Speed of Light in a Transversely Moving Body

Rajeev Kumar*

Abstract

In this paper a theory of speed of light in a transversely moving body has been presented.
Keyword : Speed of light.

1 SPEED OF LIGHT IN A MOVING BODY

Let c_2 be the speed of a light ray (or a photon) in a stationary body (medium 2) and let $c_{2,m}$ be the speed of the light ray (or a photon) in the body (medium 2) when it is moving with a velocity **v**. Then,

$$\left|\mathbf{c}_{2,m} - \mathbf{v}\right|^2 - \left|\frac{c_o}{c_1}\mathbf{c}_1 - \mathbf{v}\right|^2 = c_2^2 - c_o^2$$

where

c_o = speed of the light in vacuum

$\mathbf{c}_1$ = velocity of the light in medium 1

2 SPEED OF LIGHT IN A TRANSVERSELY MOVING BODY

Let's consider a ray of light incident perpendicularly on the interface of a body (medium 2) moving with a velocity **v** in a direction perpendicular to the direction of the incident light.

$$\left|\mathbf{c}_{2,m} - \mathbf{v}\right|^2 - \left|\frac{c_0}{c_1}\mathbf{c}_1 - \mathbf{v}\right|^2 = c_2^2 - c_o^2$$

$$\Rightarrow \left[\; c_{2,m}^2 + v^2 - 2c_{2,m} v\cos(90^o - \theta) \;\right] - \left[\; c_o^2 + v^2 - 2c_0 v\cos 90^o \;\right] = c_2^2 - c_o^2$$

$$\Rightarrow \left[\; c_{2,m}^2 + v^2 - 2c_{2,m} v\sin\theta \;\right] - \left[\; c_o^2 + v^2 \;\right] = c_2^2 - c_o^2$$

$$\Rightarrow c_{2,m}^2 - (2v\sin\theta)c_{2,m} - c_2^2 = 0 \qquad (i)$$

$$\Rightarrow c_{2,m} = \frac{2v\sin\theta + \sqrt{4v^2\sin^2\theta + 4c_2^2}}{2} = v\sin\theta + \sqrt{v^2\sin^2\theta + c_2^2}$$

$$\Rightarrow c_{2,m} = v\sin\theta + c_2\left[1 + \left(\frac{v\sin\theta}{c_2}\right)^2\right]^{\frac{1}{2}}$$

*rajeevkumar620692@gmail.com

3 PRELIMINARY ANALYSIS

Let the kinetic energy and the momentum be conserved in the x direction (parallel to the interface).

Now from (i), we have

$$c_{2,m}^2 - (2v\sin\theta)c_{2,m} - c_2^2 = 0$$

$$\Rightarrow c_{2,m}^2 = 2vc_{2,m}\sin\theta + c_2^2$$

$$\Rightarrow c_{2,m,x}^2 + c_{2,m,y}^2 = 2vc_{2,m}\sin\theta + c_2^2$$

$$\Rightarrow c_{2,m,x}^2 + c_2^2 = 2vc_{2,m}\sin\theta + c_2^2$$

$$\Rightarrow c_{2,m,x}^2 = 2vc_{2,m}\sin\theta$$

$$\Rightarrow c_{2,m}^2\sin^2\theta = 2vc_{2,m}\sin\theta$$

$$\Rightarrow \sin\theta = \frac{2v}{c_{2,m}}$$

$$\Rightarrow \frac{c_{2,m,x}}{c_{2,m}} = \frac{2v}{c_{2,m}}$$

$$\Rightarrow c_{2,m,x} = 2v$$

$$\Rightarrow \tan\theta = \frac{c_{2,m,x}}{c_{2,m,y}} = \frac{2v}{c_2}$$

$$\Rightarrow c_{2,m} = \sqrt{(2v)^2 + c_2^2} = \sqrt{4v^2 + c_2^2}$$

It should be noted that the result obtained is similar to the effect of a perfectly elastic collision of a massive body moving with a speed v with a lighter body having zero speed, along the x direction.

4 ADVANCED ANALYSIS

For a stationary body (medium 2)

$$c_{2,x} = \left(\frac{\beta_2}{\mu_2}\right) c_{o,x} \qquad \left[\beta_2 = \frac{c_2}{c_o}\right]$$

So, let the speed of a light ray (or a photon), in a moving body (medium 2), along the x direction

$$c_{2,m,x} = c_{2,x} + \left(\frac{\beta_2}{\mu_2}\right)2v = \left(\frac{\beta_2}{\mu_2}\right)2v \qquad [\because c_{2,x} = 0]$$

Now from (i), we have

$$c_{2,m}^2 - (2v\sin\theta)c_{2,m} - c_2^2 = 0$$

$$\Rightarrow c_{2,m}^2 = 2vc_{2,m}\sin\theta + c_2^2$$

$$\Rightarrow c_{2,m,x}^2 + c_{2,m,y}^2 = 2vc_{2,m}\sin\theta + c_2^2$$

$$\Rightarrow c_{2,m,x}^2 + c_{2,m,y}^2 = 2vc_{2,m,x} + c_2^2 \qquad [c_{2,m}\sin\theta = c_{2,m,x}]$$

$$\Rightarrow \left(\frac{\beta_2}{\mu_2}\right)^2 4v^2 + (c_2 + \Delta c_{2,y})^2 = \left(\frac{\beta_2}{\mu_2}\right)4v^2 + c_2^2 \qquad \left[c_{2,m,x} = \left(\frac{\beta_2}{\mu_2}\right)2v\right]$$

$$\Rightarrow (\Delta c_{2,y})^2 + 2c_2\Delta c_{2,y} = \left(\frac{\beta_2}{\mu_2}\right)\left[1 - \left(\frac{\beta_2}{\mu_2}\right)\right]4v^2 = b^2 \quad \text{(let)}$$

$$\Rightarrow (\Delta c_{2,y})^2 + 2c_2\Delta c_{2,y} - b^2 = 0$$

$$\Rightarrow \Delta c_{2,y} = \frac{-2c_2 + \sqrt{4c_2^2 + 4b^2}}{2} = -c_2 + \sqrt{c_2^2 + b^2}$$

$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{b^2}{c_2^2}\right)} - 1\right]$$

$$\Rightarrow \tan\theta = \frac{c_{2,m,x}}{c_2 + \Delta c_{2,y}} = \frac{\left(\frac{\beta_2}{\mu_2}\right)2v}{c_2 + c_2\left[\sqrt{\left(1 + \frac{b^2}{c_2^2}\right)} - 1\right]} = \frac{2v}{c_2} \times \frac{\left(\frac{\beta_2}{\mu_2}\right)}{\sqrt{\left(1 + \frac{b^2}{c_2^2}\right)}}$$

$$\Rightarrow c_{2,m} = \sqrt{(c_{2,m,x})^2 + (c_2 + \Delta c_{2,y})^2} = \sqrt{\left(\frac{\beta_2}{\mu_2}\right)^2 4v^2 + \left(1 + \frac{b^2}{c_2^2}\right)c_2^2}$$

$$\Rightarrow c_{2,m} = \sqrt{\left(\frac{\beta_2}{\mu_2}\right)^2 4v^2 + \left(c_2^2 + \left(\frac{\beta_2}{\mu_2}\right)\left[1 - \left(\frac{\beta_2}{\mu_2}\right)\right]4v^2\right)} = \sqrt{\left(\frac{\beta_2}{\mu_2}\right)4v^2 + c_2^2}$$

The coefficient of restitution for the x direction

$$e_x = \frac{c_{2,m,x} - v}{v} = \frac{c_{2,m,x}}{v} - 1 = \frac{2\beta_2}{\mu_2} - 1$$

Now the loss in kinetic energy is given as

$$|\Delta KE| = \frac{1}{2}\frac{m_a m_b}{m_a + m_b}(u_a - u_b)^2(1 - e^2)$$

So the loss in kinetic energy in the x direction

$$|\Delta KE_x| = \frac{1}{2}\frac{m_2 m_p}{m_2 + m_p}(v - 0)^2(1 - e_x^2)$$

$$\Rightarrow |\Delta KE_x| = \frac{1}{2}m_p v^2(1 - e_x^2) \qquad [m_p << m_2]$$

So the gain in kinetic energy in the y direction

$$|\Delta KE_y| = |\Delta KE_x|$$

$$\Rightarrow \frac{1}{2}m_p\left[(c_2 + \Delta c_{2,y})^2 - c_2^2\right] = \frac{1}{2}m_p v^2(1 - e_x^2)$$

$$\Rightarrow \left[(\Delta c_{2,y})^2 + 2c_2\Delta c_{2,y}\right] = v^2(1 - e_x^2)$$

$$\Rightarrow (\Delta c_{2,y})^2 + 2c_2\Delta c_{2,y} - v^2(1 - e_x^2) = 0$$

$$\Rightarrow \Delta c_{2,y} = \frac{-2c_2 + \sqrt{4c_2^2 + 4v^2(1 - e_x^2)}}{2} = -c_2 + \sqrt{c_2^2 + v^2(1 - e_x^2)}$$

$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{v^2}{c_2^2}(1 - e_x^2)\right)} - 1\right]$$

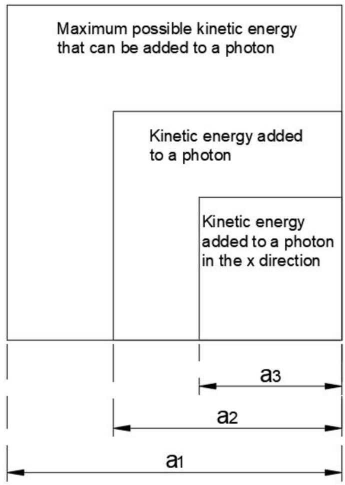
$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{v^2}{c_2^2}\left(1 - \left(\frac{2\beta_2}{\mu_2} - 1\right)^2\right)\right)} - 1\right]$$

$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{v^2}{c_2^2}\left(1 - \left(\frac{4\beta_2^2}{\mu_2^2} + 1 - \frac{4\beta_2}{\mu_2}\right)\right)\right)} - 1\right]$$

$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{v^2}{c_2^2}\left(\frac{4\beta_2}{\mu_2} - \frac{4\beta_2^2}{\mu_2^2}\right)\right)} - 1\right]$$

$$\Rightarrow \Delta c_{2,y} = c_2\left[\sqrt{\left(1 + \frac{4v^2}{c_2^2}\left(\frac{\beta_2}{\mu_2}\right)\left[1 - \left(\frac{\beta_2}{\mu_2}\right)\right]\right)} - 1\right] = c_2\left[\sqrt{\left(1 + \frac{b^2}{c_2^2}\right)} - 1\right]$$

5 ENERGY DIAGRAM



$$a_1 = \sqrt{\left(\frac{1}{2}m_p\right)}2v$$

$$a_2 = \sqrt{\left(\frac{1}{2}m_p\right)}\sqrt{\left(\frac{\beta_2}{\mu_2}\right)}2v$$

$$a_3 = \sqrt{\left(\frac{1}{2}m_p\right)}\left(\frac{\beta_2}{\mu_2}\right)2v$$

References

1. Hugh D. Young, Roger A. Freedman, Albert Lewis Ford, *“Sears’ and Zemansky’s University Physics with Modern Physics 13th edition.”*