

# Modification of Classical Electromagnetic Radiation Theory and Synchrocyclotron Without Radiation

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**Abstract** According to classical electromagnetic theory, the accelerated motion of charged particles produces electromagnetic radiation, but this is not always the case. Experiments have shown that electrons colliding with nuclei produce bremsstrahlung radiation when they are decelerated, but electrons do not radiate when they are accelerated in a uniform electric field. In synchrocyclotron, electrons radiate, but in Lawrence's Cyclotrons and induction accelerator and linear accelerators, electrons do not radiate (Blewett experiment). In this paper, the motion of charged particles in the electromagnetic field are analyzed by considering the mass-velocity formula of special relativity. It is pointed out that the accelerating motion of electrons in uniform magnetic field and strict electric field are stable, so they need not to radiate. In this way, the stability of atoms can be explained. But if the magnetic field is not uniform, or there is some kind of disturbing force, the speeds of electrons may become imaginary or faster-than-light, making the motion impossible. In this case, electrons had to radiate to change their state, making the motion possible. Therefore, acceleration is not the essential cause of radiation of charged particles, but the instability of motion is the real cause of radiation. It is proved that it is possible to eliminate the radiation of particles as long as the interference factors are eliminated so that the motion of charged particle is stable in synchrocyclotron. A simple design scheme is proposed by adding parallel current lines in the particle beam pipe of accelerator to eliminate the transverse oscillation of charge particles, to construct a new synchrocyclotron with high energy and low radiation or even no radiation.

**Key Words:** Mass-velocity formula, Classical theory of electromagnetic radiation, Antenna radiation, Bremsstrahlung, Linear accelerators, Lawrence's Cyclotrons, Electron induction accelerator, Synchrocyclotron

## 1 Introduction

There are two theories in physics that describe the radiation of charged particles in electromagnetic fields, one is the classical theory of electromagnetic radiation, another is the theory of quantum mechanics to describe the radiation and absorption of photons. This paper discusses classical electromagnetic radiation theory, does not involve quantum transition processes.

In the current high energy physics experiments, the energy of accelerator has reached its limit. Due to the severe radiation of charged particles as they approach the speed of light in a synchrocyclotron, it is very difficult to increase particle's energies, unless to build a larger accelerator by taking huge resources. Under realistic

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**The first half of this paper had been published in Applied Physics Research, Vol. 4, No. 2; 2012,  
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conditions, it can not be done. Therefore, it is the only way to seek for a new theory of high energy accelerator, overcome the defects of existing accelerator and build a new small-scale high energy accelerator.

In this paper, the classical electromagnetic radiation theory is modified. It is proved that the electromagnetic radiation of charged particles is not caused by acceleration, but by the instability of relativity motion of particles in the electromagnetic field. If the instability of motion is overcome, charged particles will not radiate. Based on it, a new scheme of high energy particle accelerator with low or even no radiation is proposed by the transform of existing accelerators.

The authors published a paper in 2012 to discuss this issue preliminary [1]. This paper make same revisions on the original paper and proposes a practical design of a radiation-free synchrocyclotron.

According to the theory of classical electromagnetic field, charged particles will radiate when they are accelerated. However, the actual experiments have shown that this is not always the case. When charged particles are accelerated, in some cases they radiate, but in some cases they do not. For example, when accelerated in a uniform electric field, charged particles generally do not radiate. In both high voltage electrostatic accelerators and electron linear accelerators, we do not observe radiation, although the electron speed is very close to the speed of light. International Linear Collider (ILC) [2], currently being planned, is just based on this fundamental fact.

Electrons accelerated in Lawrence's cyclotrons and induction accelerator do not radiate (Blewett experiment) [3, 4], but electrons accelerated in a synchrocyclotron, they radiate. In addition, synchrocyclotron radiation occurs only in the curved parts enveloped by magnets [5]. There is generally no radiation in the linear accelerating section, unless where the oscillators and torsional oscillators are used.

In the process of radio transmission, alternating voltage is used to drive the accelerating movement of electrons in the antenna, and radio waves are emitted only when the length of antenna is limited and satisfies the condition of current oscillates. If the wire is long enough, the current does not oscillate, and the electrons do not emit radio waves though their acceleration are the same.

Another simple but obvious example is the production of X-rays. When a high voltage is applied between the positive and negative electrodes of a cathode-ray tube to accelerate the electrons, we only observe the discharge of gas, without observing X-rays emitted by electrons. But if a metal target is placed between the positive and negative electrodes of the cathode tube, the fast-moving electrons collide with the target nucleus and quickly slow down, and X-rays are emitted, which is called Bremsstrahlung [6]. Although in these two cases, the accelerations of electrons are almost the same.

In addition, a more fundamental problem is why electrons orbiting atomic nuclei do not radiate electromagnetic waves. This problem had troubled Rutherford, when he proposed the structure of atom, he faced the problem of electromagnetic radiation causing atomic instability, which have not been explained well up to now.

All of these indicates that the radiation theory of classical electromagnetic is not completely consistent with the experiment, and need to be modified.

Based on the formula of mass velocity, the motion stability of charged particles in electromagnetic field is analyzed. In particular, the stability of electron motion in the antenna and the cyclotron is discussed in detail. It is proved that the accelerated motion of electrons in a uniform electric field is stable and electrons do not radiate. If the length of the wire is not limited, the accelerated motion of electron acted by the periodic electric field is also stable, and the antenna will not radiate. Only when the length of the antenna is limited and the oscillation condition is satisfied, the electrons that are accelerating in the antenna radiate radio waves.

The motion of electrons in a centered electric field may be stable or unstable. Electrons orbiting the nucleus automatically choose stable orbits that do not radiate. In this way, the famous puzzle of physics, the stability of

atomic system is explained well.

It is proved that the electron does not need to radiate when it moves in a standard circle in a uniform magnetic field. However, if the magnetic field is uniform, or there is a small disturbing force, the motion of high-speed electron will deviate from the circular orbit, the speed of electron would not only faster than the speed of light, but also may become imaginary. Such a motion is impossible, so electron had to radiate to change the of its motion state, making the motion possible.

In synchrocyclotron, due to the non-uniform magnetic field and other factors, the movement of charged particles is unstable. There is the so-called transverse oscillation phenomenon, so that charged particles need to change the motion state through radiation. This is the foundational reason for the radiation of charged particles in high-energy accelerators, resulting in a huge energy loss, so that the energy of particles can not increase continuously.

A specific design scheme is proposed in this paper by adding parallel current lines in the particle beam tube of accelerator. Through the effect of current lines on the charged particles, the transverse oscillatory motion of particle beam is eliminated, the motion of particles become stable, so that they do not need radiation. In this way, we can build a high-energy particle synchrocyclotron with low radiation loss and even no radiation loss. This new accelerator does not need to be rebuilt, but only needs to slightly modify the existing synchrocyclotron, which has the advantage of low investment and high effect, and can become the development direction of future high-energy particle accelerators.

## 2 Accelerators and Radiations

### 2.1 Electrons in linear accelerators do not radiate

The earliest particle accelerators were high voltage accelerators, in which charged particles were accelerated in a straight line under the action of high voltage. Experiments shown that although charged particles have a large acceleration, there was no radiation. Because the acceleration process of such accelerators was one-time, there are technical difficulties in generating higher voltages, and very high energies could not be obtained.

In order to solve the problem of sustainable acceleration, linear radio-frequency accelerators were later developed [5]. Its working principle was that feeding microwaves into the superconducting cavity to create a specific resonant mode of electromagnetic field. When the charged particle passes through the radio frequency cavity in a suitable phase, it is precisely affected by the positive acceleration field and is accelerated. Accelerators consist of multiple radio frequency tubes in series, which can accelerate particles to very high energies.

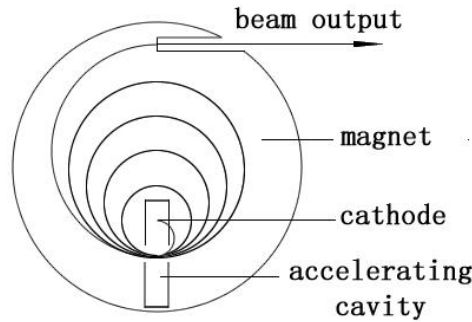
Linear radio-frequency accelerators generally emit a little or no radiation, despite the high acceleration of electrons. International Linear Collider, which is currently being implemented, uses this method to accelerate particles. In fact, if there a lot of radiations in the linear acceleration motion for electrons, there is no possibility to build ILC.

### 2.2 Electrons in Lawrence's cyclotrons do not radiate

In 1930, E.O. Lawrence proposed the principle of cyclotron. In 1931, a cyclotron with a magnetic pole diameter of 10 cm was built, and the energy of  $H_2^+$  ions was accelerated to  $8000eV$ , which confirmed the feasibility of cyclotron. Since then, cyclotron's energies have been continuously increased, and in 1939, a cyclotron with a magnetic pole diameter of 1.5 meters was built to accelerate the energy of charged particles to  $20MeV$ .

The principle of cyclotron is shown in Fig.1 [6]. The accelerator uses electromagnet to form magnetic poles,

and a vacuum chamber is placed between two magnets, and the magnetic field strength is unchanged. A cathode is placed in a vacuum chamber to produce electrons which move in a circle under the action of a uniform magnetic field. Each time a charged particle passes through accelerating cavity, it is accelerated by the action of a high frequency voltage. After the particle leaves accelerating cavity, it moves in a circle with a larger radius until it leaves the beam outlet at the end. It should be emphasized that the orbital radius of charged particles in the Lawrence accelerator is not fixed. Or, in a sense, particles move unhindered in a uniform magnetic field.

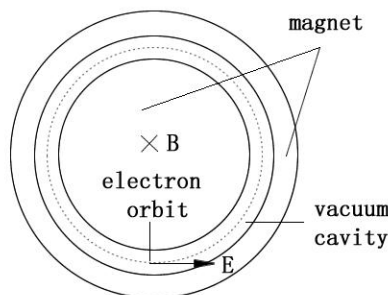


**Fig.1 Schematic diagram of cyclotron's principle**

Experiments show that in this cyclotron with uniform magnetic field, charged particles do not produce radiation when they are accelerated in a circle. According to the following analysis in this paper, the motion of charged particles in a uniform magnetic field is stable, and there is no need to produce radiation. However, due to technical difficulties in making larger uniform magnets, it is impossible to build Lawrence cyclotrons very large, limiting the energy of accelerated particles.

### 2.3 Electrons in induction accelerator do not radiate

The structure of electron induction accelerator is roughly the same with cyclotron. The magnetic field produced by the electromagnet is uniform, but the strength is not fixed, increasing with the energy of the particle. The purpose is to limit the movement of charged particle to a roughly circular orbit. At the same time, there is an alternating electric field in the tangent direction of electron orbit, and the electron accelerates under the action of the electric field force. In Fig.2, B represents the induced magnetic field perpendicular to the paper surface, and E represents the electric field corresponding to the induced magnetic field.



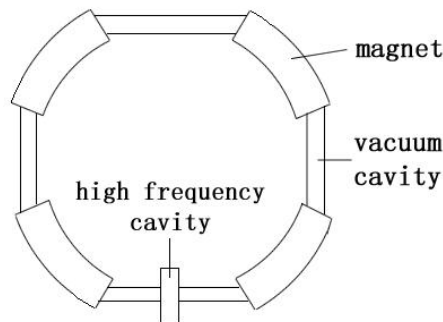
**Fig.2 Schematic diagram of induction decelerator**

General Electric has an electron induction accelerator of 100MeV in Schenectady, New York. When the physicist J. P. Blewett debugged the device in 1944, he hoped to discover the emission of electrons. Blewett used a very sensitive radio wave detector with a range  $50Hz \sim 10^8 Hz$  of frequencies to measure wavelengths from ultra-long waves to ultra-short waves. According to classical electromagnetic theory, since the electron's speed is

already close to the speed of light, the power of radiation should be quite large. However, whether Bruette placed the detector in the vacuum chamber of electron induction accelerator or outside, he never detected the radiation of electromagnetic waves [3,4]. Though Blewett also found the small contraction of electron orbitals, which would be caused by small amounts of radiation, or caused by other causes [7].

## 2.4 Electrons radiate in synchrocyclotrons

In order to solve the problem that Lawrence cyclotron could not produce higher energy, E. Macmillan and V. Wexler respectively proposed the synchrocyclotron theory in 1945 [4], the principle of which is shown in Fig.3. Unlike Lawrence cyclotron, synchrocyclotron uses discontinuous magnets, and the charged particles are confined to a fixed orbit. The magnetic field generated by the magnet change the moving direction of charged particle, and the two magnets are connected by a linear vacuum tube. Charged particles are accelerated through a high-frequency cavity when they move in a straight line part. Since synchrocyclotron can use a number of discrete magnet blocks to wrap the particle beam tube, the radius of accelerator can be made large, and the energy of particles can be greatly increased.



**Fig.3 Schematic diagram of synchrocyclotron**

In 1974, a 70MeV electron synchrocyclotron was built in the same laboratory of Blewett. H.Pollack was responsible to debug the accelerator. Because the vacuum chamber of accelerator was transparent, during the debugging process, workers inadvertently saw the radiation light [4]. This discovery caused a huge sensation at the time, and opened a new era of synchrotron radiation applications.

The frequency synchrotron radiation is continuously distributed, although mainly in visible light, but in principle there should be radiation in the radio band. Electrons in a synchrocyclotron radiate only on the turning part of track where the magnet is present. But electrons do not radiate when they are accelerated in high-frequency cavity moving in a straight line. As for why this is so, existing theories of electromagnetic radiation can not explain it.

## 2.5 The analysis of reasons that electrons do not radiate in induction accelerator

Of these four accelerators, electrons radiate only in synchrocyclotron, the other three do not. Moreover, in synchrocyclotron, electrons radiate only in the turning part of magnet, and there is no radiation in the straight part in which electrons are also being accelerated. This result is very inconsistent with the classical theory of electromagnetic radiation. It suggests that there must be some unrecognized blind spot in the principles of physics.

In induction accelerator, electrons are subjected to both magnetic and induced electric fields. The electron induction accelerator of Blewett reached an energy of 100MeV, 30MeV more than Pollack's synchrocyclotron, and electromagnetic radiation should have been observed, but no radiation was found. Because Blewett used a radio director, it was initially thought that due to the Doppler effect, when electrons move at a speed close to the speed

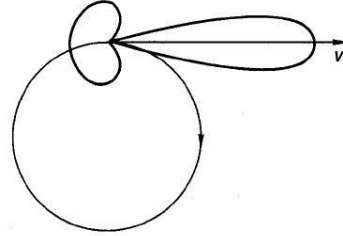
of light, the spectral frequency become several orders of magnitude greater so that no radiation could be observed at low frequencies [4].

However, things are not so simple, according to the Doppler frequency shift formula, when the light source moves, the frequency increases in some directions, and the frequency decreases in other directions. Let the light source move at speed  $V$ ,  $\nu_0$  is the natural frequency of light,  $\nu$  is the frequency measured by the stationary observer, the Doppler shift formula is [8]

$$\nu = \frac{\nu_0(1 - V \cos \varphi / c)}{\sqrt{1 - V^2 / c^2}} \quad (1)$$

When an electron moves in a circle in a cyclotron, the inter-plane distribution of electromagnetic radiation power is shown in Fig.1. According to the relativistic kinetic energy formula, the velocity of an electron with an energy 100MeV is  $V = 0.99999c$ ,  $\sqrt{1 - V^2 / c^2} = 0.0051$ . When the observer faces the direction of electron motion with  $\varphi = \pi$ , according to Eq. (1), we have  $\nu / \nu_0 = 392$ . The result shows that the spectrum is blue shifted and the frequency is 392 times larger. If measured in the direction of electron leaving the observer, there are  $\varphi = 0$  and  $\nu / \nu_0 = 0.002$ . The result is a spectral red-shift with a frequency smaller than 500 times.

The frequency range of radio wave from ultra-long wave to medium wave is  $3 \times 10^3 \text{ Hz} \sim 3 \times 10^5 \text{ Hz}$ . The frequency of short wave to ultra-short wave is  $3 \times 10^6 \sim 3 \times 10^8 \text{ Hz}$ , the frequency of microwaves is  $3 \times 10^8 \sim 3 \times 10^{11} \text{ Hz}$ , and the frequency of far-infrared light is  $3 \times 10^{11} \sim 3 \times 10^{12} \text{ Hz}$ . According to Eq.(1), in the direction of  $\varphi = \pi$  and  $\varphi = \pi/2$ , even if the frequency of low-frequency radiation increases by several hundred times, it still falls in the short-wave band, far from the frequency range of far-infrared light, let alone becoming infrared light and visible light. What's more, in the direction of  $\varphi = 0$ , the frequency of radio radiation becomes 500 times lower. The frequency range of Blewett's detector is  $50 \text{ Hz} \sim 10^8 \text{ Hz}$ , it is impossible for Blewett would not have been able to detect radiation.



**Fig. 4 Spatial distribution of radiation power for electrons in synchrocyclotron.**

Comparing induction accelerator with synchrocyclotron, there are differences. The first is that electron induction accelerator uses a uniform magnetic field, and synchrocyclotron uses a non-uniform magnetic field, so the non-uniform magnetic field may be an important reason for charged particle to produce radiation. In the following, it will be shown that charged particles accelerating in a uniform magnetic field do not radiate. In addition, accelerated electrons in the strict electrostatic field are also non-radiating, which is precisely the the key that the electrons moving around nucleus do not radiate and atoms can exist steadily.

The second is that electrons in induction accelerator are acted by both electric and magnetic fields simultaneously. In synchrocyclotron, electrons are not acted by electric and magnetic fields simultaneously. Therefore, if an electric field is also added to the magnet part of synchrocyclotron, it may be possible to eliminate radiation on the turning part of magnet. As will be seen in the following discussion, this is precisely the key to build a non-radiation synchrocyclotron.

### 3 Accelerating motion of charged particle in uniform electric field in free space

#### 3.1 The motion equation of charged particles in electromagnetic field in free space

The radiation of the accelerated charged particles is a macroscopic electromagnetic effect. This paper only discusses the problem in the scope of macroscopic physics, and does not involve quantum mechanics. The rest mass of electron is  $m_0$ , the motion mass is  $m$ , the charge of electron is  $-q$ . By considering the mass-velocity formula, the motion equation of an electron in the electromagnetic field is

$$\frac{d}{dt}(m\vec{V}) = \frac{d}{dt} \frac{m_0\vec{V}}{\sqrt{1-V^2/c^2}} = -q(\vec{E} + \vec{V} \times \vec{B}) \quad (2)$$

or

$$\frac{m_0}{\sqrt{1-V^2/c^2}} \frac{d\vec{V}}{dt} + \frac{m_0\vec{V}V/c^2}{(1-V^2/c^2)^{3/2}} \frac{dV}{dt} = -q(\vec{E} + \vec{V} \times \vec{B}) \quad (3)$$

Let  $\vec{a}$  be the acceleration of electron, we have

$$\frac{dV}{dt} = \frac{d}{dt} \sqrt{V_x^2 + V_y^2 + V_z^2} = \frac{V_x}{V} \frac{dV_x}{dt} + \frac{V_y}{V} \frac{dV_y}{dt} + \frac{V_z}{V} \frac{dV_z}{dt} = \frac{\vec{V} \cdot \vec{a}}{V} \quad (4)$$

Substituting Eq. (4) in Eq.(3), we get

$$\frac{m_0\vec{a}}{\sqrt{1-V^2/c^2}} + \frac{m_0\vec{V}(\vec{V} \cdot \vec{a})/c^2}{(1-V^2/c^2)^{3/2}} = -q(\vec{E} + \vec{V} \times \vec{B}) \quad (5)$$

#### 3.2 The motion equation of an electron in uniform electric field in free space

Suppose that an electron moves in a uniform electric field  $E$  along the straight line direction of the  $x$ -axis, the equation of motion can be simplified as

$$\frac{d}{dt} \frac{m_0V}{\sqrt{1-V^2/c^2}} = qE \quad (6)$$

By differentiating the left side of Eq.(6), we get

$$\frac{m_0a}{(1-V^2/c^2)^{3/2}} = qE \quad (7)$$

Where  $a = dV/dt$  is an acceleration. On the other hand, by considering  $V = dx/dt$  and  $dt = dx/V$ , Eq.(8) can be written as

$$V \frac{d}{dx} \frac{m_0V}{\sqrt{1-V^2/c^2}} = qE \quad (8)$$

Suppose that electron's speed  $V=0$  at the point  $x=x_0$ , to integrate the above formula by parts, we can

$$\frac{V}{c} = \sqrt{1 - \left( \frac{m_0c^2}{m_0c^2 + qE(x-x_0)} \right)^2} \quad (9)$$

When  $x \rightarrow \infty$ , we have  $V \rightarrow c$ . As discussed below, there is no singularity in the motion processes described by Eq.(9), this kind of motion is stable and possible.

### 3.3 The electron does not radiate when it is accelerated in a straight line

According to classical electromagnetic radiation theory, if the acceleration of charged particle is parallel to the velocity of motion, the radiated power is [8]

$$P_{\parallel} = \frac{q^2 a^{*2}}{6\pi\epsilon_0 c^3 (1 - V^{*2}/c^2)^3} \quad (10)$$

Where  $V^*$  is the retarded speed and  $a^*$  is the retarded acceleration. If the acceleration is perpendicular to the velocity, such as the movement of an electron in a magnetic field, the radiated power is [8]

$$P_{\perp} = \frac{q^2 a^{*2}}{6\pi\epsilon_0 c^3 (1 - V^{*2}/c^2)^2} \quad (11)$$

If taking an approximate calculation, let  $a^* = a$ , substituting Eq.(9) in Eq.(10), the radiation power of electron is

$$P_{\parallel} = \frac{q^4 E^2}{6\pi\epsilon_0 c^3 m_0^2} \quad (12)$$

Electron's static mass  $m_0 = 9.11 \times 10^{-31} / Kg$ , charge  $q = 1.60 \times 10^{-19} C$ , and  $\epsilon_0 = 8.85 \times 10^{-12}$ . Take  $E = 10^6 V/m$  be the electric field strength of liner accelerator. According to Eq.(12), the maximum radiated power of the electron is  $P_{\parallel} = 1.75 \times 10^{-19} W$ .

Assume that the average speed of electron is  $0.5c$  and the distance of acceleration is  $x - x_0 = 100$  m. According to Eq.(9), the final speed of electron is  $V = 0.999987c$ . According to the mass energy formula, electron's kinetic energy is  $E_1 = 3.15 \times 10^{-9} J$ . When an electron is accelerated from zero to nearly the speed of light, it takes time  $\Delta t_1 = 6.67 \times 10^{-7} s$  to travel 100 meters, the radiated energy is  $P_{\parallel} \Delta t = 1.17 \times 10^{-25} J$ , much less than the kinetic energy of the electron.

At this radiated power, the time required to change the electron speed from light's speed  $c$  to zero is  $\Delta t_2 = T/P_{\parallel} = 1.8 \times 10^{10} s$ , about 571 years. Assume that electron radiates red light with a wavelength  $7 \times 10^{-7} m$  and energy  $E_2 = 2.84 \times 10^{-19} J$ . We have  $E_1/E_2 = 4.12 \times 10^{-7}$ , means that the number of photons emitted by the electron during the entire acceleration process is much less than one.

Due to the quantization of the radiation light, this is equivalent to saying that the electron does not radiate during the entire acceleration process. This result is consistent with the actual experiment, which shows that the design of linear accelerator can ignore the radiation of electron. It also shows that the radiation power formula of classical electromagnetic theory is still valid in the case of linear acceleration of charged particles.

### 3.4 Electron will radiates when it is slowed down by atomic nucleus

However, in non-free space, by shooting fast-moving electrons at a metal target and slowing them down, electrons will produces X-rays, called as the bremsstrahlung [6]. The result is completely different from an electron accelerating in a straight line. Let's simply estimate the accelerations in both cases.

When electron is uniformly accelerated in a linear accelerator with an average speed of  $0.5c$ , taking  $E = 10^6 V/m$ , according to Eq.(12), the acceleration is  $a = 1.16 \times 10^{17} m/s^2$ . Let the thickness of the metal target be 1 cm, the electron shoots at the target at near the speed of light. The speed of electron decreases to zero when it leaves the target. The average speed is also  $0.5c$ . The time taken by the electron to pass the distance of 1 cm is  $t = 6.67 \times 10^{-11} s$ , and the deceleration of electron is  $a = 4.5 \times 10^{18} m/s^2$  calculated by the formula  $l = at^2/2$ .

In both cases, there are not great difference in the acceleration. However, the electron does not radiate when



it is uniformly accelerated in the linear accelerator, but produces strong radiation when it is slowed down by a metal target. Why there is such a great difference, the current radiation theory in classical electromagnetism is not able to explain.

We will prove below that whether a charged particle radiates depends on the stability of particle's motion in the electromagnetic field, and not depends on acceleration.

## 4 The essence of antenna radiation

### 4.1 The movement of electron in free space under the action of periodic electric field force

Let us first discuss the motion of electron in a periodic electric field  $E = E_0 \sin \omega t$  in free space. That is, there is no matter other than electron and electromagnetic field in the space, and the movement of electrons is not limited by boundary conditions. By considering the mass-velocity formula, the motion equation of electron is

$$\frac{d}{dt} \frac{m_0 V}{\sqrt{1 - V^2 / c^2}} = -qE_0 \sin \omega t \quad (13)$$

Taking the integral of Eq.(13), we get

$$\frac{V}{\sqrt{1 - V^2 / c^2}} - \frac{V_0}{\sqrt{1 - V_0^2 / c^2}} = \frac{qE_0}{\omega m_0} (\cos \omega t - \cos \omega t_0) \quad (14)$$

Suppose that  $V_0 = 0$  when  $t = t_0$ , the angle  $\theta_0 = \omega t_0 = \pi / 2$  with  $t_0 = \pi / (2\omega)$ , Eq.(14) can be written as

$$V = \frac{dx}{dt} = \frac{qcE_0 \cos \omega t}{\sqrt{(c\omega m_0)^2 + (qE_0)^2 \cos^2 \omega t}} \quad (15)$$

Let  $x = x_0$  when  $t = t_0$ , taking the integral again, we get

$$x - x_0 = \int_{t_0}^t \frac{qcE_0 d \sin \omega t'}{\omega \sqrt{(c\omega m_0)^2 + (qE_0)^2 (1 - \sin^2 \omega t')}} \quad (16)$$

Let

$$qE_0 \sin \omega t' = y' \quad (c\omega m_0)^2 + (qE_0)^2 = b^2 \quad (17)$$

Eq.(16) becomes

$$x - x_0 = \frac{c}{\omega} \int_{y_0}^y \frac{dy'}{\sqrt{b^2 - y'^2}} dy' = \frac{c}{\omega} \left( \arcsin \frac{y}{b} - \arcsin \frac{y_0}{b} \right) \quad (18)$$

Due to  $\omega t_0 = \pi / 2$  and  $y_0 = qE_0$ , when  $t = t_0$ , we have

$$\frac{\sin \omega t}{\sqrt{1 + (c\omega m_0 / qE_0)^2}} = \sin \left[ \frac{\omega}{c} (x - x_0) + \arcsin \frac{1}{\sqrt{1 + (c\omega m_0 / qE_0)^2}} \right] \quad (19)$$

It can be seen from Eq.(15) with  $V/c < 1$ . Since there is no limit to  $x - x_0$ , we can always find  $x$  and  $t$  to make them satisfying Eq.(19). Therefore, the motion of electrons in a periodic electric field without length boundary limit is possible, and there is no need for electron to change the motion state by radiation.

## 4.2 The radiation power of an electron in a periodic electric field

Let's calculate the radiation power of an electron accelerated in free space under the action of periodic electric field force. By the integral of the left side of Eq.(13) with respect to time, the acceleration of electron is obtained

$$a = -\frac{qE_0 \sin \omega t (1 - V^2/c^2)^{3/2}}{m_0} \quad (20)$$

Substituting Eq.(20) in Eq.(20) and let  $t^* = t$ , the radiation power of electron is

$$P_{11} = \frac{q^4 E_0^2 \sin^2 \omega t}{6\pi \epsilon_0 c^3 m_0^2} \quad (21)$$

Assuming that the electric field intensity  $E_0 = 10^3 V/m$ , the radiation frequency  $\omega = 10^5$ , calculated according to the above formula, when  $\sin \omega t = 1$ , the maximum radiation power of electron is  $P_{11} = 1.75 \times 10^{-25} W$ , the minimum power is zero, and the average radiation power is  $\bar{P}_{11} = 8.80 \times 10^{-26} W$ . According to Eq.(15), the maximum speed of electron is  $V = 0.99977c$ . According to the relativistic formula, the maximum dynamic energy of electron is  $T_m = 3.88 \times 10^{-12} J$  and the average kinetic energy is  $\bar{T} = 1.94 \times 10^{-12} J$ . The time it takes to radiate all this kinetic energy away is  $\Delta t = \bar{T} / \bar{P}_{11} = 2.20 \times 10^{13} s$ , about 700,000 years. It indicates that the electrons yet do not radiate basically in the periodic accelerated motion too.

To apply this result to the transmission process of infinite AC current, the average current strength is  $I = 1A$ , the average speed of electron is  $V = 0.5c$ , and the number of electrons passing through the cross section of the wire per unit time is  $4.17 \times 10^{10}$ . The total radiated power is  $3.67 \times 10^{-16} W$ . The radiation is so small that it can be considered that there is actually no radiation.

The experiment also proves that if the heat radiation of the antenna resistance is not taken into account, the radio wave radiation generated by the accelerated motion of electrons under the action of alternating current can be ignored during the alternating current transmission process.

However, the problem is that if the electrons are periodically accelerated in an antenna of limited length, there is a strong radiation. For example, a 1-meter-long antenna is powered by an AC transformer of  $1000V$ , and the electric field intensity is  $E_0 = 10^3 V/m$ . Let the oscillation frequency of the LC circuit be  $\omega = 10^5$  (medium wave with wavelength of 478 meters) and the current intensity be the same with  $I = 1A$ . Take the equivalent resistance of antenna radiation  $R = 50$  ohms, the radiation power is  $P = IR^2/2 = 1250W$ , which is  $3.67 \times 10^{18}$  times greater than the radiation power of electrons acted by the periodic electric field in free space!

The existing theory does not explain why under the same conditions, there is such a big difference in the radiation of electrons when the boundary is finite and when the boundary is infinite. There must be some mechanism that we don not understand at present. We discuss this problem below.

## 4.2 The reason of antenna radiation

Below we prove that the motion of electrons is limited in a bounded finite periodic electric field, the high speed electrons at the end of antenna are slowed down rapidly, producing bremsstrahlung. Therefore, the acceleration of electron in periodic electric field is not the cause of antenna radiation, but bremsstrahlung radiation caused by is the essential cause of antenna radiation. Also take the antenna length being 1 meter, electric field  $E_0 = 10^3 V/m$ , frequency  $\omega = 10^5$ , we have

$$\left(\frac{c\omega m_0}{qE_0}\right)^2 = 2.89 \times 10^{-2} \ll 1 \quad \arcsin \frac{1}{\sqrt{1+(c\omega m_0/qE_0)^2}} \sim \arcsin 1 = \frac{\pi}{2} \quad (22)$$

Eq.(19) can be simplified as

$$\sin \omega t \approx \sin\left(\frac{\omega}{c}(x-x_0) + \frac{\pi}{2}\right) \quad (23)$$

Let  $x_0$  be the middle point of antenna with  $|x-x_0| \sim 0.5m$ , we get

$$\frac{\omega}{c}|x-x_0| \leq 1.67 \times 10^{-4} \ll \frac{\pi}{2} \quad (24)$$

The value range on the left side of Eq.(23) is  $-1 \sim +1$ , but the value range on the right side is  $+1$  nearby, so Eq.(23) is generally not valid. For example, taking  $t = \pi/\omega$ , the left side of equation is equal to zero, but the right side cannot be equal to zero unless  $\omega(x-x_0)/c \sim \pm\pi/2$ . That means  $|x-x_0| = 4710m$ , well beyond the 1-meter antenna limit. Another example, taking  $t = 3\pi/(2\omega)$  with  $\sin \omega t = -1$ , according to Eq.(23), we have  $\omega(x-x_0)/c = \pm\pi$  and  $|x-x_0| = 9420m$ , greatly beyond the 1-meter limit of the antenna. Therefore, Eq.(23) cannot describe the normal motion of electrons in an antenna with a length of 1 meter.

The initial phase angle is  $\theta_0 = \pi/2$  in the above discussion. If  $\theta_0 \neq \pi/2$ , Eq.(16) cannot be integrated. However, since the physical process has nothing to do with the initial phase angle value, the above conclusions are still valid in general. For example, ignoring the second item in the right bracket of Eq.(19) and considering Eq.(24), there is

$$\sin \omega t \approx \sin\left[\frac{\omega}{c}(x-x_0)\right] = -0.0085 \sim 0.0085 \quad (25)$$

This relation is invalid in general.

However, the antennas with a length of 1 m and frequency  $\omega = 10^5$  can be manufactured practically, so the actual motion of electrons in this antenna can only be as follows. When  $\sin \omega t > 0$  and the electric field force is positive, the free electrons in the antenna are moving forward. When they reach the top of the antenna, their motion is blocked and they are rapidly slowed down, all the electrons are stationary at the same end, and the corresponding positive charge appears at the other end. Until  $\sin \omega t < 0$ , the electric field force becomes negative, all the electrons move in the opposite direction at the same time, and they end up stationary on the other side of the antenna.

This repeated formation of oscillations produces electromagnetic radiation at both ends of the antenna, the physical image of which can be simplified by describing dipole oscillations. If the antenna is long enough, electrons no longer accumulate at the end, just alternating current moving through the wire. The antenna doesn't oscillate, so it does not radiate.

### 4.3 The essence of antenna bremsstrahlung

From a microscopic point of view, electrons traveling to the end of antenna cannot leave the wire. Under the action of atomic electric field at the end of antenna, the electrons are suddenly slowed down, and the resulting radiation is actually bremsstrahlung. Since all the electrons are suddenly slowed down at the end of the wire, the resulting Bremsstrahlung is large. If the antenna is long enough, the electrons will not reach the end of antenna under the action of periodic electric field, there will be no sudden deceleration for the electrons at the end of the wire, and the radiation of antenna will be greatly reduced. According to Eq.(19), the antenna oscillates to produce

radiation, and the conditions are as follows

$$\frac{c\omega m_0}{qE_0} \ll 1 \quad \text{and} \quad \frac{\omega}{c}|x-x_0| < \frac{\pi}{2} \quad (26)$$

For the antenna with  $E_0 = 10^3 V/m$  and  $|x-x_0| = 0.5$ , the highest frequency of radio wave radiation is  $\omega = 9.42 \times 10^8$  (millimeter wave). Beyond this frequency, there is no electromagnetic radiation. For example, when  $\omega = 10^{10}$  (infrared light wave), we have

$$\begin{aligned} \frac{\omega}{c}|x-x_0| &= 16.7 & \frac{c\omega m_0}{qE_0} &= 1.71 \times 10^4 \\ \sqrt{1 + \left(\frac{c\omega m_0}{qE_0}\right)^2} &\approx 1.71 \times 10^4 & \arcsin \frac{1}{\sqrt{1 + (c\omega m_0 / qE_0)^2}} &= 5.85 \times 10^{-5} \end{aligned} \quad (27)$$

Eq.(19) becomes

$$\sin \omega t = 1.71 \times 10^4 \sin \left[ \frac{\omega}{c}(x-x_0) + 5.85 \times 10^{-5} \right] \quad (28)$$

Eq.(28) can be tenable, so there is no bremsstrahlung at the end of antenna. This is why normal radio transmitting antennas cannot radiate infrared and visible lights. Antenna radiation actually occurs mainly at the two ends of the antenna, although there are few radiation in the middle.

It can be seen that the accelerated motion of electrons in the antenna is not the cause of antenna radiation, but the bremsstrahlung occurring at the end of the antenna is the main cause of antenna radiation. It is found that if the antenna has a sharp angle, it will produce strong radiation, which is also related to the one-way movement of electrons. The specific mechanism of bremsstrahlung, as well as the stability problem of atoms, will be discussed in another paper.

## 5 The motion of charged particle in uniform magnetic field.

### 5.1 The stable motion of an electron in a uniform magnetic field

Assuming that the magnetic field is uniform and along the direction of the z axis. According to Eq.(5), the motion equation of an electron in the magnetic field is

$$m \frac{d\vec{V}}{dt} + \vec{V} \frac{dm}{dt} = -q(\vec{V} \times \vec{B}) = \vec{F}_B \quad (29)$$

The total energy of electron is  $E = mc^2 = K + m_0c^2$ , where  $K = E - m_0c^2$  is the kinetic energy of electron. Considering that the change of energy in unit time is equal to the power to do by the electromagnetic field force, as well as  $\vec{V} \perp \vec{B}$ , we get

$$c^2 \frac{dm}{dt} = \frac{dK}{dt} = \vec{F}_B \cdot \vec{V} = -q\vec{V} \cdot (\vec{V} \times \vec{B}) = 0 \quad (30)$$

For example, when electrons have no tangential force acted at a bend in the synchrotron orbit. In this case, Eq. (29) can be simplified as

$$m \frac{d\vec{V}}{dt} = \frac{m_0 \vec{a}}{\sqrt{1-V^2/c^2}} = -q(\vec{V} \times \vec{B}) = \vec{F}_B \quad (31)$$

Due to  $a = V^2/R$ , the direction of  $\vec{a}$  is the same as that of  $\vec{F}_B$ , we have

$$a = -\frac{qVB\sqrt{1-V^2/c^2}}{m_0} = \frac{F_B\sqrt{1-V^2/c^2}}{m_0} \quad (33)$$

Let  $R$  represent the radius of circle and  $p$  is the momentum of electron, we get

$$R = \frac{m_0 V}{qB\sqrt{1-V^2/c^2}} = \frac{p}{qB} \quad (32)$$

There is no singularity in Eq.(32), therefore, when an electron moves in a circular in a uniform magnetic field, its motion is stable and it does not need to radiate.

## 5.2 The instability of motion caused by perturbations

However, strict circular motion is only a mathematical ideal, which is difficult to achieve in practice. In practical processes, there exist various interfering factors, such as the non-uniformity of magnetic field, the presence of residual gas in the accelerator vacuum chamber, the electromagnetic interaction between the charged particles, and the interference of electromagnetic fields in other parts of accelerator. These interfering factors are equivalent to a perturbation force that causes the electron to move away from the circular orbit.

It is shown below that once the mass-velocity formula is taken into account, the electron's velocity may become imaginary, or exceed the speed of light in vacuum, once it deviates from its circular orbit. This is not allowed in physics, and the electrons will automatically radiate. By the damping force of radiation, the speed of electron returns to normal, making the motion possible. Assuming that there is a perturbation force  $\vec{F}_w$  when an electron moves in a uniform magnetic field, the equation of motion becomes

$$\frac{m_0 \vec{a}}{\sqrt{1-V^2/c^2}} + \frac{m_0 \vec{V}(\vec{V} \cdot \vec{a})/c^2}{(1-V^2/c^2)^{3/2}} = \vec{F}_B + \vec{F}_w = \vec{F} \quad (34)$$

Taking the dot product of  $\vec{V}$  on Eq.(34) and considering  $\vec{V} \cdot (\vec{V} \times \vec{B}) = 0$ , we get

$$\frac{m_0 \vec{V} \cdot \vec{a}}{\sqrt{1-V^2/c^2}} + \frac{m_0 V^2 (\vec{V} \cdot \vec{a})/c^2}{(1-V^2/c^2)^{3/2}} = \vec{V} \cdot \vec{F} = \vec{V} \cdot \vec{F}_w \quad (35)$$

Because the perturbation force is an external force which does work, energy is not conserved with  $dm/dt \neq 0$ . In this case, the second term on the left side of Eq.(5) is not zero. Let  $\vec{V} \cdot \vec{a} = Va \cos \theta$  and  $\vec{V} \cdot \vec{F}_w = VF_w \cos \beta$ , from Eq.(3), we have

$$\frac{m_0 a \cos \theta}{\sqrt{1-V^2/c^2}} + \frac{m_0 V^2 a \cos \theta / c^2}{(1-V^2/c^2)^{3/2}} = \frac{m_0 a \cos \theta}{(1-V^2/c^2)^{3/2}} = F_w \cos \beta \quad (36)$$

From Eq.(36), we get

$$\frac{1}{(1-V^2/c^2)^{3/2}} = \frac{F_w \cos \beta}{m_0 a \cos \theta} = \delta \quad (37)$$

$$\frac{V^2}{c^2} = 1 - \frac{1}{\delta^{2/3}} \quad (38)$$

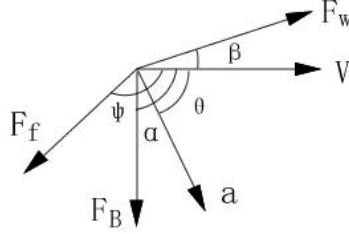
Due to  $|\vec{F}_B| \gg |\vec{F}_w|$ ,  $|m_0\vec{a}| \approx |\vec{F}_B + \vec{F}_w| \gg |\vec{F}_w|$ , we get  $0 < |\delta| \ll 1$ ,  $0 < \delta^{2/3} \ll 1$ . According to Eq.(38), we have  $V^2/c^2 \ll -1$  or  $-V^2/c^2 \gg 1$ . For example, let  $\delta=0.001$ , we have  $\delta^{2/3}=0.01$ ,  $V^2/c^2=-99$ ,  $V/c=i\sqrt{99}$ . The speed of the electron not only becomes imaginary, but also may become faster-than-light imaginary. Such a motion is obviously impossible, so the electron has to radiate, changing the state of motion by radiating damping forces to make the motion possible.

### 5.3 The radiation damping force

Let  $\vec{F}_f$  be the radiation damping force, the perturbation force caused by environment is  $\vec{F}_w$ , the real motion equation of electrons in synchrotron is

$$\frac{m_0\vec{a}}{\sqrt{1-V^2/c^2}} + \frac{m_0\vec{V}(\vec{V} \cdot \vec{a})/c^2}{(1-V^2/c^2)^{3/2}} = -q(\vec{V} \times \vec{B}) + \vec{F}_w + \vec{F}_f \quad (39)$$

In this case, the velocity is generally not perpendicular to the direction of the magnetic field. Let the angle between acceleration  $\vec{a}$  and velocity  $\vec{V}$  be  $\theta$ , the angle between perturbed force  $\vec{F}_w$  and velocity  $\vec{V}$  be  $\beta$ , and the angle between radiant damping force  $\vec{F}_f$  and velocity  $\vec{V}$  be  $\varphi$  as shown in Fig.5.



**Fig. 5 Perturbation force and radiation damping force of an electron moving in a magnetic field**

Using  $\vec{V}$  to dot multiply Eq.(39), the first term on the right side of the equation is still zero, and we obtain

$$\begin{aligned} & \frac{m_0 a \cos \theta}{\sqrt{1-V^2/c^2}} + \frac{m_0 V^2 a \cos \theta / c^2}{(1-V^2/c^2)^{3/2}} \\ &= \frac{m_0 a \cos \theta}{(1-V^2/c^2)^{3/2}} = F_w \cos \beta + F_f \cos \varphi \end{aligned} \quad (40)$$

Let

$$\frac{F_w \cos \beta + F_f \cos \varphi}{m_0 a \cos \theta} = \lambda \quad (41)$$

According Eq.(40), we get

$$\frac{V^2}{c^2} = 1 - \frac{1}{\lambda^{2/3}} \quad (42)$$

Assuming  $\vec{F}_f$  is large enough, it is possible to have  $\lambda > 1$ . According to Eq.(42), the speed of electron can be real and have  $V < c$ . Thus, by introducing the radiation damping force, it becomes possible to make the previously impossible and unstable motion of electron becoming possible and stable motion.

Using  $\vec{V}$  to cross multiply Eq.(39), and let  $\vec{V} \times \vec{F}_B = VF_B \sin \alpha$ , we get

$$\frac{m_0 a \sin \theta}{\sqrt{1-V^2/c^2}} + \frac{m_0 V^2 a \sin \theta / c^2}{(1-V^2/c^2)^{3/2}} = \frac{m_0 a \sin \theta}{(1-V^2/c^2)^{3/2}}$$

$$= F_B \sin \alpha + F_w \sin \beta + F_f \sin \varphi \quad (43)$$

From Eqs.(40) and (43), we get

$$\text{tg} \theta = \frac{F_B \sin \alpha + F_f \sin \varphi + F_w \sin \beta}{F_f \cos \varphi + F_w \cos \beta} \quad (44)$$

or

$$F_f = \frac{F_B \sin \alpha + F_w (\sin \beta - \cos \beta \text{tg} \theta)}{\cos \varphi \text{tg} \theta - \sin \beta} \quad (45)$$

As long as the radiation damping force satisfies the condition of Eq.(45), the motion of electron can continue, but the electron must radiate. The radiation of electrons in a magnetic field is a self-regulating response to this instability of motion, by which impossible motion becomes possible.

## 6 The motion of electrons in the centripetal electric field

### 6.1 The stationary motion of electrons in the centripetal electric field of nucleus

Suppose that the nucleus with charge  $Q$  is at rest, electron's charge is  $-q$ . By considering the mass-velocity formula, the motion equation of an electron in the centripetal electric field of the nucleus is

$$\frac{m_0}{\sqrt{1-V^2/c^2}} \frac{d}{dt} \left( \frac{1}{\sqrt{1-V^2/c^2}} \frac{d\vec{r}}{dt} \right) = -\frac{qQ\vec{r}}{4\pi\epsilon_0 r^3} \quad (46)$$

Assume that the electron moves in a plane. By using the pole coordinate and let  $dr/dt = \dot{r}$  and  $d\theta/dt = \dot{\theta}$ , Eq.(46) can be written as [3]

$$\frac{d}{dt} \frac{m_0 \dot{r}}{\sqrt{1-V^2/c^2}} = \frac{m_0 r \dot{\theta}^2}{\sqrt{1-V^2/c^2}} - \frac{qQ}{4\pi\epsilon_0 r^2} \quad (47)$$

$$\frac{d}{dt} \frac{m_0 r^2 \dot{\theta}}{\sqrt{1-V^2/c^2}} = 0 \quad (48)$$

Taking the integrals of Eqs.(47) and (48), the trajectory equation of an electron in the nuclear electric field is [9]

$$r = \frac{p}{1 + E \cos(\sqrt{1-\rho^2} \theta)} \quad (49)$$

Where  $E$  and  $K$  are constants with

$$E = \frac{m_0 c^2}{\sqrt{1-V^2/c^2}} - \frac{Qq}{r} \quad K = \frac{m_0 r^2 \dot{\theta}}{\sqrt{1-V^2/c^2}} \quad (50)$$

$$\rho = \frac{Qq}{Kc} \quad p = \frac{K^2 c^2 - Q^2 q^2}{QqE} \quad (51)$$

Eq.(49) describes an elliptical procession orbit, in which  $E$  is the total energy, and  $K$  is the angular momentum of an electron. As discussed in paper [1], the orbit of electron has no singularity. In this case, the motion is stable and achievable, meaning that the electron does not need to radiate.

If we do not understand in this way, the problem of atomic instability would arise according to current theory. According to Eq.(11), the radiated power of ground state electrons in hydrogen atom is:

$$P_{\perp} = \frac{q^6}{96\pi^3 \epsilon_0^3 c^3 m_0^2 r^4 (1 - V^2/c^2)} \quad (52)$$

The ground state electron energy of a hydrogen atom is  $E_1 = 13.55 eV = 2.17 \times 10^{-18} J$ , the corresponding speed of an electron is  $V = 2.57 \times 10^{-3} c$ . Let  $r = 0.53 \times 10^{-10} m$  be the first Bohr orbital radius, it can be obtained  $P_{\perp} = 4.60 \times 10^{-8} W$  from Eq.(52). According to this radiation power, we have  $E_1 / P_{\perp} = 4.72 \times 10^{-9}$ . That is, the ground state electron will lose all its energy in  $4.72 \times 10^{-9} s$  and fall onto the hydrogen nucleus. However, atoms are actually stable, and the result of formula (52) is obviously impossible.

## 6.2 The unstable motion of electron in the centripetal electric field of nucleus

The above discussion is that the nucleus is stationary, the elliptical orbital motion of an electron around the nucleus is stable. It can be done without radiation. If the electrons are not orbiting the nucleus, but shooting at the nucleus from a distance at high speed, the situation might be quite different. Since the initial position and initial speed of electron are arbitrary, coupled with the random movement of nucleus, when a high-speed electron is slowed down near the nucleus, it is possible to deviate from the stable orbit, resulting in the speed becoming imaginary or exceeding the speed of light. In this case, electron will radiate. It is analyzed below. By considering

$$\frac{dV}{dt} = \frac{d}{dt} \sqrt{V_x^2 + V_y^2 + V_z^2} = \frac{V_x}{V} \frac{dV_x}{dt} + \frac{V_y}{V} \frac{dV_y}{dt} + \frac{V_z}{V} \frac{dV_z}{dt} = \frac{\vec{V} \cdot \vec{a}}{V} \quad (53)$$

Eq.(53) can be rewritten as

$$\frac{m_0 \vec{a}}{\sqrt{1 - V^2/c^2}} + \frac{m_0 \vec{V}(\vec{V} \cdot \vec{a})/c^2}{(1 - V^2/c^2)^{3/2}} = -\frac{qQ\vec{r}}{4\pi\epsilon_0 r^3} \quad (54)$$

Using  $\vec{r}$  cross product Eq.(54), the projection of equation in the vertical direction of  $\vec{r}$  (from the page to the reader) is obtained

$$\frac{m_0(\vec{r} \times \vec{a})}{\sqrt{1 - V^2/c^2}} + \frac{m_0(\vec{r} \times \vec{V})(\vec{V} \cdot \vec{a})/c^2}{(1 - V^2/c^2)^{3/2}} = 0 \quad (55)$$

For electron's motion in the electric field of nucleus, we have  $\vec{r} \times \vec{a} \neq 0$ ,  $\vec{r} \times \vec{V} \neq 0$  and  $\vec{V} \times \vec{a} \neq 0$  in general. The situation is more complex than the motion of electron in a magnetic field. As shown in Fig. 1, let  $\vec{e}_r$  and  $\vec{e}_\theta$  represent two vertical directions in the pole coordinate system, the included angle of  $\vec{V}$  and  $\vec{e}_\theta$  is  $\alpha$ , the included angle of  $\vec{a}$  and  $\vec{e}_\theta$  is  $\beta$ , the direction of  $\vec{F}_E$  and  $\vec{e}_r$  are opposite. We have

$$\begin{aligned} \vec{V} \cdot \vec{a} &= Va \cos(\beta - \alpha) & \vec{r} \times \vec{a} &\sim ra \sin(\beta - \pi/2) = -ra \cos \beta \\ \vec{r} \times \vec{V} &= -\vec{V} \times \vec{r} \sim -Vr \sin(\pi/2 - \alpha) = -Vr \cos \alpha \end{aligned} \quad (56)$$

Substituting them in Eq.(55), the relation of electron's motion in the direction vertical to  $\vec{r}$  satisfies

$$-\cos \beta - \frac{\cos \alpha \cos(\beta - \alpha) V^2/c^2}{1 - V^2/c^2} = 0 \quad (57)$$



or 
$$\frac{V^2}{c^2} = \frac{1}{1 - \cos \alpha \cos(\beta - \alpha) / \cos \beta} \quad (58)$$

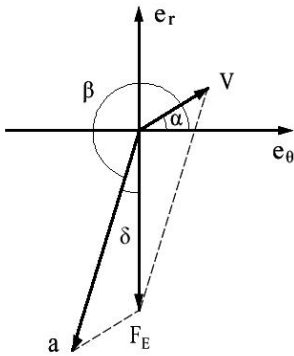
Due to  $V/c < 1$ , we should have

$$\frac{\cos \alpha \cos(\beta - \alpha)}{\cos \beta} = \cos^2 \alpha (1 + \operatorname{tg} \alpha \operatorname{tg} \beta) < 0 \quad (59)$$

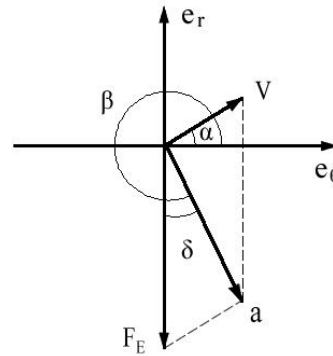
or 
$$\operatorname{tg} \alpha \operatorname{tg} \beta < -1 \quad (60)$$

When  $\alpha$  is located between the first and third quadrant, we have  $\operatorname{tg} \alpha > 0$ . When  $\alpha$  is located between the second and fourth quadrants, we have  $\operatorname{tg} \alpha < 0$ . If  $\alpha$  is located at the first and third quadrants, in order to make Eq.(60) tenable,  $\beta$  should be located in the second and third quadrants. At the same time, Eq.(60) should be satisfied. Otherwise, electron's motion is still unstable and electron still radiate.

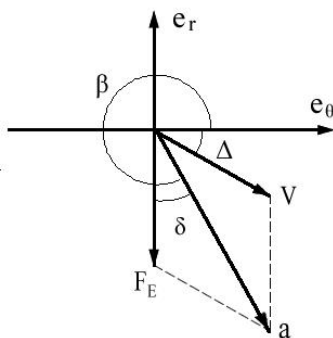
Suppose that  $\alpha$  is located at the first quadrant and  $\beta$  is located at the second quadrant as shown in Fig.6, electron's motion is unstable, it will radiate. If  $\alpha$  is located at the first quadrant and  $\beta$  is located at the fourth quadrant as shown in Fig.7, the electron's motion is possible, but should still satisfy Eq.(60). Let  $\beta = 270^\circ + \delta$  and  $\operatorname{tg} \beta = -\operatorname{ctg} \delta$ , Eq.(60) becomes  $\operatorname{tg} \alpha \operatorname{ctg} \delta > 1$  or  $\operatorname{tg} \alpha > \operatorname{tg} \delta$ , i.e.,  $\alpha > \delta$ . In this case, electron does not radiate. If  $\alpha < \delta$ , electron's motion is still unstable and it will radiate.



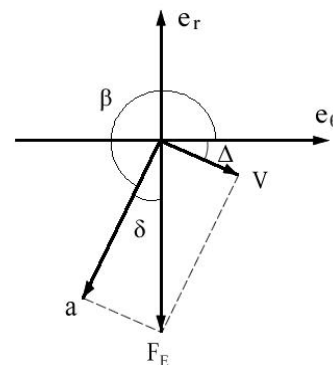
**Fig.6 Impossible motion form 1 for an electron in central electric field, it will radiate.**



**Fig.7 Possible motion form 1 for an electron in central electric field, it may not radiate.**



**Fig.8 Impossible motion form 2 for an electron in central electric field, it will radiate.**



**Fig.9 Possible motion form 2 for an electron in central electric field, it may not radiate.**

Suppose that  $\alpha = -\Delta$  is located at the fourth quadrant and  $\beta$  is also located at the fourth quadrant as

shown in Fig.8, the motion of electron is impossible and it will radiate. In Fig.9,  $\alpha = -\Delta$  is located at the fourth quadrant and  $\beta$  is also located at the third quadrant. The motion of electron's motion is possible, but it should satisfies Eq.(60). Let  $\beta = 270^\circ - \delta$  and  $\alpha = -\Delta$ , we have  $tg\beta = ctg\delta$  and  $tg\alpha = -tg\Delta$ . Eq.(60) becomes  $tg\Delta ctg\delta > 1$  or  $tg\Delta > tg\delta$ , i.e.,  $\Delta > \delta$ . In this case, electron does not radiate. If  $\Delta < \delta$ , electron's motion is still unstable, it will radiate.

Besides, if  $\vec{V} \cdot \vec{a} = 0$ , electron moves in a circle and does not radiate. Because there are two freedom degree of  $\delta$  and  $\Delta$  for electron to choose, it's motion is possible in the central force field, it need not to radiate. This further proves that atoms are generally stable, and electrons moving around the nucleus do not radiate.

In the synchronous light source of accelerator, oscillator and torsional pendulum are used to produce radiations. Oscillators and torsional pendulum must use alternating magnetic fields [7], and cannot use alternating electric fields, which is why the reason. Applying a periodic electric field in the direction perpendicular to the velocity of high electron will also cause the orbital motion of electron to wobble. However, as mentioned earlier, the movement of electrons in free space under the action of periodic electric fields is stable, so there is no need for radiation. It is impossible to make free electrons produce laser light by alternating electric fields.

## 7 Synchrocyclotron without radiation loss

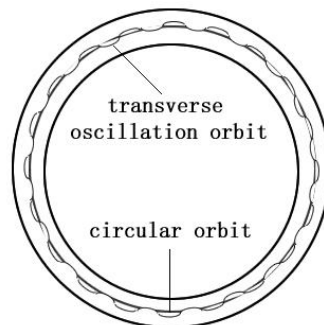
### 7.1 The reason causing radiation in synchrocyclotron

Existing high energy particle synchrocyclotron uses variety of sector magnets, such as straight edge sector, spiral sector and separate sector magnets, which produce non-uniform magnetic fields. When an electron is injected into a cyclotron, the deviation of angle will also cause the deviation of circular orbit, and so on.

These factors will cause the change of beam orbit of accelerated particle, causing the so-called transverse oscillation of the orbit [10], such as the wavy orbit shown in Fig.10.

When a charged particle is traveling very close to the speed of light, a small deviation from the orbit of circle can cause the particle's speed becoming imaginary or exceed the speed of light. The particle had to correct its orbit by radiation, which is the fundamental reason why electrons moving in the synchrocyclotron radiate, while electrons moving in the linear accelerator and induction accelerator generally do not radiate.

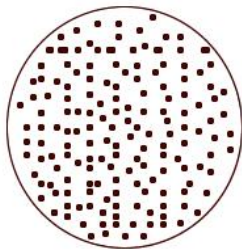
Experiments show that in the circular storage ring of the synchrocyclotron, when the particle beam is injected at the beginning, it is dispersed on the cross section of the ring, as shown in Fig.11. Due to the existence of transverse oscillatory motion, strong synchrotron radiation appears. Under the action of the magnetic force, the amplitude of transverse oscillation gradually decreases, the synchrotron radiation also weakens, and the motion orbit of particle is closer to the circular orbit in the middle.



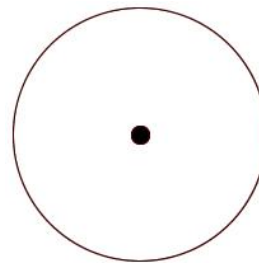
**Fig.10 Transverse oscillatory orbital motion of electrons in a synchrocyclotron**

The transverse area of particle beam in the storage ring decreases and the brightness of particle beam increases. When the cross-section of particle beam is reduced to a certain extent, the particle beam forms into clusters and the density is greatly increased, as shown in Fig.12, and basically no longer radiates. After that, new particles are injected into the storage ring again. In this way, the particles are bunched into clusters. After accumulating a large number of particle clusters, they are drawn out for collision [10].

According to the existing radiation theory, the radiation of charged particles is determined by the acceleration, the transverse oscillation amplitude of particles in the storage ring is reduced, and after moving along the standard circle, due to the constant centripetal acceleration, a large amount of radiation still exist. Therefore, due to the loss of energy, the orbit of particle is still unstable, and eventually they will collide with the wall of the beam tube and disappear. The charged particles can not be formed into clusters and stable existence in the beam tube, or the particles can not be stored. However, this is not the case. The fact is that the storage ring of synchrocyclotron can store particles just shows that the particles do not radiate as they move in a standard circle in the synchrocyclotron.



**Fig.11 Dispersed distribution of electrons which are just injected into the storage ring with strong radiation**

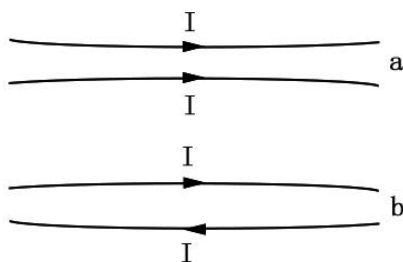


**Fig. 12 Radiation is greatly reduced after electron beam is formed in the ring**

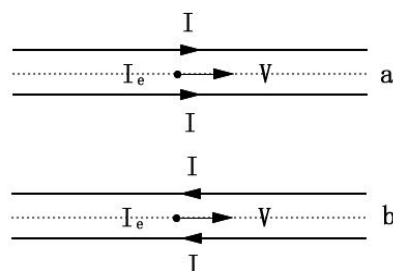
Therefore, it can be imagined that if we introduce some force to replace the radioactive damping force in the synchrocyclotron to satisfy Eq.(45), or more directly, eliminate the interfering force in the accelerator so that the electron can moves in a standard circular orbit, the electron does not have to radiate. As long as they are bound by a sufficiently strong magnetic force, they can be accelerated to the desired high-energy state without limit.

### 7.2 The design of synchrocyclotron without radiation loss

As discussed above, the uneven magnetic field, as well as other interference factors, will cause the deviation of particle's trajectory from the circular orbit, resulting in the radiation of charged particles. Therefore, the key is to eliminate the interference factor, so that the charged particles move in a standard circular orbit. In this way, we can design a synchrocyclotron with low or even no radiation.



**Fig.13 Mutual attraction and repulsion of two current lines**



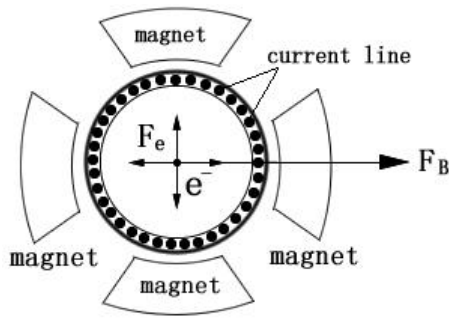
**Fig.14 Action of two fixed current lines on a moving charged particle**

In electromagnetism, it is known that two current lines repel each other when the current directions are the same (Fig.13 a), and attract each other when the current directions are opposite (Fig.13 b). The reason is that the linear current creates a magnetic field around the current, which exerts a force on the other moving current.

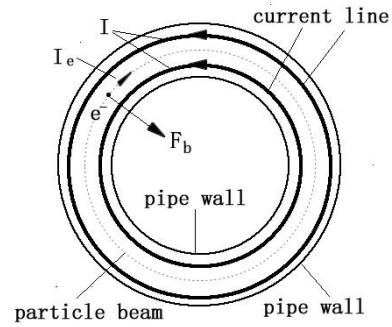
Suppose that there are two fixed parallel current lines  $I$  with the same current direction. There is a charged particle moving in a direction parallel to the current line, which can be regarded as the current  $I_e$  as shown in Fig.6. The distance between  $I$  and  $I_e$  is  $b$ , and the action force of two currents is expressed as follows [11]

$$F_e = \frac{\mu_0 I \cdot I_e}{2\pi b} \quad (61)$$

If  $I_e$  is just in the middle position of two current lines  $I$ , no matter whether either  $I_e$  is at the same direction or at the opposite directions with  $I$ , the charged particle is acted by two forces which are equal value but opposite direction so that they cancel each other out. When the motion of charged particle deviates from the intermediate line, if the directions of  $I_e$  and  $I$  are the same (Fig.14 a), the charged particle will move closer and closer to the current line which is closer to it, and finally collide with the current line and disappear. If the directions of  $I_e$  and  $I$  are opposite (Fig.14 b), the two current lines  $I$  will create a combined force to bring the charged particle back to its equilibrium position in the middle.



**Fig.15 Section diagram of parallel current line of particle beam tube of non-radiation synchrocyclotron**



**Fig. 16 Plane diagram of parallel current line of particle beam tube of non-radiation synchrocyclotron**

Using this principle, the orbital oscillations of particles in synchrocyclotron can be eliminated, and make the accelerated particle moving in a stable circular orbit. Fig.15 shows a cross-section diagram of particle beam tube of a synchrocyclotron modified in this way. Some direct current lines parallel to the pipe wall are arranged along the circumference of pipe wall. The accelerated electron  $e^-$  moves in the center of pipe in a direction perpendicular to the paper surface. Fig.16 shows the top plane view diagram of non-radiation synchrocyclotron. The current lines are arranged along the inner walls of ring pipe, and the current intensity is  $I$ . The charged particle stream in the middle of beam tube can be regarded as a current line, and the current intensity is  $I_e$ .

A charged particle moving perpendicular to the surface of the paper at the middle point as shown in Fig. 8 is acted by forces up and down, or right and left. If the particle deviates from the intermediate point, the balance of motion is disrupted, and the current lines up and down (or left and right), create a resultant force that pulls the particle back to the intermediate point, eventually achieving a balanced and stable motion.

The mass of electron is  $m_0 = 9.11 \times 10^{-31} \text{ Kg}$ , the charge is  $e = 1.60 \times 10^{-19} \text{ C}$ . Suppose that an electron moves in a closer speed of light in the tube, the current intensity is  $I_e = ec$ . Taking  $\mu_0 = 1.26 \times 10^{-6}$ ,  $b = 0.1m$ , according to Eq.(61), we get  $F_e = 9.63 \times 10^{-17} I$ . On the other hand, considering the circular motion of electron, the radius of circle is  $R = 100m$ . The centripetal force or the magnetic field binding force acted on the

electron is  $F_b = m_0 c^2 / R = 8.19 \times 10^{-16} N$ . To make  $F_e$  be the same with  $F_b$ , we only need to take  $I = 8.52 A$ , this is easy to achieve.

In Fig.16, assume that at a certain moment, the accelerated particle moves to the left away from its equilibrium position, the distance from the current line on the left is  $b_1$ , and the distance from the corresponding current line on the right is  $b_2$  with  $b_2 > b_1$ . The combined force of these two current lines on the accelerated particle is

$$\Delta F_e = \frac{\mu_0 I \cdot I_e}{2\pi} \left( \frac{1}{b_1} - \frac{1}{b_2} \right) = \frac{\mu_0 I \cdot I_e}{2\pi} \frac{b_2 - b_1}{b_1 b_2} > 0 \quad (61)$$

$\Delta F_e$  is a positive force which makes the particle moves toward light side. In this way, the motion of particle can be fine-tuned to reduce the oscillatory motion of the accelerated particle, so that the charged particles moves along the standard circle in synchrocyclotron.

If the current intensity of the current line is large enough, the transverse oscillation motion of particle in the tube can be completely eliminated. Therefore, it is possible to build a miniaturized, low-radiation, or even basically non-radiation high-energy synchrocyclotrons.

## 7 Conclusions

According to classical electromagnetic theory, the accelerated motion of charged particles produces electromagnetic radiation, but this is not always the case. Experiments show that charged particles radiate sometime but not radiate sometime during the following acceleration processes, which cannot be explained by the existing theory.

1. Charged particles are accelerated in a uniform electric field but not radiate or few radiation even they reach the speed of light. However, when a high-speed charged particle is slowed down by a metal target, it emits intense X-ray radiation, although the absolute values of electron's accelerations are basically the same in both cases.

2. In the radio antenna, if the parameters are not properly selected, the electrons that do periodic acceleration motion do not radiate. Only when the condition of antenna oscillates is satisfied, they radiate.

3. The electron does not radiate when it is accelerated in the circle orbit around the atomic nucleus, and the atom is stable.

4. In electron induction accelerator and , electrons do not radiate, only the charged particles moving in the synchrocyclotron, they radiate.

All these facts show that acceleration is not the essential cause of radiation produced by charged particles. The radiation theory of classical electromagnetism need to be revised. We need to reconsider the physical mechanism of charged particle radiation.

This paper considers the mass velocity formula and analyses the stability of charged particle moving in the electromagnetic field. It is proved that the radiation of charged particle is determined by the stability of motion, not by the acceleration. Under certain conditions, the motion of charged particles is theoretically impossible, and the motion of practice is unstable. The charged particle must then change its state through radiation to achieve a stable motion.

It is proved that when an electron moves in a magnetic field, if the magnetic field is not uniform, or if there is a small interfering force, the motion of high-speed electron may be unstable. In this case, the speed of electron may not only exceed the speed of light, but also become imaginary. In order to make the motion be possible, electrons have to radiate. This is the essential reason of charged particles radiate in synchrocyclotrons, which decreased the energy of particles.

On the other hand, as long as the appropriate electromagnetic force is applied, the interference force can be counteracted, the motion of electrons in the magnetic field can become stable, so that the charged particles do not radiate. In this way, we can obtain a logically consistent theory which is coincident with the experiment on the radiation of charged particles.

According to this understanding, a design scheme of synchrocyclotron without radiation loss is proposed in this paper. As long as we lay parallel current lines in the particle beam tube and eliminate the factors that make the particle motion unstable, we can build a high energy and low radiation loss synchrocyclotron. Only spending relatively little money to renovate the existing small and medium-sized synchrocyclotron, we can make them reach and exceed the high energy of present large accelerators. There is no need to build another large particle collider with a circumference of hundred kilometers.

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