

Norwegian Dictionaries by H. Scavenius and The Graphical Law

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Abstract

We study the Norwegian language head entries of two Norwegian Dictionaries composed by H. Scavenius. We draw the natural logarithm of the number of the Norwegian language head entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised/unnormalised. We find that the Norwegian head entries underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field.

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I. INTRODUCTION

In this paper, we study the Norwegian language. We study the head entries as those appear in two Norwegian Dictionaries,[[1],[2]]. Looking for the graphical law, we count one by one all the Norwegian head entries in the dictionary, McKay's Modern English-Norwegian and Norwegian-English Dictionary(Gyldendal's) by B. Berulfsen and H. Scavenius, David McKay Company Inc., New York (Gyldendal Norsk Forlag, Oslo 1949), [1]. In this dictionary, [1], Norwegian-English part is composed by H. Scavenius. These have few entries more, in general, for each letter compared to that in the Gyldendal's Ordbøker, Norsk-Engelsk ved H. Scavenius, Gyldendal Norsk Forlag, Oslo 1933, [2]. In the copy of the dictionary, [1], we have used, from p161 to p192 are misplaced, corresponding to the end part of the letter K through to the part for the letter M. Hence we have resorted to counting for the misplaced part, from p162 to p193 of the Gyldendal's Ordbøker, Norsk-Engelsk ved H. Scavenius, Gyldendal Norsk Forlag, Oslo 1933, [2].

We have started considering magnetic field pattern in [3], in the languages we converse with. We have studied there, a set of natural languages, [3] and have found existence of a magnetisation curve under each language. We have termed this phenomenon as the Graphical Law. Then, we moved on to investigate, [4], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [5] to [91].

The planning of the paper is as follows. We give an introduction to the standard curves of magnetisation of Ising model in the section II. In the section III, we describe the analysis of the head entries of the Norwegian language, [[1],[2]]. Sections IV and V are Acknowledgment and Bibliography respectively.

II. MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is

more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N}\sum_i\sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment, M is $\mu\sum_i\sigma_i$ or, $\mu(N_+ - N_-)$ or, μNL , $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$. $\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[92], for the lattice of spins, setting μ to one, is $-\epsilon\sum_{n,n}\sigma_i\sigma_j - H\sum_i\sigma_i$, where n.n refers to nearest neighbour pairs. The difference ΔE of energy if we flip an up spin to down spin is, [93], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $\exp(-\frac{\Delta E}{k_B T})$, [94]. In the Bragg-Williams approximation,[95], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$\ln \frac{1 + L}{1 - L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [96]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [93]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [92],[93],[94],[95],[96], due to Bethe-Peierls, [97], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, I, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.1. Empty spaces in the table, I, mean corresponding point pairs were not used for plotting a line.

BW	BW(c=0.01)	BP(4,βH = 0)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE I. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours .

C. Bethe-peierls approximation in presence of four nearest neighbours, in presence of external magnetic field

In the Bethe-Peierls approximation scheme , [97], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{e^{\frac{2\beta H}{\gamma}} factor^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula Ala [97] is given in the appendix of [8].

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{factor-1}{e^{\frac{2\beta H}{\gamma}} factor^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

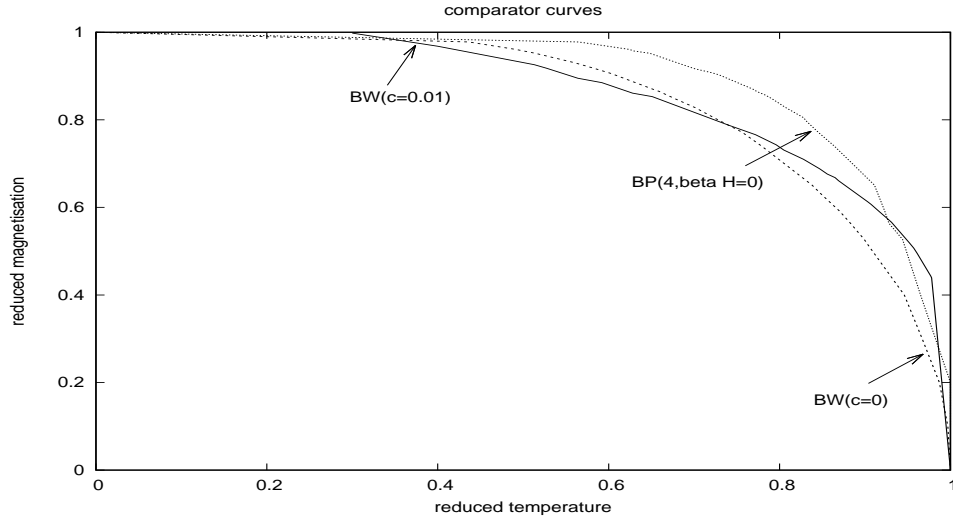


FIG. 1. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

In the following, we describe data s in the table, II, generated from the equation(4) and curves of magnetisation plotted on the basis of those data s. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.2. Empty spaces in the table, II, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
			1.00	0.964	0.513
				1.00	0.500
					0.400
					0.300
					0.200
					0.100
					0

TABLE II. Bethe-Peierls approx. in presence of little external magnetic fields

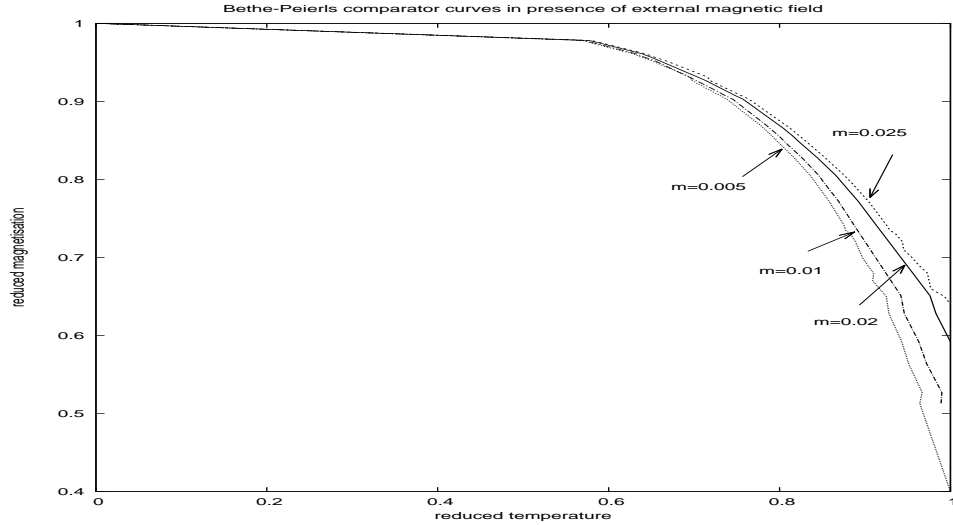


FIG. 2. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Spin-Glass

In the case coupling between (among) the spins, not necessarily n.n, for the Ising model is (are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[101–103], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[104]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [105], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [106], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida etal, [107], Gray and Moore, [108],finally Parisi, [109], [110] improved and gave final touch, [111], to their line of work. Parisi and collaborators, [112]-[116], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher etal,[117–119], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [120, 121], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [122]-[128], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [129].

Here, in our work to follow, spin-glass refers to spin-glass phase of a system with infinite range random interactions.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	R	S	T	U	V	W	X	Y	Z	Æ	Ø	Å
980	1481	61	655	509	1851	933	1072	703	205	1806	1102	1385	632	1086	1300	963	3428	1482	1488	1038	17	5	56	8	28	127	104

TABLE III. Norwegian head entries

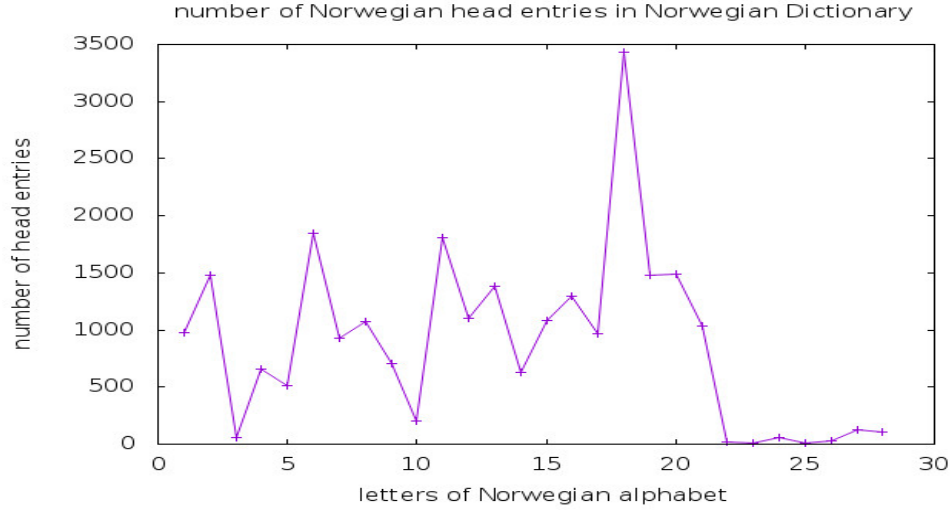


FIG. 3. Vertical axis is number of the Norwegian head entries, $[[1],[2]]$, and horizontal axis is respective letters. Letters are represented by the sequence number in the alphabet or, dictionary sequence, $[[1],[2]]$.

III. ANALYSIS OF THE NORWEGIAN HEAD ENTRIES

The Norwegian language alphabet is composed of twenty eight letters. Counting all the head entries, $[[1],[2]]$, one by one from the beginning to the end, starting with different letters, we obtain the table, III.

Highest number of head entries, three thousand four hundred twenty eight, starts with the letter S followed by head entries numbering one thousand eight hundred fifty one beginning with F, one thousand eight hundred six beginning with the letter K etc. To visualise we plot the number of head entries against respective letters in the dictionary sequence, $[[1],[2]]$, in the figure fig.3.

For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by f and the respective rank, denoted by k . k is a positive integer starting from one. The lowest value of f is five for the letter X. Hence

k	lnk	lnk/lnk _{lim}	f	lnf	lnf/lnf _{max}	lnf/lnf _{n-max}	lnf/lnf _{2n-max}	lnf/lnf _{3n-max}	lnf/lnf _{4n-max}	lnf/lnf _{5n-max}
1	0	0	3428	8.140	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.205	1851	7.523	0.924	1	Blank	Blank	Blank	Blank
3	1.10	0.326	1806	7.499	0.921	0.997	1	Blank	Blank	Blank
4	1.39	0.412	1488	7.305	0.897	0.971	0.974	1	Blank	Blank
5	1.61	0.478	1482	7.301	0.897	0.970	0.974	0.9994	1	Blank
6	1.79	0.531	1481	7.300	0.897	0.970	0.973	0.9993	0.9999	1
7	1.95	0.579	1385	7.233	0.889	0.961	0.965	0.990	0.991	0.991
8	2.08	0.617	1300	7.170	0.881	0.953	0.956	0.982	0.982	0.982
9	2.20	0.653	1102	7.005	0.861	0.931	0.934	0.959	0.959	0.960
10	2.30	0.682	1086	6.990	0.859	0.929	0.932	0.957	0.957	0.958
11	2.40	0.712	1072	6.977	0.857	0.927	0.930	0.955	0.956	0.956
12	2.48	0.736	1038	6.945	0.853	0.923	0.926	0.951	0.951	0.951
13	2.56	0.760	980	6.888	0.846	0.916	0.919	0.943	0.943	0.944
14	2.64	0.783	963	6.870	0.844	0.913	0.916	0.940	0.941	0.941
15	2.71	0.804	933	6.838	0.840	0.909	0.912	0.936	0.937	0.937
16	2.77	0.822	703	6.555	0.805	0.871	0.874	0.897	0.898	0.898
17	2.83	0.840	655	6.485	0.797	0.862	0.865	0.888	0.888	0.888
18	2.89	0.858	632	6.449	0.792	0.857	0.860	0.883	0.883	0.883
19	2.94	0.872	509	6.232	0.766	0.828	0.831	0.853	0.854	0.854
20	3.00	0.890	205	5.323	0.654	0.708	0.710	0.729	0.729	0.729
21	3.04	0.902	127	4.844	0.595	0.644	0.646	0.663	0.663	0.664
22	3.09	0.917	104	4.644	0.571	0.617	0.619	0.636	0.636	0.636
23	3.14	0.932	61	4.111	0.505	0.546	0.548	0.563	0.563	0.563
24	3.18	0.944	56	4.025	0.494	0.535	0.537	0.551	0.551	0.551
25	3.22	0.955	28	3.332	0.409	0.443	0.444	0.456	0.456	0.456
26	3.26	0.967	17	2.833	0.348	0.377	0.378	0.388	0.388	0.388
27	3.30	0.979	8	2.079	0.255	0.276	0.277	0.285	0.285	0.285
28	3.33	0.988	5	1.609	0.198	0.214	0.215	0.220	0.220	0.220
29	3.37	1	1	0	0	0	0	0	0	0

TABLE IV. Norwegian head entries: ranking, natural logarithm, normalisations

we attach a limiting number of head entries equal to one. The corresponding limiting rank, k_{lim} or, k_d is twenty nine. As a result both $\frac{lnf}{lnf_{max}}$ and $\frac{lnk}{lnk_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table, IV and plot $\frac{lnf}{lnf_{max}}$ against $\frac{lnk}{lnk_{lim}}$ in the figure fig.4. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table, IV and redo the plot, normalising the $lnfs$ with next-to-maximum lnf_{n-max} , and starting from $k = 2$ in the figure fig.5. This program then we repeat up to $k = 6$, resulting in figures up to fig.9.

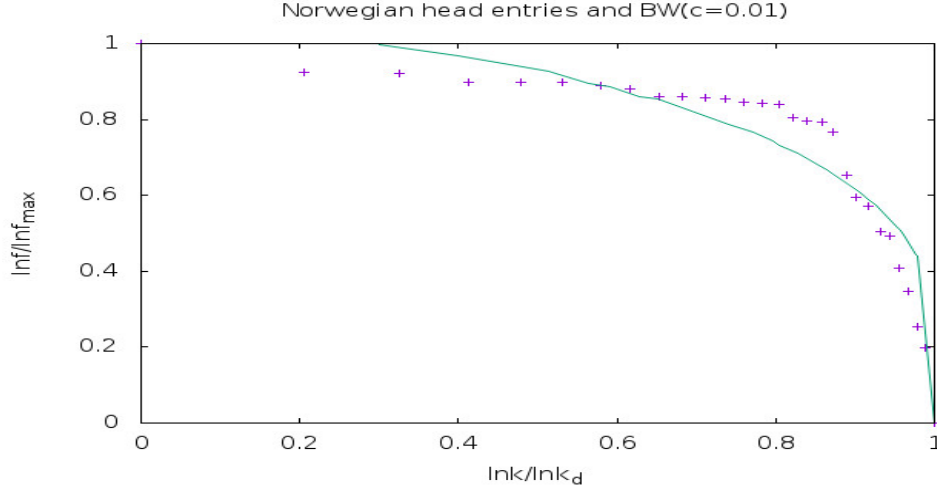


FIG. 4. The vertical axis is $\frac{\ln f}{\ln f_{max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve being the Bragg-Williams curve, $BW(c=0.01)$, in the presence of external magnetic field.

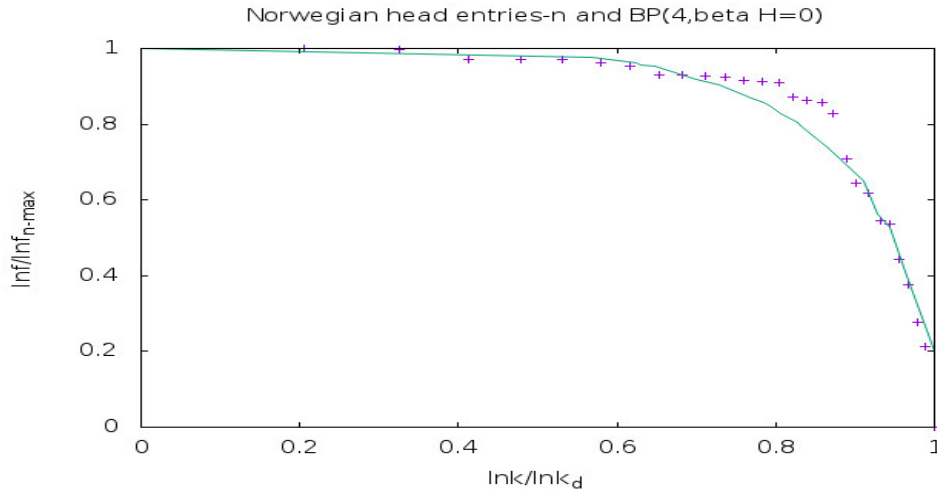


FIG. 5. The vertical axis is $\frac{\ln f}{\ln f_{n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve, $BP(4, \beta H = 0)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and in the absence of external magnetic field, $m = 0$ or, $\beta H = 0$, of the Ising Model.

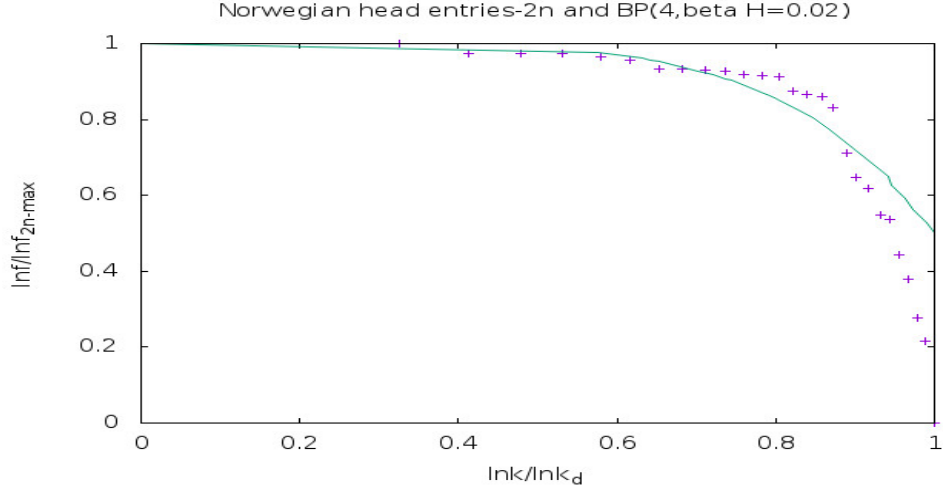


FIG. 6. The vertical axis is $\frac{\ln f}{\ln f_{2n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve, $BP(4, \beta H = 0.02)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$, of the Ising Model.

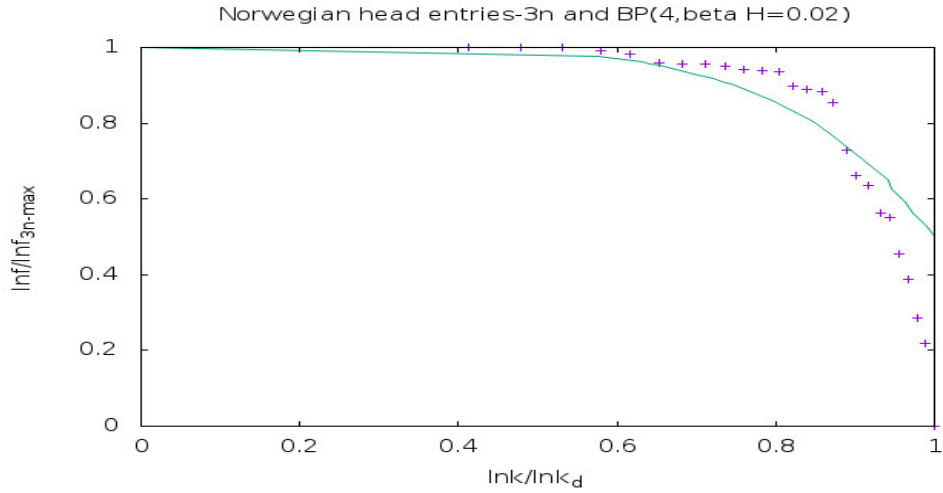


FIG. 7. The vertical axis is $\frac{\ln f}{\ln f_{3n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve, $BP(4, \beta H = 0.02)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$, of the Ising Model.

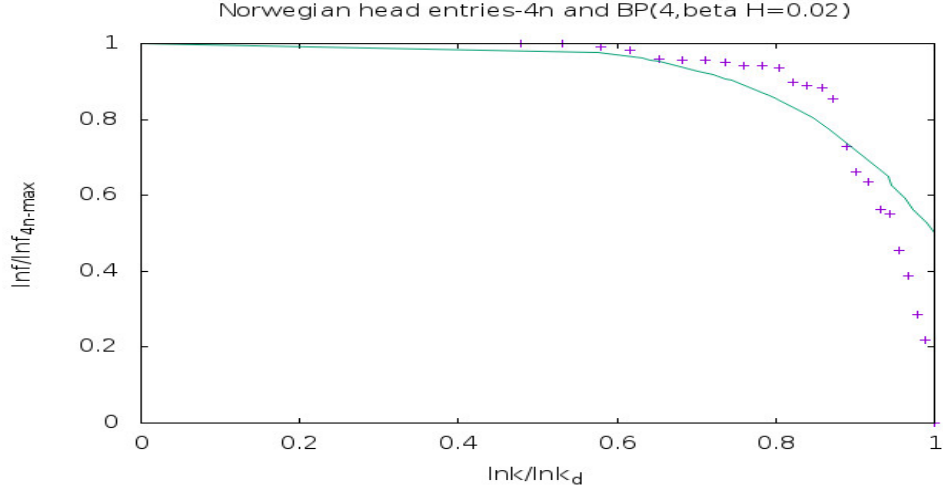


FIG. 8. The vertical axis is $\frac{\ln f}{\ln f_{4n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve, $BP(4, \beta H = 0.02)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$, of the Ising Model.

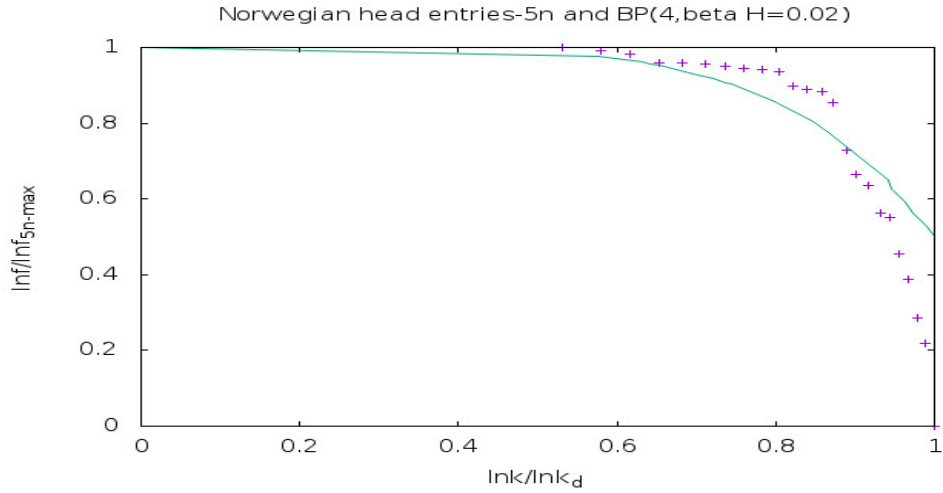


FIG. 9. The vertical axis is $\frac{\ln f}{\ln f_{5n-max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the head entries of the Norwegian language with the fit curve, $BP(4, \beta H = 0.02)$, being the Bethe-Peierls curve in the presence of four nearest neighbours and little external magnetic field, $m = 0.01$ or, $\beta H = 0.02$, of the Ising Model.

A. tentative conclusion

Matching of the plots in the figures fig.(4-9), with comparator curves i.e. the magnetisation curves of the Ising Model in various approximations, are with dispersions and dispersions do not reduce over higher orders of normalisations.

To explore for possible existence of spin-glass transition, in the presence of little external magnetic field, $\frac{\ln f}{\ln f_{max}}$, $\frac{\ln f}{\ln f_{n-max}}$, $\frac{\ln f}{\ln f_{2n-max}}$ and $\frac{\ln f}{\ln f_{3n-max}}$ are drawn against $\ln k$ in the figures fig.10-fig.13.

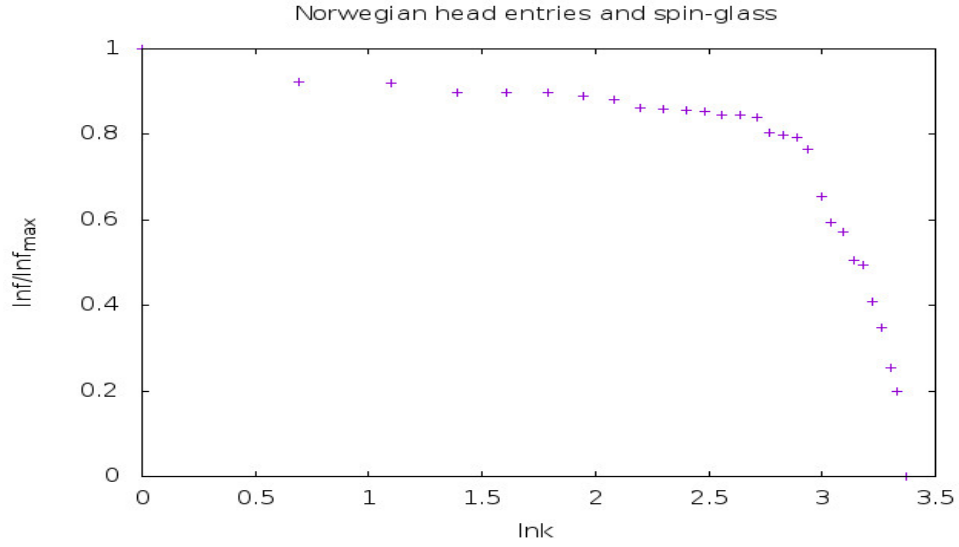


FIG. 10. The vertical axis is $\frac{\ln f}{\ln f_{\max}}$ and the horizontal axis is $\ln k$. The + points represent the head entries of the Norwegian language.

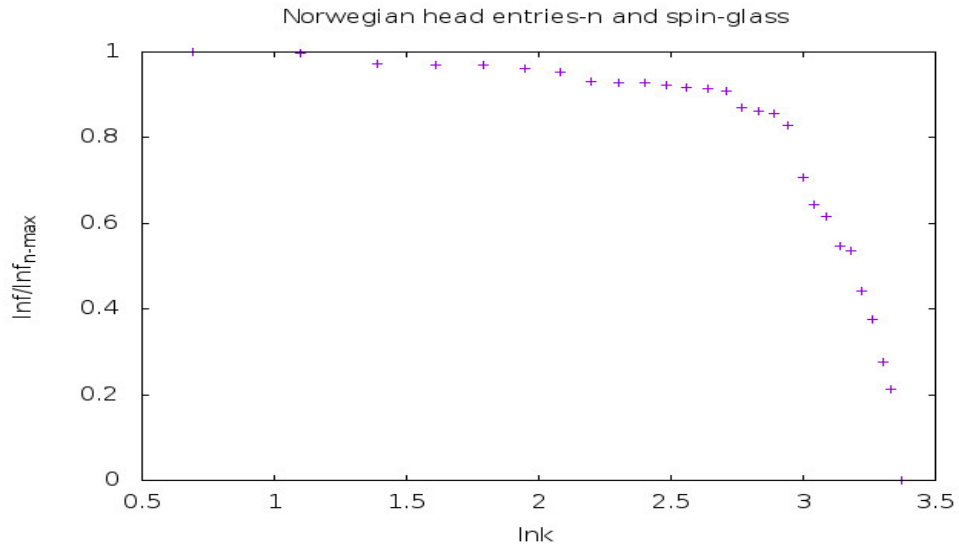


FIG. 11. The vertical axis is $\frac{\ln f}{\ln f_{n-\max}}$ and the horizontal axis is $\ln k$. The + points represent the head entries of the Norwegian language.

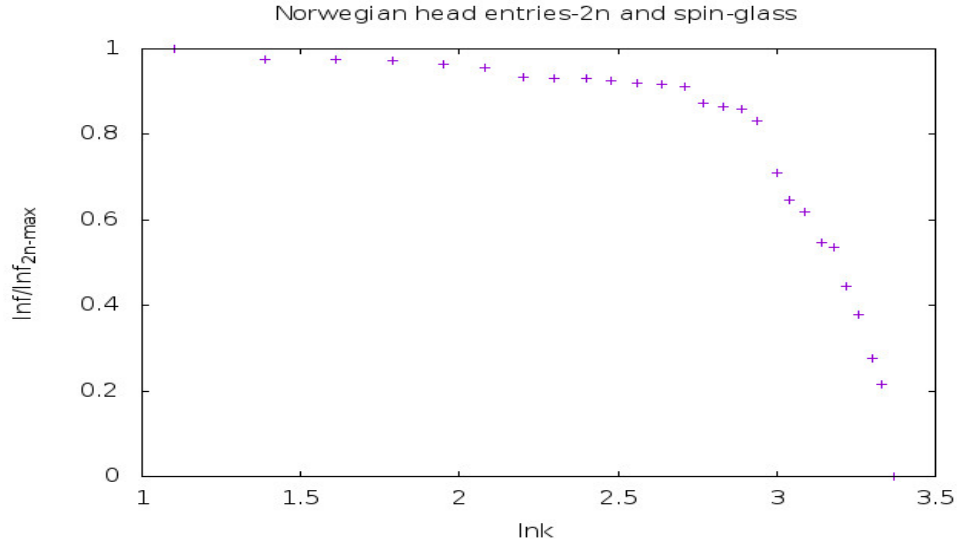


FIG. 12. The vertical axis is $\frac{\ln f}{\ln f_{2n-max}}$ and the horizontal axis is $\ln k$. The + points represent the head entries of the Norwegian language.

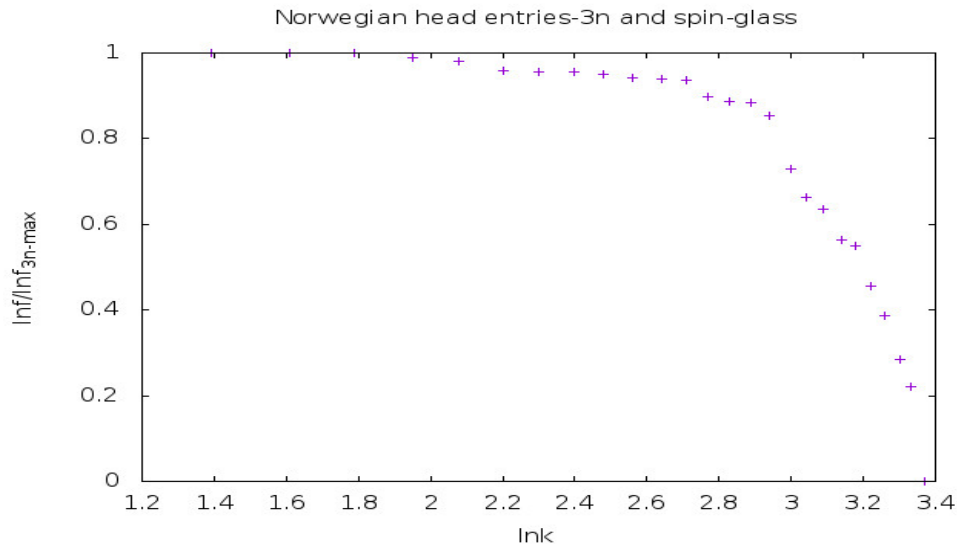


FIG. 13. The vertical axis is $\frac{\ln f}{\ln f_{3n-max}}$ and the horizontal axis is $\ln k$. The + points represent the head entries of the Norwegian language.

B. conclusion

In the figures Fig.10-Fig.13, the points has a smoothed transition, [116]. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-line rises like the branch of a rectangular hyperbola. Hence, the Norwegian head entries, [[1],[2]], are well-suited to be described by a Spin-Glass magnetisation curve, [101], in the presence of little external magnetic field. Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{n-max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$\ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [131].

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