Incorporation of Imaginary and Complex Numbers into a 3D Coordinate System

Author
Sigrid M.-L. Obenland

Abstract

In the scientific literature, complex numbers comprising a real and an imaginary part are represented as a vector in a Gaussian number plane spanned by one coordinate axis representing the imaginary numbers and another orthogonal coordinate axis representing the real numbers. In the following, I show how the imaginary axis and the real axis can be incorporated into a three-dimensional real coordinate system, thereby creating a fused coordinate system of both, real and complex numbers.

Comments

Fig. 1

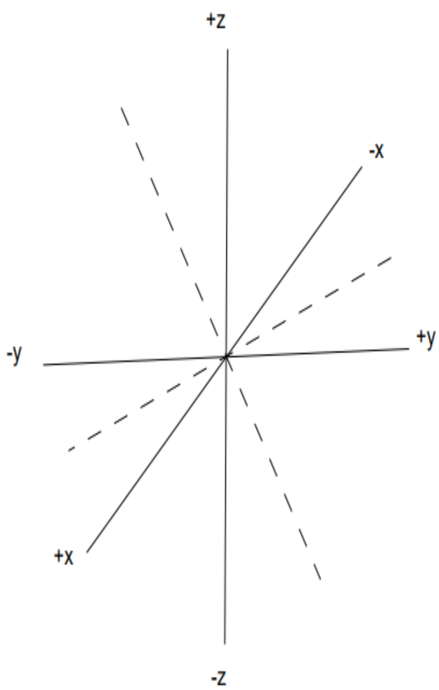


Figure 1 shows the angle bisector lines of the -y/+x, the -x/+y, the -x/-y and the +y/+y quadrants of a Cartesian coordinate system.

Fig. 2

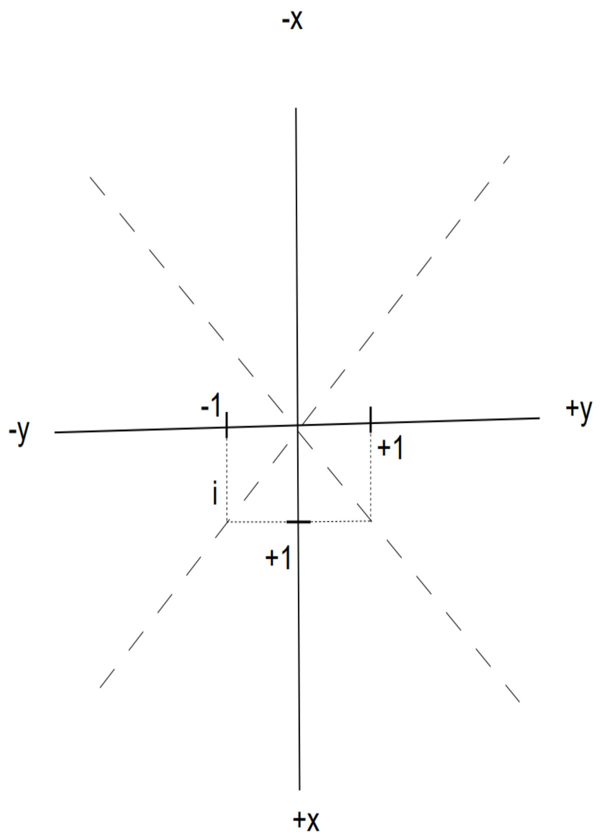


Figure 2 shows the same angle bisectors in top-view. A square in the -y/+x and the -x/+y quadrant has the oriented area of -1. The square root of this square, i. e. one side of it, is √(-1), which is commonly designated "i" in mathematics. The diagonal of the square is √(-2), which can be rewritten as √(2) √(-1) = √(2) i. Therefore, i can be viewed as a vector in the direction of the diagonal of the square -1y/+1x (positive i) or in the direction of the diagonal of a square -1x/1y (negative i). The calibration of this diagonal to |1| is the diagonal divided by √(2). The angle bisectors of the quadrants +x/+y and -x/-y, which are orthogonal to the imaginary angle bisectors, can in a similar manner be calibrated to the real numbers +1 and -1 by dividing the diagonals of the squares with area 1 by √2.

We thus get an orthogonal coordinate system that has the same properties as the Gaussian complex plane.

Fig. 3

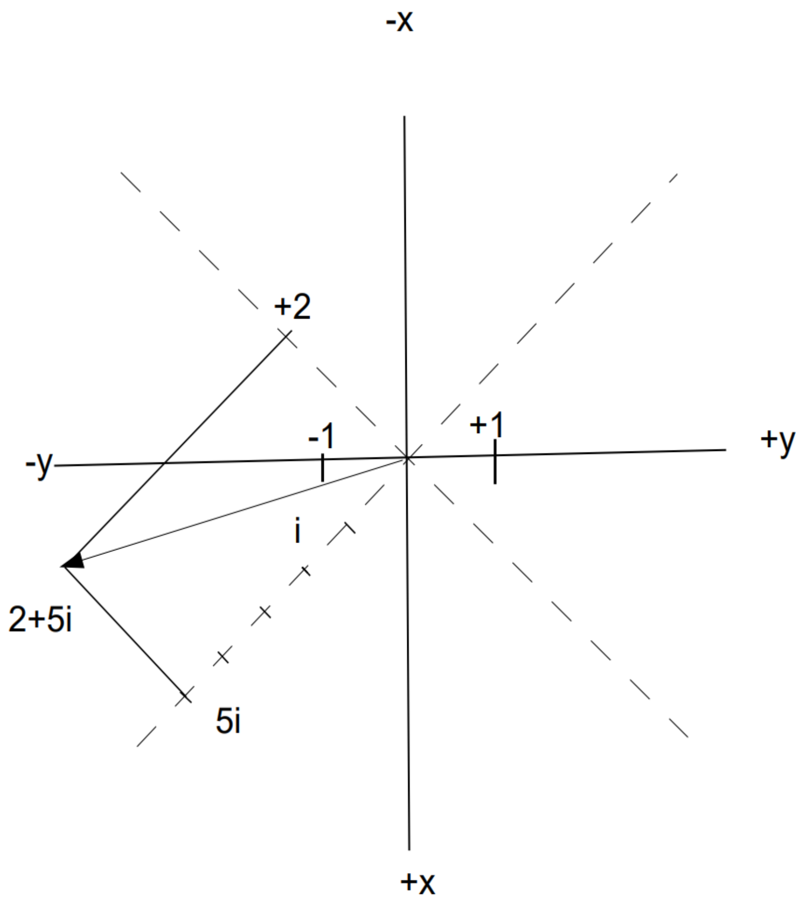


Figure 3 shows how complex numbers (e.g. 2 +5i) are incorporated into this plane. The coordinate of the imaginary part is plotted on the angle bisector of -y/x or -x/y, and the real part is plotted on the angle bisector of x/y or -y/-x. Then the perpendicular on these two points is erected, and the intersection is the complex number represented as a vector.

This way of incorporating the complex Gaussian plane of complex numbers into a 3D coordinate system has certain advantages. For example, the real and the imaginary part of parabolas and other second order polynomials can be visualized in a single 3D coordinate system. This is shown in figure 4.

Fig 4

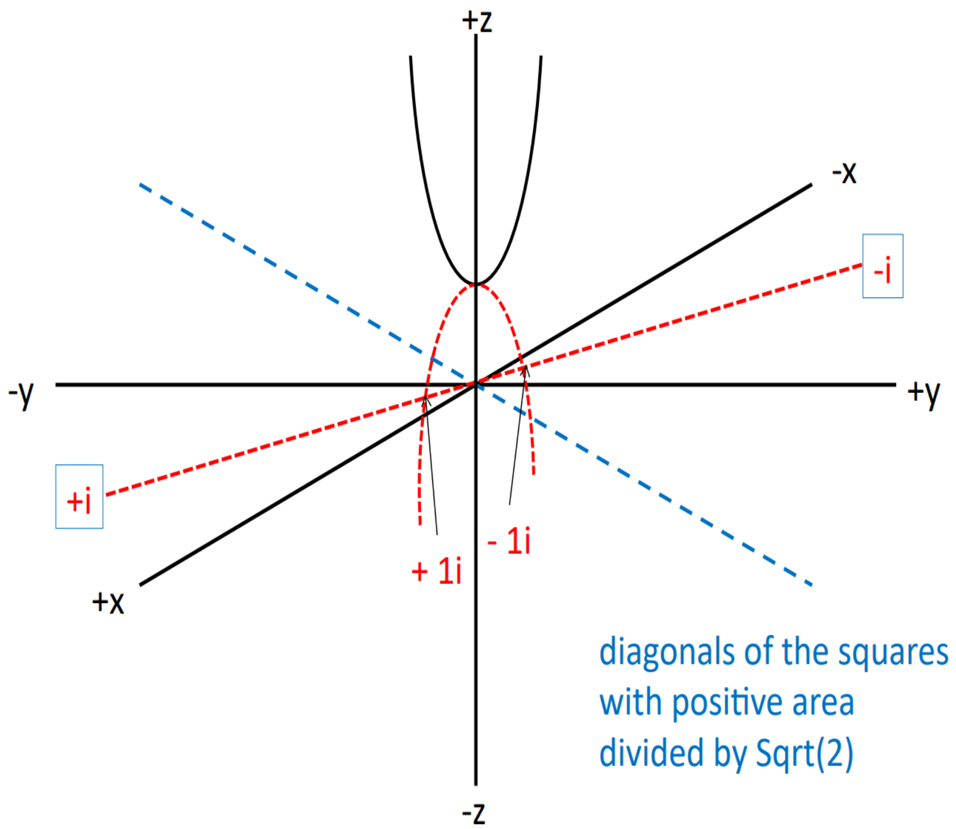
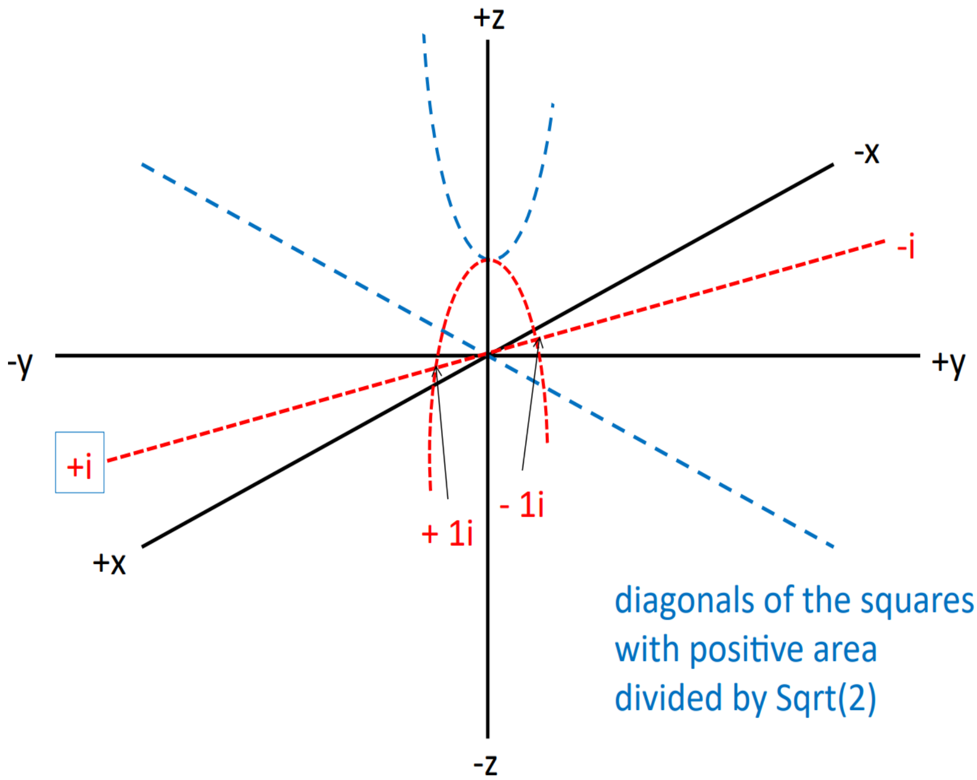


Figure 4 shows the diagram of the polynomial $F_1(x) = x^2 + 1$. The roots of this function are imaginary ($x_{1/2} = +/- \sqrt{(-1)} = +/- i$).

In figure 4 the parabola drawn in black is based on the coordinates -/+ x (usual presentation). Then a duplicate is rotated by 45° and thereafter turned upside down. This yields the imaginary parabola drawn in red is with the imaginary roots $-/+ \sqrt{(-1)} = -/+ i$. In this way one gets a two-fold parabola comprising the black and red parabola with a co-domain of $- \infty$ to $+ \infty$. In this way, three-dimensional complex numbers can be created, namely a two-dimensional complex number in the Gaussian number plane as shown in Fig. 3 forming an intersection point, to which a parallel line to the z-axis of any deliberate length is added as the third dimension. It should be mentioned that in certain cases, the lower part of the z-axis may also be imaginary instead of real. One then gets complex numbers with two imaginary parts and one real part.

Fig 5



In figure 5 the initial polynomial $F(x) = x^2 + 1$ shown in blue is drawn over the diagonals of the squares with positive area divided by √(2). In there, a rotation of 90° of the initial parabola and turning it upside down gives the imaginary parabola (drawn in red). Of course, the diagonals have exactly the same measure as the x, y, and z-axis.

Also polynomials with a linear member, such as $F_2(x) = x^2 + x + 1$, can be drawn in such a D3 coordinate system. Here, the apex is not any longer on the z-axis. For symmetry reasons, it is therefore best to draw the original polynomial over the diagonal of the positive squares, then rotate a duplicate by 90° and turn it upside down (unfortunately, my drawing tools are to poor to show this is a acceptable way). The complex roots of $F_2(x)$ are $x_{1/2} = ½ +/- √(3)i$. The apex of the parabola is at x= -1/2 and z= ¾. The value of z at x=0 is 1. When you rotate a duplicate as drawn above by 90° and turn it upside down, it can be seen that the intersecting points of the two branches of the imaginary parabola with the Gaussian number plane, as defined in Fig.3 , are ½ +/- √(3)i.