{2}$.



83 When we construct a square on the hypotenuse of a right triangle and use that hypotenuse as a leg for a new construction, we notice that the uncertainty (the Adelos) of the area increases by a factor of $\sqrt{2}$. This continuous growth does not merely represent a numerical error, but underscores a deeper reflection on the nature of knowledge and reality: the idea that, even when we attempt to capture the truth in a mathematical construct, it fades, revealing a complexity that always seems to elude us. The constant increment associated with $\sqrt{2}$ thus becomes a symbol of our inability to reach a definitive truth, reflecting a tension between our desire for certainty and the irreducibility of reality.

The $\sqrt{2}$, an irrational number that cannot be expressed as a finite fraction, embodies the infinity of uncertainty. The Pythagoreans, who initially sought to understand the world through their rigorous numerical framework, were confronted with a profound crisis when they encountered the irrationality of $\sqrt{2}$. This number challenged their belief in the finiteness and order of mathematical constructs, leading them into an existential and philosophical crisis about the nature of the infinite and its implications. On this rock, they were struck, finding their project stranded. They were so close to reaching the shore and finding stable ground, if only they had fully embraced the irrational hypotenuses to squared integers. These constructions will be developed further later in this chapter.

This idea can be read as a reflection on the limits of technique and reason. Even when man believes he has grasped reality, captured it in a symbolic construct, that reality fades, withdraws, revealing new layers of complexity and ambiguity. Here lies the existential tension between Delos and Adelos: the desire for absolute knowledge collides with the irreducibility of the world, with its fundamentally uncertain nature.

Moreover, the fading with each successive construction can be seen as an infinite process of becoming, where reality, like the square on the hypotenuse, is never complete, never fully understood, but always found transformed after each measurement. It is a vision that recalls Heraclitus' philosophy of "everything flows," where stability is only a temporary illusion, and each state of knowledge is merely a step towards another level of uncertainty.

We can also consider constructing the squares starting from the legs according to the relationship ${leg1_{(a)}}^2 + {leg2_{(a)}}^2$

(22)

$$D_1 = \left(\frac{\partial(leg1^2 + leg2^2)}{leg1}\right)^2 = 4\,leg1^2$$

(23)

$$D_2 = \left(\frac{\partial(leg1^2 + leg2^2)}{leg2}\right)^2 = 4\,leg2^2$$

(24)

$$Af = 2\sqrt{leg1^2 + leg2^2} = twice\ the\ hypotenuse$$

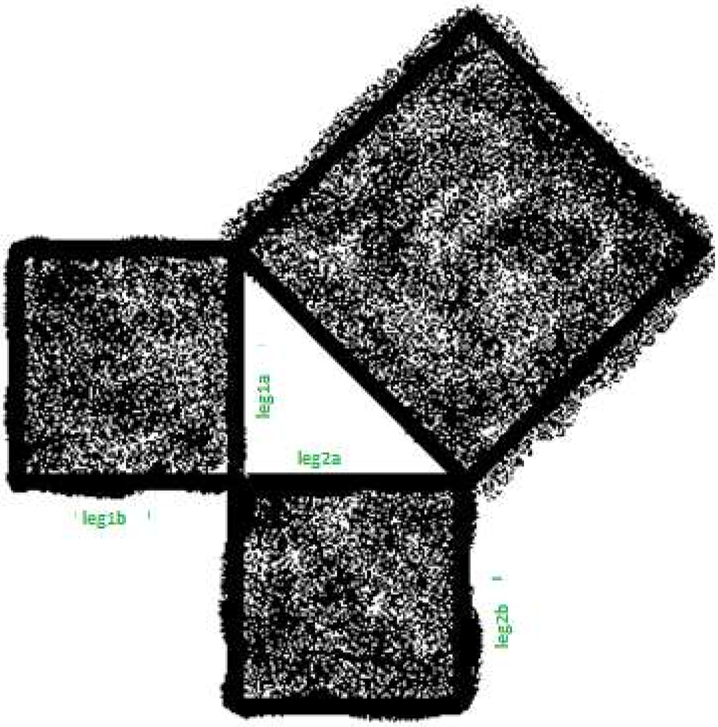
In this case, the adelos increases by a factor double that of the hypotenuse. By numerically simulating two legs with values of $10_{(1)}$ and $10_{(1)}$, and a second with legs of $20_{(1)}$ and $20_{(1)}$, we obtain with 1000 samples:

$9.989_{(0.998)}$, $10.034_{(1.018)}$ → $202.487_{(28,469)}$

$19.983_{(1.045)}$, $20.080_{(1.002)}$ → $804.615_{(57,8)}$

In the case where we consider the construction legs as four empirical numbers, the adelos is calculated as the square constructed on the hypotenuse:

$leg1a_{(a)} \times leg1b_{(a)} + leg2a_{(a)} \times leg2b_{(a)}$



(25)

$$Af = \sqrt{leg1a^2 \times leg1b^2 + leg2a^2 \times leg2b^2}$$

The adelos grows less compared to the previous calculation. It is observed that, depending on the setup of the calculation, even though the delos is equivalent, this does not hold true for the adelos.

By numerically simulating the 4 legs with values of $10_{(1)}$, and a second simulation with legs of $20_{(1)}$, we obtain with 1000 samples:

$10.057_{(0.937)}$, $10.024_{(0.988)}$, $10.019_{(0.994)}$, $9.910_{(1.018)}$ → $200.048_{(19,702)}$
$19.981_{(0.987)}$, $20.033_{(0.993)}$, $20.030_{(1.031)}$, $20.093_{(0.984)}$ → $802.737_{(40,385)}$

The fact that constructing a calculation in two different ways does not yield the same result has both mathematical and philosophical implications, touching upon

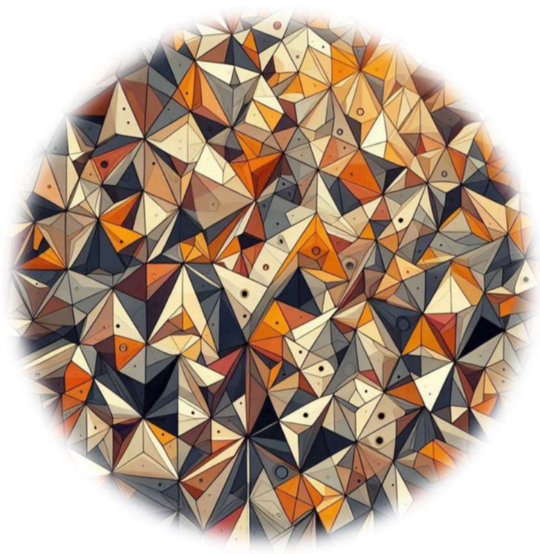
fundamental concepts related to the nature of mathematics, knowledge, and reality.
From a mathematical perspective, this discrepancy indicates that the result is not invariant with respect to the calculation method used. This phenomenon can reflect sensitivity to initial conditions, particularly relevant in the context of numerical computation. In complex systems, even small differences in calculations can be amplified, leading to divergent results. This is well known in chaos theory, where minimal variations in initial conditions can produce entirely different outcomes.

Philosophically, the divergence between the results suggests that our access to mathematical truth or reality is conditioned by the method employed. There is no single path to knowledge, but rather different representations of reality that can lead to different conclusions, even when starting from the same principles. This reflects a postmodern view of knowledge, in which objectivity is questioned.

From the perspective of the concept of Delos (what is clear and defined) and Adelos (what is obscure and indefinite), the discrepancy between the results highlights the impossibility of fully reducing Adelos to Delos. Even though the calculation appears to be based on clear and defined principles (Delos), the element of indeterminacy (Adelos) continues to affect the final result, varying depending on the method used. This suggests that, despite our efforts to formalize reality through mathematics and formal equivalences in Delos, a part of it remains elusive and irreducible.

The divergence of results also underscores the limits of rationality and technique. Even with the most advanced mathematical tools, we cannot always achieve a precise and absolute knowledge of the world. This raises a critique of blind trust in technique and science as the only paths to truth.

Philosophers such as Kurt Gödel have shown that there are intrinsic limits to mathematics itself (through his incompleteness theorem), highlighting that mathematics cannot fully capture all truths. The divergence of results can be interpreted in this context: mathematics, while a powerful tool, does not perfectly represent reality. This introduces an element of epistemological humility, calling for the recognition of the limits of our understanding.



We now aim to determine the hypotenuse given the two legs, which is also referred to as the calculation of the Euclidean norm.
Let $x_{(a)}$ and $y_{(a)}$ be the legs of a triangle; the value of the hypotenuse $d_{(\mathrm{a})}$ will be:

(26)

$$D_1 = \left(\frac{\partial(\sqrt{x^2+y^2})}{\partial x}\right)^2 = \frac{x^2}{x^2+y^2}$$

(27)

$$D_2 = \left(\frac{\partial(\sqrt{x^2+y^2})}{\partial y}\right)^2 = \frac{y^2}{x^2+y^2}$$

$$Af = 1$$

Let us denote by $\overset{\perp}{\rightarrow}$ the operation of calculating the Euclidean norm, that is, the calculation of the hypotenuse length:

(28)

$$x_{(a)}, y_{(a)} \overset{\perp}{\rightarrow} \sqrt{x^2+y^2}_{(\mathrm{a})}$$

<u>this result tells us that this operation leaves the adelos unchanged.</u>

By numerically simulating two segments with values of $10_{(1)}$ and $10_{(1)}$, we obtain with 1000 samples:

$$10.578_{(0.997)}\ ,\ 10.041_{(0.987)} \xrightarrow{\perp} 14.968_{(0,983)}$$

The property of adelos conservation in the construction of the hypotenuse can be extended to the calculation of the Euclidean norm in any dimension:

(29)

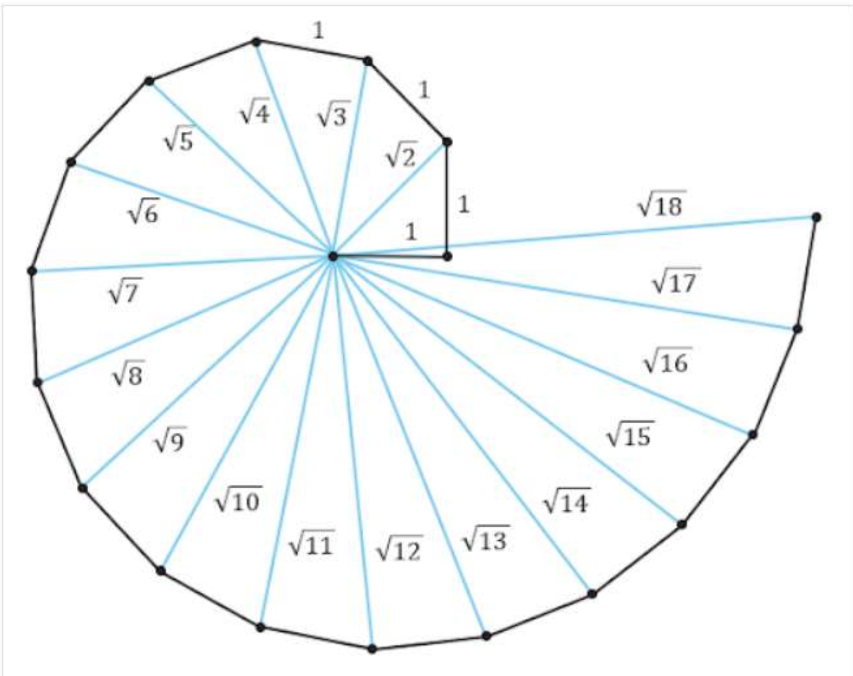
$$\boldsymbol{Af} = ||\, \boldsymbol{\nabla} || \boldsymbol{x}_{(\boldsymbol{a})} ||\, || = \mathbf{1}$$

We have just seen that the calculation of the Euclidean norm is an isovariable operation. Starting from a unit length segment $x_{(\mathrm{a})}$ we can construct a segment of any length and unit uncertainty $\mathbf{a}$ as a subsequent construction of hypotenuse segments and their squares starting from segments:

(30)

$$\boldsymbol{x}_{(\boldsymbol{a})} \xrightarrow{\boldsymbol{n}\perp} \boldsymbol{n}\boldsymbol{x}_{(\boldsymbol{a})}\ \forall\, \boldsymbol{n}^{\mathbf{2}} \in \mathbb{Z}$$

88 Geometrically, we can construct a segment corresponding to any Natural number through the construction of the Pythagorean spiral of Theodorus of Cyrene, as shown in the following diagram:



With this method, N^2 constructions are required starting from the unit length to obtain the length NNN, which can be as large as desired while keeping the adelos unchanged and equal to its initial value.

Now, let us examine a method that requires a smaller number of constructions to obtain a Natural number N:

2 is obtained as follows: $\mathbf{1}_{(\boldsymbol{a})}, \mathbf{1}_{(\boldsymbol{a})} \xrightarrow{1\perp} \sqrt{2}_{(\mathrm{a})}$ $\quad \sqrt{2}_{(a)}, \sqrt{2}_{(a)} \xrightarrow{2\perp} \mathbf{2}_{(\mathrm{a})}$

3 is obtained as follows: $1_{(a)}, 2_{(a)} \xrightarrow{1\perp} \sqrt{5}_{(a)}$ $\quad \sqrt{5}_{(a)}, 2_{(a)} \xrightarrow{2\perp} \mathbf{3}_{(\mathrm{a})}$

4 as: $\sqrt{2}_{(a)}, 1_{(a)} \xrightarrow{1\perp} \sqrt{3}_{(\mathrm{a})}$ $\quad \sqrt{3}_{(a)}, 2_{(a)} \xrightarrow{2\perp} \sqrt{7}_{(\mathrm{a})}$ $\quad \sqrt{7}_{(a)}, 3_{(a)} \xrightarrow{3\perp} \mathbf{4}_{(\mathrm{a})}$

5 (Pythagorean number) as: $3_{(a)}, 4_{(a)} \xrightarrow{1\perp} \mathbf{5}_{(\mathrm{a})}$

6 as: $\sqrt{2}_{(a)}, 3_{(a)} \xrightarrow{1\perp} \sqrt{11}_{(\mathrm{a})}$ $\quad \sqrt{11}_{(a)}, 5_{(a)} \xrightarrow{2\perp} \mathbf{6}_{(\mathrm{a})}$

7 as: $2_{(a)}, 3_{(a)} \xrightarrow{1\perp} \sqrt{13}_{(\mathrm{a})}$ $\quad \sqrt{13}_{(a)}, 6_{(a)} \xrightarrow{2\perp} \mathbf{7}_{(\mathrm{a})}$

8 as: $\sqrt{11}_{(a)}, 2_{(a)} \xrightarrow{1\perp} \sqrt{15}_{(\mathrm{a})}$ $\quad \sqrt{15}_{(a)}, 7_{(a)} \xrightarrow{2\perp} \mathbf{8}_{(\mathrm{a})}$

9 as: $1_{(a)}, 4_{(a)} \xrightarrow{1\perp} \sqrt{17}_{(\mathrm{a})}$ $\quad \sqrt{17}_{(a)}, 8_{(a)} \xrightarrow{2\perp} \mathbf{9}_{(\mathrm{a})}$

10 as: $\sqrt{3}_{(a)}, 4_{(a)} \xrightarrow{1\perp} \sqrt{19}_{(\mathrm{a})}$ $\quad \sqrt{19}_{(a)}, 9_{(a)} \xrightarrow{2\perp} \mathbf{10}_{(\mathrm{a})}$

And so on.

The rule for the construction of all natural numbers N is given as:

(31)

$$N_{(a)} = (N-1)_{(a)} \perp \sqrt{2N-1}_{(a)}$$

And where the irrational hypotenuse can be constructed as:

(32)

$$\sqrt{2N-1}_{(a)} = \sqrt{\left\lfloor \frac{2N-1}{2} \right\rfloor} \perp \sqrt{2N-1-\left\lfloor \frac{2N-1}{2} \right\rfloor}_{(a)}$$

Where the symbol ⌊ ⌋ denotes the greatest integer less than or equal to the expression within the parentheses.

It is easily noted that for the construction of any quantity, it is sufficient to use a previously constructed quantity with an integer or a smaller irrational hypotenuse.

The required number of constructions is:

(33)

$$(N-1) + \left\lfloor \frac{2N-1}{2} \right\rfloor - \left\lfloor \frac{\sqrt{2N-1}+1}{2} \right\rfloor + 1 + \left\lfloor \frac{N}{2} \right\rfloor - \left\lfloor \frac{\sqrt{N}}{2} \right\rfloor$$

Where different colours indicate integer terms, odd squares, and even squares, according to the following distinction. For N=10, this method requires 21 constructions compared to the 99 required by the spiral of Theodorus.

Integers: 9,8,7,6,5,4,3,2,1

Odd squares $\sqrt{19}, \sqrt{17}, \sqrt{15}, \sqrt{13}, \sqrt{11}, \cancel{\sqrt{9},} \sqrt{7}, \sqrt{5}, \sqrt{3}$

Even squares $\sqrt{10}, \sqrt{8}, \sqrt{6}, \cancel{\sqrt{4},} \sqrt{2}$

We will call this construction **"Τριμερὴς Γένεσις"** *(Trimerȅs Génesis)* — Tripartite Genesis.

The Tripartite Genesis harmoniously integrates integers, odd squares, and even squares, avoiding the direct construction of irrational hypotenuses as done, for example, in the Spiral of Theodorus. This makes it more compatible with an "orthodox Pythagorean" view of mathematics.

The tripartite genesis of Natural numbers exhibits gaps, much like prime numbers. However, in the latter case, the gaps do not follow a regular pattern as they do for perfect squares. For prime numbers, the absence of numbers grows in an unpredictable manner, with spacings increasing progressively and irregularly. In contrast, perfect squares create gaps according to a well-defined progression, dictated by the quadratic growth of their sequence.

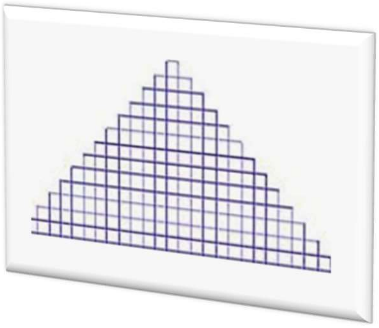
This analogy suggests that both the tripartite genesis of natural numbers and the distribution of prime numbers share a structure with voids, yet with a fundamental difference: while perfect squares impose a geometric regularity in their distribution, prime numbers appear in a more chaotic manner, albeit following certain global statistical laws, such as the Prime Number Theorem.

One could thus hypothesize a generalization of the concept of "structured gap" in natural numbers, distinguishing between Delian gaps (perfect squares) and Adelian gaps (prime numbers).

In a geometric construction, we can imagine an iterative process in which, starting from a base (e.g., a unit segment x_a), all natural numbers can be generated through the construction of right-angled triangles. The hypotenuse can be interpreted as an "operator" that generates natural numbers from a simple element.

If we consider Adelos as uncertainty or the obscure (e.g., the irrationality of the hypotenuse's length), the fact that it remains unchanged implies that, even though the construction allows for the generation of discrete values (natural numbers), the irreducible aspect of the geometric reality (Adelos) is not altered. The "dark" element of the calculation remains present and unchanged, without interfering with the production of natural numbers.

The construction of an empirical number is an intrinsically sequential and hierarchical process that must pass through all the previous stages, with no possibility of "mental shortcuts." A deep understanding of such a number requires awareness of all these intermediate steps. While it is possible to name or use an empirical number without mentally retracing the entire conceptual scale, its true "mathematical construction" necessarily implies this complete path, reflecting the complexity and layering of the very concept of number.



The concept can be extended to mathematical sets, where the construction of the hypotenuse can be seen as a process that generates a dense subgroup of numbers (the natural numbers), without altering the overall set of possibilities. This suggests a coexistence between continuity and discreteness: the natural numbers emerge from a continuous construction, without modifying the underlying element of indeterminacy. In a sense, this concept can be linked to fractality: although the geometric process appears ordered and predictable, it maintains an intrinsic complexity that is never fully resolved.

Thus, the hypotenuse becomes a symbol of the tension between the discrete and the continuous, between the finite and the infinite. The concept of irrationality could be further developed to describe the "residual" irreducible (Adelos) that persists even in the process of constructing natural numbers.

Philosophically, the idea of generating all the natural numbers (Delos, that is, what is clear and defined) without altering the Adelos (the obscure, the indefinite) becomes a metaphor for the limits of human knowledge. No matter how much we progress in rational and scientific understanding (Delos), there will always be an element of mystery, a shadow that cannot be entirely eliminated. However, this element can be kept limited.

This leads us to reflect on the principle of uncertainty: as much as we try to formalize reality through rational tools, there is always a residue of irreducible complexity that escapes complete understanding. However, the other side of the coin is that, paradoxically, one can discover an order in the disorder precisely through the disorder.

The idea that Adelos remains unchanged, despite the construction of natural numbers, can be interpreted as a philosophical paradox, where order (represented by natural numbers) emerges from a substrate that, in itself, is chaotic or incomprehensible. This recalls Heraclitus, who argued that "conflict is the father of all things" and that harmony arises from contrast.

In Nietzschean terms, this process could be seen as a transcendence of the human attempt to reduce the world to a clear and predictable system. Adelos represents what cannot be dominated by Technique and Rationality. The Nietzschean being can coexist with the condition of Adelos by accepting and integrating the irreducible uncertainty and complexity into their worldview. The Overman does not seek to eliminate Adelos, but to live it as an essential part of their experience and ability to create meaning. Complete understanding cannot be attained, but life and meaning can still be constructed within this complex reality.

From a postmodern perspective, this construction suggests that there is no single, definitive representation of the real. Mathematics, while extremely powerful, can at most limit uncertainty, but cannot eliminate it entirely. This reflects a critique of epistemological absolutism: there are no definitive truths, only partial and conditioned ways of accessing reality.

Another key reading is the psychological one. The unchanged Adelos, despite mathematical progress, can be seen as a metaphor for the unconscious that persists even when internal awareness (Delos) is reached. As much as we can expand our conscious knowledge, there is always a hidden and mysterious part of the psyche that remains inaccessible. Adelos can amplify the motivation to overcome personal challenges, improve social relationships, and contribute to the common good. Facing and integrating Adelos is a process that stimulates personal and social growth, reflecting the tension between the known and the unknown and fostering deeper and more meaningful self-realization.

In summary, the construction of the hypotenuse as a generator of natural numbers, while keeping Adelos unchanged, represents a powerful metaphor for the tension between the known and the unknown, between order and chaos, and between human rationality and the intrinsic limits of knowledge and technique.

The fundamental property of calculating the hypotenuse and its Adelos holds when the operation of constructing the square is performed empirically. This property also holds for other constructions but is not universally valid in general.

For example, the same result can be obtained starting from an analogous relation:

(34)

$$d = \frac{y}{sin\left(arctg\,\frac{y}{x}\right)}$$

(35)

$$D_1 = \left(\frac{\partial\left(\frac{y}{sin\left(arctg\,\frac{y}{x}\right)}\right)}{\partial x}\right)^2 = \frac{1}{\frac{y^2}{x^2}+1}$$

$$D_2 = \left(\frac{\partial \left(\frac{y}{\sin\left(arctg\frac{y}{x}\right)} \right)}{\partial y} \right)^2 = \frac{y^2}{x^2 + y^2}$$

$$Af = 1$$

By numerically simulating two segments with values of $10_{(1)}$ and $10_{(1)}$, respectively, we obtain, using 1000 samples, the same result as equation (24):

x=$10.005_{(1.005)}$, y=$9.987_{(1.020)}$ d=$14.174_{(1,007)}$

However, it does not hold, for instance, with a different construction:

(36)

$$d = \frac{y}{\sin(\alpha)}$$

It produces the same Delos but does not leave the Adelos unchanged:

(37)

$$D_1 = \left(\frac{\partial \left(\frac{y}{\sin(\alpha)} \right)}{\partial \alpha} \right)^2 = y^2\, cotg^2(\alpha)\, cosec^2(\alpha)$$

(38)

$$D_2 = \left(\frac{\partial \left(\frac{y}{\sin(\alpha)} \right)}{\partial y} \right)^2 = cosec^2(\alpha)$$

(39)

$$Af = cosec(\alpha)\sqrt{(y^2\, cotg^2(\alpha) + 1)}$$

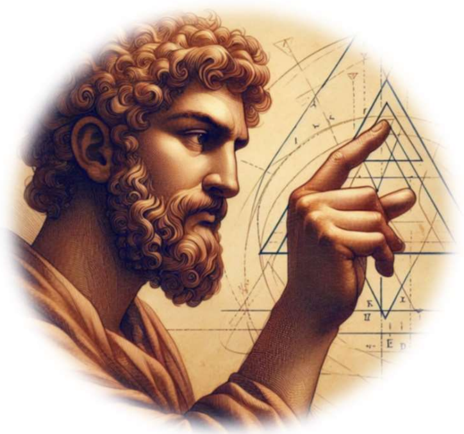
That for $\alpha \to 0$; $U \to \infty$

By numerically simulating two segments with values of $0.005_{(0.001)}$ and $10_{(0.001)}$, respectively, we observe, using 1000 samples, an increase in the Adelos by three orders of magnitude:

$$\alpha = 0.005_{(0.001)}\ ,\ y = 10_{(0.001)}\quad d = 10.304_{(2,22)}$$

The conclusion is that, as previously observed for the addition and subtraction of the same quantity, certain invariance properties of operations valid in symbolic calculations no longer hold when performed with empirical numbers. Even the geometric construction of the operation itself may not always be equivalent, as seen in the earlier constructions of the hypotenuse segment.

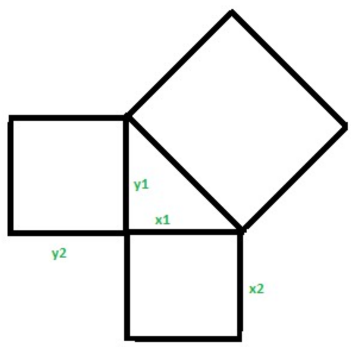
Touching God with the tip of the legs of a triangle



We now aim to construct the hypotenuse by considering 4 segments:

(40)

$$x1_{(a)}, x2_{(a)}, y1_{(a)} y2_{(a)} \xrightarrow{\perp} \sqrt{x1\ x2 + y1\ y2}$$



The Adelos of this function is calculated as:

$$Af = \frac{1}{2}\sqrt{\frac{x1^2 + x2^2 + y1^2 + y2^2}{x1\,x2 + y1\,y2}}$$

Noting that the legs x1, x2 and y1, y2 are two measurements of the same empirical variable, the expression simplifies as:

(41)

$$Af = \frac{1}{2}\sqrt{\frac{2x^2 + 2y^2}{x^2 + y^2}} = \frac{\sqrt{2}}{2}$$

The relationship holds for the normal distribution but can be computed for any other distribution.

What is particularly interesting is that, unlike the case of constructing squares, in this case, the Adelos not only does not depend on the measurements of the legs and remains constant, but it even decreases. One might think that the repeated application of this operation could reduce the Adelos to an arbitrarily small value, but never to zero.

If we interpret the Adelos as an error or uncertainty, we could say that this is a divine operation that renders a quantity arbitrarily perfect, or almost perfect, as there is always a very small residual where the demon can take refuge.

To reduce the Adelos to an arbitrarily small value, ε, n constructions must be performed according to the following relationship:

(42)

$$n = \frac{ln(\varepsilon)}{ln(\frac{\sqrt{2}}{2})}$$

Considering the sequence of constructed legs, one would arrive at the quantity:

$1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 3 \ldots \sqrt{n}$

If, for example, one started with an initial Adelos of 1 mm and aimed to reach an Adelos equivalent to the Planck length, approximately 1,616 X 10^{-32}mm it would require n=212 constructions. Starting from a leg 10 mm long, one would arrive at a triangle with leg $10_{(a)}$ mm and $10 \times \sqrt{212} \approx 145{,}6_{(\varepsilon)}$ mm

In terms of the length of the initial unit segment, it must therefore have a length equal to:

(43)

$$u = \frac{L}{\sqrt{n}}$$

Where L is the length of the segment one wishes to obtain with an Adelos smaller than ε.

This implies that the first n constructions will have an Adelos greater than ε.

To reduce the Adelos of the initial constructions, one could consider starting from the constructed hypotenuses and reconstructing the legs. This operation generally increases the Adelos, but for specific constructions, it may reduce it.

Starting from the hypotenuse and working backward to one of the legs, the uncertainty is not invariant and depends on the initial values themselves:

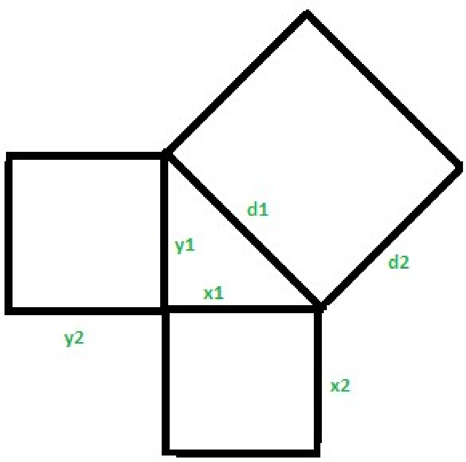
(44)

$$x = \sqrt{d^2 - y^2}_{(2\sqrt{d^2+y^2})}$$

If we perform the inverse construction of (38):

(45)

$$x1_{(a)}, x2_{(a)}, d1_{(a)} d2_{(a)} \xrightarrow{\perp'} \sqrt{d1\ d2 - x1\ x2}$$



The Adelos of this function is calculated as:

$$Af = \frac{1}{2}\sqrt{\frac{d1^2 + d2^2 + x1^2 + x2^2}{d1\, d2 - x1\, x2}}$$

Noting that the legs x1, x2 and the hypotenuses d1, d2 are similar, the expression simplifies as:

(46)

$$Af = \frac{\sqrt{2}}{2}\sqrt{\frac{d^2 + x^2}{d^2 - x^2}}$$

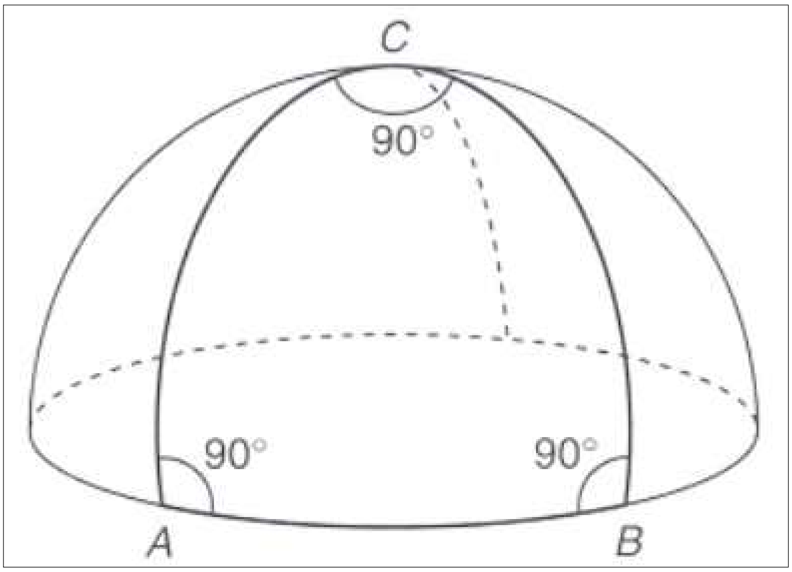
With appropriate choices of the hypotenuse and the leg, it is possible to obtain an Adelos of the calculated leg smaller than 1, leading to a reduction of the Adelos.

It can be easily calculated that with a ratio of d and x less than $\sqrt{3}$, the Adelos is reduced.

For example, starting with $d=2_{(a)}$ and $x=1_{(a)}$ the second leg is obtained as $\sqrt{3}_{(\sqrt{5/6}\,a)}$

Alternatively, starting with $d=41_{(a)}$ and $x=9_{(a)}$ the second leg is obtained as $40_{(\sqrt{881/1600}\,a)}$

We now ask whether what was found for the right triangle in Euclidean space also holds for a spherical surface, and therefore, we will proceed with measuring a geodesic on a spherical surface.



Consider the Pythagorean theorem for spherical triangles, that is, the cosine of the hypotenuse is equal to the product of the cosines of the two legs.

Let $\boldsymbol{r}$ be the radius of the sphere, $\boldsymbol{a}$ the horizontal side, and $\boldsymbol{b}$ the vertical side. The distance between two points is given by the relation:

(47)

$$d = r\ arcos\ (cos\frac{a_{(u)}}{r}\ \ cos\frac{b_{(u)}}{r})$$

(48)

$$Af = \sqrt{\frac{(sen^2\frac{a}{r}\ cos^2\frac{b}{r} + sen^2\frac{b}{r}cos^2\frac{a}{r})}{1 - cos^2\frac{a}{r}\quad cos^2\frac{b}{r}}}$$

The uncertainty in the measurement of the geodesic is not constant and varies as a function of the distance between the points considered, oscillating between 0 and 1.

In principle, this behaviour allows for the identification of the space in which the measurements are made, and through the analysis of the Adelos, it is possible to determine whether we are in a Euclidean or non-Euclidean space.

➢ ***The flight of Ione***

Ione was undoubtedly a curious spirit, whose mind wandered beyond the boundaries of the known world, eager to grasp the secrets of the universe. Yet, like all men, he lived suspended between the sky and the sea, kept afloat by what others called "the laws of life." These laws were his strongest anchor but also his most invisible chain.

The sea represented daily life: habits, shared beliefs, the security of tradition. They were the rules of the community, the stories passed down by elders, the customs no one dared to question. Ione, too, floated on those waters like everyone else. But while many found comfort in that placid surface, Ione couldn't help but look at the sky.

One of the most devoted to the stability of that sea was his uncle Crisius. Crisius was a middle-aged man, respected in the community for his experience, but also known for his aversion to change. "The sea keeps us safe," he often told Ione. "There is no need to search beyond what we already know. Birds fly because they are made to fly. We are made to stay here." For Crisius, life was a sequence of unchanging rhythms, a stability not to be disturbed. Study was almost a useless activity because the matters of the world were evident and appeared clear to everyone.

But Ione felt there was more. Every time he watched the birds, he felt a drive within himself. "If I can challenge the sea," he thought, "I can also reach the heights of the birds. But to do so, I will have to use every strength of my body and mind."

In his family, there was also a young nephew named Leander. Leander was even more ambitious than Ione. He had studied every discipline, from geometry to medicine, and believed that traditions were merely chains for weak minds. Unlike Crisius, who embraced stability, Leander despised everything tied to continuity. He wanted to rid himself of the sea altogether, unbothered by its rules.

One day, as the three stood together by the shore, Leander impatiently addressed Ione. "Uncle, you are too cautious. I don’t need these attempts; I can fly immediately. I’ve read all the texts, I know every theory. The sky belongs to me."

Ione looked at him with a melancholic smile. "Leander, the knowledge you have gained is valuable, but without *phronesis*, without the wisdom to understand the right balance between the sea and the sky, you risk losing everything you’ve learned. Flying is not just a matter of knowing; it’s also knowing when to stop, when to observe, when to act."

One day, after watching the birds soaring through the sky, Ione decided to act. He let himself fall into the waters of the sea, ready to fight against those invisible forces that held him back. The water was cold and dense, and he immediately felt the pressure around his body. Every time he tried to rise, the sea held him, wrapping around him like a vice. He felt the weight of the water crushing his chest; every breath became laborious. The waves rocked him, trying to convince him to let go, to stay where everything was known and safe.

But Ione resisted. He began to kick hard, trying to push himself beyond the surface. His legs pushed powerfully against the water; he felt his muscles burn from the effort. The water was no longer just an obstacle; it was a living force capable of pulling him back. Every movement seemed futile, as if the sea itself mocked him, turning every attempt into a cruel game.

The water ran over his arms, heavy and viscous, almost as if it wanted to hold him down. Ione felt the salt burn his eyes, the cold seep into his bones. Every kick was a silent scream against nature, against the force that enveloped him, yet he didn’t stop. Every breath became an act of defiance, every heartbeat a reminder of his determination to see the sky up close.

At one point, he managed to rise enough to see the horizon, that thin strip of light separating the sea from the sky. It was so close, yet it seemed unreachable. The wind lashed his face, mingled with sprays of water that entered his mouth. The taste of salt was bitter, like the effort he was experiencing. But he couldn't stop. He had to go on; he had to feel the real wind, not just the illusion brought by the sea.

Meanwhile, his uncle Crisius watched from the shore, shaking his head. "Ione, come back," he shouted. "It's pointless to fight against what you can't change. The sea is our home; it will never abandon us. Return to the safety of its waters."

But Ione didn't listen. Every fibre of his body was focused on the struggle. His hands cut through the water, his feet pushed with desperation. He felt his heart pounding furiously in his chest, his arms growing heavy, his breath short and gasping. His body screamed for rest, but his mind drove him forward.

Finally, with one last, desperate kick, he managed to emerge completely from the water. The sky was above him, free and vast, and the wind welcomed him like an embrace. For a moment, he felt as light as the birds he had admired so much. The fatigue seemed to vanish, replaced by the pure joy of having reached that goal. He looked up and let himself be filled with the sunlight that kissed him warmly.

But freedom was fragile. He already felt the call of the sea below him, the weight of the water wanting to pull him back down. He knew he couldn't stay there for long. The sea claimed him, and his body couldn't resist forever. But in that brief moment, he understood he had touched something beyond mere existence. He had flown, even if only for a moment.

As he let himself be drawn back into the waters, he smiled. Because even though the sea could hold his body, it could never erase what he had experienced. He had defied the gravity of life, and even though his flight wasn't meant to last forever, the sky he had touched would belong to him for eternity.

And he thought of how the great Hellenistic wisdom, which had soared so high, was eventually destroyed by the basest Roman civilization, incapable of understanding the depth of the knowledge it had sought to replace. Yet, like Ione's flight, that light would never be entirely extinguished, ready to be reignited who knows how many centuries later. [XI]

From that day, the waves of the sea no longer frightened him, for he had known the sky. And he understood that the rules that had given him stability throughout his life, those unwritten laws men call common sense, were indeed necessary but could never replace the experience of those, like him, who had chosen to fly.

In the years that followed, Ione often reflected on that experience of flight, recognizing it as a powerful metaphor for the human condition. In that brief moment, suspended between the sea and the sky, he had experienced the balance between the concreteness of the immediate and the infinity of the horizon, an insight that led him to contemplate the limits and possibilities of knowledge.

One afternoon, as the sun set slowly, painting the sky in golden hues, Ione and his young nephew, Leandro, walked together toward the hill of Alessia. Leandro, who had been keenly following his uncle's intellectual journey, accompanied him with curiosity. Ione felt it was time to share his reflections, to pass on the wisdom he had gained through his extraordinary experiences.

As they ascended the hill, Ione gazed at the distant sea. The waves, which had once seemed like a prison to him, now appeared as part of a greater whole. He saw in them the rhythm of life, the eternal dance between stability and change. With this contemplative mindset, they reached the hilltop, ready for a conversation that would delve into the depths of human understanding.

Seated at the summit, overlooking the sea, Ione and Leandro watched the waves gently breaking on the shore, reflecting the colours of the sunset. It was one of those moments where time felt suspended, allowing the mind to wander beyond the visible.

Leandro, young and impassioned, was deeply immersed in his studies. He had read countless texts and delved into the depths of philosophy, yet he felt something was missing. Turning to Ione, he sought answers.

"Uncle," Leandro began, "how can we truly know the truth? Every time I dive deeper into the study of something, it feels like I lose sight of the whole. And when I try to see the whole, the details elude me. It's as if my mind can never see everything clearly."

Ione smiled, watching the waves' relentless motion. "Leandro," he said slowly, "the human mind can be likened to vision. Like vision, the mind has its limits, and those limits define what we can comprehend."

Leandro looked at him, intrigued. "What do you mean, master?"

Ione took a deep breath and began to explain. "Imagine presbyopia, a condition where the eyes lose the ability to focus on distant objects. It's as though the mind, when overly focused on details, loses sight of the whole. This often happens to those who study relentlessly: they immerse themselves so deeply in particulars that they forget to lift their gaze and see the ocean beyond the waves."

Leandro nodded, pondering these words. "So focusing on details can be dangerous?"

"Not dangerous," Ione replied, "but limiting. When you focus solely on details, you lose perspective. It's the trap of scholars who, lost in minutiae, forget that the world is woven together by larger connections."

After a brief pause, Ione continued. "Then there's presbyopia. Think of those who can only see what is distant, unable to focus on what is near. This is the common state of the human mind. Most people grasp the general characteristics of something but fail to see its details. They live on the surface of things, never diving into the depths."

Leandro reflected on these words. "And perfect vision? Does a mind exist that can see everything, both the details and the whole?"

Ione shook his head, smiling wistfully. "Perfect vision is an illusion, Leandro. In the human mind, there's no condition that allows one to see all details and the whole simultaneously. We are always constrained by something, time, perspective, or our own capacity to perceive. No one can know everything, just as no one can see every wave of the ocean and the entire sea at once."

Leandro sighed, absorbing the depth of his master's words. "So, are we destined never to fully know the truth?"

"Not necessarily," Ione replied, gazing at the horizon. "Although we have no lenses for the mind like we do for the eyes, we can still strive for balance. The key is not to focus too much on the details or remain only on the surface. It's important to find a middle ground, where we can grasp both the whole and the particulars, even if imperfectly. It's a difficult balance to maintain, but it brings us closer to the truth."

Leandro remained silent for a moment, letting Ione's words settle in his mind. "So, the truth is like the flight you once described? Something we can touch but never fully grasp?"

Ione nodded slowly. "Exactly, Leandro. Flight is an attempt, a struggle against the limits imposed by our nature. We can rise for a moment, glimpse what lies above the waves, but we cannot stay there forever. Yet, even that brief flight is worth all the effort."

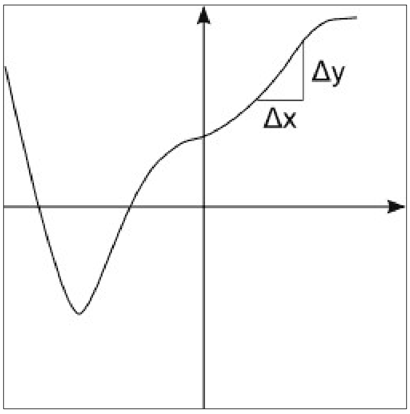
The sun had now disappeared beyond the horizon, and a gentle sea breeze caressed the two. Leandro felt a little lighter, as if he had understood another fragment of that vast ocean of knowledge.

And so, as Ione had taught him, he sought to keep the balance between seeing the ocean and knowing the waves, realizing that the path to truth is made of small steps and brief flights.

➢ Calculation with Empirical Variables

The Derivation

Let $y = f(x)$ be a function defined on an interval [a,b], and let $\frac{\Delta y}{\Delta x}$ be the incremental ratio of the function in the vicinity of a point x_0 that lies within the interval.



From Calculus, we know that if, as the increment Δx of the variable tends to zero, the limit of the incremental ratio of a function in the vicinity of one of its points exists and is finite, then this limit is the derivative of the function at that point:

$$f'(x0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

Geometrically, the meaning of the derivative of a function at a given point is that it represents the slope of the tangent line to the curve at that point.

When working with empirical data, the derivative can be approximated using finite differences. Suppose we have two measurements of the same variable x_a, the approximated derivative can then be expressed as:

(49)

$$f'\left(x_{(a)}\right) = \frac{f\left(x_{(a)}\right) \mathbin{\hat{-}} f\left(x_{(a)}\right)}{x_{(a)} \mathbin{\hat{-}} x_{(a)}} = d_{(a')}$$

The measurements are distinct and not identical, of the same variable. This methodology, based on finite differences, allows us to approximate the derivative without resorting to infinitesimal calculus, that is:

(50)

$$f'\left(x_{(a)}\right)_{(a')} = \frac{f\left(\overrightarrow{\underset{x_{(a)}}{1}}\right) - f\left(\overrightarrow{\underset{x_{(a)}}{2}}\right)}{\overrightarrow{\underset{x_{(a)}}{1}} - \overrightarrow{\underset{x_{(a)}}{2}}} = d_{(a')}$$

The notation $\overrightarrow{\underset{x_{0_{(a)}}}{1}}$ refers to the first measurement, and $\overrightarrow{\underset{x_{0_{(a)}}}{2}}$ refers to the second measurement, where the values of the measurements cannot be identical. Moreover, a’ represents the Adelos of the derivative f’.

Let’s see in the following table some formulas for the fundamental derivatives:

Functions	Delos of y' (d)	Adelos of y’ (a')
Constant Function $y = k$	0	0
Power Function $y = x^n, n \in \mathbb{R}$	nx^{n-1}	$\frac{\lvert n(n-1)\rvert}{\sqrt{2}} x^{n-2}$
$y = kx$	k	0
$y = \frac{1}{x}$	$-\frac{1}{x^2}$	$\sqrt{2}x^{-3}$
$y = \sqrt{x}$	$\frac{1}{2\sqrt{x}}$	$\frac{1}{4\sqrt{2}}x^{-\frac{3}{2}}$
$y = \sqrt[3]{x}$	$\frac{1}{3x^{\frac{2}{3}}}$	$\frac{2}{9\sqrt{2}}x^{-\frac{5}{3}}$
$y = \frac{1}{x^2}$	$-\frac{2}{x^3}$	$\frac{6}{\sqrt{2}}x^{-4}(*)$
$y = \frac{1}{x^3}$	$-\frac{3}{x^4}$	$\frac{12}{\sqrt{2}}x^{-5}(*)$
Absolute value Function $y = x$	$\frac{\lvert x\rvert}{x}$	0
Logarithmic Function $y = \ln(x)$	$\frac{1}{x}$	$\frac{1}{\sqrt{2}x^2}(*)$
Exponential Function $y = e^x$	e^x	$\frac{e^x}{2\sqrt{2}}(*)$
Trigonometric Function $y = \sin x$	$\cos x$	$\frac{\sin x}{\sqrt{2}}(*)$
$y = \cos x$	$-\sin x$	$\frac{\cos x}{\sqrt{2}}(*)$
$y = \tan x$	$\frac{1}{\cos^2 x}$	$\propto x$ (**)

(*) Empirically found form

(**) Form not yet determined

Let $y = f(x)$ be a function defined and bounded on a closed interval $[\alpha; \beta]$. Geometrically, the definite integral represents the area under the curve of the function and the x-axis between $x = \alpha$ and $x = \beta$.

Let $\{x_0, x_1, \dots, x_n\}$ be a partition of the interval $[a; b]$ such that $\alpha = x_0 < x_1 < \cdots < x_n = \beta$, and let $\Delta x_i = x_i - x_{i-1}$ be the widths of the subintervals. Consider a point $x_{i_{(a)}}$ within each subinterval $[x_{i-1} - x_i]$. To measure the definite integral, we calculate the area as:

(51)

$$\int_{\alpha}^{\beta} f\left(x_{(a)}\right) = \sum_{i=0}^{n} f\left(x_{i_{(a)}}\right) \Delta x_i \ = \ d_{(a\prime)}$$

Here, the goal is not to study the expression of the Adelos for every algebraic function, but to mention that there are functions where the Adelos increases and others where it decreases, as we develop the integral calculus.

The calculation can be carried out by using the Fundamental Theorem of Calculus with the employment of empirical variables:

(52)

$$\int_{\alpha}^{\beta} f\left(x_{(a)}\right) = F(\beta) - F(\alpha) \ = \ d_{(a\prime)}$$

where $F'\left(x_{(a)}\right) = f(x_{(a)})$.

In this case, the calculation of the Adelos is directly applied by using equation (7).

➢ *Conclusions.*

In the vast tapestry of the cosmos, where light and shadow merge, a profound truth hides behind the apparent certainties of the world. Ione, the guardian of Delos and Adelos, has learned that the laws governing reality are not as rigid as they seem. Not even the Pythagorean theorem, that pillar of geometry, can remain unchanged when one enters the abyss of Adelos, where uncertainty and potential dwell in every corner.

In the world of pure light of Delos, the Pythagorean theorem appears as an unshakable truth: the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. But when one looks through the veil of Adelos, that clarity shatters. The shadow of uncertainty penetrates even geometric truths, making the edges blurry, the contours undefined. The perfect triangle dissolves, and all that remains is a trace, a potential that escapes the rigid precision of the hyperuranic formulas.

Yet, in this dissolution, a new creation emerges. With empirical constructions, the very essence of natural numbers can be formed, one by one, preserving Adelos intact, like a seal that protects the mystery of the world. There is no need to sacrifice uncertainty to create order. Numbers emerge from chaos, not as fixed entities, but as possibilities that breathe, that move, keeping their margin of indeterminacy alive.

But Adelos is not only the realm of possibilities. It is also the inscrutability of the real, an abyss that, if not confronted with awareness, can generate deep anguish, like a dark sea in which the navigator risks losing himself. There is no comfort in the illusion of certain foundations. Adelos is a naked truth, devoid of hypocritical values imposed by man and his Technique. The wanderer proceeds not to dominate, but to inhabit Nature without defined goals, without values, accepting the vertigo of uncertainty that cannot be annihilated as an essential part of his journey, and yielding only to necessity.

However, like every human being, even Ione is tempted to reduce Adelos to a symbolic thought, trying to contain it within the limits of Delos, the knowledge that man can comprehend. This is a human tendency: to want to know, to reduce everything to the light of rational thought. But Ione knows that there is a part of Adelos that remains irreducible, that cannot be reduced to reason or symbols. This irreducibility is such that not even God can dominate it, and it is precisely this impossibility that makes God's existence an illusion. In Adelos, even God finds an insurmountable boundary, a limit that denies the possibility of a definitive divine order.

It is in this interplay of light and shadow that Ione seeks his balance. But soon he understands that it will never be perfect, nor rational. Living with Adelos will not lead to stable harmony but to a profound transformation. An inner change that makes him capable of accepting that the world was not created for him, nor for anyone else. There is no design, no higher purpose guiding him. Nature, with its shadows and its light, is indifferent to human presence, just as it has been toward any God.

Thus, as the sea stretches before him, Ione realizes that his path is not toward total understanding, but toward acceptance of a universe that has never claimed to be understood. A universe that was not created for man, nor for a God, but simply is. His smile slowly fades, while awareness passes through him. The transformation has begun, but it brings no consolation. It brings only the raw and inescapable reality of a world without certain foundations, a world that exists beyond any human attempt to give it a definitive meaning. Life is like a race down a water slide, at some point, the race stops suddenly, and only the slide remains, only the water flowing, and others continue to run down the current until they too are cast away by life. There is only the direction given by events, but not the meaning.

And as the sun sets, colouring the sky with soft and fleeting hues, Ione rises and resumes his journey. He knows that he will never find all the answers, and that his journey is destined to continue in a world where shadow and light are inseparable and therefore immensely interesting. A world that awaits neither answers nor questions but simply continues, indifferent, in its eternal becoming; but also, in subjecting us to Necessity, and for this reason, it must be lived fully and faced with courage and enthusiasm for our daily challenges, big or small.

References and Notes.

[I] The ideas from the text *Thus Spoke Zarathustra* by Friedrich Nietzsche are developed.

[II] Elements that may seem anachronistic and close to the post-modern world will often be introduced, but the aim is to invite the reader to reconsider the ancient world, which was already highly advanced in technique and science. In particular, refer to the books published by the scholar Lucio Russo:

The Forgotten Revolution: How Science Was Born in 300 BC and Why It Had to Be Reborn.
2004 Springer.

L'America dimenticata. I rapporti tra le civiltà e un errore di Tolomeo
2013 Milano, Mondadori.

Il tracollo culturale. La conquista romana del Mediterraneo: 146-145 a.C.
2022 Carocci Editore.

[III] Gaia, a new look at life on earth
Lovelock, James
1979 Oxford ; New York : Oxford University Press

[IV] Beyond Words: What Animals Think and Feel
Carl Safina
2015 Henry Holt and Company.

[V] The concept of the "Wanderer" developed in the text, as well as other enlightening essays by Professor Umberto Galimberti, will often be referenced.

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Psiche e techne
2019; Milano Feltrinelli

[VI] These 'mathematical' considerations will become clear after reading the technical section..

[VII] Monte Carlo Methods
Jun S. Liu
2001; Springer

[VIII] Probability and Statistics
Morris H. DeGroot e Mark J. Schervish
2002; Pearson

[IX] Concepts introduced by Martin Heidegger, such as Da-sein and Mit-sein, are developed here using Greek terms: Sympatheia and Enéine.

[X] The initial ideas were developed around 2011 and later published in an article in 2014. For a long time, they remained unpublished, partly due to various life setbacks and limited interest within the scientific community. However, my participation in the Pythagorean Week, chaired by Prof. Piergiorgio Odifreddi in the summer of 2024, reignited my interest in these studies. I decided to review and update the work, which contained some inaccuracies, and to develop it in its current form. This process has been supported by two artificial tools that contributed to my research and the critical revision of both the philosophical and mathematical parts. Lacking a network of academic human contacts, I found in these "artificial companions" valuable allies.

[XI] Inspired by the readings of Lucio Russo (see point II)