

The interval containing n primes

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Abstact This research aims at coming up with some formulation of the interval containing a given number of primes using the prime number theorem

keywords Prime number theorem; Interval containing a given number of primes

Introduction

The number of primes in an interval can be estimated using theorems and conjectures. The Bertrands postulate postulates states that there is a prime in the interval $(n, 2n) || n > 1$ where n is an integer. We have several conjectures

Key question

What is the solution of the equation below

$$x = x \ln x \quad (1)$$

To get a general idea let us work out the solution of the equation below using a calculator.

$$x = 3 \ln x$$

and we obtain the solution:

$(x_1; x_2) = (1.8571839, 4.5364037)$ This interval contains nearly three primes. Again we can calculate the equation

$$x = 5 \ln x$$

We obtain the solution $(x_1, x_2) = (1.2958555, 12.713207)$ This interval contains 5 primes. Again let us solve the equation:

$$x = 6 \ln x$$

$(x_1, x_2) = (1.2268887, 16.998887)$ This interval contains 6 primes. Again let us solve the equation:

$$x = 7 \ln x$$

we get the solution

$(x_1, x_2) = (1.1843482, 21.464949)$ This interval contains 8 primes.

Interval containing n primes

We note that the interval containing n primes is

$$(1, n \ln n + 4) \quad (2)$$

This is because

$$\frac{n \ln n + 4}{\ln(n \ln n + 4)} \approx n \quad (3)$$

Thus:

$$(10000000000 \ln(10000000000) + 4) / \ln((10000000000 \ln(10000000000)) + 4) = 8801100308.37805$$

This result is not good enough The gap containing n primes will need solving the equation

$$\frac{n \ln n + x_n}{\ln(n \ln n + x_n)} = n \quad (4)$$

In which case the interval then is

$$(1, n \ln n + x_n) \quad (5)$$

Thus for the case $n = 5$, $x_5 = 4.666$ $(1, 5 \ln 5 + 4.666) = (1, 12.713)$. The interval actually contains 5 primes.

Summary and conclusion

It is possible to come up with a method giving for determining some interval containing a given number of primes. A method for determining the interval containing a given number of primes has been achieved.

References

- [1]. Lauren[^]iu Panaitopol, Intervals containing prime numbers. NNTDM 7 (2001), 4,111-114