PROOF OF "COLLATZ'S CONJECTURE" IN MULTI-DIMENSIONAL SPACE

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Abstract. We found why "Eternal Loop" can't exist in Multi-Dimensional space, and why any number can't diverge. And "Collatz Tree" includes most things in the world as indexed. It seems like a sleeping lion.

1. Introduction

The Collatz's Conjecture is "Increasing the step, any selected Odd and Even natural number (≥ 1) can arrive(or converge) to 1 by Collatz's Equation" as below.

$$f(x) = \begin{cases} 3x+1, & \text{(when "x" is Odd natural number)} \\ x\ /\ 2, & \text{(when "x" is Even natural number)} \end{cases} \tag{1.1}$$

For example, $(13) \to 40 \to 20 \to 10 \to (5) \to 16 \to 8 \to 4 \to 2 \to (1) \to 4 \to 2 \to \dots$

Because any Even number converges to Odd number, so we can focus on "Odd" number only. And "Odd" number equation is more complicated than "Even" number.
We can define the equation from "Parent" and "Child" relation for proof.
And we prepare the tool equation for proof.

Definition 1.1 (Equivalent Equation with "Odd" number)**.**

$$A_{K+1} = \frac{3A_K + 1}{2^{Z_K}} \quad (Z_K\ (\geq 1)\ is\ integer,\ all\ A_K(\geq 1)\ is\ Odd,\ -\infty < K < \infty)$$

Proof.
By Collatz Equation, any Even number "x" converge to Odd number.

$$x = 2^P Y \quad (P \geq 1,\ Y\ is\ Odd)$$
$$f(x) = (2^P Y)/2 = 2^{P-1} Y \quad (\because x\ is\ Even)$$
$$f(f(x)) = f^2(x) = (2^{P-1}Y)/2 = 2^{P-2}Y \quad (\because 2^{P-1}Y\ is\ Even\ when\ P-1 \geq 1)$$
$$\dots$$
$$f^P(x) = Y \quad (Y\ is\ Odd)$$

So, Any Even number "x" converges to Odd number.

And Odd number A_K can converge to next Odd number A_{K+1}.
Because A_K is Odd, next number is $3A_K+1$ ($Even$).
$3A_K+1$ converge to Odd with multiple(≥ 1) times of ($x/2$).

So, $A_{K+1} = \dfrac{3A_K+1}{2^{Z_K}} \quad (Z_K \geq 1)$

□

Example 1.2 (Equivalent Equation with "Odd" number).
For example, from Odd number "17".
By Collatz Equation,

$$(17) \to 52 \to 26 \to (13) \to 40 \to 20 \to 10 \to (5) \to 16 \to 8 \to$$
$$4 \to 2 \to (1) \to 4 \to 2 \to (1) \to 4 \to 2 \to (1) \to \dots$$

By Equivalent Equation,

$$A_0 = 17,\ A_1 = \frac{3(17)+1}{2^2} = 13,\ A_2 = \frac{3(13)+1}{2^3} = 5,$$
$$A_3 = \frac{3(5)+1}{2^4} = 1,\ A_4 = \frac{3(1)+1}{2^2} = 1,\ A_5 = \frac{3(1)+1}{2^2} = 1$$
$$\text{So, } A = (17,\ 13,\ 5,\ 1,\ 1,\ 1,\ \dots)$$

Theorem 1.3 (If $A_K = 1 \ \ (K \geq 0)$ from any Odd A_0 by Equivalent Equation, then "Collatz's Conjecture" is true).

Proof.
Because the result of "Collatz Equation" and "Equivalent Equation" is same in only Odd number.
So, if any A_0 (≥ 1) can converge to "1" by "Equivalent Equation", then "1" is exist in the result of "Collatz Equation" and "Collatz's Conjecture" is true.

□

Definition 1.4 (Modulus Symbol. "X mod Y" = $M_Y(X)$).
For example, X mod 4 = $M_4(X)$. $M_3(7) = 7 \ mod \ 3 = 1$

Definition 1.5 ($(Parent\ of\ A_K) = A_{K+1} = P(A_K)$).
Any Odd number A_K (>= 1) have "Next Odd" number A_{K+1} (>= 1).
Define "Parent of (X)" as $P(X)$

$$A_{K+1} = P(A_K) = \frac{3A_K+1}{2^{Z_K}}$$

$$(1.2) \qquad 2^{Z_K} = \begin{cases} 2^1 & (\text{when } M_4(A_K) = 3) \\ 2^X & (\text{when } M_4(A_K) = 1,\ X \geq 2) \end{cases}$$

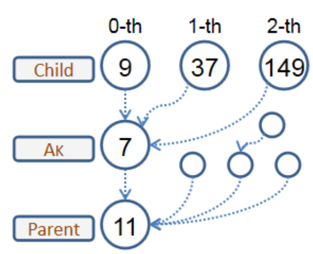

FIGURE 1. Parent and Child of A_K

Example 1.6 ($(Parent\ of\ A_K) = A_{K+1} = P(A_K)$).

$$\text{When } M_4(A_K) = 3,\ A_{K+1} = \frac{3(3)+1}{2^1} = 5,\ A_{K+1} = \frac{3(15)+1}{2^1} = 23.$$
$$\text{When } M_4(A_K) = 1,\ A_{K+1} = \frac{3(9)+1}{2^2} = 7,\ A_{K+1} = \frac{3(21)+1}{2^6} = 1.$$

Theorem 1.7 ($P(A_K) = A_K\ (A_K = 1),\ P(A_K) > A_K\ (A_K \geq 3,\ 2^{Z_K} = 2^1),\ P(A_K) < A_K\ (A_K \geq 3,\ 2^{Z_K} \geq 2^2)$).

Proof.

For $A_{K+1} = P(A_K) = A_K,\ A_K = \dfrac{3A_K+1}{2^{Z_K}}$

$(2^{Z_K} - 3)A_K = 1$, so $A_K = 1,\ 2^{Z_K} = 2^2$.

When $2^{Z_K} = 1,\ A_{K+1} = \dfrac{3A_K+1}{2_1} = \dfrac{3}{2^1}A_K + \dfrac{1}{2^1} > A_K$

When $2^{Z_K} \geq 2,\ A_{K+1} = \dfrac{3A_K+1}{2_{Z_K}} = \dfrac{3}{2^{Z_K}}A_K + \dfrac{1}{2^{Z_K}} < A_K$

□

Definition 1.8 ("Brother coefficient" $\Omega_N(X)$).

Define "Brother Coefficient" as

$$\Omega_N(X) = \begin{cases} 4^N(X) + \Omega_{N-1}(1) & (X \geq 2,\ N \geq 0) \\ (4^{N+1} - 1)/3 & (X = 1,\ N \geq -1) \end{cases}$$

$\Omega_{-1}(1) = 0,\ \Omega_0(1) = 1,\ \Omega_1(1) = 5,\ \Omega_2(1) = (4^{(2)+1} - 1)/3 = 21,$

And it has good visible in "base-4" as $\Omega_3(1) = 1111_4,\ \Omega_3(3) = 3111_4,\ \Omega_3(11) = 23111_4$. Because this equation can be used in "Brother" relation, so we can call it as "Brother coefficient".

Theorem 1.9 ($\Omega_{N+1}(X) = 4\ \Omega_N(X) + 1 \quad (X \geq 1,\ N \geq 0)$).

$\Omega_N(X)$ is Odd (when $N > 0$)

$\Omega_0(X) = X,\ \Omega_1(X) = 4^1X + 1,\ \Omega_2(X) = 4^2X + 5,\ \Omega_3(X) = 4^3X + 21$

Proof.

$$\begin{aligned} \Omega_{N+1}(1) &= (4^{(N+1)+1} - 1)/3 \qquad (N \geq 1) \\ &= (4 * 4^{N+1} - 1 - 3)/3 + 1 = 4(4^{N+1} - 1)/3 + 1 = 4\Omega_N(1) + 1 \\ \Omega_0(1) &= (4 - 1)/3 = 1 \\ \Omega_{N+1}(X) &= 4^{N+1}X + \Omega_N(1) = 4 * 4^N X + 4\Omega_{N-1}(1) + 1 \qquad (N \geq 0) \\ &= 4(4^N X + \Omega_{N-1}(1)) + 1 = 4\Omega_N(X) + 1 \\ \Omega_0(X) &= 4^0(X) + \Omega_{-1}(1) = X + 0 = X \end{aligned}$$

$\Omega_N(X)$ is Odd (when $N > 0$) because $\Omega_N(X) = 4\Omega_{N-1}(X) + 1$

□

Definition 1.10 ("$S_{th}Child$" of A_{K+1} $(0 \le S < \infty)$).
Any A_{K+1} (A_{K+1} <> $6x+3$) can get infinite "Child", and we can divide them with index.
Define "S_{th}Child of A_{K+1}" as

$$A_K = C_S(A_{K+1}) = \begin{cases} not\ exist & (\text{when } M_3(A_{K+1}) = 0) \\ (A_{K+1}Z_K - 1)/3 \quad (Z_K = 2^2 * 4^S) & (\text{when } M_3(A_{K+1}) = 1) \\ (A_{K+1}Z_K - 1)/3 \quad (Z_K = 2^1 * 4^S) & (\text{when } M_3(A_{K+1}) = 2) \end{cases}$$

Proof.
In Reverse equation $A_K = \dfrac{2^{Z_K}A_{K+1} - 1}{3}$, we can get smallest "Child" A_K as below.

$$A_K = not\ exist \qquad (\text{when } M_3(A_{K+1}) = 0)$$
$$A_K = (A_{K+1} * 2^2 - 1)/3 \qquad (\text{when } M_3(A_{K+1}) = 1)$$
$$A_K = (A_{K+1} * 2^1 - 1)/3 \qquad (\text{when } M_3(A_{K+1}) = 2)$$

When $A_K = 4X+3$, $A_{K+1} = \dfrac{3A_K+1}{2^1}$ $(X \ge 0)$

$$A_{K+1} = \frac{3(4X+3)+1}{2^1} = 6X+5,\ M_3(A_{K+1}) = 2$$

When $A_K = 4X+1$, $A_{K+1} = \dfrac{3A_K+1}{2^{2+Z}}$ $(Z \ge 0)$

$$A_{K+1} = \frac{3(4X+1)+1}{2^{2+Z}} = \frac{3X+1}{2^Z}$$

Because $M_3(2^Z) = 1$, $Z = 2y+0$. $(\because M_3(2^Z) = M_3(2^{2y}) = M_3((3+1)^y) = 1)$
Because $M_3(A_{K+1}2^Z) = 1$, $M_3(A_{K+1}) = 1$. So, smallest $2^{Z_K} = 2^{2+Z} = 2^2$.

And, when $M_3(A_{K+1}) = 0$ then $\dfrac{2^{Z_K}(3x)-1}{3} = 2^{Z_K}x - \dfrac{1}{3}$ can't be integer.

Let the smallest "Child" as "0_{th}Child".
We can get $S_{th}Child$ as below.

$$A_K = C_0(A_{K+1}) = \frac{2^X A_{K+1} - 1}{3} \quad (X = 1\ or\ 2)$$

Because $S_{th}Child$ is made from A_{K+1}. so we can set

$$A_K = C_S(A_{K+1}) = \frac{2^{X+P}A_{K+1} - 1}{3} \quad (X = 1\ or\ 2,\ P \ge 1)$$

When P is Odd $(= 2y+1)$, $M_3(2^{2y+1}) = M_3(2 * 4^y) = M_3(2 * (3+1)^y) = 2$
Because $M_3(2^X A_{K+1}) = 1$, $M_3(2^{X+(2y+1)}A_{K+1} - 1) = 1$,
So A_K can't be integer.
When P is Even $(= 2y)$, $M_3(2^{2y}) = M_3(4^y) = M_3((3+1)^y) = 1$
Because $M_3(2^X A_{K+1}) = 1$, $M_3(2^{X+(2y)}A_{K+1} - 1) = 0$,

$$\text{So "}S_{th}Child \text{ of } A_{K+1}\text{"} = \frac{2^X 4^S A_{K+1} - 1}{3} = 4^S \frac{2^X A_{K+1} - 1}{3} + \frac{4^S - 1}{3}$$
$$= \Omega_S(A_K) \quad (\Omega_0(A_K) = A_K\ is\ Odd,\ so\ \Omega_S(A_K)\ is\ Odd)$$

Z_K of "$0_{th}Child$" is $2^X 4^0$, Z_K of "$1_{th}Child$" is $2^X 4^1$ □

Theorem 1.11 (If $A_K = C_S(A_{K+1})$, then $C_{S+Y}(A_{K+1}) = \Omega_Y(A_K)$ $(S \geq 0,\ Y \geq 0)$).

Proof.

Let $C_S(A_{K+1}) = \dfrac{2^X 4^S A_{K+1} - 1}{3}$, then $C_{S+Y}(A_{K+1}) = \dfrac{2^X 4^{S+Y} A_{K+1} - 1}{3}$

$(X = 1\ or\ 2)$

$$C_{S+Y}(A_{K+1}) = \frac{2^X 4^{S+Y} A_{K+1} - 1}{3} = 4^Y \frac{2^X 4^S A_{K+1} - 1}{3} + \frac{4^Y - 1}{3}$$
$$= 4^Y C_S(A_{K+1}) + \Omega_{Y-1}(1) = \Omega_Y(C_S(A_{K+1})).$$

□

Theorem 1.12 (If $M_3(C_K(X)) = 0$, then $M_3(C_{K+1}(X)) = 1,\ M_3(C_{K+2}(X)) = 2$).

$M_3(C_K(Y)) = M_3(C_{K+3}(Y))$

Proof.

Let $C_K(X) = 3a + 0$. $\quad (M_3(C_K(X)) = 0)$

$C_{K+1}(X) = 4(C_K(X)) + 1 = 4(3a + 0) + 1 = 3(4a) + 1 \quad (M_3(C_{K+1}(X)) = 1)$

$C_{K+2}(X) = 4(C_{K+1}(X)) + 1 = 4(3(4a) + 1) + 1 = 3(4^2 a) + 5 \quad (M_3(C_{K+2}(X)) = 2)$

For $C_{K+3}(Y) = \Omega_3(C_K(Y)) = 4^3 C_K(Y) + 21$

$M_3(C_{K+3}(Y)) = M_3((3 + 1)^3 C_K(Y) + 3 * 7) = M_3(C_K(Y))$

So, $M_3(C_K(Y)) = M_3(C_{K+3}(Y))$

□

Example 1.13 (If $M_3(C_K(X)) = 0$, then $M_3(C_{K+1}(X)) = 1,\ M_3(C_{K+1}(X)) = 2$).

$For\ A_{K+1} = 5,$	$For\ A_{K+1} = 13$
$C_0(A_{K+1}) = 3,\ M_3(3) = 0,$	$C_0(A_{K+1}) = 17,\ M_3(17) = 2,$
$C_1(A_{K+1}) = 13,\ M_3(13) = 1,$	$C_1(A_{K+1}) = 69,\ M_3(69) = 0,$
$C_2(A_{K+1}) = 53,\ M_3(53) = 2,$	$C_2(A_{K+1}) = 277,\ M_3(277) = 1,$
$C_3(A_{K+1}) = 213,\ M_3(213) = 0,$	$C_3(A_{K+1}) = 1109,\ M_3(1109) = 2,$
$C_4(A_{K+1}) = 853,\ M_3(853) = 1,$	$C_4(A_{K+1}) = 4437,\ M_3(4437) = 0,$
$C_5(A_{K+1}) = 3413,\ M_3(3413) = 2,$	$C_5(A_{K+1}) = 17749,\ M_3(17749) = 1,$

2. Collatz Hole

For any Odd number A_0, A_K always have Parent A_{K+1} $(K \geq 0)$.
In this Sequence, the worst problem that we can imagine is $A_K = A_0$ $(K > 0)$.
Then A_0 can not exodus from "Eternal Loop".

$$(A_0 \to \ldots \to A_{K-1} \to A_0 \to \ldots \to A_{K-1} \to \ldots)$$

And a researcher already have announced "Very Long Loop exist!", and "Collatz's Conjecture is not true".

So, we must check whether "Eternal Loop can exist?".
If "Eternal Loop" exist, the size is infinite.
But, by putting "Number" on multi-dimension space, we can prove "Eternal Loop can't exist".

Definition 2.1 (Collatz Hole).
Define "Circle Loop" that is consist of N (≥ 2) count of "Number" (≥ 3) with "Parent-Child" relation as "Collatz Hole".

$A = \{A_0,\ \dots,\ A_{N-2},\ A_{N-1}\} \quad (A_{K+1} = P(A_K),\ A_N = A_0)$

$A_i <> A_k\ (0 \leq i < k < N),\ A_k <> 6x + 3\ \ (It\ can't\ have\ Child)$

Let A_0 is smallest "Number" in Sequence, and it is no matter because "Circle Loop".

So $A_0 < A_K\ (0 < K < N)$

And in "Collatz Hole", $A_K = A_{K+N}$ and $Z_K = Z_{K+N}$ because it is "Circle Loop".

"1" is also "Eternal Loop" because $P(1) = 1$ and $P(P(1)) = 1$, but it is not the "Loop" that we want to find. "1" is just the goal of "Collatz's Conjecture".

And for A_0 (≥ 3), there not exist A_0 that is $P(A_0) = A_0 = \dfrac{3A_0 + 1}{2^{Z_0}}$. So, "Collatz Hole" ($N = 1$) can't exist.

Because numbers that can't exodus "Loop" is infinite ($C_S(A_0)$ also can't exodus and S is infinite), so the calling as "Hole" seems to be better than "Loop".

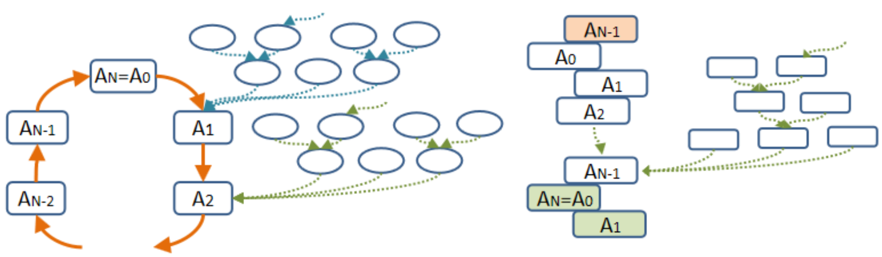

FIGURE 2. Collatz Hole

Theorem 2.2 (In "Collatz Hole", $2^{Z_0} = 2^1$ and $2^{Z_{N-1}} \geq 2^2$).

Proof.
In "Collatz Hole" $A = (A_0, A_1, \dots, A_{N-2}, A_{N-1})$ $(A_N = A_0)$, $A_0 < A_K$ $(0 < K < N)$ by definition.
Because $A_1 > A_0$ and $P(A_0) = A_1$, so $P(A_0) > A_0$ and $2^{Z_0} = 2^1$.
Because $A_{N-1} > A_0$ and $P(A_{N-1}) = A_0$, so $P(A_{N-1}) < A_{N-1}$ and $2^{Z_{N-1}} \geq 2^2$.

□

Theorem 2.3 (A_N Equation with A_0 and Z_K).

$$2^{Z_0+Z_1+Z_2+\dots+Z_{N-1}} A_N = 3^N A_0 + 3^{N-1} + 3^{N-2} 2^{Z_0} + 3^{N-3} 2^{Z_0+Z_1} + \dots + 3^1 2^{Z_0+Z_1+\dots+Z_{N-3}} + 3^0 2^{Z_0+Z_1+\dots+Z_{N-3}+Z_{N-2}},$$

$$(from\ 2^{Z_K} A_{K+1} = 3A_K + 1)$$

Proof.

$$2^{Z_0}A_1 = 3A_0 + 1$$
$$2^{Z_1}A_2 = 3A_1 + 1 \qquad 2^{Z_0+Z_1}A_2 = 3^2A_0 + 3^1 + 3^0 2^{Z_0}$$
$$2^{Z_2}A_3 = 3A_2 + 1 \qquad 2^{Z_0+Z_1+Z_2}A_3 = 3^3A_0 + 3^2 + 3^1 2^{Z_0} + 3^1 2^{Z_0+Z_1}$$
$$\cdots$$
$$2^{Z_{N-1}}A_N = 3A_{N-1} + 1 \qquad 2^{Z_0+\cdots+Z_{N-1}}A_N = 3^N A_0 + 3^{N-1} + 3^{N-2}2^{Z_0} + 3^{N-3}2^{Z_0+Z_1} + \cdots + 3^1 2^{Z_0+Z_1+\cdots+Z_{N-3}} + 3^0 2^{Z_0+Z_1+\cdots+Z_{N-3}+Z_{N-2}}$$

□

Example 2.4 (A_N Equation with A_0 and Z_K)**.**
For $A_0 = 9,\ A_1 = 7,\ A_2 = 11,\ A_3 = 17.$
$2^{Z_0} = 2^2,\ 2^{Z_1} = 2^1,\ 2^{Z_2} = 2^1.$

$$2^{Z_0}A_1 = 3^1A_0 + 3^0 \qquad 2^2 7 = 3^1(9) + 3^0$$
$$2^{Z_0+Z_1}A_2 = 3^2A_0 + 3^1 + 3^0 2^{Z_0} \qquad 2^{2+1}11 = 3^2(9) + 3^1 + 3^0 2^2$$
$$2^{Z_0+Z_1+Z_2}A_3 = 3^3A_0 + 3^2 + 3^1 2^{Z_0} + 3^0 2^{Z_0+Z_1} \qquad 2^{2+1+1}17 = 3^3(9) + 3^2 + 3^1 2^2 + 3^0 2^{2+1}$$

Theorem 2.5 (Any "Collatz Hole" must have N ($N \geq 4$) count of Number.)**.**

Proof.
Assume there exist "Collatz Hole" ($N = 2$).
$2^{Z_0+Z_1}A_2 = 3^2A_0 + 3^1 + 3^0 2^{Z_0}$
$2^{1+Z_1}A_0 = 3^2A_0 + 3^1 + 2^1$ ($\because A_2 = A_0,\ 2^{Z_0} = 2^1$)
$(2^{1+Z_1} - 3^2)A_0 = 5 = 1 * 5$
Only $A_0 = 5$ have possibility for answer. But Z_1 can't exist ($\because 2^{1+Z_1} = 10$).
$\therefore$ "Collatz Hole of ($N = 2$)" can't exist.

Assume there exist "Collatz Hole" ($N = 3$).
$2^{Z_0+Z_1+Z_2}A_3 = 3^3A_0 + 3^2 + 3^1 2^{Z_0} + 3^0 2^{Z_0+Z_1}$
$2^{1+Z_1+Z_2}A_0 = 3^3A_0 + 3^2 + 3^1 2^1 + 2^{1+Z_1}$ ($\because A_3 = A_0,\ 2^{Z_0} = 2^1$)
$(2^{1+Z_1+Z_2} - 3^3)A_0 = 15 + 2^{1+Z_1}$
$2^{1+Z_1+Z_2} - 3^3 > 0.\ \ 2^{Z_1+Z_2} > 13 + 1/2.\ \ \therefore 2^{Z_1+Z_2} \geq 2^4$

$2^{Z_0+Z_1}A_2 = 3^2A_0 + 3^1 + 3^0 2^{Z_0} > 2^{Z_0+Z_1}A_0$ ($\because A_2 > A_0$)
$3^2A_0 + 3 + 2^1 > 2^{Z_0+Z_1}A_0$ ($2^{Z_0} = 2^1$)
$3^2 + 5/A_0 > 2^{Z_0+Z_1}$
So, $2^4 \leq 2^{Z_0+Z_1} < 2^3 + 5/A_0 < 15$

We can't find Z_0 and Z_1. So, "Collatz Hole of ($N = 3$)" can't exist.

$\therefore$ "Collatz Hole" must have N (≥ 4) if exist.

□

Definition 2.6 ("Collatz Hole" Vector $\vec{H_K}$, and "Normal" Vector $\vec{n}$).
In "Collatz Hole" $A = (A_0, A_1, \ldots, A_{N-2}, A_{N-1})$ $(A_N = A_0)$, we can get "N" count of equation for A_{K+N} with A_K *and* Z_x $(0 \le K < N)$.

$$2^Z A_{K+N} = 3^N A_K + 3^{N-1} + 3^{N-2}2^{Z_K} + 3^{N-3}2^{Z_K+Z_{K+1}} + \ldots + 3^0 2^{Z_K+Z_{K+1}+\cdots+Z_{K+(N-3)}+Z_{K+(N-2)}}$$
$$(\ Z = Z_K + Z_{K+1} + \cdots + Z_{K+(N-2)} + Z_{K+(N-1)}, \quad A_{K+N} = A_K\ )$$
$$0 = 3^N A_K + 3^{N-1} + 3^{N-2}2^{Z_K} + 3^{N-3}2^{Z_K+Z_{K+1}} + \ldots + \{3^0 2^{Z_K+Z_{K+1}+\cdots+Z_{K+(N-3)}+Z_{K+(N-2)}} - 2^Z A_K\}$$
$$(or\ 2^{Z_K+Z_{K+1}+\cdots+Z_{K+(N-3)}+Z_{K+(N-2)}} = 2^Z/2^{Z_{K+(N-1)}})$$

We can devide all equation as "Inner Product" of 2 vectors.

$$\vec{H_K} \cdot \vec{n} = 0 \quad (\vec{H_0} \cdot \vec{n} = \vec{H_1} \cdot \vec{n} = \ldots\ = \vec{H_{N-1}} \cdot \vec{n} = 0)$$

We can define $\vec{H_K}$ and $\vec{n}$ as

$$\vec{H_K} = \begin{bmatrix} A_K \\ 1 \\ 2^{Z_K} \\ 2^{Z_K+Z_{K+1}} \\ \ldots \\ 2^{Z_K+Z_{K+1}+\cdots+Z_{K+(N-3)}} \\ 2^{Z_K+Z_{K+1}+\cdots+Z_{K+(N-3)}+Z_{K+(N-2)}} - 2^Z A_K \end{bmatrix} \quad \vec{n} = \begin{bmatrix} 3^N \\ 3^{N-1} \\ 3^{N-2} \\ 3^{N-3} \\ \ldots \\ 3^1 \\ 3^0 \end{bmatrix}$$

Any value of axis in $\vec{H_K}$ and $\vec{n}$ is not zero because $2^Z/2^{Z_{K+(N-1)}} <> 2^Z A_K$.
$\vec{H_K}$ and $\vec{n}$ is in (N+1)-dimensional space, and all $\vec{H_K}$ is on One hyperplane of the space. (0, 0, ...,0) is on that hyperplane, $\vec{n}$ is "Normal" vector of that hyperplane.

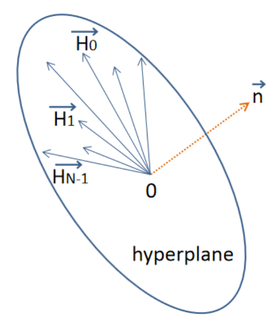

FIGURE 3. "Collatz Hole" vector

Example 2.7 ("Collatz Hole" Vector $\vec{H_K}$, and "Normal" Vector $\vec{n}$).
For example, assume "Collatz Hole" $= (A_0, A_1, A_2)$

$$2^Z A_0 = 3^3 A_0 + 3^2 + 3^1 2^{Z_0} + 3^0 2^{Z_0+Z_1}\ ,$$
$$2^Z A_1 = 3^3 A_1 + 3^2 + 3^1 2^{Z_1} + 3^0 2^{Z_1+Z_2}\ ,$$
$$2^Z A_2 = 3^3 A_2 + 3^2 + 3^1 2^{Z_2} + 3^0 2^{Z_2+Z_0}\ ,$$
$$0 = 3^3 A_0 + 3^2 + 3^1 2^{Z_0} + 3^0(2^{Z_0+Z_1} - 2^Z A_0)$$
$$0 = 3^3 A_1 + 3^2 + 3^1 2^{Z_1} + 3^0(2^{Z_1+Z_2} - 2^Z A_1)$$
$$0 = 3^3 A_2 + 3^2 + 3^1 2^{Z_2} + 3^0(2^{Z_2+Z_0} - 2^Z A_2)$$

$$\vec{B_0} = \begin{bmatrix} A_0 \\ 1 \\ 2^{Z_0} \\ 2^{Z_0+Z_1} - 2^Z A_0 \end{bmatrix} \vec{B_1} = \begin{bmatrix} A_1 \\ 1 \\ 2^{Z_1} \\ 2^{Z_1+Z_2} - 2^Z A_1 \end{bmatrix} \vec{B_2} = \begin{bmatrix} A_2 \\ 1 \\ 2^{Z_2} \\ 2^{Z_2+Z_0} - 2^Z A_2 \end{bmatrix}$$

$$\vec{n} = \begin{bmatrix} 3^3 \\ 3^2 \\ 3^1 \\ 3^0 \end{bmatrix} \quad Z = Z_0 + Z_1 + Z_2, \quad \vec{H_K} \cdot \vec{n} = 0$$

In 4-dimensional space, all $\vec{H_K}$ must be on One hyperplane because $\vec{H_K} \cdot \vec{n} = 0$.

Theorem 2.8 (In any "Collatz Hole", any $\vec{H_P}$ and $\vec{H_Q}$ ($P <> Q$) is "Linearly Independent".).

Proof.
In "Collatz Hole" $A = (A_0, A_1, \ldots, A_{N-2}, A_{N-1})$ $(A_N = A_0,\ N \geq 4)$,

$$\vec{H_P} = \begin{bmatrix} A_P \\ 1 \\ \cdots \\ \cdots \end{bmatrix} \qquad \vec{H_Q} = \begin{bmatrix} A_Q \\ 1 \\ \cdots \\ \cdots \end{bmatrix} \qquad (P <> Q)$$

For $\vec{H_P}$ and $\vec{H_Q}$ is "Linearly Dependent", it must be $\vec{H_P} = \alpha \vec{H_Q}$

$$A_P = \alpha A_Q \quad 1 = \alpha * 1,\ so\ A_P = A_Q$$

It is contradiction because $A_P <> A_Q$, so any $\vec{H_P}$ and $\vec{H_Q}$ is "Linearly Independent".

□

Theorem 2.9 ("Collatz Hole" can not exist).

Proof.
In "Collatz Hole" $A = (A_0, A_1, \ldots, A_{N-2}, A_{N-1})$ $(A_N = A_0,\ N \geq 4)$,
All "Collatz Hole" Vector $\vec{H_K}$ must be in One hyperplane.
$2^{Z_0} = 2^1$ and there must exist at least One "S" for $2^{Z_S} = 2^1$ $(0 < S \leq N-2)$ because $N \geq 4$ and $2^{Z_{N-1}} \geq 2^2$.
Assume there not exist $2^{Z_S} = 2^1$, then

$$2^{Z_0+Z_1+Z_2} A_3 = 3^3 A_0 + 3^2 + 3^1 2^{Z_0} + 3^0 2^{Z_0+Z_1}$$
$$2^{1+Z_1+Z_2} A_3 = 3^3 A_0 + 3^2 + 3^1 2^1 + 2^{1+Z_1}$$
$$2 * 2^{Z_1+Z_2} A_3 = 27A_0 + 15 + 2 * 2^{Z_1}$$
$$A_3 = \frac{27A_0 + 15}{2 * 2^{Z_1+Z_2}} + \frac{1}{2^{Z_2}}$$

When $2^{Z_1} = 2^2$ and $2^{Z_2} = 2^2$, A_3 have maximum value because $2^{Z_K} \geq 2^2$ $(0 < K < N)$ by assumation.

$$A_3 = \frac{27A_0 + 15}{2 * 2^{2+2}} + \frac{1}{2^2} = \frac{27A_0 + 15}{32} + \frac{1}{4} = \frac{27}{32} A_0 + \frac{23}{32}$$

By definition, it must be $A_3 > A_0$ and only $A_0 = 3$ is available.
But "3" converge to "1". ∴ For "Collatz Hole" exist, $2^{Z_S} = 2^1$ must exist.

Because all $\vec{H_K}$ is in One hyperplane and any $\vec{H_K}$ is "Linearly Independent" with others and all factor of $\vec{H_K}$ is not zero. So, it must be

$$\vec{H_K} = \beta * \vec{H_0} + \gamma * \vec{H_S} \quad (for\ 0 \leq K < N)$$

So, β and γ must exist for $\vec{H_{N-1}}$.

$$\vec{H_{N-1}} = \beta * \vec{H_0} + \gamma * \vec{H_S}$$

$$\vec{H_{N-1}} = \begin{bmatrix} A_{N-1} \\ 1 \\ 2^{Z_{N-1}} \\ \cdots \\ \cdots \end{bmatrix} = \beta \vec{H_0} + \gamma \vec{H_S} = \beta \begin{bmatrix} A_0 \\ 1 \\ 2^{Z_0} \\ \cdots \\ \cdots \end{bmatrix} + \gamma \begin{bmatrix} A_S \\ 1 \\ 2^{Z_S} \\ \cdots \\ \cdots \end{bmatrix}$$

We can get 2 equation as below.

$$1 = \beta * 1 + \gamma * 1 \qquad 2^{Z_{N-1}} = \beta * 2^{Z_0} + \gamma * 2^{Z_S}$$

$$\text{So, } 2^{Z_{N-1}} = 2^1, \ \textit{because } 2^{Z_0} = 2^1 \textit{ and } 2^{Z_S} = 2^1$$

It is contradiction, because it must be $2^{N-1} \geq 2^2$.

So, any Set "A" ($N \geq 2$) can't be "Collatz Hole", because "Collatz Hole" vector $\vec{H_K}$ of Set "A" can not be on one Hyperplane.

$\therefore$ "Collatz Hole" can't exist.

□

Example 2.10 ("Collatz Hole" can not exist)**.**
Because we proved "Collatz Hole" can't exist, so it is not easy to make "Collatz Hole" sample. But we can try making sample extremely.

Assume $A_0 = 811$ and "Set" A is "Collatz Hole" ($N = 100,\ A_0$ is smallest).
Let $A_{N-1} = A_{-1} = C_0(A_0) = \dfrac{2^2 811 - 1}{3} = 1081.\ 2^{Z_{N-1}} = 2^2$
$A_1 = P(A_0) = \dfrac{3 * 811 + 1}{2^1} = 1217,\ 2^{Z_0} = 2^1$
So, $A = (811,\ 1217,\ A_2,\ A_3,\ \ldots,\ A_{98},\ 1081)$
There must be $2^{Z_S} = 2^1$ $(S > 0)$. Assume $2^{Z_S} = 2^{Z_{20}} = 2^1$.
Then, we can get "Collatz Hole Vector" $\vec{H_0}$ and $\vec{H_S} = \vec{H_{20}}$ and $\vec{H_{N-1}} = \vec{H_{99}}$.

$$\vec{H_0} = \begin{bmatrix} 811 \\ 1 \\ 2^{Z_0} = 2^1 \\ \ldots \\ 2^Z/2^{Z_{0-1}} - 2^Z 7 \end{bmatrix} \qquad \vec{H_{20}} = \begin{bmatrix} A_{20} \\ 1 \\ 2^{Z_{20}} = 2^1 \\ \ldots \\ 2^Z/2^{Z_{20-1}} - 2^Z A_{20} \end{bmatrix}$$

$$\vec{H_{99}} = \begin{bmatrix} 1081 \\ 1 \\ 2^{Z_{99}} = 2^2 \\ \ldots \\ 2^Z/2^{Z_{99-1}} - 2^Z A_{99} \end{bmatrix} \qquad \vec{n} = \begin{bmatrix} 3^{100} \\ 3^{99} \\ 3^{98} \\ \ldots \\ 3^1 \\ 3^0 \end{bmatrix}$$

Any $\vec{H_X}$ and $\vec{H_Y}$ $(X <> Y)$ is Linear Independent, and all factor of $\vec{H_K}$ is not zero.
And because all $\vec{H_K}$ is on One hyperplane, so β and γ must exist for $\vec{H_{99}} = \beta * \vec{H_0} + \gamma * \vec{H_{20}}$.

$$\vec{H_{99}} = \begin{bmatrix} A_{99} \\ 1 \\ 2^2 \\ \ldots \\ 2^Z/2^{Z_{99-1}} - 2^Z A_{99} \end{bmatrix} = \beta \vec{H_0} + \gamma \vec{H_{20}} = \beta \begin{bmatrix} A_0 \\ 1 \\ 2^1 \\ \ldots \\ \ldots \end{bmatrix} + \gamma \begin{bmatrix} A_{20} \\ 1 \\ 2^1 \\ \ldots \\ \ldots \end{bmatrix}$$

We can use 2 equation as below.

$$1 = \beta + \gamma \qquad 2^2 = 2^1\beta + 2^1\gamma \qquad \text{Then, } 2^2 = 2^1, \text{ it is contradiction.}$$

So, all $\vec{H_K}$ is not on One hyperplane. And, Set "A" is not "Collatz Hole".

3. Proof of Collatz's Conjecture

After proof of "Collatz Hole" done, any "Invented Model of Collatz" in history seems to be acceptable because we got rid of big wall.
And now we prove "Why any A_0 converge to '1' ?" by using a Model.
In $A_K = \frac{3A_{K-1}+1}{2^{Z_{K-1}}}$, $C_S(A_K)$ of Odd number A_K (≥ 3) can be divided as 3 "Types" as below.

$C_0(C_S(A_K))$ not exist when $M_3(C_S(A_K)) = 0$
$C_0(C_S(A_K)) > C_S(A_K)$, when $M_3(C_S(A_K)) = 1$
$C_0(C_S(A_K)) < C_S(A_K)$, when $M_3(C_S(A_K)) = 2$

For example, for $A_K = 5$. $C_0(A_K) = 3,\ C_1(A_K) = 13,\ C_2(A_K) = 53$
$C_0(C_0(A_K))$ *not exist* $(M_3(C_0(A_K)) = 0)$
$C_0(C_1(A_K)) = C_0(13) = 17 > 13 = C_1(A_K)$ $(M_3(C_1(A_K)) = 1)$
$C_0(C_2(A_K)) = C_0(53) = 35 < 53 = C_2(A_K)$ $(M_3(C_2(A_K)) = 2)$

And this "Types" is same in $C_X(A_K)$ and $C_{X+3}(A_K)$, because $M_3(C_X(A_K)) = M_3(C_{X+3}(A_K))$
For example, for $A_K = 5,\ C_3(5) = 213,\ C_4(5) = 853,\ C_5(5) = 3413$
$C_{3X+0}(A_K) = (3,\ 213,\ \dots)$ can NOT have "Child"
$C_{3X+1}(A_K) = (13,\ 853,\ \dots)$ can have "$0_{th}Child$" $(> C_{3X+1}(A_K))$
$C_{3X+2}(A_K) = (53,\ 3413,\ \dots)$ can have "$0_{th}Child$" $(< C_{3X+2}(A_K))$

So we treat 3 Child (that is in neighbor) together for easy to analysis.

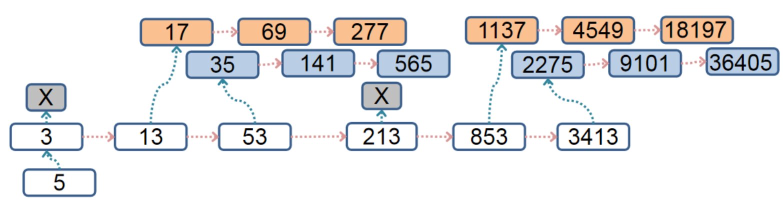

Figure 4. Child-of-Child of 5

Assume we can use only $(0_{th},\ 1_{th},\ 2_{th})Child$ of A_K (for all K).

For example, $A_2 = 5$, we can find all A_1 and A_0 that $A_K = C_S(A_{K+1})$ $(S < 3)$,
From $A_2 = 5,\ A_1 = 3$ *or* 13 *or* 53 $(C_2(5) = 53)$.
From $A_1 = 13,\ A_0 = 17$ *or* 69 *or* 277 $(C_2(13) = 277)$
From $A_1 = 53,\ A_0 = 35$ *or* 141 *or* 565 $(C_2(53) = 565)$

We can express "Count" of revealed A_K as rough.
$n(A_2) = 1,\ n(A_1) = 3 = 3 * 2^0,\ n(A_0) = 6 = 3 * 2^1$
And we can imagine more Count. Because all "3 Brother that is neighbor" can make 6 Child. "1 of them" can't have Child and "2 of them" can have each 3 Child.
$n(A_{-1}) = 3 * 2^2,\ n(A_{-2}) = 3 * 2^3,\ n(A_{-3000}) = 3 * 2^{3001}$
Then, we can have a question "Can $n(A_X)$ $(X > 0)$ will be 1 even if $n(A_0)$ is very big?".

Definition 3.1 (Brother Group).

From any Odd number A_0, we can get A_{K+1} $(0 \leq K < \infty)$ by $A_{K+1} = \dfrac{3A_K + 1}{2^{Z_K}}$.

Define "Limitation of Brother Index of A_0" as $L(A_0)$.

$2^{6(L(A_0)-1)} < 2^{Max(Z_K)} \leq 2^{6L(A_0)}$ $(L(A_0) \geq 1 \; as \; integer, \; 0 \leq K < \infty)$.

We can notes $L(A_0)$ as "L" for simple when A_0 is obvious.

Then $S < 3L$, in $C_S(A_{K+1}) = A_K$.

Define "Brother-of-Brother" relation as

"X" $(X \geq 1)$ and "Y" $(Y \geq 1)$ is "Brother-of-Brother" relation when $P^T(X) = P^T(Y)$ $(T \geq 1)$ without $P(1) = 1$ relation.

ex) 11 and 23 is "Brother-of-Brother", $P^3(11) = 5 = P^3(23)$.
$11 \to 17 \to 13 \to 5, \quad 23 \to 35 \to 53 \to 5$

ex) 17 and 13 is NOT "Brother-of-Brother", $P^2(17) = 5 = P^1(13)$.
$17 \to 13 \to 5, \quad 13 \to 5 \qquad (2 <> 1)$

ex) 113 and 85 is NOT "Brother-of-Brother", $P^2(113) = 1 = P^2(85)$
$113 \to 85 \to 1, \quad 341 \to 1 \to 1$ (includes $P(1) = 1$).

Define "K_{th} Brother Group of A_0" as "$G_{A_0}(K)$" $(K \; is \; integer)$.

$G_{A_0}(K)$ includes A_K and all "Brother-of-Brother of A_K"

We can notes as "$G(K)$" for simple when A_0 is obvious.

Notes "Count of numbers in $G(K)$" as $n(G(K))$.

Proof.

For any A_{K+1} $(A_{K+1} <> 6k+3)$, $2^{Z_K} = 2^1 \; or \; 2^2$ when $A_K = C_0(A_{K+1})$

So, $2^{Z_K} = 2^{1+2*S} \; or \; 2^{Z_K} = 2^{2+2*S}$ when $A_K = C_S(A_{K+1})$ $(S \geq 0)$

From $2^{Z_K} \leq 2^{6L}$

$1 + 2S \leq 6L$ or $2 + 2S \leq 6L$

$S \leq (6L-1)/2$ or $S \leq 3L - 1$

$\therefore S < 3L$ when $2^{6(L-1)} < 2^{Max(Z_K)} \leq 2^{6L}$ $(L \geq 1 \; , \; 0 \leq K < \infty)$.

□

Example 3.2 (Brother Group).

For example, for $A_0 = 17$, $A_1 = 13$ $A_2 = 5$, $A_3 = 1$, $A_4 = 1$, $Z = (2, 5, 4, 2, 2, \dots)$,
$2^{6*(1-1)} < 2^{Max(Z_K)} = 2^5 \leq 2^{6*1}$, $L = 1$

$G(0) = (17, \; 35, \; 69, \; 141, \; 277, \; 565)$	$n(G(0)) = 6$
$G(1) = (3, \; 13, \; 53)$	$n(G(1)) = 3$
$G(2) = (5, \; 21)$	$n(G(2)) = 2$
$G(3) = (1)$	$n(G(3)) = 1$
$G(4) = (\;\;)$	$n(G(4)) = 0$

For example, for $A_0 = 909,\ A_1 = 341,\ A_2 = 1,\ Z = (3, 10, 2, 2, \dots)$,
$2^{6*(2-1)} < 2^{Max(Z_K)} = 2^{10} \le 2^{6*2},\ L = 2$

$$G(0) = (3,\ 13,\ 53,\ 213,\ 853,\ 3413,\ 113,\ 453,\ 1813,\ 7253,\ 29013,\ 116053,\ 227,\ 909,\ 3637,\ 14549,\ 58197,\ 232789) \quad n(G(0)) = 18$$
$$G(1) = (5,\ 21,\ 85,\ 341,\ 1365) \quad n(G(1)) = 5$$
$$G(2) = (1) \quad n(G(2)) = 1$$

Theorem 3.3 (For any A_0 (≥ 1), $n(G(K))$ is as below.).

$$n(G(K)) \begin{cases} = 0 & (when\ A_x = 1\ (x < K)) \\ = 1 & (when\ A_K = 1) \\ = 3L(A_0) - 1 & (when\ A_{K+1} = 1) \\ \quad G(K) \ni C_x(1)\ \ (1 \le x < 3L) & \\ \ge 3L(A_0) & \end{cases}$$

$$n(G(K+1)) < n(G(K)) \quad (n(G(K)) > 0)$$
If $G(K) \ni Y$*, then it can't be* $G(x) \ni Y \quad (x <> K)$

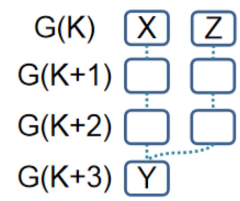

FIGURE 5. in "Collatz Hole"

Proof.
For any A_0 (≥ 3), $L(A_0)$ and $G_{A_0}(K)$ exist even if huge.
First of all, because "Collatz Hole" can't exist, any Odd number (≥ 3) in $G(K)$ can't be in another $G(x)$ $(x <> K)$. ($\because$ "Brother Group" is made by "Parent-Child" relation).
In "Figure", $X <> Y$ because "Collatz Hole" can't exist.
And if $X = Z$, then $X \to Y$ and $Z \to Y$ is same thing.

For any Odd X in $G(K)$, X can have "Child" in $G(K-1)$ as
"$0_{th}Child, \dots, (3L-1)_{th}Child$" ($X \ge 3$, Count = "$3L$")
or "$1_{th}Child, \dots, (3L-1)_{th}Child$" ($X = 1$, Count = "$3L-1$").
So, $n(G(K)) < n(G(K-1))$ $(n(G(K)) > 0)$ ($\because 3L \ge 3,\ 3L-1 \ge 2$).
"Count=$3L-1$" is only when "$X = 1$", because it is not $G(K-1) \ni$ "$C_0(1) = 1$".
$\because P(1) = 1$ relation can't be "Brother Group" .
And if $n(G(K)) < 3L$, then $G(K)$ can't have number Y ($Y \ge 3$ *and* $Y <> C_S(1)$).

If $G(K)$ have "1", $n(G(K)) = 1$ because $P^T(1) = 1 = P^T(X)$ $(X \ge 3)$ use $P(1) = 1$.
So, it must be $G(K-1) \ni C_S(1)$ $(S > 0)$ only. And $G(x) = 0$ $(x > K)$.
Because $n(G(K+1)) = 0$ in $n(G(K)) = 1$, so $n(G(K+1)) < n(G(K))$ $(n(G(K)) > 0)$.
Because $n(G(K+1)) < n(G(K))$, "1" $(n(G) = 1)$ can't be in another $G(x)$ $(x <> K)$.
$\therefore$ Any Odd number X (≥ 1) in $G(K)$ can't be in another $G(x)$ $(x <> K)$ □

Theorem 3.4 (For any A_0 (≥ 3), $n(G(K+1)) = \dfrac{n(G(K))}{2L}$ $(n(G(K+1)) \ge 3L)$).

Proof.
When $n(G(K+1)) \ge 3L$, $G(K+1)$ don't have 1 or "Child of 1".
So all numbers of $G(K+1)$ is as "$C_0(x),\ \dots,\ C_{3L-1}(x)$", $n(G(K+1)) = 3L * Y$.
And every "$C_0(x),\ \dots,\ C_{3L-1}(x)$" can have "$2L * 3L$" Child in $G(K)$.
So, $n(G(K)) = 3L * Y * 2L$. $\quad \therefore n(G(K)) = 2L * n(G(K+1))$ □

Theorem 3.5 (For any A_0, it can be $G(K) < 3L \ \ (K \geq 0)$).

$$K > \frac{\log_2(n(G(0)) \ / \ 3L)}{\log_2 2L}$$

Proof.
For Odd number A_0 ($A_0 \geq 3$), there exist $n(G(0))$ and let $n(G(0)) = g$ (even if $n(G(0))$ is huge). When $g \geq 3L$, we can get $G(-X) \ \ (X > 0)$. In this time, $2^{Z-X} \leq 2^{6L}$.

$$n(G(-1)) = (2L)g. \qquad \text{so, } n(G(0)) = g = \frac{n(G(-X))}{(2L)^X} \quad (X > 0)$$

$$\text{So, } \lim_{X\to\infty} \frac{n(G(0))}{n(G(-X))} = \lim_{X\to\infty} \frac{g}{(2L)^X g} = \lim_{X\to\infty} \frac{1}{(2L)^X} = 0$$

So, $n(G(0)) \ll n(G(-X))$ when "X" is big.
This means $n(G(0))$ is very smaller than whole scale.

So, we can find "X" for $n(G(X)) < 3L$ even in huge "g".

$$\frac{g}{(2L)^X} < 3L, \text{ so } X > \frac{\log_2(g/3L)}{\log_2 2L}$$

□

Theorem 3.6 (Any Odd number A_0 converge to "1").

$$n(G(K)) = \begin{cases} 3L(2L-1)(2L)^Q & A_{K+Q+2} = 1 \quad (\textit{when } n(G(K)) \geq 3L, \ Q \geq 0) \\ 3L-1 & A_{K+1} = 1 \quad (\textit{when } 1 < n(G(K)) < 3L) \\ 1 & A_K = 1 \quad (\textit{when } n(G(K)) = 1) \\ 0 & A_x = 1 \ \ (x < K) \quad (\textit{when } n(G(K)) = 0) \end{cases}$$

Proof.
For any A_0 (≥ 3), $L(A_0)$ and $G_{A_0}(K)$ exist even if huge.
And it can be $G(K) < 3L$ in $K > \dfrac{\log_2(n(G(0)) \ / \ 3L)}{\log_2 2L}$.

So for $\dfrac{\log_2(n(G(0)) \ / \ 3L)}{\log_2 2L} < K \leq \dfrac{\log_2(n(G(0)) \ / \ 3L)}{\log_2 2L} + 1$,

it must be $n(G(K)) > 1$, because $n(G(K-1)) = 3L-1 < 3L$ is contradiction when $n(G(K)) = 1$. It must be $n(G(K-1)) \geq 3L$.
Then $1 < n(G(K)) < 3L$, so $n(G(K)) = 3L - 1$. And $A_{K+1} = 1$.
$\therefore$ Any Odd number A_0 converge to "1".

And now, we can get "General Equation" of $n(G(K))$ $(n(G(K)) \geq 3L)$.
Let $G(X) = 3L - 1$, then $G(X) = (C_1(1), \ \ldots, \ C_{3L-1}(1))$.
Because it is not $G(X) \ni C_0(1)$,
$G(X-1)$ have "Child" of "$C_1(1), \ C_2(1)$" and "$C_3(1), \ldots, \ C_{3L-1}(1)$ ".

$C_{2+3x}(1)$ $(x \geq 0)$ can't have Child, $C_2(1) = 21, \ C_5(1) = 1365$.
So, $n(G(X-1)) = \{1\}(3L) + \{(3L-3) * \dfrac{2}{3}\}(3L) = 3L(2L-1)$.

$$n(G(X-2)) = 3L(2L-1) * (2L)$$
$$n(G(X-3)) = 3L(2L-1) * (2L)^2$$

$\therefore n(G(K)) = 3L(2L-1)(2L)^Q \quad (\textit{when } n(G(K)) \geq 3L, \ Q \geq 0)$

Then we can get "X" for $A_X = 1$.

$$n(G(K+Q)) = 3L(2L-1) \quad (3L \leq 3L(2L-1) < 3L*2L)$$
$$n(G(K+Q+1)) = 3L-1 \quad (1 < 3L-1 < 3L)$$
$$n(G(K+Q+2)) = 1, \text{ so } A_{K+Q+2} = 1$$

□

Example 3.7 (Any Odd number A_0 converge to "1")**.**
In "Example 3.2",
$A_0 = 17,\ L = 1,\ n(G(0)) = 6 = 3L(2L-1)*(2L)^1$.
$n(G(1)) = 3L(2L-1)(2L)^0 = 3,\ n(G(2)) = 3L-1 = 2,\ n(G(3)) = 1$.
$A_0 = 909,\ L = 2,\ n(G(0)) = 18 = 3L(2L-1)(2L)^0$.
$n(G(1)) = 3L-1 = 5,\ n(G(2)) = 1$.

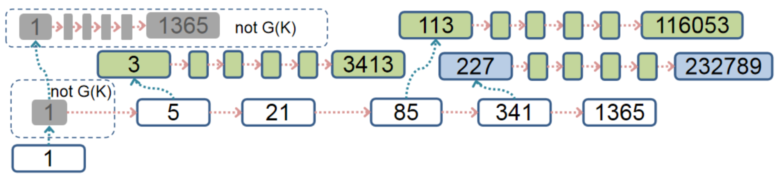

FIGURE 6. Child-of-Child of 1 $(L = 2)$

Theorem 3.8 (Collatz's Conjecture is true)**.**

Proof.
It seems the main reason of that we can prove is "Collatz Hole can't exist". Because of that, we can count the number exactly without worrying about "Isolated Loop", and estimate the logic exactly.
Any Odd A_0 can converge to "1" by "Equivalent Equation", so "Collatz's Conjecture" is true.

□

4. Conclusion

By proving that "Collatz Loop (Eternal Loop)" can't exist, we proved that "Collatz's Conjecture" is true. And the connection of all Number is shaped as "Tree".

In short, we can say that "The Collatz Tree is huge indexed Tree (by $Z = (Z_0, Z_1, \dots)$) structure.".
This "Tree" can includes all binary data in the world, specially all is already indexed (or having position).

Remark 4.1 (The specification of Collatz Tree.).
1. Tree is consist of all "Odd" and "Even" number (≥ 1).
2. From 1, we can reach to any Natural number (> 1) by reverse equation.
3. Path Set "Z" that includes "Z_K ($0 \leq K < N$)" from a Odd number A_0 (≥ 3) to $A_N = 1$ ($A_{N-1} > 1$) is unique.
(For Even number, it needs one more variable.)
4. Another number set (such as "Integer") can be used in "Collatz Tree" with proper mapping.
For example, integer T ($-\infty < T < \infty$).
$x = (2 * T + 1) \quad (when\ T \geq 0)$
$x = (2 * (-T) + 0) \quad (when\ T < 0)$

Remark 4.2 (Hope with Collatz Tree).
"Collatz Tree" seems to be the good tool for solving another "Question" of mathematics because it include all cases of "Multi-Dimensional Choice (by Z)".
Anything that can be changed to natural number can be a member of "Collatz Tree".
And maybe we can find an "Operator" of it within not so long time.

References

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