New Tricks For Memorizing Trigonometric Identities

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Abstract

We give a way to memorize all 27 standard trigonometric identities, including the product to sum (P2S) and sum to product (S2P) trigonometric identities. These are generally not memorized, but puzzled out using the sum of two angles for the P2S and then P2S for S2P. We show here a faster way to recall these using what might be called heuristic generalizations. All 27 of the standard identities [1] are reviewed in light of this type of trick. Links to a TI-84 program and a youtube video are given.

Introduction

For some odd reason our minds (at least my mind) finds it simple to remember $\cos^2 a + \sin^2 a = 1$, the Pythagorean trigonometric identity. Other such identities are not so easy to recall. If we had a way (read tricks) to go from this identity to others that might help. Here's an example of such a trick. Generalize

$$\cos(a - a) = 1 = \cos(a)\cos(a) + \sin(a)\sin(a)$$

by making the second as bs, as in

$$\cos(a - b) = \cos(a)\cos(b) + \sin(a)\sin(b), \tag{2}$$

one of the standard identities.[1]

Of course $2 + 3 = 5$ and $1 + 4 = 5$ does not imply $2 = 1$ and $3 = 4$, but this heuristic generalization works in this situation.

[1] The equation number references a list, Table 1, of the 27 standard trig identities [1].

$\pm$

Here's another example of the idea. We know

$$0 = \sin(a - a) = \sin(a)\cos(a) - \cos(a)\sin(a)$$

is true enough and with a heuristic generalization this is

$$\sin(a - b) = \sin(a)\cos(b) - \cos(a)\sin(b). \tag{4}$$

Knowing that $\sin$ is an odd function, $\sin(-b) = -\sin(b)$, yields two more identities and we have four of the six $\pm$ identities:

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

and

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b).$$

2a, R, and a/2

Its helpful for some other identities to gloss cosine as homogeneous and negative and sine as heterogeneous and positive. This makes double angles (2a) for these two functions stand out:

$$\cos 2a = \cos^2 a - \sin^2 a = \cos\cos\,\text{(same)} - \sin\sin\,\text{(same)} \tag{7}$$

and

$$\sin 2a = 2\sin a\cos a = \sin\cos\,\text{(not same)} + \cos\sin\,\text{(not same)}. \tag{10}$$

One can remember the reduction formulas (R) using another appeal to the Pythagorean identity: that is

$$1 = \cos^2 a + \sin^2 a = \frac{1 + \cos 2a}{2} + \frac{1 - \cos 2a}{2}$$

heuristically suggests

$$\cos^2 a = \frac{1 + \cos 2a}{2},\ \sin^2 a = \frac{1 - \cos 2a}{2}, \tag{12, 13}$$

and

$$\tan^2 a = \frac{1 - \cos 2a}{1 + \cos 2a}. \tag{14}$$

This one equation works for half angle formulas. Start with a change in arguments giving

$$\cos^2 a/2 = \frac{1+\cos a}{2} \text{ and } \sin^2 a/2 = \frac{1-\cos a}{2}$$

and then take square roots:

$$\cos a/2 = \pm\sqrt{\frac{1+\cos a}{2}} \text{ and } \sin a/2 = \pm\sqrt{\frac{1-\cos a}{2}}. \qquad (15, 16)$$

The three identities for $\tan a/2$ follow:

$$\tan(a/2) = \pm\sqrt{\frac{1-\cos(a)}{1+\cos(a)}}, \frac{1-\cos(a)}{\sin(a)}, \frac{\sin(a)}{1+\cos(a)}, \qquad (17, 18, 19)$$

A way to remember all three half angle identities for $\tan$ is to immediately pencil in or ditto to the right the $1-\cos(a)$ and $1+\cos(a)$ and fill in what remains in the fraction with $\sin(a)$.

What about $\tan$?

The identities for $\sin 2a$ (10) and $\cos 2a$ (7) imply

$$\tan(2a) = \frac{\sin 2a}{\cos 2a} = \frac{2\sin a\cos a}{\cos^2 a - \sin^2 a}\frac{\frac{1}{\cos^2 a}}{\frac{1}{\cos^2 a}} = \frac{2\tan a}{1-\tan^2 a},^{2}$$

giving

$$\frac{\tan a + \tan a}{1 - \tan a \tan a}. \qquad (11)$$

With a heuristic generalization, we arrive at

$$\tan(a \pm b) = \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b}. \qquad (5, 6)$$

Two more

We've touched on all six $(\pm)$ and all but 2 of the $(2a)$ identities. To go from $\cos 2a = \cos^2 a - \sin^2 a$ to

$$\cos 2a = 2\cos^2 a - 1 \qquad (8)$$

[2] No pop here!

and

$$\cos 2a = 1 - 2\sin^2 a \tag{9}$$

just remember the 2 goes in front of the one present and a 1 replaces the one not present. It's another scratch out and replace idea, especially easy with pen and pencil.

S2P (24,25,26,27)

Sum to product identities are easy to immediately recall with more of the same type of tricks. For *all sine* recall $\sin$ is associated with heterogeneous and plus. Also *adding* suggests *doubling*:

$$\underline{\sin a + \sin a} + \overline{\sin a - \sin a} = \underline{2\sin\frac{a+a}{2}\cos\frac{a-a}{2}} + \overline{2\cos\frac{a+a}{2}\sin\frac{a-a}{2}}.$$

For *all cosine* recall $\cos$ is homogeneous and negative:

$$\underline{\cos a + \cos a} + \overline{\cos a - \cos a} = \underline{2\cos\frac{a+a}{2}\cos\frac{a-a}{2}}\overline{-2\sin\frac{a+a}{2}\sin\frac{a-a}{2}}.$$

These yield all S2P identities with heuristic generalizations. Homogeneous (all sines, all cosines) sums and differences beget mixed products ($\sin\cos$, $\cos\sin$, $\cos\cos$, and $\sin\sin$).

P2S (20,21,22,23)

This *sums of all sines and cosines* begets *mixed products* reverses. Mixed products beget all sines and cosines. There's an added twist though: we use the identity for $\sin(2a)$ and once again the Pythagorean identity:

$$\sin(a+a) = \underline{\sin a\cos a} + \overline{\cos a\sin a} \tag{1}$$

$$= \underline{\frac{1}{2}(\sin(a+a) + \sin(a-a))} + \overline{\frac{1}{2}(\sin(a+a) - \sin(a-a))} \tag{2}$$

and

$$1 = \underline{\cos a\cos a} + \overline{\sin a\sin a} \tag{3}$$

$$= \underline{\frac{1}{2}(\cos(a-a) + \cos(a+a))} + \overline{\frac{1}{2}(\cos(a-a) - \cos(a+a))}. \tag{4}$$

and of course you need to apply heuristic generalizations. Note: you can see reduction formulas for $\cos^2$ and $\sin^2$ in (4).

Basic Identities (27)

No	Id	Code
1	$\cos(a+b)=\cos a\cos b-\sin a\sin b$	COSP
2	$\cos(a-b)=\cos a\cos b+\sin a\sin b$	COSM
3	$\sin(a+b)=\sin a\cos b+\cos a\sin b$	SINP
4	$\sin(a-b)=\sin a\cos b-\cos a\sin b$	SINM
5	$\tan(a+b)=\frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$	TANP
6	$\tan(a-b)=\frac{\tan(a)-\tan(b)}{1+\tan(a)\tan(b)}$	TANM
7	$\cos(2a)=\cos^2 b-\sin^2 b$	CDB1
8	$\cos(2a)=2\cos^2 a-1$	CDB2
9	$\cos(2a)=1-2\sin^2 a$	CDB3
10	$\sin(2a)=2\sin a\cos a$	SDOU
11	$\tan(2a)=\frac{2\tan(a)}{1-\tan^2(a)}$	TDOU
12	$\cos^2(a)=\frac{1+\cos(2a)}{2}$	CRED
13	$\sin^2(a)=\frac{1-\cos(2a)}{2}$	SRED
14	$\tan^2(a)=\frac{1-\cos(2a)}{1+\cos(2a)}$	TRED
15	$\cos(a/2)=\pm\sqrt{\frac{1+\cos(a)}{2}}$	CHAF
16	$\sin(a/2)=\pm\sqrt{\frac{1-\cos(a)}{2}}$	SHAF
17	$\tan(a/2)=\pm\sqrt{\frac{1-\cos(a)}{1+\cos(a)}}$	THF1
18	$\tan(a/2)=\frac{1-\cos(a)}{\sin(a)}$	THF2
19	$\tan(a/2)=\frac{\sin(a)}{1+\cos(a)}$	THF3
20	$\sin(a)\sin(b)=\frac{1}{2}[\cos(a-b)-\cos(a+b)]$	S*SS
21	$\cos(a)\cos(b)=\frac{1}{2}[\cos(a-b)+\cos(a+b)]$	C*CS
22	$\sin(a)\cos(b)=\frac{1}{2}[\sin(a+b)+\sin(a-b)]$	S*CS
23	$\cos(a)\sin(b)=\frac{1}{2}[\sin(a+b)-\sin(a-b)]$	C*SS
24	$\sin(a)+\sin(b)=2\sin(\frac{a+b}{2})\cos(\frac{a-b}{2})]$	S+SP
25	$\sin(a)-\sin(b)=2\sin(\frac{a-b}{2})\cos(\frac{a+b}{2})$	S-SP
26	$\cos(a)+\cos(b)=2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$	C+CP
27	$\cos(a)-\cos(b)=-2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2})$	C-CP

Table 1: 27 Trigonometric Identities

TI84-CE Program

Each trig id is given a four letter abbreviation (see Table 1). These are jammed into Str0, the string variable 0; see lines 002-003 in Figure 1. The next lines of code, lines 004-031 then build the menu items 1 to 6; see Figure 2. A download of the program file is at: Randomization program and here's a link to a youtube video: (right click, new tab): Drill Videos.

```
ØØ1 randIntNoRep(1,27)→L1
ØØ2 "SINPSINMCOSPCOSMTANMTANPCDB1CDB2CDB3SDUB"→StrØ
ØØ3 StrØ+"TDUBCREDSREDTREDCHAFSHAFTHF1THF2THF3S*SS"→StrØ
ØØ4 StrØ+"C*CSS*CSC*SSS+SPS-SPC+CPC-CP"→StrØ
ØØ5 sub(StrØ,4(L1(1)-1)+1,4)+" "→Str1
ØØ6 Str1+sub(StrØ,4(L1(2)-1)+1,4)+" "→Str1
ØØ7 Str1+sub(StrØ,4(L1(3)-1)+1,4)+" "→Str1
ØØ8 Str1+sub(StrØ,4(L1(4)-1)+1,4)+" "→Str1
ØØ9 Str1+sub(StrØ,4(L1(5)-1)+1,4)+" "→Str1
Ø1Ø sub(StrØ,4(L1(6)-1)+1,4)+" "→Str2
Ø11 Str2+sub(StrØ,4(L1(7)-1)+1,4)+" "→Str2
Ø12 Str2+sub(StrØ,4(L1(8)-1)+1,4)+" "→Str2
Ø13 Str2+sub(StrØ,4(L1(9)-1)+1,4)+" "→Str2
Ø14 Str2+sub(StrØ,4(L1(1Ø)-1)+1,4)→Str2
Ø15 sub(StrØ,4(L1(11)-1)+1,4)+" "→Str3
Ø16 Str3+sub(StrØ,4(L1(12)-1)+1,4)+" "→Str3
Ø17 Str3+sub(StrØ,4(L1(13)-1)+1,4)+" "→Str3
Ø18 Str3+sub(StrØ,4(L1(14)-1)+1,4)+" "→Str3
Ø19 Str3+sub(StrØ,4(L1(15)-1)+1,4)→Str3
Ø2Ø sub(StrØ,4(L1(16)-1)+1,4)+" "→Str4
Ø21 Str4+sub(StrØ,4(L1(17)-1)+1,4)+" "→Str4
Ø22 Str4+sub(StrØ,4(L1(18)-1)+1,4)+" "→Str4
Ø23 Str4+sub(StrØ,4(L1(19)-1)+1,4)+" "→Str4
Ø24 Str4+sub(StrØ,4(L1(2Ø)-1)+1,4)→Str4
Ø25 sub(StrØ,4(L1(21)-1)+1,4)+" "→Str5
Ø26 Str5+sub(StrØ,4(L1(22)-1)+1,4)+" "→Str5
Ø27 Str5+sub(StrØ,4(L1(23)-1)+1,4)+" "→Str5
Ø28 Str5+sub(StrØ,4(L1(24)-1)+1,4)+" "→Str5
Ø29 Str5+sub(StrØ,4(L1(25)-1)+1,4)→Str5
Ø3Ø sub(StrØ,4(L1(26)-1)+1,4)+" "→Str6
Ø31 Str6+sub(StrØ,4(L1(27)-1)+1,4)+" "→Str6
Ø32 "RANDOMIZE"→Str7
Ø33 Menu("27 TRIG IDS",Str1,A,Str2,B,Str3,C,Str4,D,Str5,E,Str6,F,Str7,G,"QUIT",Q)
Ø34 Lbl A:Lbl B:Lbl C:Lbl D:Lbl E:Lbl F:Lbl G:prgmA:Lbl Q:Stop
```

Figure 1: Code for a TI84-CE program that randomizes our 27 trig ids.

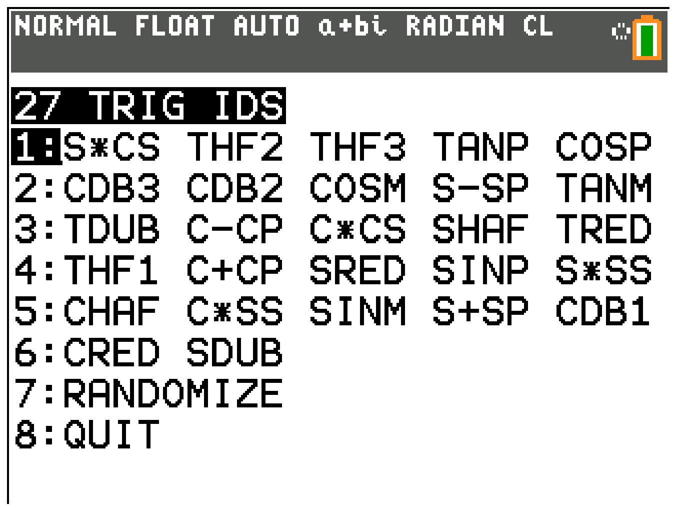

Figure 2: Screen capture showing 27 trig ids randomized (see Code column in Table 1).

Conclusion

I think I have finally memorized all 27 trig ids. I of course knew a long time ago the typical textbook presentations of these identities. They prove each one in turn, as if that should be enough to really get them into ones working memory as tools to be applied. It doesn't work. Complex numbers helps a lot to recall some ids, but not all of them.

This all begs the question. What is the best presentation of math formulas – math – that makes for the fastest retention. This doesn't seem to be ever brought up as a pedagogical topic.

This is in contrast to the use of statistics, taste tests and user groups that tweak burgers and word processing software. We can make hyperpalatable food and user friendly software, but at this juncture making memorable math escapes us. We don't even try. The fruit of making math sink in might be the elevation of human kind out of a default animal kingdom that has us eating and polluting till we choke: obesity and global warming are real.

References

[1] Blitzer, R. (2010). *Algebra and Trignometry*, 3rd ed., Pearson.