WHAT'S WRONG WITH THE LAW OF UNIVERSAL GRAVITATION?
What should this law actually look like?

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Abstract. *The evolution of the law of universal gravitation is shown from the verbal formula proposed by Newton in 1687 to the present day. The law $F_N \propto mM/r^2$ discovered by Newton was not an exact law of gravitation. Newton only indicated the proportional dependence of force on masses and did not attribute any numerical value to the gravitational force. In an unfinished form, Newton's law $F_N \propto mM/r^2$ existed for almost 200 years (!), until the constant G appeared in it. But even in its modern formulation, Newton's law $F = GMm/r^2$ gives only a part of the force of universal gravitation. Newton's law does not work at large distances. Newton's law is not applicable on the scale of the Universe. The formula for Newton's law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula for the law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies. Here it is shown that in addition to Newton's law there are two more laws of gravitation that remained undiscovered for over 300 years. A new law of gravitation is presented that describes the attraction of all bodies in the Universe. The obstacle to the discovery of this law of gravitation was the unsolved N-body problem. Here we present a solution to this problem.*

Keywords: *Newton's law; N-body problem; law of universal gravitation; parameters of the observable universe; dark matter; galaxy rotation curve; cosmological constant Λ; Pioneer anomaly.*

1. Introduction

Newton formulated his law of gravity in 1687 in verbal form. It was not an exact law of gravity. Newton only indicated the proportional dependence of force on masses and did not attribute any numerical value to the gravitational force. The symbolic form of the original law proposed by Newton is as follows:

$$F_N \propto \frac{mM}{r^2} \qquad (1)$$

Where: F_N is the gravitational force, m, M are the masses of bodies, r is the distance, $\propto$ is the proportionality sign.

As we can see, Newton did not present the world with an exact formula for the law of gravitation. In this unfinished form, Newton's law existed for almost 200 years. It turned into the exact equality $F_N = GmM/r^2$ only after the gravitational constant G was introduced into it [1 - 6]. This happened in several stages (Fig. 1).

J. A. M. Pereira in [1] showed the history of the appearance of the constant G. In 1803, S. D. Poisson presented the formula for the law of universal gravitation $F = f\,mM/r^2$ with the coefficient *f*, which later became the constant G [1, 2]. In 1873, A. Cornu and J. B. Baille also used the symbol (*f*) for the coupling constant in Newton's law of gravitation (1 - 4). The law of universal gravitation in the form familiar to us $F = GmM/r^2$ was presented by A. K ̈onig, F. Richarz, [5, 6], J. H. Poynting [1, 7].

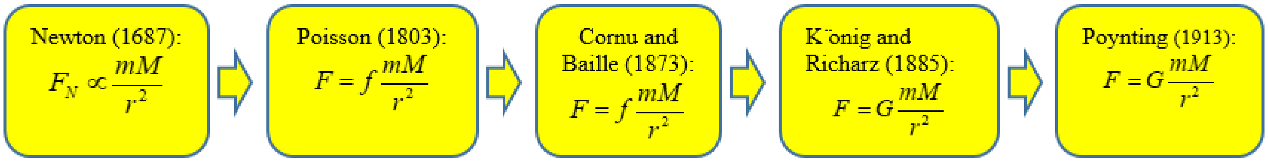


Fig.1 Step-by-step refinement of Newton's law of universal gravitation.

But even the refined Newton's law F = GMm/r^2 gives only a part of the force of universal gravitation. Newton's law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula for Newton's law F = GMm/r^2 "*does not see*" a significant part of the gravitational force. It describes gravitation only to one local source of attraction "M" and does not take into account that bodies simultaneously gravitate to all other bodies. For more than 300 years, it has not been possible to derive a formula for the law of gravitation that should describe gravitation to all N bodies in the Universe. The incompleteness of Newton's law of universal gravitation is indicated by significant discrepancies between observations and the predictions of the law. This applies to the shift in the perihelion of planets, to the rotation curves of galaxies, to the Pioneer anomaly [8 - 12]. The revealed gravitational anomalies in the dynamics of stars show that at large distances Newton's law is not fulfilled to a significant extent and has significant discrepancies with observations [10, 11].

The term "*law of universal gravitation*" implies taking into account all acting gravitational forces. Newton's law F = GMm/r^2 "does not see" all acting gravitational forces. A law of gravitation is needed that should show not only the force of gravitational interaction of two bodies, but also describe the attraction to all N bodies in the Universe. Such a law of gravitation should be sought beyond the framework of Newtonian dynamics.

2. The N-body problem and the problem of obtaining the equation of gravitational force for all N bodies in the Universe

Newton's refined law F = GMm/r^2 does not reflect the reality of gravity in the Universe. Newton's law does not work at large distances. The formula of the law includes two point masses "M" and "m". This law is applicable to the gravitational interaction of two bodies. It describes the attraction only to one local source of attraction. The real picture of gravity is different. Bodies simultaneously gravitate to all other bodies, and not just to one local source of attraction. There are a huge number of bodies in the Universe. But their parameters are not included in the formula F = GMm/r^2 .

Newton's law gives a good approximation for the gravitational problem of two bodies. Even if it were possible to solve the problem of 3, 4, 5, etc. bodies, the description of gravity would still be incomplete until the gravitational action of the entire Universe is taken into account. The problem of discovering the law of gravitational force for all N bodies in the Universe has an external resemblance to the N-body problem. Identification of the differential N-body problem and the problem of obtaining the equation of gravitational force for all N bodies in the Universe may indicate the futility of solving the problem. It is known that the N-body problem has no analytical solution [13]. As applied to the

Universe, the value of N is very large. Even numerical methods for solving it for large N are powerless. The absence of an analytical solution to the N-body problem can serve as an argument to justify the impossibility of obtaining an equation of gravitational force that takes into account the gravitation of a huge number of bodies in the Universe. The pairwise application of Newton's law to many bodies in order to obtain the equation of gravitational force for all N bodies in the Universe obviously has no analytical solution, just like the N-body problem. The problem is comparable in complexity to the N-body problem, since N is very large. Direct application of Newton's law to N bodies requires an additive representation of the total force as a sum of forces. This runs into the insurmountable problem of "*bad infinity*" (die Schlecht-Unendliche) [15].

Newton was convinced of the impossibility of obtaining an exact law of gravity that takes into account the gravitational action of many bodies. This is indicated by the text from Newton's manuscript of 1684 [14]: "... *the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the action of all these on each other. But to consider simultaneously all these causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind*."

Below we show that there is a solution and present a new law of cosmological force.

3. The integral problem of N bodies as a reduction of the differential problem of N pairwise forces to the problem of two bodies.

To find the gravitational force that takes into account the action of N bodies, there is no need to solve the differential problem of N pairwise forces. It is proposed to obtain the equation of the gravitational force for a system of N bodies instead of searching for a direct solution to the differential problem of N pairwise forces. To do this, it is necessary to move on to solving the integral problem for the system of N bodies. As a result, the differential problem of determining N forces from many pairs of bodies can be reduced to a two-body problem, in which the central body is represented by a system consisting of N bodies.

The system of N bodies in the integral gravitational problem is considered as the Universe with known parameters. As a result, the problem of obtaining the equation of force from the action of N bodies has a solution even for $N \to \infty$.

4. Inverse N-body problem.

Newton solved the two-body gravitational problem. The inverse two-body problem, namely, the problem of finding the law of gravitation based on the known trajectory of a body's motion, had not yet been formulated at that time. Such a problem appeared in the late 1870s. It was formulated by Bertrand J. [16-18]. The first and second Bertrand gravitational problems are known [16-18]. The first Bertrand problem was formulated for trajectories that are conic sections. The second Bertrand problem was formulated for trajectories that are closed curves. In the general case, for trajectories represented by algebraic curves, this problem is known as the Koenigs problem [16-18]. The solution to these problems yields two well-known laws: Hooke's law and Newton's law. The Bertrand and Koenigs problems are inverse two-body problems.

The problem inverse to the N-body problem is not represented in physics. Possibly, because the direct N-body problem has no solution. Here we show that the inverse N-body problem has a solution and leads to a new law of gravity. We formulate the inverse N-body problem and give its solution. This is a new problem of finding the law of gravitational force from known integral characteristics of the N-body system. The N-body system is considered as an integral object of gravitational interaction.

We propose the following formulation of the gravitational problem of the N-body system: "*Knowing the integral characteristics of the N-body system, find the law of the force of gravitational interaction of a body of mass m with the N-body system*".

The scheme for solving the gravitational problem of the N-body system is as follows:

1. N individual bodies are considered as a single object - as a system of N bodies.
2. The problem of N pairwise forces is reduced to the problem of two bodies.
3. The central body is a system of N bodies.
4. The Universe with known parameters can be considered as a system of N bodies.
5. Selecting an integral parameter that is a characteristic for the system of N bodies.
6. Obtaining an acceleration formula for the central body represented by the system of N bodies.
7. Obtaining an equation for a new law of gravity.
8. Obtaining a new formula for the law of universal gravitation.

If the Universe is a system of N bodies, then the problem has several solutions depending on the choice of the integral characteristic of the Universe (Mu, Ru, Λ, Tu): F = (mc^2)/Ru, F = GmMu/Ru^2, F = (mc^2)√Λ, F = mc/Tu. Of all the parameters of the Universe, the cosmological constant Λ is available for measurement. A practically applicable solution to the gravitational problem of the N-body system is the law of gravitational force of the form: F = (mc^2)√Λ.

5. The law of cosmological force.

The transition from the differential problem of N Newtonian pair forces to the integral problem of a system of N bodies gives a solution to the problem of gravitational interaction of N bodies. In the integral problem of N bodies, a holistic object is considered — the Universe consisting of N bodies. Thus, the problem of obtaining the equation of the total gravitational force is reduced to the two-body problem, in which the central body is represented by a system consisting of N bodies. To be precise, then (N-2) bodies. For the Universe, we can accept the condition: (N-2) ≈ N. It remains to solve the problem of choosing a parameter that uniquely characterizes the system of N bodies. The Universe, as a holistic object, has such an integral parameter — the cosmological constant Λ. This constant is an integral characteristic of the system of all bodies in the Universe. The next step is to find the acceleration for the central body represented by the system of N bodies. The constant Λ allows us to represent the cosmological acceleration Ao, which is caused by all bodies in the Universe, as:

$$\mathbf{A_0 = c^2\sqrt{\Lambda}} \qquad (2)$$

Accordingly, the gravitational force with which the system of N bodies acts on a test body of mass "m" is expressed by the formula $\mathbf{F_{Cos} = mA_0 = (mc^2)\sqrt{\Lambda}}$. Fig. 2 shows the law of cosmological force.

$$F_{Cos} = mc^2\sqrt{\Lambda}$$

Fig. 2. The law of cosmological force. Where: $\mathbf{F_{Cos}}$ is the cosmological force, **m** is the mass of the body, **c** is the speed of light in vacuum, **Λ** is the cosmological constant.

The unknown cosmological force is due to the gravitational effect of all bodies in the Universe on a test body of mass "m". Since all bodies in the Universe are distributed in space, the formula for the law of cosmological force does not include the inverse square law. This new law of gravitation reveals an unknown cosmological force that acts on any body in the Universe. This is the part of the gravitational force that Newton's law "does not see". This is the part of the gravitational force with which bodies simultaneously gravitate toward all other bodies. This is the part of the gravitational force that makes the law of universal gravitation complete.

Instead of the gravitational constant G, the law of cosmological force contains the cosmological constant Λ. The new law of gravity shows that any body of mass m experiences a cosmological force proportional to the mass of the body and the cosmological constant Λ. The law of the cosmological force operates beyond the applicability of Newton's law of gravity. It applies to the gravitational interaction in the Universe. The cosmological force has a linear dependence on the mass of the body and does not obey the inverse square law.

On small scales, the additional cosmological force is much smaller than the Newtonian force. For example, on Earth, F_{Cos} is ~10^10 times lessthan the Newtonian force. As we can see, within the solar system, Newton's law of gravity has a highaccuracy. At small distances, the main part of the universal gravitational force is the Newtonian force.

On the scale of the Universe, the cosmological force is enormous. At large distances, it exceeds the Newtonian force. At large distances, the main part of the force of universal gravitation is the cosmological force F_{Cos}.

The study of the equation of the new law of the cosmological force shows that the value of the cosmological force in the limit is equal to the Planck force:

$$\lim_{m \to M_U} F_{Cos} = \lim_{m \to M_U} mc^2\sqrt{\Lambda} = 1.21027 \bullet 10^{44}\, N = \frac{c^4}{G} \qquad (3)$$

The theoretical limit of the cosmological force at m → M_U reaches the enormous value c^4/G = 1.21027 x 10^{44} N.

6. New formula for the law of universal gravitation.

The completed law of universal gravitation can be represented as a combination of the force of gravitational interaction of two bodies and the cosmological force. Based on the additivity of gravitational forces, the new formula for the law of universal gravitation (Fig. 3) combines Newton's law $F = GMm/r^2$ and the law of cosmological force F_{Cos}=(mc^2)√Λ.

$$F_U = G\frac{mM}{r^2} + mc^2\sqrt{\Lambda}$$

Fig. 3. New formula of the law of universal gravitation.

The force of universal gravitation is represented by the vector sum of two forces: the Newtonian gravitational force F_N and the cosmological gravitational force F_{Cos}:

$$\vec{F}_U = \vec{F}_N + \vec{F}_{Cos} \qquad (4)$$

The value of the resulting force of universal gravitation is in the range of values from F_U = GmM/r^2 - (mc^2)√Λ to F_U = GmM/r^2 + (mc^2)√Λ (Fig. 4).

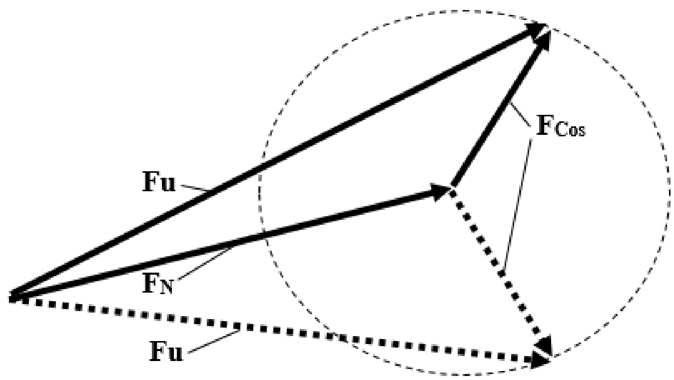


Fig. 4. The force of universal gravitation F_U as a vector sum of two forces: F_N and F_{Cos}.

7. Hooke-Kepler's law of gravity.

The force of gravitational interaction between two bodies was represented by just one simple formula: Newton's law F_N=GmM/r^2. At the same time, Newton's formula is not the only possible formula for the law of gravitational interaction between two bodies. The law of gravitational interaction between two bodies can be represented without the constant G (Fig. 5):

$$F_{H-K} = \frac{mR^3}{T^2 r^2}$$

Fig. 5. Hooke-Kepler's law of gravity. Where: m is the mass of the body, R and T are orbit parameters, r is the distance [19].

The Hooke-Kepler law does not include the coefficient 4π2. This constant appears in the special case of a circular orbit. Real orbits are not circular. The Hooke-Kepler law includes the parameters of an elliptical orbit and the inverse square law. This perfect law of gravitation was hinted at by Robert Hooke in his correspondence with Newton. Robert Hooke pointed out that the law of gravitation must take into account the elliptical orbits of the planets and the inverse square law.

The formula includes the Kepler ratio R^3/T^2 and the inverse square law. The Hooke-Kepler's law of gravitation does not include the central mass. The Hooke-Kepler's law is an exact and complete law of gravitational interaction between two bodies.

Using the Hooke-Kepler's law, the equivalent formula for the law of universal gravitation would be:

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda} \qquad (5)$$

8. Two equivalent formulas of the law of universal gravitation.

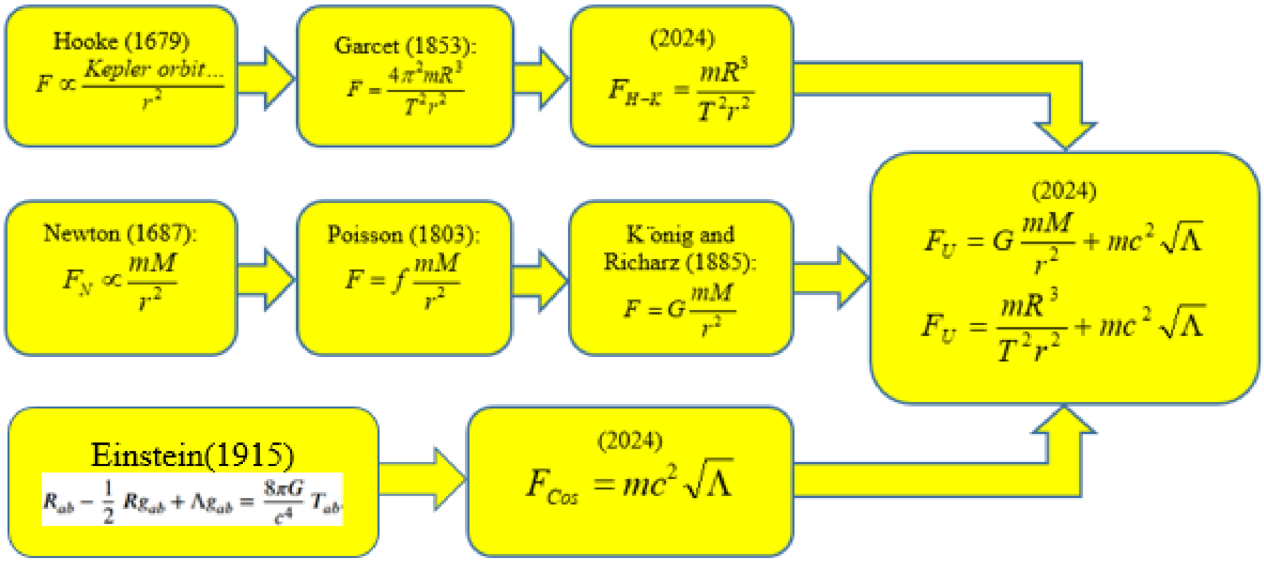
Newton's law $F = GMm/r^2$ does not give a complete description of the gravitational interaction. The law sees the force of gravity of two bodies, but does not take into account that the bodies simultaneously gravitate to all other bodies. Hooke-Kepler's law $F = mR^3/T^2r^2$ also sees the force of gravity of two bodies, but does not take into account that the bodies simultaneously gravitate to all other bodies. The missing force is given by the law of cosmological force: $F_{Cos} = mc^2\sqrt{\Lambda}$. The total force of universal gravitation is given by two equivalent formulas (Fig. 6):

$$F_U = G\frac{mM}{r^2} + mc^2\sqrt{\Lambda}$$

$$F_U = \frac{mR^3}{T^2 r^2} + mc^2\sqrt{\Lambda}$$

Fig. 6. Two equivalent formulas of the completed law of universal gravitation.

The first term in the formulas is the gravitational force of two bodies. The second term is the force of gravitational interaction with all N bodies in the Universe. As we can see, the law of universal gravitation can be represented both with the constant G and without this constant. The law of universal gravitation turned out to be more complex than Newton's law. Newton's law is not enough for a complete description of gravity. Three particular laws are needed: $F = GMm/r^2$, $F_{Cos} = mc^2\sqrt{\Lambda}$, $F_{H\text{-}K} = mR^3/T^2r^2$, which are combined into two equivalent formulas of the law of universal gravitation (Fig. 7).



.Fig. 7. Three laws of gravity as necessary and sufficient "components" of the law of universal gravitation.

9. The long road to the complete law of universal gravitation.

The road to the complete law of universal gravitation turned out to be very long (Fig. 7). More than 300 years ago, Newton's law was discovered in the form of a proportional dependence of force on two masses $F_N \propto mM/r$^2. Newton's law $F_N \propto mM/r$^2 turned into an exact law of gravitational interaction of two bodies many years later with the appearance of the gravitational constant ($F = fMm/r^2$ (1803), $F = fMm/r^2$ (1873), $F = GMm/r^2$ (1885)). But even in the modern formulation, Newton's law F = GMm/r^2 gives only a part of the force of universal gravitation.

Since 1853, Newton's law has not been the only law of gravitation. There is an alternative law of gravitation, which does not include the gravitational constant G. This alternative law of gravitation is less well known. In 1853, H. Garcet presented Newton's gravitational force as: $\boldsymbol{F} = \boldsymbol{4\pi^2}(R^3/\boldsymbol{T^2})(\boldsymbol{m/r^2})$ [1, 20]. This formula directly follows from Kepler's third law. This formula for the law of gravitation remained unnoticed by scientists for a long time.

H. Garcet was a professor of mathematics in Paris who published "Leçons nouvelles de cosmographie", a textbook on astronomy. H. Garcet is a cousin of Jules Verne. Jules Verne used his cousin H. Garcet's knowledge of astronomy when writing his books [21].

The law of gravitation proposed by H. Garcet differed favorably from Newton's law of gravitation. The alternative law of gravity $\boldsymbol{F} = \boldsymbol{4\pi^2}(R^3/\boldsymbol{T^2})(\boldsymbol{m/r^2})$ does not include the gravitational constant G and the central mass M. It includes the parameters of the Keplerian orbit R^3 and $\boldsymbol{T^2}$. This is its great advantage. Distances and periods are known from observations with great accuracy. This cannot be said about the masses that are included in Newton's law. H. Garcet obtained this formula for the special case of a circular orbit. This is indicated by the coefficient $\boldsymbol{4\pi^2}$.

In essence, H. Garcet proposed a more accurate law of gravity than Newton's law of gravity back in 1853. At that time, Newton's law was presented as a verbal formula. A revised Newton's law with a known value of the constant G appeared later in 1873.

In [19] the alternative law of gravity is presented as: $F_{H\text{-}K} = mR^3/T^2r^2$. I call this law of gravity the Hooke-Kepler law. The first verbal formulation of this alternative law of gravity appeared in the correspondence between Robert Hooke and Newton back in 1679. This happened 8 years before Newton discovered the law of gravity. Robert Hooke indicated in his letter that the law of gravity should take into account the elliptical orbits of the planets and the inverse square law [19]. Robert Hooke took Kepler's laws as a basis. It is known that Kepler's laws do not include masses. Contrary to Hooke's proposal, Newton introduced mass into his law of gravity. As a result, instead of the exact law of gravity $F = mR^3/T^2r^2$, Newton proposed an approximate proportionality law $F_N \propto mM/r^\wedge 2$. What prevented Newton from using Hooke's suggestion and giving the world an accurate law of gravity in the form of $F = mR^3/T^2r^2$? This remains a mystery. At that time, both Kepler's laws and the inverse square law were known, and there was even a hint from Hooke. In reality, the opportunity was missed, back in 1687, to give the world an accurate law of gravity. Even today, the Hooke-Kepler law $F_{H\text{-}K} = mR^3/T^2r^2$ is a more perfect law of gravity than Newton's law $F = GMm/r^2$. This is due to the unsolved problem of obtaining a more accurate value of the gravitational constant G ($G = 6.674\ 30(15) \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$). But even two laws of gravity ($F = mR^3/T^2r^2$ and $F = GMm/r^2$) are not enough for a complete law of universal gravitation. The formula $F = GMm/r^2$ and the formula $F = mR^3/T^2r^2$ "see" the gravitational force of two bodies, but do not take into account the force of gravitational interaction of all N bodies in the Universe. For a complete law of universal gravitation, the third law of gravitational interaction is needed. The third law of gravitational interaction gives the force of gravitational interaction caused by all N bodies in the Universe. This additional force is "not seen" by Newton's law. This additional force is "not seen" by Hooke-Kepler's law. The unknown additional force is given by the law of cosmological force: $F_{Cos} = mc^2\sqrt{\Lambda}$.

Fig. 8 shows the long path from Newton's law of universal gravitation to the completed law of universal gravitation.

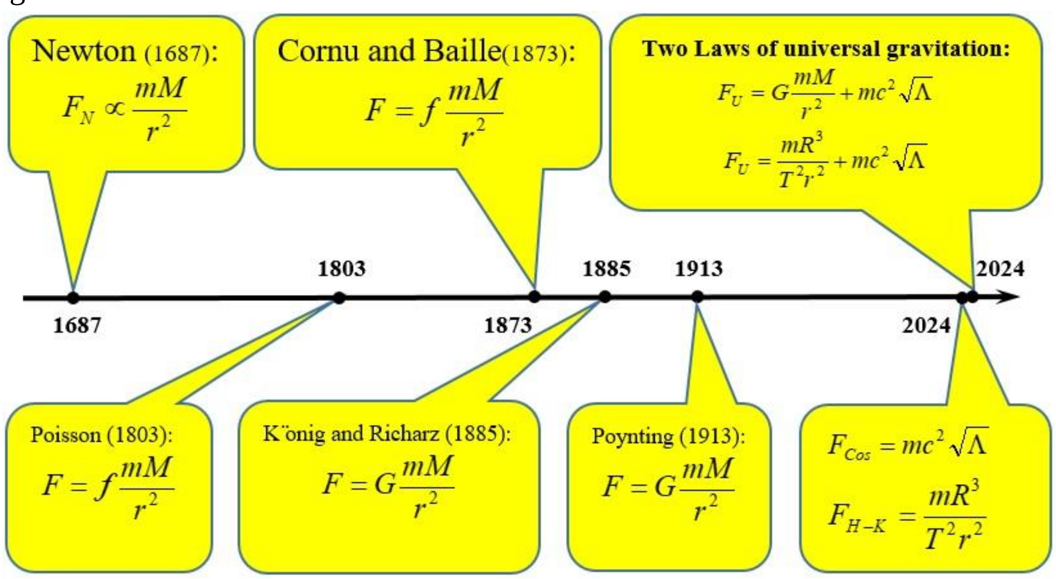


Fig. 8. Evolution of the law of universal gravitation from Newton's verbal formula to two equivalent formulas of the law of universal gravitation.

10. Laws of Gravity

It is a common belief in physics that the gravitational force can only be represented by Newton's law. Here we have shown that this is not the case. The complete "set" of physical laws for describing gravity is shown in Fig. 9.

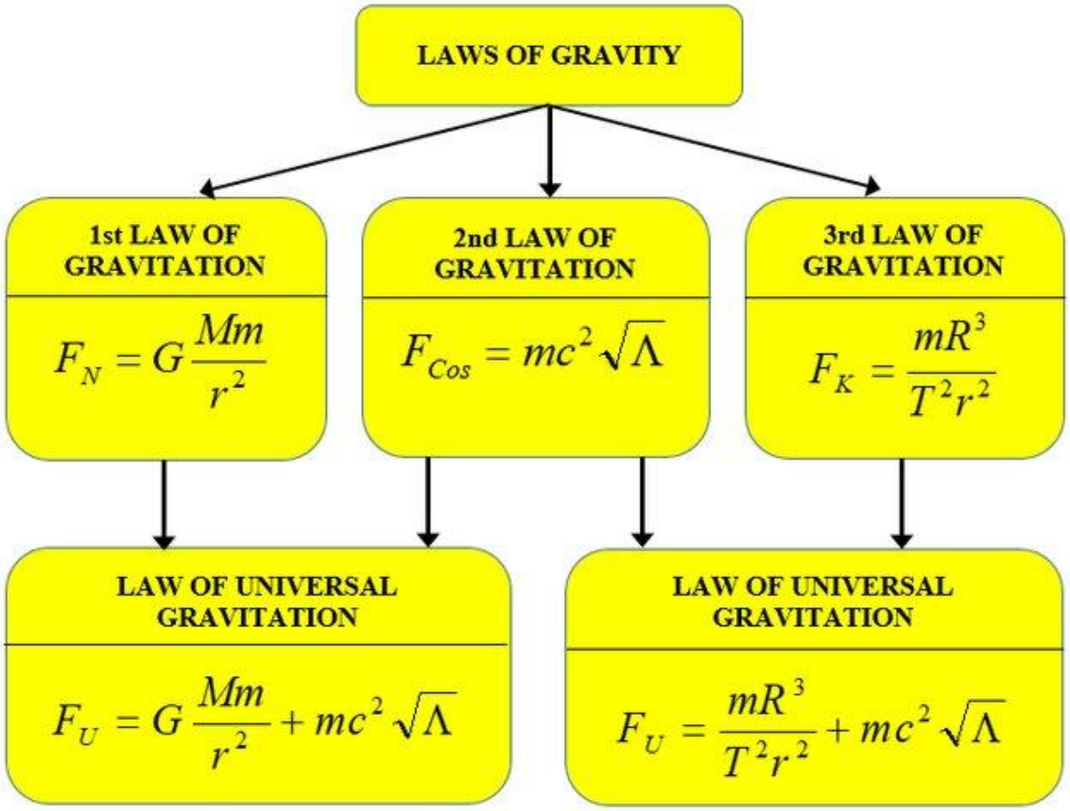


Fig. 9. Laws of gravitation for a complete description of gravitational interaction in the Universe.

The law of universal gravitation turned out to be more complex than Newton thought. The law of universal gravitation can be represented by two equivalent formulas: $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$, $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$ (fig. 9). Newton's law of gravitation $F = GMm/r^2$ is included as a component in only one equivalent formula of the law of universal gravitation. The other equivalent formula of the law of universal gravitation includes the Hooke-Kepler law of gravitation $F_{H\text{-}K} = mR^3/T^2r^2$ as a component. The law of cosmological force $F_{Cos}=mc^2\sqrt{\Lambda}$ completes both equivalent formulas of the law of universal gravitation.

11. Cosmological constant Λ in the law of universal gravitation

The equivalent formulas of the law of universal gravitation include the cosmological constant Λ. The cosmological constant Λ is an integral characteristic of the N-body system, where the N-body system is represented by all the bodies of the Universe. The value of Λ can be obtained with high accuracy from systems of cosmological equations (Fig. 10). This is possible due to coincidences of large Weyl-Eddington-Dirac numbers. Combinations of the parameters of the Universe and fundamental physical constants lead to large Weyl-Eddington-Dirac numbers. These combinations give a large number of cosmological equations. From cosmological equations, it is easy to compose systems of algebraic equations that contain the cosmological constant Λ. In Fig. 10, as an example, 4 systems of algebraic equations of the Universe are given, containing the cosmological constant Λ.

$$\begin{cases} G\hbar/r_e^3 A_0 = c^1 \\ 1/T_U^2\Lambda = c^2 \\ M_U A_0 G = c^4 \\ M_U R_U A_0 G/T_U = c^5 \\ M_U R_U A_0^2 G = c^6 \end{cases}$$

a)

$$\begin{cases} M_U \Lambda G = A_0 \\ c^5 r_e^3/M_U G^2 = \hbar \\ M_U G T_u^2 = R_u^3 \\ G M_U = c^2 R_U \\ M_U \Lambda G T_u^2 = R_U \end{cases}$$

b)

$$\begin{cases} \dfrac{c^5 r_e^3}{M_U G^2} = \hbar \\ M_U \Lambda c r_e^3 = \hbar \\ \dfrac{R_U \Lambda c^3 r_e^3}{G} = \hbar \\ \dfrac{c r_e^3 A_0}{G} = \hbar \\ \dfrac{M_U r_e^3}{R_U T_U} = \hbar \end{cases}$$

c)

$$\begin{cases} \dfrac{M_U G^2 m_e}{c^4 r_e^2} = \alpha, \\ \dfrac{m_e}{M_U \Lambda r_e^2} = \alpha, \\ \dfrac{G m_e}{R_U \Lambda c^2 r_e^2} = \alpha, \\ \dfrac{G m_e}{r_e^2 A_0} = \alpha, \\ \dfrac{G m_e T_U}{r_e^2 c} = \alpha. \end{cases}$$

d)

Fig. 10. Systems of cosmological equations for calculating the parameters of the observable Universe. Where: **α** is the fine structure constant, **ħ** is Planck's constant, M_U is the mass of the observable Universe, G is Newton's gravitational constant, Λ is the cosmological constant, R_U is the radius of the observable Universe, T_U is time, A_0 is the cosmological acceleration, $\mathbf{r_e}$ is the classical radius of the electron; **c** is the speed of light in a vacuum; $\mathbf{m_e}$ is the mass of the electron.

All systems of cosmological equations (Fig. 10) give the same value of the cosmological constant Λ and other parameters of the Universe (Fig. 11). Here we introduce a new important parameter of the Universe - the cosmological acceleration A_0. The obtained value of the cosmological acceleration (A_0 = 10.4922...x10^{-10} m/s^2) turned out to be very close to the prediction of the MOND theory [22, 23].

$$M_U = 1.15348... \bullet 10^{53}\, kg$$
$$R_U = 0.856594... \bullet 10^{26}\, m$$
$$T_U = 2.85729... \bullet 10^{17}\, s$$
$$\Lambda = 1.36285... \bullet 10^{-52}\, m^{-2}$$
$$A_0 = 10.4922... \bullet 10^{-10}\, m/s^2$$

Fig. 11. Values of the parameters of the Universe obtained from the systems of cosmological equations.

12. How Newton's law of gravity (the law of two-body gravity) came to be called the Law of Universal Gravitation.

The first verbal formula for the law of universal gravitation in the history of science was given by Robert Hooke in a letter to Newton [19]. This occurred in 1679, seven years before the publication of the Principia.

Even before 1679, Hooke spoke of "*universal attraction*." He understood gravity as the general attraction of everything to everything else. He distinguished between the local gravity of two bodies and universal attraction. Hooke believed that the force of universal gravitation was "*encoded*" in the orbital motion of the planets. Hooke shared this knowledge with Newton in 1679. Newton did not accept Hooke's hint [19]. Newton did not use orbital parameters in his law of gravitation; he was seeking a solution to the two-body gravitational problem.

In his letter to Newton, Hooke expected Newton to formulate a universal law of gravitation that took into account the orbital motion of bodies. But instead of a law of universal gravitation based on the orbital motion of bodies, Newton proposed a law of two-body gravitation based on the mass of the gravitating body. The two-body law of gravitation did not demonstrate the universal gravitational force that actually acts on bodies.

Hooke first discussed gravity as a universal force that gives celestial bodies their spherical shape in 1665 in his book Micrographia. In Micrographia, he does not yet use the precise formulation "*universal attraction*," but he lays the foundation for this idea by discussing gravity as a universal property of matter to aggregate into spheres. In his "Observation VI", Hooke makes a bold proposition for his time: "*...Attraction is common not only to the Earth, Sun, Moon, and Planets, but to every minute body in the Universe*..."

In 1674, in his work "*An Attempt to Prove the Motion of the Earth from Observations*," Hooke already wrote about "*universal attraction*." He asserted that all celestial bodies exert a force of attraction toward their centers and that they also attract all other celestial bodies. Instead of "*universal gravitation*," he used the term "*universal attraction*."

Newton found a solution to the two-body gravity problem that only approximately describes the actual gravity of the universe. In the universe, there is no isolated gravitational interaction between two bodies. All bodies in the universe participate in gravitational interaction. This discovery is due to Robert Hooke. The gravitational force of two bodies always "adds" to the gravitational forces of other bodies in the universe. The law $F = GmM/r^2$ "doesn't see" these "additions." The law $F = GmM/r^2$ is applicable only on small scales, where the contribution of the force F to the universal gravitational attraction is greatest.

To Newton's credit, he did not call his proportional relationship $F_N \propto mM/r^2$ the law of universal gravitational attraction. How then did the laws of two-body gravity ($F_N \propto mM/r^2$, $F = GmM/r^2$), which do not take into account the gravity of all bodies in the Universe, come to be called the laws of universal gravitation?

Newton didn't do this. Others did. Roger Cotes, the editor of the second edition of Philosophiae Naturalis Principia Mathematica (1713), did. In the preface, he emphasizes the universality of Newton's law of universal gravitation. Voltaire, in his Philosophical Letters (1734), explained Newton's ideas using Hooke's concept of "*universal attraction*." Voltaire failed to mention that "*universal attraction*" is Robert Hooke's term, and it refers not to the law of universal gravitation between two bodies, but to the attraction between all bodies in the universe. Thus, Voltaire also authored the historical injustice that kept Hooke's contribution in the shadows for over three centuries.

Thus, due to Roger Cotes's words, the law of two-body gravitation, $F_N \propto mM/r^2$, and later $F = GmM/r^2$, was unjustifiably called the law of universal gravitation. However, the true law of universal gravitation, which takes into account the fact that every body in the universe attracts every other body, as Robert Hooke claimed, was never discovered.

Thus, by unjustifiably calling the law of two-body gravitation, $F = GmM/r^2$, the law of universal gravitation, we are not repeating Newton's words, but rather the words of Roger Cotes, who was a great admirer of Newton.

13. Conclusion.

The law of universal gravitation has come a long way from Newton's verbal formula to the full formula of the law of universal gravitation. For more than 300 years, the gravitational force was represented by only one formula $F = GMm/r^2$. This formula gives only a part of the force of universal gravitation. The formula $F = GMm/r^2$ is quite accurate on the scale of the Solar System. But it is not applicable on the scale of the Universe. The formula of Newton's law $F = GMm/r^2$ shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. The formula of Newton's law describes gravitation only to one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies.

The differential problem of N paired Newtonian forces creates the illusion of the impossibility of obtaining an exact law of gravitation describing the gravitation of all bodies in the Universe. It is shown here that in order to find the gravitational force taking into account the action of all bodies in the Universe, it is not necessary to solve the differential problem of N paired Newtonian forces. The problem of N-paired Newtonian forces is reduced to the two-body problem, where the central body is represented by a system of N bodies.

The transition from the differential problem of N-paired Newtonian forces to the integral problem for a system of N bodies leads to a new law of gravitation: $F_{Cos}=(mc\hat{}2)\sqrt{\Lambda}$. The complete law of universal gravitation combines Newton's law $F = GMm/r^2$ and the law of the cosmological force $F_{Cos}=(mc\hat{}2)\sqrt{\Lambda}$. Newton's law is included as a component in the complete law of universal gravitation.

It is shown here for the first time that in addition to Newton's law, there are two new laws of gravitation. One new law of gravitation follows directly from Kepler's law. This is the new law of gravitational interaction of two bodies. It does not contain the constant G and the central mass: $F = mR^3/T^2r^2$. Another new law of gravitation $F_{Cos} = mc^2\sqrt{\Lambda}$ gives an additional force of gravitational interaction with all bodies in the Universe. The complete set of laws of gravitation leads to two equivalent formulas of the law of universal gravitation $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$, $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$.

The new formulas for the law of universal gravitation give a complete and consistent description of the gravitational interaction in the Universe. As a result, many problems of cosmology are solved, including the problem of the cosmological constant Λ and the problem of dark matter [16].

14. Conclusions

1. The law discovered by Newton $F_N \propto mM/r\hat{}2$ was not the exact law of gravity. Newton only pointed out the proportionality and did not attribute any numerical value to the gravitational force.

2. Newton's law $F = GMm/r^2$, refined almost 200 years later, does not give the full force of universal gravitation. The law shows the force of gravitational interaction of only two bodies out of all N bodies in the Universe. It describes the attraction to only one local source of attraction and does not take into account that bodies simultaneously gravitate to all other bodies.

3. The similarity between the N-body problem and the problem of obtaining the total force of universal gravitation may create the illusion that it is impossible to obtain an exact law of gravitation describing the gravitation to all N bodies in the Universe.

4. To find the gravitational force taking into account the action of N bodies, there is no need to solve the differential problem of N Newtonian forces of pairwise interaction. To do this, it is necessary to move on to solving the integral problem, in which N individual bodies are replaced by one system consisting of N bodies.

5. The differential problem of obtaining N Newtonian forces from pairwise interaction can be reduced to the two-body problem, in which the central body is represented by one system of N bodies.

6. The integral parameter of the system of N bodies as applied to the Universe is the cosmological constant Λ. This constant is a characteristic of the system that includes all the bodies of the Universe.

7. The integral problem for the system of N bodies leads to a new law of gravitation: $F_{Cos}=mc^2\sqrt{\Lambda}$. This is the part of the gravitational force that Newton's law "does not see". This is the part of the gravitational force with which bodies simultaneously gravitate to all other bodies in the Universe. This is the part of the gravitational force that makes the law of universal gravitation complete.

8. The law of the cosmological force $F_{Cos}=mc^2\sqrt{\Lambda}$ shows that any body of mass m is acted upon by a cosmological force proportional to the mass of the body and the cosmological constant Λ. The cosmological force is linearly dependent on the mass of the body and does not obey the inverse square law.

9. Based on the additivity of gravitational forces, the complete law of universal gravitation combines Newton's law $F = GMm/r^2$ and the law of the cosmological force $F_{Cos} = mc^2\sqrt{\Lambda}$. The formula for the complete law of universal gravitation $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$ includes two constants: the gravitational constant G and the cosmological constant Λ.

10. On small scales, the additional cosmological force is much less than the Newtonian force. On small scales, the Newtonian force makes up the bulk of the force of universal gravitation. On large scales (galaxies...) the cosmological force exceeds the Newtonian force. On large scales the main part of the universal gravitational force is the cosmological force F_{Cos}.

11. The complete law of universal gravitation is represented by two equivalent formulas: $F_U = GMm/r^2+mc^2\sqrt{\Lambda}$, $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$

12. The law of universal gravitation, presented by two equivalent formulas ($F_U = GMm/r^2+mc^2\sqrt{\Lambda}$, $F_U = mR^3/T^2r^2+mc^2\sqrt{\Lambda}$), gives a complete and consistent description of gravitational interaction in the Universe.

13. With the advent of the law of universal gravitation, which takes into account both the gravity of two bodies and the gravity of the universe, the following problems of astrophysics and cosmology are solved:

- the rotation curve of the Galaxies;

- the Pioneer anomaly;
- the problem of dark matter;
- the problem of the cosmological constant Λ ($\Lambda = 1.36285... \cdot 10^{-52}$ m^{-2});
- the value of the cosmological acceleration A_0 ($A_0 = 10.4922... \cdot 10^{-10}$ m/s^2).

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