# Transcendental Signals: Word and Number

Tim Xia
University of Texas at Austin
timtx@utexas.edu

**Abstract**

We summarize major historical currents in the study of signals, and offer an alternative perspective that is centered around the meaning of signals, a topic left unanswered by Shannon since the 1948. Despite the seeming variety of number systems, we suspect that the geometry often found in the eigen-spectrum of numerical signal system is independent of the local dynamics and exists rather out there. The implication then is that same geometry can be implemented by different numerical representations, this is well-known through the principle of computational equivalence. A less obvious implication is whether different geometry could underlie the same numerical pattern. We discuss examples in which this discrepancy of underlying geometry could come in to curious effect. And we provide a mathematical description of ectropic process wherein some hidden algebra could come into effect in the creation of efficacy and mathematical probability.

## Introduction

Nothing has changed for about a century. What progress has been made after Godel's bombshell? There is a stagnation in our understanding of words. To this today we look at the word in the same gloomy picture made out by Wittgenstein: the words are like a muddy edifice hanging in the air, where symbols get dragged out of symbols in an illogical and confusing way. This idea of words as a pile of warm and buzzing confusion seems to be a major inconvenience of humanity and an undesired effect of the enlightenment project, rather than an advantageous feature. There are two alternative view points as to how to understand word systems function; one of them with much precedent, the other without.

Word centric view: some words that pass as signals may admit more information than that is locally decomposable or measurable.

Model centric view: things can be decompose into kolmogorov programs, and such that one program is in effect no different than another, i.e. computationally equivalent description systems.

The model centric view represents the philosophy that also underlies contemperoary measure theory and axiomatic probabilities [8], the scheme in which we can totally deconstruct the system of words into primitive components and thats the end of it. This is the Shannon or Kolmogorvan approach where the treatment of information is local only; a study of communications in which the primary concern is not meaning, but the efficacy of channels [11].

The other approach has remained hitherto rather unknown: the idea that words that pass as signals can have meanings outside of the locally well-analyzed axiomatic signal channel system. That beyond the immediately backdrop of its communication context, a word may travel well above its locality and take effect in a downstream context that is not known in immediacy.

A closer examination of the problems revolving around the understanding of words reveals a problem of numerics. For we can always assign numbers to words, as is down in modern studies of communication. The problem then arises: can we really be sure that 1 equals to 1, in the backdrop that we cannot really distinguish 1 from 0; as it was found in Godel's theorem: a machine that produces truths can produce untruths.

In what follows, we show that given two seemingly equivalent schemes of representation of signals, one may encode different algebraic structures in the decimal points of the communication system in a way that allows one to have computationally equivalent non-equivalent representations.

We discuss this phenomenon in the context of quantumn two slit experiement and characterize its ectropic effects. Finally, connections are made with mathematical conjectures concerning operator algebra and with properties of transcendental numbers expressible as irreducible polynomials.

# Mathematical Signal Systems

We provide the mathematical principle underlying signal systems using the real numbers [4]. As we will show, the technical use of real numbers is not a constraint, similar systems using the p-adic numbers or complex numbers could capture the same geometrical information. The main point of signal analysis using dynamic network is the separation of pattern from local dynamics.

## Cognitive Network

A **cognitive network** is a directed graph $G = (V, E)$, where:

- $V$ is the set of **agents**.
- $E$ is the set of **directed edges** representing interactions between agents.

Each agent $v \in V$ has:

- A **state** $f_v \in \mathcal{F}_v$, where $\mathcal{F}_v$ is the state space of agent $v$.
- A **signal** $x_e \in X$ for each edge $e \in E$, where $X$ is the signal space.

## Propagation Dynamics

A complex network is modeled as a directed graph $G = (V, E)$, where $V$ is the set of agents and $E$ is the set of directed edges. Each agent $v \in V$ has incoming edges $T(v)$ and outgoing edges $F(v)$.

The state of each agent is determined by its input signals, and the propagation dynamics govern the spread of information:

$$x_{F_v} = f_v(x_{T_v})$$

where $x_{T_v}$ are the signals received by agent $v$, and $x_{F_v}$ are the signals it transmits. The **propagation dynamics** describe how signals spread across the network. Let $x_E \in X^{|E|}$ be the vector of all signals. The propagation dynamics are governed by a flow $\Gamma^t$:

$$\Gamma^t : \mathcal{F}_V \times X^{|E|} \to X^{|E|},$$

where $\mathcal{F}_V = \prod_{v \in V} \mathcal{F}_v$ is the joint state space of all agents. The propagation dynamics converge to a **fixed point** $z_E = p(f_V)$, which represents the **aggregate** of macro-level information. The fixed point satisfies:

$$z_{Fv} = f_v(z_{Tv}) \quad \forall v \in V,$$

where $z_{Fv}$ is the output signal of agent $v$. $z_{Tv}$ is the input signal to agent $v$.

## Pattern Dynamics

The **pattern dynamics** describe how agents adjust their states over time to form patterns. The pattern dynamics are governed by a flow $\psi_V^t$:

$$\psi_V^t : \mathcal{F}_V \to \mathcal{F}_V.$$

Each agent $v$ adjusts its state $f_v$ to maximize a **utility function** $q_v$, which depends on its input and output signals:

$$\psi_v^t(f_v) = \arg\max_{f'_v \in \mathcal{F}_v} \left\{ q_v(z_{Tv}, f'_v(z_{Tv})) : d_v(f_v, f'_v) \leqslant Lt \right\},$$

where:

- $d_v$ is a metric on $\mathcal{F}_v$.
- $L$ is a Lipschitz constant that limits the rate of change of $f_v$.

Patterns emerge through **feedback loops** in the network. A **feedback loop** is a closed walk $C = (V_C, E_C)$ in the graph $G$, where:

- Agents in $V_C$ reinforce each other's signals through **positive feedback**.
- The signals along the loop stabilize, forming a **pattern**.

A pattern is **coherent** if every part of the pattern depends on the rest, and it is **stable** if the aggregate signals $z_{E_C}$ are **Lyapunov stable**.

On longer timescales, agent states evolve through pattern dynamics:

$$\frac{df_v}{dt} = \psi_v(f_v, x_{T_v})$$

where $\psi_v$ describes the continuous evolution of agent function $f_v$, subject to the Lipschitz constraint:

$$d_v(f_v, \psi_v^t(f_v)) \leqslant L|t|$$

Patterns emerge as stable structures in the signal space. A pattern is a stable feedback loop, meaning the signals remain at a fixed point:

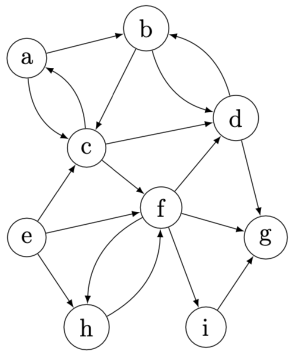


Figure 1: A directed graph. As an example of the notation, $T_b = \{(a,b),(d,b)\}$ and $F_b = \{(b,c),(b,d)\}$.

$$z_E = p(f_V), \quad \text{where} \quad p : F_V \to X^{|E|}$$

ensuring that for any agent $v$,

$$z_{F_v} = f_v(z_{T_v}).$$

The key to cognition in this framework is the separation of pattern from local dynamics achieved via the separation of timescales:

- Fast Timescale (Propagation Dynamics): Signals propagate and converge to a fixed point $z_E = p(f_V)$.
- Slow Timescale (Pattern Dynamics): Agents adjust their states $f_V$ based on the aggregate $z_E$, forming patterns.

The separation is maintained by ensuring that the propagation dynamics converge faster than the pattern dynamics evolve.

## Metrical Interpretation and The Geometry of Patterns

The quality of being a signal system imply a notion of metric, namely that some signals are closer to each other than others. This equivalence is not hard to spell out, simply consider the equivalence between neural networks and signal systems [4], and see neural networks are a product of ultra-metrics [7]. The metrical interpretation leads us to a discussion of the geometry and its connection to patterns: what kind of patterns can be instantiated in signal systems and what are the corresponding degrees of computability of these patterns? It was shown in [12, 13] that riemannian metrics can be endowed with arbitary degrees of computability. That:

> There are 'pits' or 'basins' in the graph of diameter with depth of the magnitude roughly equal to the 'halting function' [i.e., settling function] for $A_2$ and spaced at intervals growing slightly faster than their depth. These are merely bumps in the basins of the (spaced much further apart) much deeper basins that correspond to $A_1$. And even these huge basins are merely bumps in the basins corresponding to $A_0$. And so on.

It is also worth mentioning Gromov's theorem on the correspondance between the complexity of word problem and existence of infinitely contractible geodesics that correspond to the degree of the unsolvability of the fundamental word problem [12].

# Effect of Ectropy

To what extent can more complex mathematical description of the same thing can actualize in reality? In this section we show the possibility of emergence of physical effect due to alternative construal of probabilities implied by the signal system. In the following two examples we see that alternative construal of statements yield observational measurement effects that are "physically" impossible.

The mathematical effect of construal on realization of probabilities. [8]

### Setup of the Two-Slit Experiment

A source emits quantum particles (e.g., photons or electrons) that pass through two slits, $e_1$ and $e_2$, and then hit a screen $S'$. The experiment consists of two scenarios:

- **Both slits open**: An interference pattern appears on the screen.
- **One slit closed**: No interference pattern is observed.

Let $\Psi$ represent the quantum state when both slits are open. The probability distribution of the particles hitting the screen $S'$ is given by $P_\Psi(X)$, where $X$ is a region on the screen.

When only slit $e_1$ is open, the quantum state is $e_1$, and the probability distribution is $P_{e_1}(X)$. Similarly, when only slit $e_2$ is open, the state is $e_2$, and the distribution is $P_{e_2}(X)$.

If particles behaved classically, the probability distribution when both slits are open would be the sum of the probabilities when each slit is open individually:

$$P_\Psi(X) = P(e_1)P_{e_1}(X) + P(e_2)P_{e_2}(X), \qquad (1)$$

where $P(e_1)$ and $P(e_2)$ are the probabilities that a particle passes through slit $e_1$ or $e_2$.

However, in quantum mechanics, the probability distribution includes an interference term due to the wave-like nature of quantum particles:

$$P_\Psi(X) = |\psi_1(X) + \psi_2(X)|^2 = |\psi_1(X)|^2 + |\psi_2(X)|^2 + 2\mathrm{Re}(\psi_1(X)\psi_2^*(X)), \qquad (2)$$

where $\psi_1(X)$ and $\psi_2(X)$ are the wavefunctions for particles passing through slits $e_1$ and $e_2$, respectively. The term $2\mathrm{Re}(\psi_1(X)\psi_2^*(X))$ is responsible for the observed interference pattern.

The paradox arises when attempting to interpret the experiment using classical conditional probabilities. If we assume that particles pass through one slit or the other, we expect classical probability addition, which does not hold.

The key issue is that the quantum states $e_1$ and $e_2$ are not independent. The interference term indicates that particles exist in a superposition of states. Measuring which slit a particle passes through destroys the interference pattern.

In quantum mechanics, the conditional probabilities $P(e_1|X)$ and $P(e_2|X)$ (the probability that a particle passed through slit $e_1$ or $e_2$ given that it hit region $X$) cannot be defined in the classical sense, as the particles do not have a definite path until measured.

The paradox is resolved by recognizing that quantum particles do not have definite paths until measured. The wavefunction $\Psi$ describes a superposition of states, leading to the interference pattern.

Classical conditional probabilities break down in the quantum regime because particles do not behave like classical objects with well-defined trajectories. Instead, they exhibit wave-particle duality and obey quantum mechanics.

### Interpretation of Two-Slit Experiment

When interpreted as a signal system the two-slit experiment highlights the necessity of superposition and wavefunctions in explaining interference patterns. This means that oddities that exist in quantumn two slit type experiments is a result of the necessity to account for multiple possible construals, otherwise known as different parts of the unrealized wave function. The interference pattern cannot be explained by assuming that particles pass through one slit or the other in a classical sense. Instead, it requires quantum superposition, reinforcing the fundamental principles of quantum mechanics. Modern varieties of the quantumn two-slit experiment reflect this effect of differential construal of probability of the measured signal [1]. As long as there is added complexity in the construal of the probability, one can expect effects of unrealized reality on the measurement outcome.

### Classical Hermeneutics

The connection to classical study of hermaneutics is straightforward. The act of interpretation of text is to draw out the connection to potential motives to the manifestation of text. which is in effect psychoanalytic. In the setting of particles, we experimentally posit two motives (entry points) and measure the effect of outcome of the particle motion. and we see that, depending on the postulated motive, the opening of the slit, the particle behaves in a wave like or particle like way. Same may happen in the case of the analysis of the manifestation of words. For example, if Alice interprets a piece of text after it has been written through the lens of the oedipal complex, then surely alice is to find the opulence of the oedipal motive all over text. However, if Alice were to postulate some other hidden motive, say oedipal 2, then upon examination of the same text, Alice will be again sure to find the effect of the postulated motive all over the place again, intervening with the original postulated oedipal in a wave-like manner.

# Modern Experimental manifestations

If we interpret the manifestation and material realization of ectropy is due to an artifact in the differential construal of information, that seemingly impossible alternatives can be realized by way of construal, how real is this effect, and how can it possibly be measured?

To what extent is this effect of ectropy due to differential information construal real in real life? and where does the differential information come from?

### Less is More

Hertwig and Todd [6] did a study and found an interesting effect of some symbols emitting more information than they could accounted for, and they relegated this effect to the efficacy of heuristical logic. In actuality, some symbols propagate "further" and others "shorter", this is simply an effect of some symbols admitting more possibility of local elaborations. Some symbols admit more readily elaborable possible construals in different local contexts.

### Ectropy in Large Nets

Here we examine the efficacy of large nets in creating information through differential construal of the wave-function in signal systems [10]. One sees in this context that the by construing states in a way that is more than the computational minimal, one could induce more applicability of construal in downstream signal processes. The overall effect manifests itself in faster leanring and more accuracy compared to a straightforwardly trained neural network. This is not an isolated phenomenon and has been noticed in connection with graphical neural networks [9], wherein the degree of computability of a well-trained net that includes transcendental geometry could be transferred through fine tuning to another net used for a different purpose.

# Conclusions

We propose a word-centric interpretation of signals. This perspective of words is supported by quantumn experiemnts as well as recent results from large language models. We explain the effect of differential construal of information and how that could create ectropic effects in downstream processes. We propose that a measure of ectropy that is simply the difference in entropy between two alternative construals J(x) = xlogx - ylogy. This is an analogue of the measure of reality of quantumn events as found in [2].

## Appendix: Polynomials, Transcendence, and General Ramanujan Theorem

Here we make connections between properties of signals being passed in signal systems to polynomial expressions and transcendental numbers.

I wish to make the argument that the existence of transcendental numbers necessitates the existence of non-sofic Lie-like groups. Consider the Mahler approach to transcendental numbers. One could classify numbers into four separate classes.

### Mahler's Classification and Relationship to Polynomial Expressions

Mahler's classification divides all complex numbers into four distinct categories—A-numbers, S-numbers, U-numbers, and T-numbers—based on their *approximation by* *algebraic numbers*. This classification is defined using the function $\omega(\xi)$, which measures how well a complex number $\xi$ can be approximated by algebraic numbers.

For a given transcendental number $\xi$, consider polynomials of the form:

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$$

where $a_i \in \mathbb{Z}$. The classification is based on how well $\xi$ is approximated by the roots of such polynomials.

### A-numbers (Algebraic Numbers)

- If $\omega(\xi) = 0$, the number $\xi$ is an **A-number**.
- This implies that $\xi$ is algebraic, meaning it satisfies a polynomial equation with integer coefficients.
- These numbers are simply algebraic, meaning they are exactly roots of some polynomial with integer coefficients.

### S-numbers (Singular Transcendental Numbers)

- If $0 < \omega(\xi) < \infty$, the number $\xi$ is an **S-number**.
- S-numbers can be approximated exceptionally well by algebraic numbers but remain transcendental.
- Their approximation is governed by specific bounds on the algebraic degree and the size of the coefficients of the approximating polynomials.
- These numbers satisfy polynomial equations with integer coefficients but do not admit particularly strong approximations by algebraic numbers of small degree. There exist sequences of polynomials with controlled coefficients whose roots approach $\xi$, but not excessively well.

### U-numbers (Ultratranscendental Numbers)

- If $\omega(\xi) = \infty$ and there exists a positive integer $N$ such that $\omega(\xi, N) \neq \infty$, then $\xi$ is a **U-number**.
- U-numbers lie in an intermediate category. Their approximation is less regular than S-numbers, but they can still be approximated within certain constraints.
- U-numbers often have a degree $m$, indicating how well they can be approximated by algebraic numbers of degree at most $m$.
- For these numbers, there exist infinitely many polynomials with small integer coefficients that vanish at $\xi$ up to an extremely small error. This means that for each small $\varepsilon > 0$, there exists a polynomial $P(x)$ of relatively small degree $n$ such that:

  $$|P(\xi)| < e^{-n^{1+\varepsilon}}$$

  for some sequence of polynomials with rapidly shrinking errors. This is an indicator of an unusually strong approximation by algebraic numbers.

### T-numbers (Totally Transcendental Numbers)

- If $\omega(\xi) = \infty$ and $\omega(\xi, N) = \infty$ for all $N$, then $\xi$ is a **T-number**.
- T-numbers are the hardest to approximate using algebraic numbers. For these numbers, no degree of approximation is achievable within finite bounds.
- Numbers like $e$ and $\pi$ are examples of T-numbers.
- These numbers are the hardest to approximate algebraically. Unlike U-numbers, there are no sequences of polynomials with small coefficients that approximate $\xi$ particularly well. In other words, for a T-number, the polynomials that approximate it have relatively large coefficients and do not provide good rational approximations.

### Connection to Polynomial Expressions

The classification is fundamentally related to the nature of polynomials $P(x) \in \mathbb{Z}[x]$ that have $\xi$ as an approximate root. Specifically, Mahler's classification is determined by the behavior of such polynomials in terms of their degrees and coefficients.

This leads us to the world of polynomials, and what is possible therein. We know that certain class of polynomials are called periods, and can be made to correspond to transcendental numbers, and the well known transcendental $\pi$ can be expressed in Feynman integral terms. Feynman integrals are particle paths traveling within an Hamiltonian system that is expressible in terms of high dimensional Euclidean space or its complex equivalent high dimensional Hilbert space.

We know that the behavior of groups can be matricized and converted to groups [5]. This also reminds us of Pestov mentioned that sofic groups are groups that admit matrix models. The existence of transcendental numbers imply then that there are indeed dynamics that manifest at a nonsofic way. In the sense that these expressions are not compltely reducible to finite expressions of numbers.

Another way of looking at this is the following. We know that the matrix algebra in neural networks can be decomposed into prime components from the generalized ramanujan conjecture. We can also see that the well known identity of Euler:

$$\prod_{p \text{ prime}} \left(1 - \frac{1}{p^2}\right)^{-1} = \frac{\pi^2}{6}$$

We thus see that in the polynimal arithmetic of the neural network, prime components can be arithmetically easily connected to express transcendental like patterns.

The provided equations discuss the distribution of primes and almost primes in the context of certain polynomial values. Here is the explanation of these equations in relation to the **Generalized Ramanujan Conjecture (GRC)**[3]: The GRC underpins the theoretical framework for studying the distribution of primes and almost primes in polynomial values. By providing rigorous spectral gap bounds and controlling error terms, the GRC ensures the validity of prime number theorme analogue in generalized contexts, such as higher-degree polynomials and sequences with additional algebraic structure.

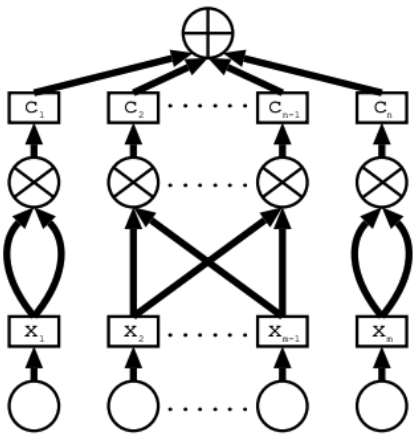


Figure 2: A network for $c_1x_1^2 + c_2x_2x_{m+1} + \cdots + c_{n-1}x_2x_{m-1} + x_m^2$

**Distribution of Primes**

$$|\{p \in \mathbb{P} : p \leqslant t\}| \sim \frac{t}{\log t}, \quad \text{as } t \to \infty.$$

This is the **Prime Number Theorem**, which describes the asymptotic density of prime numbers less than or equal to $t$. It quantifies how primes become less frequent as $t$ increases.

**Prime Number Theorem Analogue for Polynomials**

$$|\{x \in \mathbb{Z} : f(x) \in \mathbb{P}_r, |x| \leqslant t\}| \geqslant \text{const.} \cdot \frac{t}{\log t}.$$

Here:

- $f(x)$ is an irreducible polynomial with integral coefficients and a positive leading term.
- $\mathbb{P}_r$ represents the set of numbers with at most $r$ prime factors (also known as $r$-almost primes).
- This inequality states that for any such $f(x)$, there exists a positive lower bound for the count of $r$-almost primes generated by $f(x)$, which is proportional to $\frac{t}{\log t}$.

The difficulty lies in proving the infinitude of primes (or almost primes) in the values of $f(x)$ for higher-degree polynomials, as their structure is more complex than that of linear polynomials.

# References


[1] J. S. Araújo et al. *Quantum Reality Erasure with Spacelike-Separated Operations.* 2024. arXiv: 2407.07185 [quant-ph]. URL: https://arxiv.org/abs/2407.07185.

[2] A. L. O. Bilobran and R. M. Angelo. "A measure of physical reality". In: *EPL (Europhysics Letters)* 112.4 (Nov. 2015), p. 40005. ISSN: 1286-4854. DOI: 10.1209/0295-5075/112/40005. URL: http://dx.doi.org/10.1209/0295-5075/112/40005.

[3] Alexander Gorodnik and Amos Nevo. "Quantitative ergodic theorems and their number-theoretic applications". In: *Bulletin of the American Mathematical Society* 52.1 (2015), pp. 65–113.

[4] Jack Hall. *Cognition in Dynamical Systems, Second Edition.* 2022. arXiv: 1805.00787 [cs.MA]. URL: https://arxiv.org/abs/1805.00787.

[5] Michael Hauser and Asok Ray. "Principles of riemannian geometry in neural networks". In: *Advances in neural information processing systems* 30 (2017).

[6] Ralph Hertwig and Peter M Todd. "More is not always better: The benefits of cognitive limits". In: *Thinking: Psychological perspectives on reasoning, judgment and decision making* (2003), pp. 213–231.

[7] Anders Karlsson. "Generalized Lyapunov Exponents and Aspects of the Theory of Deep Learning". In: *arXiv preprint arXiv:2212.13097* (2022). URL: https://arxiv.org/pdf/2212.13097.

[8] Andrei Khrennikov. *Non-Archimedean Analysis: Quantum Paradoxes, Dynamical Systems and Biological Models.* Vol. 427. Mathematics and Its Applications. Dordrecht: Kluwer Academic Publishers, 1997. ISBN: 978-0-7923-4800-3. DOI: 10.1007/978-94-015-8840-4. URL: https://doi.org/10.1007/978-94-015-8840-4.

[9] Thien Le and Stefanie Jegelka. *Limits, approximation and size transferability for GNNs on sparse graphs via graphops.* 2023. arXiv: 2306.04495 [cs.LG]. URL: https://arxiv.org/abs/2306.04495.

[10] Machel Reid, Yutaro Yamada, and Shixiang Shane Gu. *Can Wikipedia Help Offline Reinforcement Learning?* 2022. arXiv: 2201.12122 [cs.LG]. URL: https://arxiv.org/abs/2201.12122.

[11] Claude E. Shannon. "A Mathematical Theory of Communication". In: *Bell System Technical Journal* 27.3 (1948), pp. 379–423. DOI: 10.1002/j.1538-7305.1948.tb01338.x. URL: https://doi.org/10.1002/j.1538-7305.1948.tb01338.x.

[12] Robert I Soare. "Computability theory and differential geometry". In: *Bulletin of Symbolic Logic* 10.4 (2004), pp. 457–486.

[13] Shmuel Weinberger. "Computers, Rigidity, and Moduli: The Large-Scale Fractal Geometry of Riemannian Moduli Space". In: (2020).