

To Demonstrate Goldbach's Strong Conjecture by Deduction using $4x \pm 1$ Equations in Loops and Gaps of 4.

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Abstract.

After defining the writing of even numbers with equations $4x \pm 1$; the article shows that odd numbers obey loops of 4 numbers to infinity that obey the equations $4x \pm 1$. Each odd number in the loop can be prime (P) or composite (C), but for an even number E to be $E = P + P'$ such that $P' > P$ and such that $P' > E/2$ and $P < E/2$, P and P' must belong to two loops symmetrical with respect to E/2 and occupy the same positions in them and specific unit digits. The article shows that the three possible sums of an even number E are $E = C + C'$; or $E = P + C$; or $E = P + P'$ (5 is composite and P is prime). The article demonstrates that $E = C + C' \leftrightarrow E = C + P \leftrightarrow E = P + P'$; and that one sum can be converted into another by subtracting and adding gaps of $4n$ from or to the two terms of addition. This is a deductive demonstration of Goldbach's strong conjecture shown here for the first time.

Keywords. Goldbach' strong conjecture. Primes. Composites. Primes $4x \pm 1$. Gaps of 4. Addition. Subtraction. Evens. Odds. Loops.

Abbreviations. GSC : Goldbach's strong conjecture. P : primes. C : composite.

Introduction.

Using $6x \pm 1$ equations there are three types of evens including $6x$; $6x + 2$ and $6x + 4$. Using the same equations, we can convert an even $6x$ in sum of two primes one of which is $6x - 1$ and the other is $6x + 1$ while $6x + 2$ is a sum of two primes $6x + 1$. Finally, $6x + 4 = 6x - 2$ is a sum of two primes $6x - 1$ [1-2]. However there are only two evens with $4x \pm 1$ equations including $4x$ and $4x + 2$ which reduces the number of cases and allows to investigate in a new way the Goldbach's strong conjecture stating that every even denoted $E \geq 4$ is a sum of two primes. The aim of this article is to demonstrate Goldbach's conjecture using the $4x \pm 1$ equations; the gaps between the $4x \pm 1$ primes; and predict the positions of the equidistant $4x \pm 1$ primes, which are the only ones that count for verifying this conjecture by calculation.

Results

1. The subsets of even and primes numbers (P)

By the equation $6x \pm 1$ there are three subsets of even numbers denoted Se such that $Se1 = \{6x\}$; $Se2 = \{6x + 2\}$ and $Se3 = \{6x + 4\}$. On the other hand, there are two subsets of even numbers denoted Sf such that $Sf1 = \{4x\}$ and $Sf2 = \{4x + 2\}$ (see also **Figure 1**). P are divided into two subsets Spn including $Spn1 = \{4x - 1\}$ and $Spn2 = \{4x + 1\}$ (**Table 1**).

Table 1 : The two subsets $Spn1$ and $Spn2$ of Prime Numbers (P) by the equations $4x \pm 1$.

Spn1	Spn2
$\{4x - 1\}$	$\{4x + 1\}$
3	1
7	5
11	9
15	13
19	17
23	21
27	25
31	29
35	33
39	37
43	41
47	45
51	49
55	53
59	57
63	61
67	65
71	69
75	73
79	77
83	81
87	85
91	89
95	93
99	97
$\rightarrow +\infty$	$\rightarrow +\infty$

2. Other subsets of integers

Let SO_{3n} be the subset of odd numbers that are multiples of 3 and therefore

$$SO_{3n} = \{3, 9, 15, 21, 27, 33, 39, \dots \rightarrow +\infty\}.$$

We can see that the numbers $4x + 1$ and $4x - 1$ alternate in the subset SO_{3n} . For example $3 + 6 = 9 = 4x + 1$; $9 + 6 = 15 = 4x - 1$; $15 + 6 = 21 = 4x + 1$; $21 + 6 = 27 = 4x - 1$ to infinity.

Therefore we have $SO_{3n}(4x + 1)$ and $SO_{3n}(4x - 1)$.

Let SO_{pc} be the subset of all P and composite *odd* numbers (C) that are not $3n \rightarrow$

$$\{SO_{3n}\} \cap \{SO_{pc}\} = \emptyset.$$

Let Se_{2n} be the subset of all even integers $2n$ such that $n > 0$ so $Se_{2n} = \{2, 4, 6, 8, 10, 12, \dots \rightarrow +\infty\}$.

A natural number denoted $n \geq 3$ is either even or odd. If even, it is $4x$ or $4x + 2$. If odd, it is $4x + 1$ or $4x + 3$, knowing that $4x + 3 \leftrightarrow 4x - 1$.

3. Conversion of an even ≥ 4 in sum of two primes according to Goldbach's strong conjecture (GSC)

GSC means $2n = P + P'$ such that $P' > P$ and such that $n - PN_1 = PN_2 - n$. PN_1 and PN_2 are said to be equidistant at n . How then can we obtain P s from n ? It all depends on whether the even $2n$ is $4x$ or $4x + 2$; whether n is odd or even; and whether n is $3n$ or not. These are the rules by which the GSC operates.

« Even numbers $4x$ are always the sum of one P $4x - 1$ and another P $4x + 1$ according to GSC. On the other hand, $4x + 2$ evens are the sum of either two P $4x - 1$ or two P $4x + 1$. This is because $4x - 2 \leftrightarrow 4x + 2$. And so, only even numbers $4x$ are the sum of P s belonging to both Spn_1 and Spn_2 whereas even numbers $4x + 2$ derive from either Spn_1 or Spn_2 but never from both Spn (Table 1)».

3a. The even $2n$ is non $3n$

3a1. The even $2n$ is $4x$

3a1-1. n is even and is $4x$

In this case $n - \{SO_{3n}\}(4x + 1) = \{Spn_1\}(4x - 1)$ and $n - \{SO_{3n}\}(4x - 1) = \{Spn_2\}(4x + 1)$.

$n + \{SO_{3n}\}(4x + 1) = \{Spn_2\}(4x + 1)$ and $n + \{SO_{3n}\}(4x - 1) = \{Spn_1\}(4x - 1)$.

<p>Two Examples of $n - \{SO_{3n}\}$ and $n + \{SO_{3n}\}$</p> <p>$80 = 2 \times 40$.</p> <p>$40 - 3 = 37$ such that 3 is $4x - 1$ and 37 $4x + 1$.</p> <p>$40 + 3 = 43$ such that 3 is $4x - 1$ and 43 is $4x - 1$</p> <p>In this case $80 = 37 + 43$ and 37 and 43 are equidistant at 40.</p> <p>$40 - 33 = 7$ such that 33 is $4x + 1$ and 7 is $4x - 1$.</p> <p>$40 + 33 = 73$ such that 33 is $4x + 1$ and 73 is $4x + 1$</p> <p>In this $80 = 7 + 73$ and 7 and 73 are equidistant at 40.</p>
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To get all the possible sums we calculate with both $\{SO_{3n}\}$ and $\{SO_{pc}\}$ which means all odds except prime factors of the tested number. We thus calculate $n - \{SO_{pc}\}$ and $n + \{SO_{pc}\}$ together with $n - \{SO_{3n}\}$ and $n + \{SO_{3n}\}$. But note that even if we do not have them all we can get them by using the available sums $E = P + P' = (P \pm 4n) + (P' \pm 4n) = P'' + P'''$ and so on. In other words $E = P + P' = (P + 4n) + (P' - 4n) = P'' + P'''$; or $E = P + P' = (P - 4n) + (P' + 4n) = P'' + P'''$.

Example. $80 = 2 \times 40$.

$40 - 3 = 37$; $40 - 7 = 33$; $40 - 9 = 31$; $40 - 11 = 29$; $40 - 13 = 27$; $40 - 17 = 23$; $40 - 19 = 21$; $40 - 21 = 19$; $40 - 23 = 27$; $40 - 27 = 13$; $40 - 29 = 11$; $40 - 31 = 9$; $40 - 33 = 7$; $40 - 37 = 3$.

$40 + 3 = 43$; $40 + 7 = 47$; $40 + 9 = 49$; $40 + 11 = 51$; $40 + 13 = 53$; $40 + 17 = 57$; $40 + 19 = 59$; $40 + 21 = 61$; $40 + 23 = 63$; $40 + 27 = 67$; $40 + 29 = 69$; $40 + 31 = 71$; $40 + 33 = 73$; $40 + 37 = 77$.

We have $80 = 37 + 43 = 19 + 61 = 13 + 67 = 7 + 73$. In all cases $80 = P(4x + 1) + P'(4x - 1)$ $x > 0$

3a1-2. n is even and is $4x + 2$

In this case $n - \{SO3n\}(4x + 1) = \{Spn2\}(4x + 1)$ and $n - \{SO3n\}(4x - 1) = \{Spn1\}(4x - 1)$.

$n + \{SO3n\}(4x + 1) = \{Spn1\}(4x - 1)$ and $n + \{SO3n\}(4x - 1) = \{Spn1\}(4x + 1)$.

Example $100 = 2 \times 50$ such that 50 is $4x + 2$.

$50 - 9 = 41$ such that both 9 and 41 are $4x + 1$.

$50 + 9 = 59$ such that 9 is $4x + 1$ and 59 is $4x - 1$.

$100 = 41 + 59$ such that 41 and 59 are equidistant at 50.

$50 - 39 = 11$ such that both 11 and 39 are $4x - 1$.

$50 + 39 = 89$ such that 39 is $4x - 1$ and 89 is $4x + 1$.

$100 = 11 + 89$ such that 11 and 89 are equidistant at 50.

Here are all the sums of two primes by $n - \{Sopc\}$ and $n + \{Sopc\}$ together with $n - \{SO3n\}$ and $n + \{SO3n\}$.

$50 - 3 = 47$; $50 - 7 = 43$; $50 - 9 = 41$; $50 - 11 = 39$; $50 - 13 = 37$; $50 - 17 = 33$; $50 - 19 = 31$; $50 - 21 = 29$; $50 - 23 = 27$; $50 - 27 = 23$; $50 - 29 = 21$; $50 - 31 = 19$; $50 - 33 = 17$; $50 - 37 = 13$; $50 - 39 = 11$; $50 - 41 = 9$; $50 - 43 = 7$; $50 - 47 = 3$.

$50 + 3 = 53$; $50 + 7 = 57$; $50 + 9 = 59$; $50 + 11 = 61$; $50 + 13 = 63$; $50 + 17 = 67$; $50 + 19 = 69$; $50 + 21 = 71$; $50 + 23 = 73$; $50 + 27 = 77$; $50 + 29 = 79$; $50 + 31 = 81$; $50 + 33 = 83$; $50 + 37 = 87$; $50 + 39 = 89$; $50 + 41 = 91$; $50 + 43 = 93$; $50 + 47 = 97$.

We have $100 = 47 + 53 = 41 + 59 = 29 + 71 = 17 + 83 = 11 + 89 = 3 + 97$.

$100 = PN(4x + 1) + PN(4x - 1)$ $x > 0$

Here is another example with all the sums of two primes by $n - \{Sopc\}$ and $n + \{Sopc\}$ together with $n - \{SO3n\}$ and $n + \{SO3n\}$.

Example $140 = 4 \times 35$ and $140 : 2 = 70$ and $70 = 4 \times 17 + 2 (4x + 2)$.

In this case $n = (4x + 1) + (4x + 1)$ or $n = (4x - 1) + (4x - 1) \leftrightarrow n - (4x + 1) = 4x + 1$ and $n - (4x - 1) = 4x - 1$. Exampe $70 - 5(4x + 1) = 65(4x + 1)$ and $70 - 7(4x - 1) = 63(4x - 1)$.

$70 - 3 = 67$; $70 - 5 = 65$; $70 - 7 = 63$; $70 - 9 = 61$; $70 - 11 = 59$; $70 - 13 = 57$; $70 - 15 = 55$; $70 - 17 = 53$; $70 - 19 = 51$; $70 - 21 = 49$; $70 - 23 = 47$; $70 - 25 = 45$; $70 - 27 = 43$; $70 - 29 = 41$; $70 - 31 = 39$; $70 - 33 = 37$; $70 - 35 = 35$; $70 - 37 = 33$; $70 - 39 = 31$; $70 - 41 = 29$; $70 - 43 = 27$; $70 - 45 = 25$; $70 - 47 = 23$; $70 - 49 = 21$; $70 - 51 = 19$; $70 - 53 = 17$; $70 - 55 = 15$; $70 - 57 = 13$; $70 - 59 = 11$; $70 - 61 = 9$; $70 - 63 = 7$; $70 - 65 = 5$; $70 - 67 = 3$

$70 + 3 = 73$; $70 + 5 = 75$; $70 + 7 = 77$; $70 + 9 = 79$; $70 + 11 = 81$; $70 + 13 = 83$; $70 + 15 = 85$; $70 + 17 = 87$; $70 + 19 = 89$; $70 + 21 = 91$; $70 + 23 = 93$; $70 + 25 = 95$; $70 + 27 = 97$; $70 + 29 = 99$; $70 + 31 = 101$; $70 + 33 = 103$; $70 + 35 = 105$; $70 + 37 = 107$; $70 + 39 = 109$; $70 + 41 = 111$; $70 + 43 = 113$; $70 + 45 = 115$; $70 + 47 = 117$; $70 + 49 = 119$; $70 + 51 = 121$; $70 + 53 = 123$; $70 + 55 = 125$; $70 + 57 = 127$; $70 + 59 = 129$; $70 + 61 = 131$; $70 + 63 = 133$; $70 + 65 = 135$; $70 + 67 = 137$.

We have $140 = 67 + 73 = 61 + 79 = 43 + 97 = 37 + 103 = 31 + 109 = 13 + 127 = 3 + 137$.

Conclusion : An even $E = 4n$ such that n is even and whether $n = 4x$ or $n = 4x + 2$

$$E = 4n = P(4x + 1) + P(4x - 1) \quad x > 0$$

3a1-3. n is odd and is $4x + 1$ or $4x - 1$

It is impossible to get an even $2n$ which is $4x$ from $2 \times$ (Odd numbers) that are either $4x + 1$ or $4x - 1$. Because $2 \times (4x + 1) \neq 4x$ and $2 \times (4x - 1) \neq 4x$.

3a2. The even $2n$ is $4x + 2$

3a2-1. First method

In this case $n \neq 4x$ and either $n = 4x + 1$ or $n = 4x - 1$. However, an even is either $4x$ or $4x + 2$. So n cannot be even in this case and is absolutely odd. In this case we have two cases either $4x + 2 = (4x + 1) + (4x + 1)$ or $4x + 2 = (4x - 1) + (4x - 1) \rightarrow n = 4x + 1$ or $n = 4x - 1$.

- If $n = 4x - 1$ then $n - (4x) = 4x - 1$ and $n + (4x) = 4x - 1$.

Example $70 = 2 \times 35$

$$70 = 4 \times 17 + 2 = 4x + 2.$$

$$35 = 36 - 1 = 4x - 1.$$

$35 - 4 = 31 - 4 = 27 - 4 = 23 - 4 = 19 - 4 = 15 - 4 = 11 - 4 = 7 - 4 = 3$ (31 ; 27 ; 23 ; 19 ; 15 ; 11 ; 7 ; 3 are all $4x - 1$).

$35 + 4 = 39 + 4 = 43 + 4 = 47 + 4 = 51 + 4 = 55 + 4 = 59 + 4 = 63 + 4 = 67$ (39 ; 43 ; 47 ; 51 ; 55 ; 59 ; 63 and 67 are all $4x - 1$).

Therefore $70 = 23 + 47$; $70 = 11 + 59$ knowing that 23 et 47 ; 11 and 59 are all equidistant at 35 thus satisfying the GSC.

- If $n = 4x + 1$ then $n - (4x) = 4x + 1$ and $n + (4x) = 4x + 1$.

Example $122 = 2 \times 61$

$$122 = 4 \times 30 + 2$$

$$61 = 4 \times 15 + 1.$$

$61 - 4 = 57 - 4 = 53 - 4 = 49 - 4 = 45 - 4 = 41 - 4 = 37 - 4 = 33 - 4 = 29 - 4 = 25 - 4 = 21 - 4 = 17 - 4 = 13 - 4 = 9 - 4 = 5$ (57 ; 53 ; 49 ; 45 ; 41 ; 37 ; 33 ; 29 ; 25 ; 21 ; 17 ; 13 ; 9 ; and 5 are all $4x + 1$).

$61 + 4 = 65 + 4 = 69 + 4 = 73 + 4 = 77 + 4 = 81 + 4 = 85 + 4 = 89 + 4 = 93 + 4 = 97 + 4 = 101 + 4 = 105 + 4 = 109 + 4 = 113 + 4 = 117$ (65 ; 69 ; 73 ; 77 ; 81 ; 85 ; 89 ; 93 ; 97 ; 101 ; 105 ; 109 ; 113 ; 117 are all $4x + 1$).

$122 = 61 + 61$; $122 = 13 + 109$ such that 13 and 109 are equidistant at 61.

- The limitation of this method is that it does not give all possible sums of two Ps of the number being tested. But note that even if we do not have them all we can get them by using the available sums this way $E = P + P' = (P \pm 4n) + (P' \pm 4n) = P'' + P'''$.

3a2-2. Second method

The other method is to calculate $n - 2n(n = 1)$ and $n + 2n(n = 1)$

Example $70 = 2 \times 35$; $70 = 4 \times 17 + 2 = 4x + 2$ and $35 = 36 - 1 = 4x - 1$.

$35 - 2 = 33 - 2 = 31 - 2 = 29 - 2 = 27 - 2 = 25 - 2 = 23 - 2 = 21 - 2 = 19 - 2 = 17$ and so on.

$35 + 2 = 37 + 2 = 39 + 2 = 41 + 2 = 43 + 2 = 45 + 2 = 47 + 2 = 49 + 2 = 51 + 2 = 53$ and so on.

We will have $70 = 3 + 67 = 11 + 59 = 17 + 53 = 23 + 47 = 29 + 41$ and so $70 = (4x - 1) + (4x - 1)$ or $70 = (4x + 1) + (4x + 1)$.

Example $122 = 2 \times 61$; $122 = 4 \times 30 + 2$ and $61 = 4 \times 15 + 1$.

$61 - 2 = 59$; $61 - 4 = 57$; $61 - 6 = 55$; $61 - 8 = 53$; $61 - 10 = 51$; $61 - 12 = 49$; $61 - 14 = 47$ and so on.

$61 + 2 = 63 + 2 = 65 + 2 = 67 + 2 = 69 + 2 = 71 + 2 = 73 + 2 = 75 + 2 = 77 + 2 = 79 + 2 = 81$ and so on.

We will have $122 = 61 + 61 = 43 + 79 = 19 + 103 = 13 + 109$. And so $122 = P(4x + 1) + P'(4x + 1)$ or $122 = P(4x - 1) + P'(4x - 1)$.

- The great advantage of this method is that it gives all possible sums of two Ps of the number being tested. Indeed, $(4x - 1) - 2 = 4x - 3 = 4x + 1$ while $(4x - 1) + 2 = 4x + 1$. Whereas $(4x + 1) - 2 = 4x - 1$ and $(4x + 1) + 2 = 4x + 3 = 4x - 1$. On the other hand either $4x - 1 (\pm 4n) = 4x - 1$; or $4x + 1 (\pm 4n) = 4x + 1$.

Conclusion : An even $2n$ which is $4x + 2$ or $E(4x+2) = P(4x - 1) + P'(4x - 1)$; or $E(4x+2) = P(4x + 1) + P'(4x + 1)$.

3b. The even $2n$ is $3n$ and is $4x$

In this case $n = (4x - 1) + (4x + 1)$ or $n = (4x + 1) + (4x - 1)$.

Because $2n$ is $3n$, n is $3n$ and to get primes we have to calculate $n - \{SOpc\} = \{Spn1\}$ or C and $n + \{SOpc\} = \{Spn2\}$ or C . In this case $n - \{Spn1\} = \{Spn2\}$ and $n - \{SPn2\} = \{Spn1\}$.

Note that we exclude prime factors of the tested number and $\{SO3n\}$. ***The major difference between evens that are $3n$ and non $3n$ is that we exclude $\{SO3n\}$ from the former and we cannot calculate $n - \{SO3n\} = 3n$ and $n + \{SO3n\} = 3n$ which are not primes.***

Example. $96 = 4 \times 24$; 96 is $3n$ and $96 : 2 = 48 = 4x$ which is $3n$.

Calculate $48 - \{Sopc\}$ by excluding $\{SO3n\}$.

$48 - 5 = 43$; $48 - 7 = 41$; $48 - 11 = 37$; $48 - 13 = 35$; $48 - 17 = 31$; $48 - 19 = 29$; $48 - 23 = 25$;
 $48 - 25 = 23$; $48 - 29 = 19$; $48 - 31 = 17$; $48 - 35 = 13$; $48 - 37 = 11$; ; $48 - 41 = 7$; $48 - 43 = 5$.

Note $48 - 5(4x + 1) = 43(4x - 1)$; and $48 - 7(4x - 1) = 41(4x + 1)$.

$48 + 5 = 53$; $48 + 7 = 55$; $48 + 11 = 59$; $48 + 13 = 61$; $48 + 17 = 65$; $48 + 19 = 67$; $48 + 23 = 71$;
 $48 + 25 = 73$; $48 + 29 = 77$; $48 + 31 = 79$; $48 + 35 = 83$; $48 + 37 = 85$; ; $48 + 41 = 89$; $48 + 43 = 91$.

We have $96 = 43 + 53 = 37 + 59 = 29 + 67 = 23 + 73 = 17 + 79 = 13 + 83 = 7 + 89$.

We therefore have all possible sums of two primes.

For the rest the same rules apply to both $2n$ that is $3n$ or not after excluding $\{SO3n\}$ with the former ($3n$).

4. Demonstration of Goldbach's strong conjecture following equations $4x \pm 1$ and gaps of $4n$.

1. First, let's clarify the relationship between odd numbers and $4x \pm 1$. The $4x \pm 1$ equations always run in a 4-step loop involving 4 successive odd numbers. For example, 3 5 7 9 11 13 15 17 we have 3 is $4x - 1$; 5 is $4x + 1$; 7 is $4x - 1$; and 9 ($4x + 1$) (the first loop) and we have 11 is $4x - 1$; 13 is $4x + 1$; 15 is $4x - 1$ and 17 is $4x + 1$ (the second loop) and so on to infinity. On the other hand, there are 5 types of odd numbers ending in the unit digit 1; 3; 5; 7 and 9. Composite numbers end with any of these 5 unit digits, but a prime number never ends with 5 (except 5 itself), which is one of the reasons why composite numbers outnumber primes (in addition to multiples of 3 which are also composites). Recall that E is any even number ≥ 4 and $E/2$ is any integer ≥ 2 . **Figure 1** shows how even numbers are formed by the equations $4x \pm 1$. An even number $E = 2n$ ($E/2 = n$) is either $4x$ or $4x + 2$. In the first case, $E/2 = 4x$ or $E/2 = 4x + 2$. In the second case, $E/2 = 4x - 1$ or $E/2 = 4x + 1$. This means that when an even number is $4x$, it will be the sum of two primes or two composite numbers, one of which is $4x - 1$ and the other $4x + 1$. Whereas when it is $4x + 2$; it will be the sum of two primes or two composite numbers, both of which are at the same time $4x - 1$ or $4x + 1$. Remember that an odd number $4x - 1$ or $4x + 1$ is either prime or composite. Neither can be predicted by this equation, unless primality tests or factorization tests are performed.

2. **Figure 2** shows all possible $4x \pm 1$ loops that occur before and after $E/2$. Be $E = 4x$ and so $E/2 = 4x$ (**Panels 2A + 2B**) or $E/2 = 4x + 2$ (**Panels 2C + 2D**). For instance the first odd number that preceded $E/2 = 4x$ is $4x - 1$ and the one that follows it is $4x + 1$ and to go from one to another, we subtract or add 2. In the case where $E = 4x + 2$ we have either $E/2 = 4x + 1$ or $E/2 = 4x - 1$. **Figure 2E + 2F** shows the loops that occur in the former case and **Figure 2G + 2H** shows the latter case. These are the loops that always occur, and if E tends to infinity and $E/2$ tends to infinity, the loops repeat ad infinitum. The greater the even number, the greater the number of loops, and the greater the total number of possible sums of two prime numbers. A loop requires 8 numbers, 4 of them odd in their natural order; so to find the approximate total number of loops for an even number E , simply calculate $E/8$. To find the one before or after $E/2$, calculate $E/16$. Note that you can have half-loops when the number is not a multiple of 4. For example, the even number 10 has two half-loops $<$ and > 5 . We have 1 2 3 4 5 6 7 8 9 10 and the first half-loop is $3 - 1$ (to exclude 1) and the second is $7 - 9$. Knowing that 3 is $4x - 1$ and 7 is $4x - 1$ so $3 + 7 = 4x - 2$ which is also $4x + 2$ and in fact 10 is either $4x + 2$ or $4x - 2$. ***A major rule postulates that if two prime numbers P and P' are such that $P + P' = E$, then P and P' occupy the same positions in two symmetrical loops, one of which is $< E/2$ and the other $> E/2$.*** This applies to any even numbers $4x$ and $4x + 2$ enough large to have at least two loops.
3. An even number denoted E ends in 0; 2; 4; 6; and 8 units digits. In the rest of this section, even numbers will be denoted as NX with X the unit digit. An even number ending in 0 is therefore denoted $E = N0$. In this case, $E/2 = N0$ (**Figure 3A + 3B**) or $N/2 = N5$ (**Figure 3C + 3D**). Figure 3 then shows the unit digits of the odd numbers within the loops $< E/2$ and $> E/2$. Two odd numbers occupying symmetrical positions must be added together to obtain the unit digit of the even number. In the case of $E = N0$ and $E/2 = N5$, we can see that the 4 odd numbers in symmetrical loops $< E/2$ and $> E/2$ add up for $E = N0$. For example $9 + 1$; $7 + 3$; $1 + 9$ and $7 + 3$, which shows that $N0$ even numbers have a high probability of being formed by the addition of two prime numbers. Note that the difference between odd numbers within the loop is 2 or 4. In fact, 4 is used to exclude 5, which cannot be the unit figure of a prime number. However, in some cases, the P number 5 is re-used. We have the same result with $E = N0$ and $E/2 = N5$ (**Figure 3C and 3D**). In the case of $E = N2$ and $E/2 = N1$ (**Figure 3E + 3F**), we have the possible sums of two primes ending with the unit digits $9 + 3$; $3 + 9$; and $1 + 1$. In the case of $E = N2$ and $E/2 = N1$ we have the possible sums of two primes ending with the unit digits $9 + 3$; $3 + 9$; and $1 + 1$. The remainder of Figure 3, i.e. Figure 3G-L, gives the cases of evens ending with the unit digits 4; 6 and 8. With evens ending 6, it was necessary to use the unit digit 5 (evens if the corresponding number is not prime) to get all possible sums of 6 including two primes $3 + 3$; $7 + 9$ and $9 + 7$.
- Note that this loop system allows us to predict the positions of the two potential equidistant primes $P < E/2$ and $P' > E/2$ such that $E = P + P'$.*** In addition, the unit digits is also a good too to determine if they can add up to form an even with a given unit digit.

- **Figure 4** shows that loops $< E/2$ and those $> E/2$ give three possible sums of E including $E = C + C'$; $E = C + P$; and $E = P + P'$. **A second major rule is that these three forms of E -sums are inter-convertible or inter-deductible, so you can switch from one to the other and so we have $E = C + C' \leftrightarrow E = C + P \leftrightarrow E = P + P'$.** **Figure 4** shows that by subtracting and adding $4n$ to both addition terms, we can convert one sum into another.

As **Table 1** shows, there's always a gap of $4n$ between two primes, either $4x - 1$ or $4x + 1$; but there's a variable gap of $2n$ between $4x - 1$ and $4x + 1$ primes. As is well known, there's a gap of 6 between the prime numbers $6x - 1$ or between those $6x + 1$, but a variable gap of $2n$ between the $6x - 1$ and the $6x + 1$ primes. *Similarly, there is a gap of $4n$ between a composite odd number $C = 4x - 1$ and prime number $P = 4x - 1$ or a composite odd numbers $C = 4x + 1$ and prime number $P = 4x + 1$.* There is though a variable $2n$ gaps between $C = 4x - 1$ numbers and $P = 4x + 1$ primes and reciprocally. Here are demonstrations of the inter-conversions $E = C + C' \leftrightarrow E = C + P \leftrightarrow E = P + P'$. First, note that $E = 4x$ or $E = 4x + 2$. In both cases, $4x \pm 4n = 4X$ and $(4x + 2) \pm 4n = 4X + 2$. **Adding or subtracting $4n$ to or from an even number E does not change its initial writing to $4x$ or $4x + 2$.**

$$E = C + C' = (P \pm 4n) + (P' \pm 4n) \rightarrow E \pm 4n \pm 4n = P + P' \rightarrow E \pm 4N = P + P'$$

$$\text{If } E = 4x \rightarrow E \pm 4N = 4x \pm 4N = 4X \rightarrow \mathbf{4X = P + P'}$$

$$\text{If } E = 4x + 2 \rightarrow E \pm 4N = (4x + 2) \pm 4N \rightarrow \mathbf{4X + 2 = P + P'}$$

$$E = P + C = (P'' \pm 4n) + (P''' \pm 4n) \rightarrow E \pm 4N = P'' + P'''$$

$$\text{If } E = 4x \rightarrow E \pm 4N = 4x \pm 4N = P'' + P''' \rightarrow \mathbf{4X = P'' + P'''}$$

$$\text{If } E = 4x + 2 \rightarrow E \pm 4N = (4x + 2) \pm 4N = 4X + 2 = P'' + P''' \rightarrow \mathbf{4X + 2 = P'' + P'''}$$

$$E = P + P' = (P'' \pm 4n) + (P''' \pm 4n) \rightarrow E \pm 4n \pm 4n = P'' + P''' \rightarrow E \pm 4N = P'' + P'''$$

$$\text{If } E = 4x \rightarrow E \pm 4N = 4x \pm 4N = P'' + P''' \rightarrow \mathbf{4X = P'' + P'''}$$

$$\text{If } E = 4x + 2 \rightarrow E \pm 4N = (4x + 2) \pm 4N = 4X + 2 = P'' + P''' \rightarrow \mathbf{4X + 2 = P'' + P'''}$$

Discussion

The theorem « Between n and $2n$ and $n > 6$, there is at least one prime in $4k - 1$ and at least one in $4k + 1$ - Proven by Erdős. Example between 7 and 14 we have $7 = 4 \times 2 - 1$; $11 = 4 \times 3 - 1$; $13 = 4 \times 3 + 1$ » *doesn't predict the position of all equidistant primes neither their units digits*. It is well known that primes can be $4x - 1$ or $4x + 1$ but what is needed is to predict their positions. This article shows that for each even $E = 4x$ or $E = 4x + 2$ there are repeating loops of 8 numbers where the prime or composite numbers always occupy the same positions $4x - 1$ ou $4x + 1$ before or after $E/2$. If $E = P + P'$; P and P' must be in the same loop and in the same positions symmetrically before and after $E/2$. This finding is new and can be exploited to design new computer programs that reproduce these loops in order to note the positions of the primes and test their equidistance (being in the same loop before and after $E/2$ and occupying the same positions). This will make it possible to convert an even number into all possible sums of two primes; or even test other types of conjectures. This article offers a unique tool for investigating prime numbers in loops of 8 numbers $< E/2$ and $> E/2$.

The accuracy of the data in this article can be seen in the symmetry before and after $E/2$ and in the consistency between the different cases of numbers. Examining the unit digits of even numbers helps refine the search for equidistant primes whose two-by-two sums give E .

The article offers for the first time a deductive proof of Goldbach's strong conjecture based on the equivalence between the sums $E = C + C' \leftrightarrow E = C + P \leftrightarrow E = P + P'$. This inter-conversion depends on the gaps $4n$ between the primes P ; or between primes P and composite odd numbers C . It is certain that the prime numbers have a critical density in the set N of natural integers that makes this inter-conversion possible in all cases of even numbers. In fact composite or prime numbers are all $4x \pm 1$ (or $6x \pm 1$ [2]) which makes this interconversion always possible therefore this article demonstrates Goldbach's strong conjecture.

By using the $4x \pm 1$ equations in a novel way, this article finally offers a clue to the solution of the age-old Goldbach conjecture. The interconversion of possible sums of E is a valid demonstration and will certainly mark future research on this conjecture. What's more, the system of $4x \pm 1$ loops provides fertile ground for computer science aimed at deciphering the secret of prime numbers or studying their infinite distribution.

References

1. Bahbouhi, B. (2025). Verification of Goldbach's Strong and Weak Conjectures at Infinity Using Basic and Accessible Mathematics. J Robot Auto Res, 6(1), 01-11.
2. Bahbouhi, B. (2025). Calculation rules and new methods for the strong and weak conjectures of Goldbach to be verified. J Robot Auto Res, 6(1). 01 - 36.

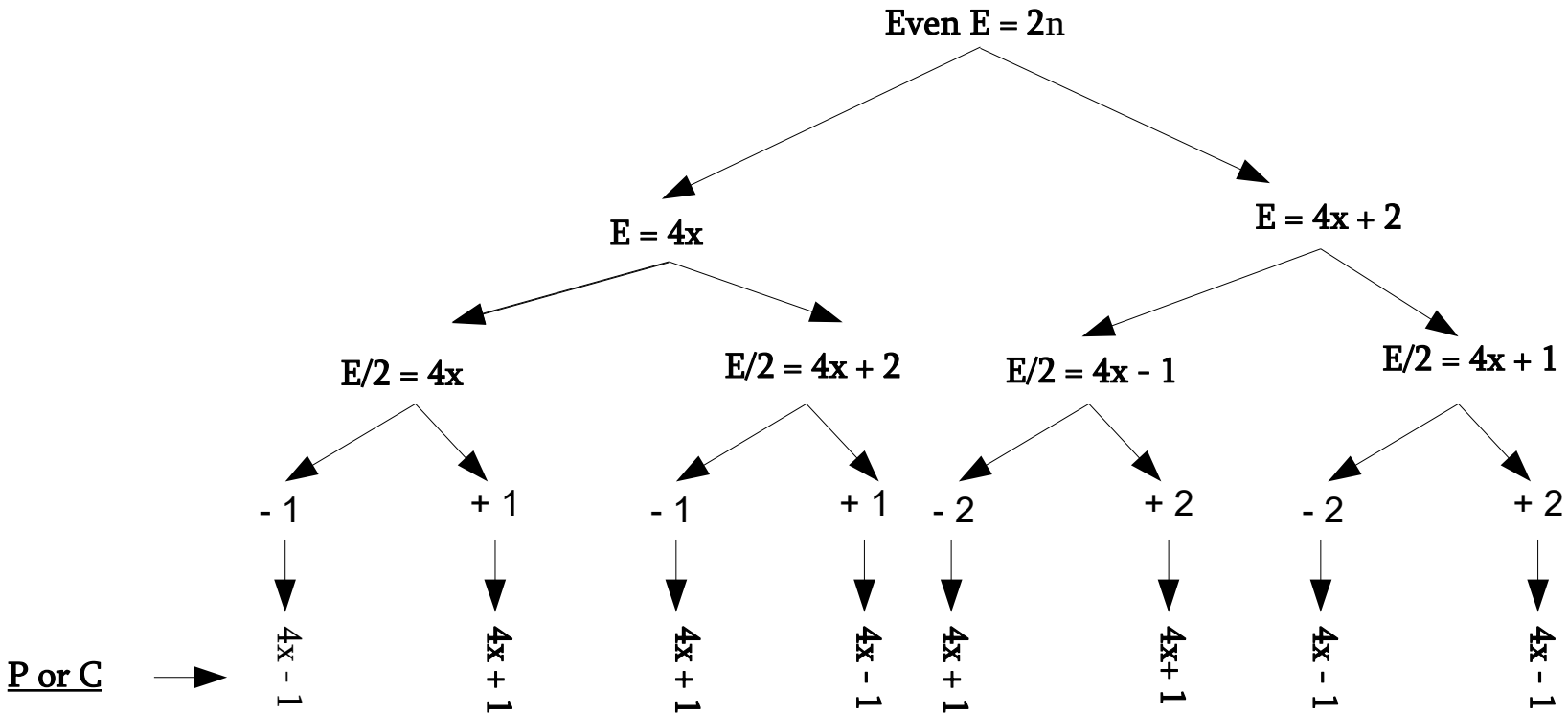


Figure 1: How to obtain prime numbers (P) and composite numbers (C) according to the equations of even numbers (E) such that $E = 4x$ and $E = 4x + 2$. This is true to infinity. Note P or C is $4x \pm 1$.

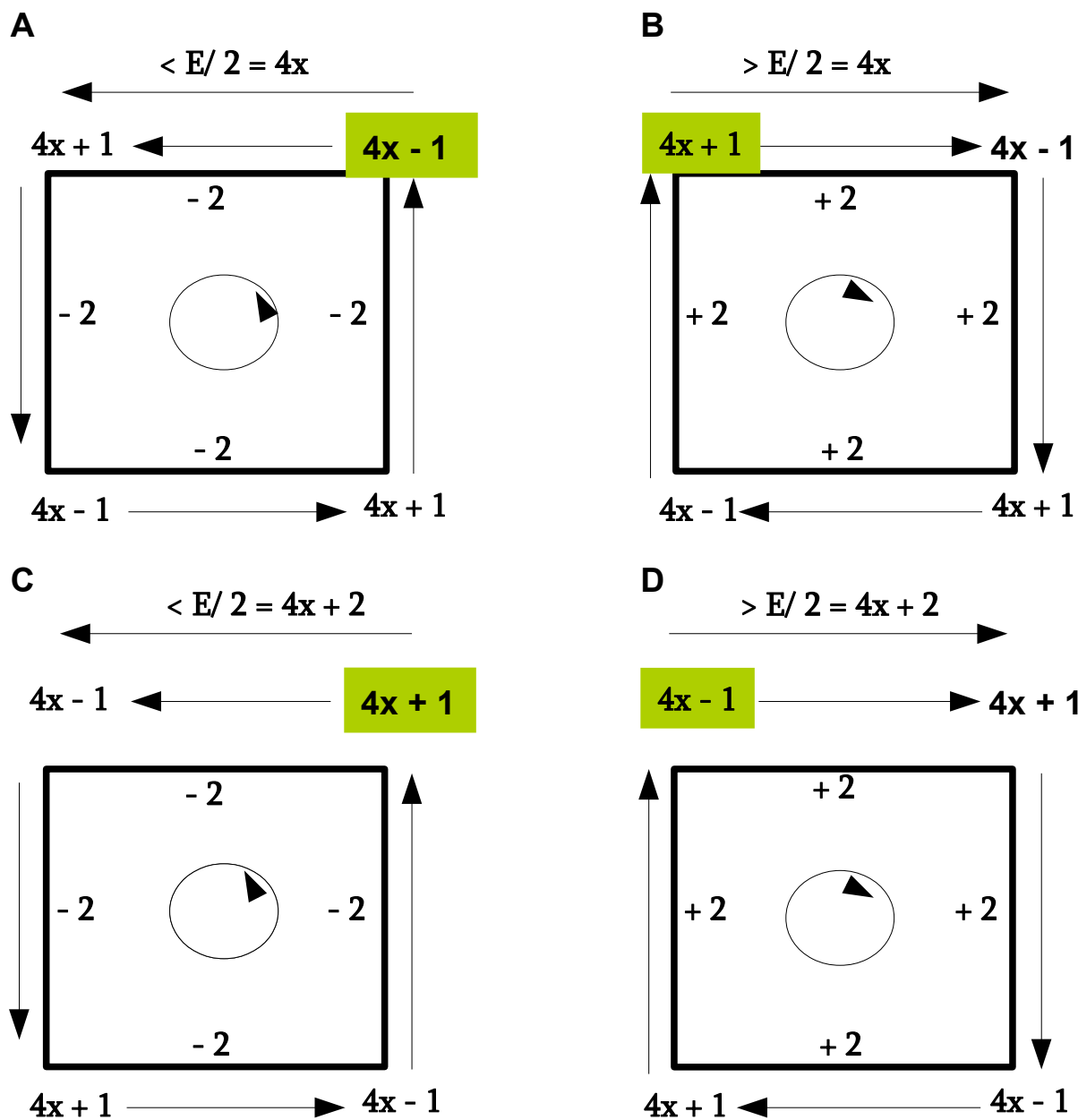


Figure 2. Odd umbers P or C that are $4x - 1$ or $4x + 1$ appear in loops of 8 numbers that repeat in the same way from $E/2$ to 0 and from $E/2$ to E . This loop takes place every 8 integers from $E/2$ in the direction towards 0 and in the opposite direction from $E/2$ towards E . The two loops of 8 $< E/2$ and $> E/2$ are symmetrical. The two loops of 8 are shown for $E = 4x$ even numbers with either $E/2 = 4x$ (A + B) or $E/2 = 4x + 2$ (C + D). This is true to infinity. In the figure only odd numbers are shown separated of a gap = 2.

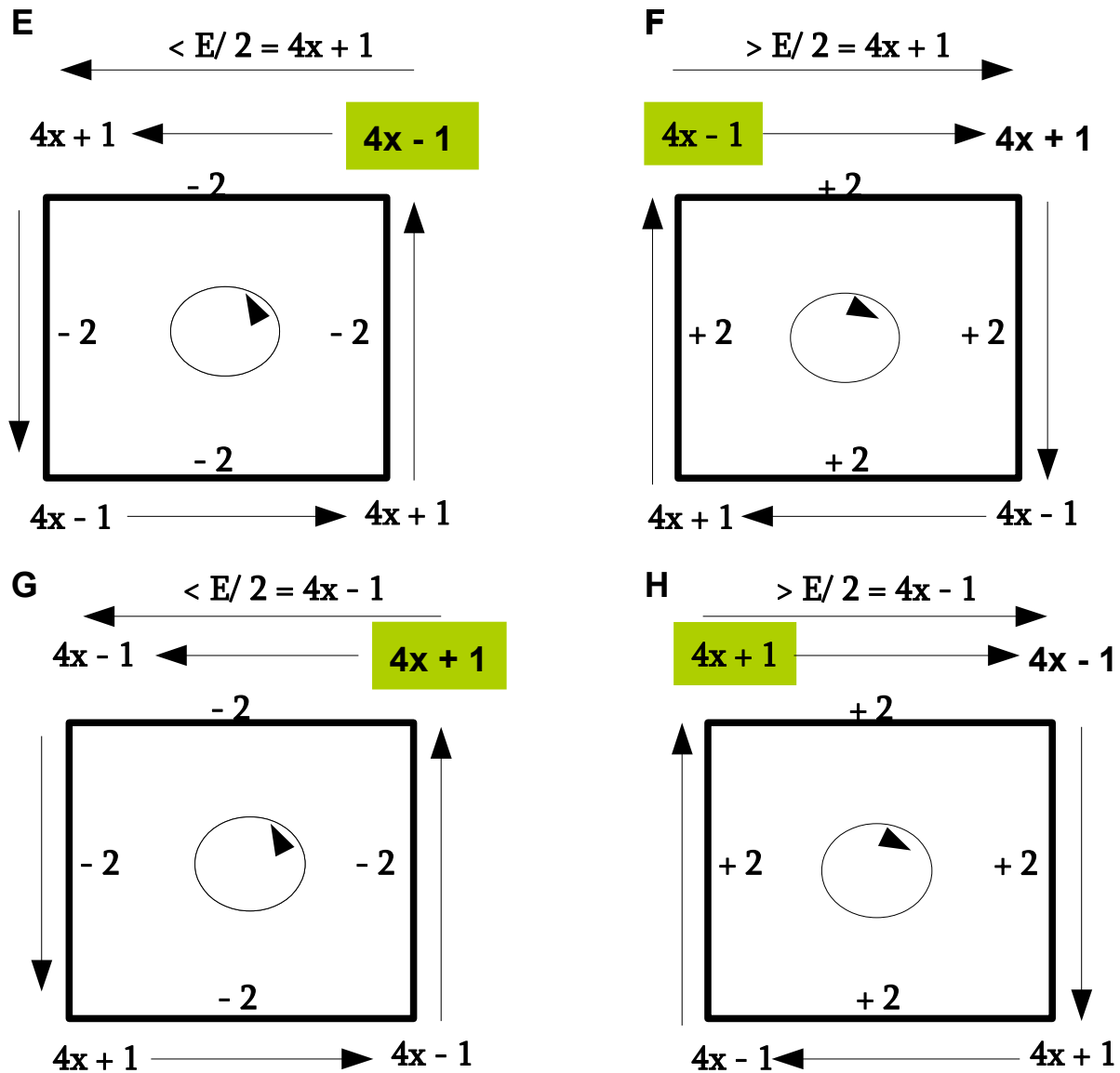


Figure 2 continued. Case of Evens $E = 4x + 2$ and therefore $E/2 = 4x + 1$ (Panels E and F) or $E/2 = 4x - 1$ (Panels G and H).

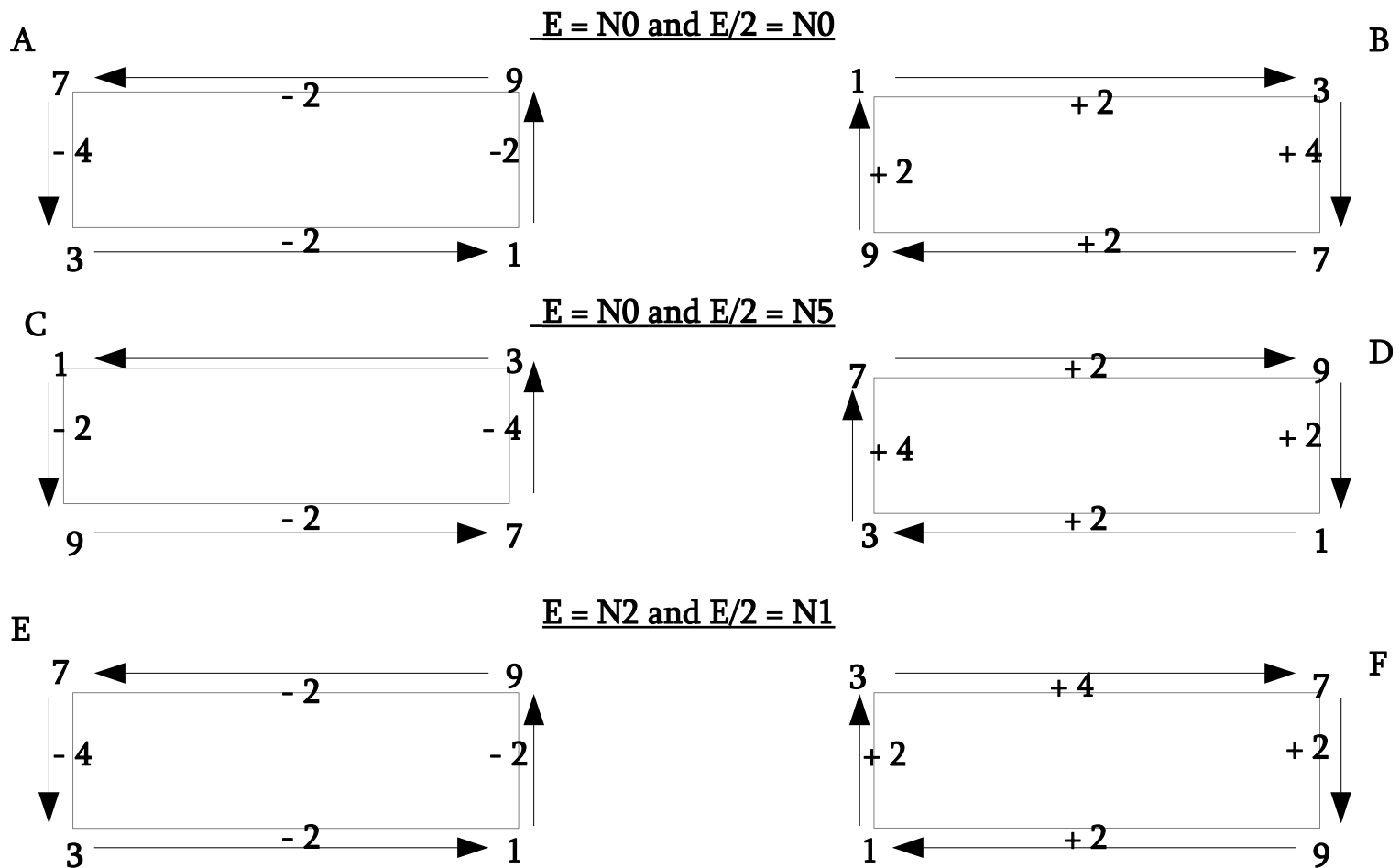


Figure 3A-F. Unit digits of primes (P) or composite (C) numbers $< E/2$ and $> E/2$ in a loop of 10 from 0 to E. If E tends to infinity, the loops of 10 repeat ad infinitum. The first number of the loop either $< E/2$ or $E/2$ is calculated by $E/2 - n$ ($n > 0$) to obtain all possible sums of units digits that correspond to E. The number 5 is omitted when necessary because no prime number P ends with 5 except the number 5. The Gaps of 2 or 4 preceded by the sign - ($< E/2$) and + ($> E/2$) are used to find the numbers P which add up to give the unit figure of E in accordance with the GSC. E or E/2 are written as N followed by the unit digit. **Panels A-B** (E ending with 0 unit digit) ; **Panels C-D** (E ending with 0 unit digit) ; **Panels E - F** (E ending with 2 unit digit). Note that the sum of two odd numbers $< E/2$ and $> E/2$ equidistant from E/2 gives E by obeying the rules for adding their unit digits.

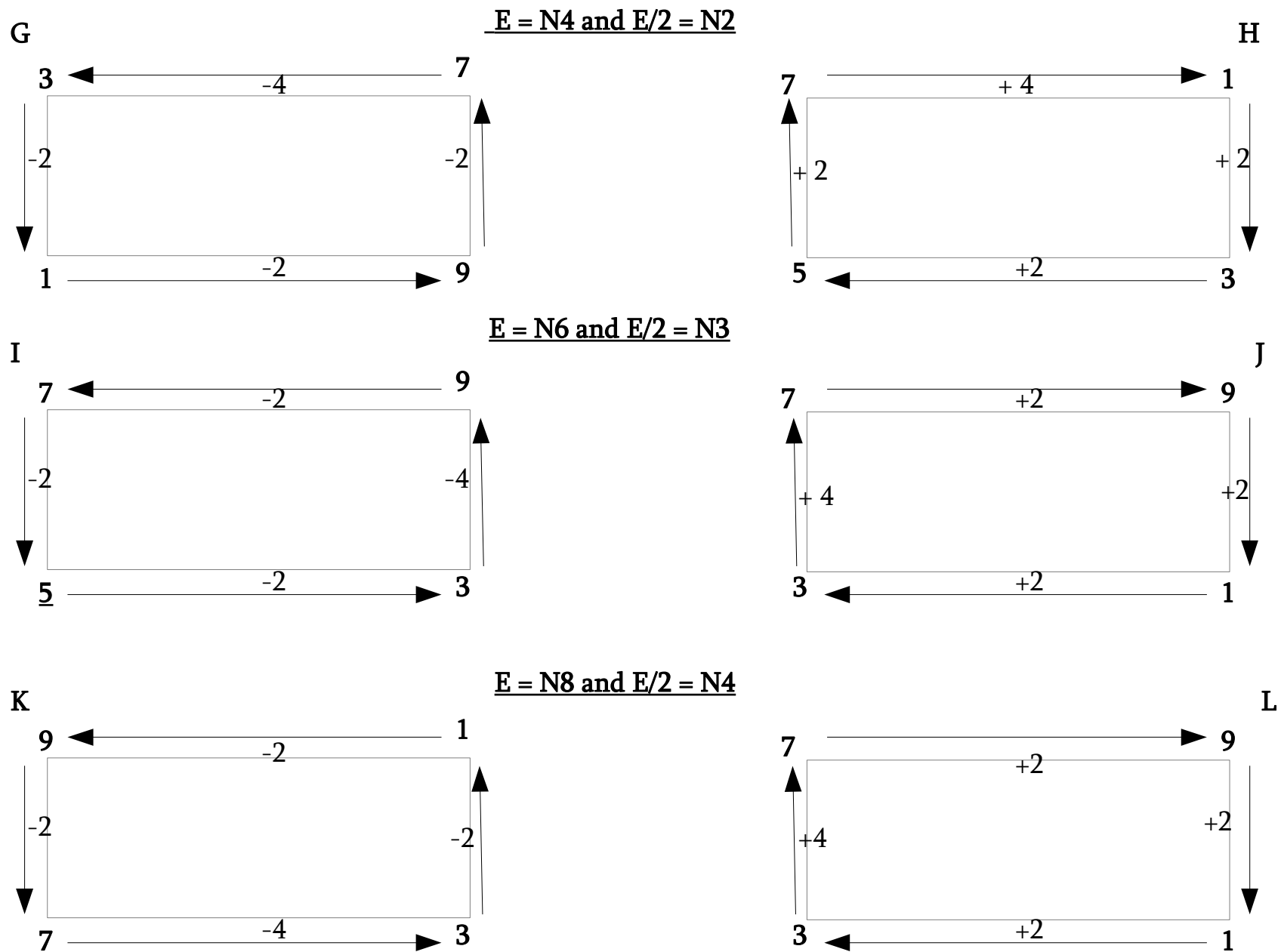


Figure 3 continued (Panels G-L). Unit digits of primes (P) or composite (C) numbers $< E/2$ and $> E/2$ in a loop of 10 from 0 to E. **Panels G-H** (E ending with 4 unit digit) ; **panel I-J** (E ending with 6 unit digit) ; **panels K-L** (E ending with 8 unit digit).

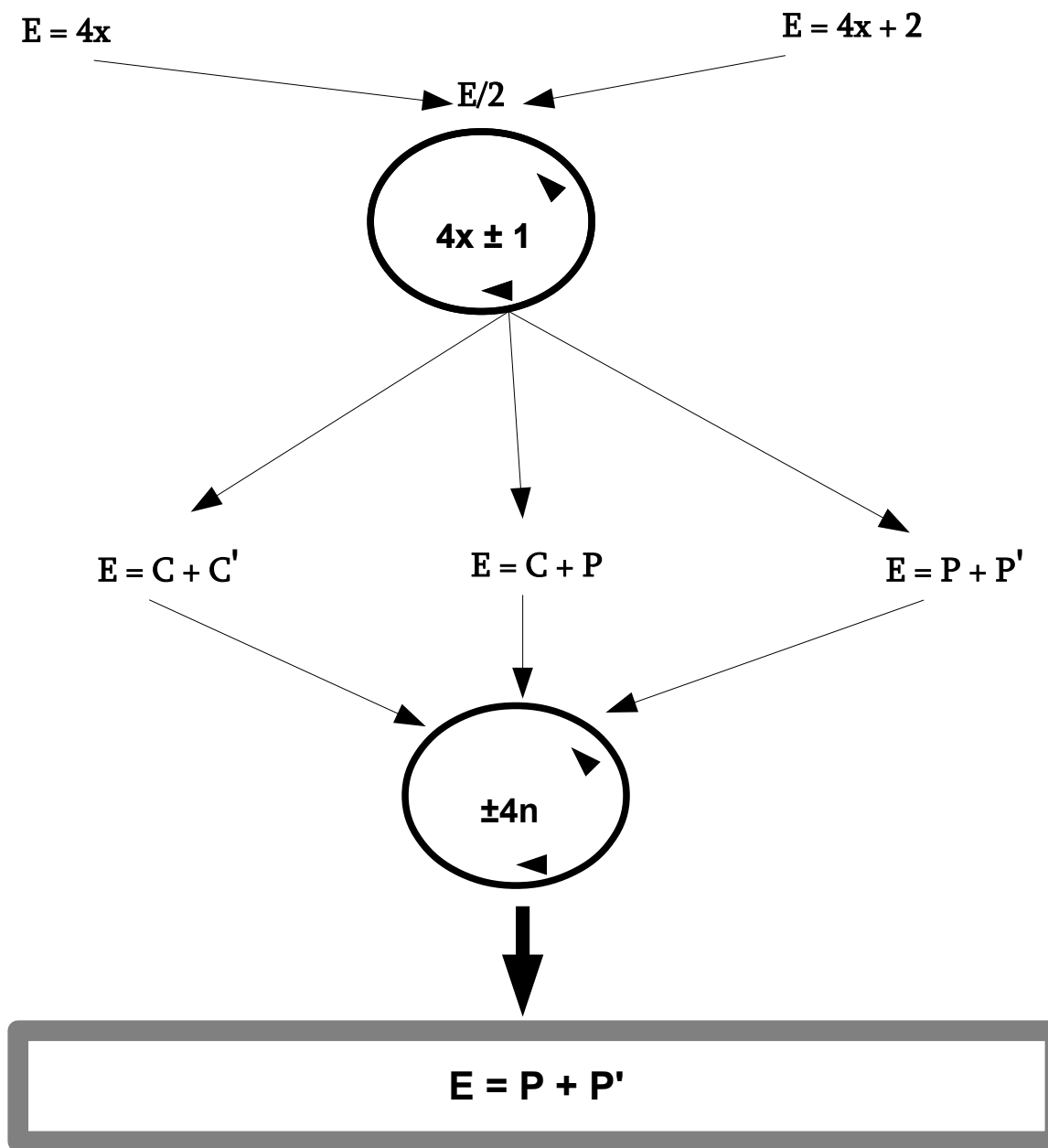


Figure 4. The deductive reasoning that starts with the even numbers $E = 4x$ and $E = 4x + 2$ and leads to their conversion into the sum of two prime numbers P and P' such that $E = P + P'$. Goldbach's strong conjecture is then demonstrated by this reasoning, proving that it is provable by a deductive chain calculus.