

Fizeau Experiment Revisited

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Abstract

In this paper an alternative explanation of the result of the Fizeau experiment with moving water has been presented.

Keyword : Fizeau experiment.

1 SPEED OF LIGHT IN A MOVING BODY

Let c_2 be the speed of a light ray (or a photon) in a stationary body (medium 2) and let $c_{2,m}$ be the speed of the light ray (or a photon) in the body (medium 2) when it is moving with a velocity \mathbf{v} . Then,

$$\left| \mathbf{c}_{2,m} - \mathbf{v} \right|^2 - \left| \frac{c_o}{c_1} \mathbf{c}_1 - \mathbf{v} \right|^2 = c_2^2 - c_o^2$$

where

c_o = speed of the light in vacuum

c_1 = velocity of the light in medium 1

2 FIZEAU EXPERIMENT

Let's calculate $\Delta c_{2,f}$ for a forward moving ray of light (moving along the direction of water flow) and $\Delta c_{2,b}$ for a backward moving ray of light (moving against the direction of water flow).

$$\begin{aligned} & \left| \mathbf{c}_{2,fm} - \mathbf{v} \right|^2 - \left| \frac{c_o}{c_1} \mathbf{c}_1 - \mathbf{v} \right|^2 = c_2^2 - c_o^2 \\ \Rightarrow & (c_{2,fm} - 7.059)^2 - (299792458 - 7.059)^2 = \left(\frac{3}{4} \times 299792458 \right)^2 - (299792458)^2 \\ \Rightarrow & c_{2,fm} = 224844341 \text{ ms}^{-1} \\ \Rightarrow & \Delta c_{2,f} = c_{2,fm} - c_2 = 224844341 - 224844343.5 = -2.5 \text{ ms}^{-1} \end{aligned}$$

$$\left| \mathbf{c}_{2,bm} - \mathbf{v} \right|^2 - \left| \frac{c_0}{c_1} \mathbf{c}_1 - \mathbf{v} \right|^2 = c_2^2 - c_o^2$$

$$\Rightarrow (c_{2,bm} + 7.059)^2 - (299792458 + 7.059)^2 = \left(\frac{3}{4} \times 299792458 \right)^2 - (299792458)^2$$

$$\Rightarrow c_{2,bm} = 224844346 \text{ ms}^{-1}$$

$$\Rightarrow \Delta c_{2,b} = c_{2,bm} - c_2 = 224844346 - 224844343.5 = 2.5 \text{ ms}^{-1}$$

Let

L = total distance traversed in water by a light ray (or a photon) in going from the source to an interference plane

t_{sw} = time spent in stationary water by a light ray (or a photon) in going from the source to the interference plane

c_{sw} = speed of the light ray (or a photon) in stationary water

λ_{sw} = wavelength of the light ray (or the stream of photons) in stationary water

Now, the time spent in air and glass (of the water tube) by a light ray (or a photon) in going from the source to the interference plane will be same for both a forward moving ray and a backward moving ray, so its effect gets cancelled out.

So, the fringe shift (additional optical path difference) for moving water as compared to stationary water in the Fizeau experiment

$$\delta = \frac{2|\Delta c_2| t_{sw}}{\lambda_{sw}}$$

$$\Rightarrow \delta = \frac{2}{\lambda_{sw}} \times |\Delta c_2| \times \frac{L}{c_{sw}}$$

$$\Rightarrow \delta = \frac{2}{\left(\frac{\lambda_o}{\mu_w} \right)} \times |\Delta c_2| \times \frac{L}{\left(\frac{c_o}{\mu_w} \right)}$$

$$\Rightarrow \delta = \frac{2L\mu_w^2 |\Delta c_2|}{\lambda_o c_o} = \frac{2 \times 2.974 \times \left(\frac{4}{3} \right)^2 \times 2.5}{(526 \times 10^{-9}) \times 299792458}$$

$$\Rightarrow \delta = 0.17$$

But the experimental fringe shift value, i.e., $\delta_e = 0.23$

The difference in the experimental value and the calculated value can be attributed to many factors such as :

- Lack of required precision in the measurement of the other parameters.
- Non-uniformity in the speed of the water flowing in the tube.
- Calculation limitation of the calculator.
- Lack of knowledge of the actual value of the speed ratio.

3 RESOLUTION BY SPEED RATIO

Let's define absolute speed ratio index as

$$\beta_i = \frac{c_i}{c_o}$$

$$\Rightarrow \frac{c_2}{c_1} = \left(\frac{\beta_2}{\beta_1} \right) = \beta_{21}$$

and $\beta_{\text{vacuum}} = 1$

Let's calculate $\Delta c_{2,f}$ for a forward moving ray of light and $\Delta c_{2,b}$ for a backward moving ray of light.

$$\left| \mathbf{c}_{2,fm} - \mathbf{v} \right|^2 - \left| \frac{c_0}{c_1} \mathbf{c}_1 - \mathbf{v} \right|^2 = c_2^2 - c_o^2$$

$$\Rightarrow (c_{2,fm} - 7.059)^2 - (299792458 - 7.059)^2 = (\beta_w \times 299792458)^2 - (299792458)^2$$

$$\Rightarrow \Delta c_{2,f} = c_{2,fm} - c_2 = c_{2,fm} - (\beta_w \times 299792458)$$

$$\left| \mathbf{c}_{2,bm} - \mathbf{v} \right|^2 - \left| \frac{c_0}{c_1} \mathbf{c}_1 - \mathbf{v} \right|^2 = c_2^2 - c_o^2$$

$$\Rightarrow (c_{2,bm} + 7.059)^2 - (299792458 + 7.059)^2 = (\beta_w \times 299792458)^2 - (299792458)^2$$

$$\Rightarrow \Delta c_{2,b} = c_{2,bm} - c_2 = c_{2,bm} - (\beta_w \times 299792458)$$

Now

$$\Delta c_{2,fm} \approx -\Delta c_{2,bm} \quad [\because v \ll c]$$

$$\Rightarrow |\Delta c_{2,fm}| = |\Delta c_{2,bm}| = |\Delta c_2|$$

So

$$\delta = \frac{2}{\lambda_{sw}} \times |\Delta c_2| \times \frac{L}{c_{sw}} = \frac{2}{\beta_w \lambda_o} \times |\Delta c_2| \times \frac{L}{\beta_w c_o}$$

$$\Rightarrow \delta = \frac{2L |\Delta c_2|}{\beta_w^2 \lambda_o c_o} = \frac{2 \times 2.974 \times |\Delta c_2|}{\beta_w^2 \times (526 \times 10^{-9}) \times 299792458}$$

$$\Rightarrow \delta = 0.038 \times \frac{|\Delta c_2|}{\beta_w^2}$$

Letting

$$\delta \approx \delta_e$$

$$\Rightarrow 0.038 \times \frac{|\Delta c_2|}{\beta_w^2} \approx 0.23$$

$$\Rightarrow \frac{|\Delta c_2|}{\beta_w^2} \approx 6.05$$

Now

$$\begin{aligned} (c_{2,fm} - 7.059)^2 - (299792458 - 7.059)^2 &= (\beta_w \times 299792458)^2 - (299792458)^2 \\ \Rightarrow (\beta_w \times 299792458 - 6.05\beta_w^2 - 7.059)^2 - (299792458 - 7.059)^2 &= (\beta_w \times 299792458)^2 - (299792458)^2 \\ \Rightarrow \beta_w &= 0.7 \end{aligned}$$

$$\begin{aligned} (c_{2,bm} + 7.059)^2 - (299792458 + 7.059)^2 &= (\beta_w \times 299792458)^2 - (299792458)^2 \\ \Rightarrow (\beta_w \times 299792458 + 6.05\beta_w^2 + 7.059)^2 - (299792458 + 7.059)^2 &= (\beta_w \times 299792458)^2 - (299792458)^2 \\ \Rightarrow \beta_w &= 0.7 \end{aligned}$$

$$\Rightarrow |\Delta c_2| \approx 6.05 \times 0.7^2 = 3 \text{ ms}^{-1}$$

It should be noted that β_{21} need not be equal to μ_{12} because :

1. $\beta_{21} = \mu_{12}$, as derived by Christiaan huygens' wave theory, is valid for a mechanical wave, but light is not a mechanical wave.
2. $\beta_{21} = \mu_{12}$, as derived in Fermat's principle of least time, need not be applicable for the path of a light ray as the concept of the path of least time is meaningless unless the destination point is predefined.

References

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