Rearranging the perfect binary tree: a proof of the Collatz Conjecture

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Abstract

The Collatz conjecture considers recursively sequences of positive integers where n is succeeded by $\frac{n}{2}$, if n is even, or $\frac{3n+1}{2}$, if n is odd. The conjecture states that for all starting positive integers n the sequence eventually reaches the trivial cycle 1, 2, 1, 2……The inverted Collatz sequences can be represented as a Collatz tree with 1 as its root node. In order to prove the Collatz conjecture, one must demonstrate that a Collatz tree covers all positive integers. In this paper, we construct a Collatz tree with 1 as its root node by rearranging the perfect binary tree. We prove that a Collatz tree is a connected tree and covers all positive integers.

1. Introduction

The Collatz conjecture considers recursively sequences of positive integers where n is succeeded by $\frac{n}{2}$, if n is even, or $\frac{3n+1}{2}$, if n is odd. The conjecture states that for all starting positive integers n the sequence eventually reaches the trivial cycle 1, 2, 1, 2……The inverted Collatz sequences can be represented as a Collatz tree with 1 as its root node. In order to prove the Collatz conjecture, one must demonstrate that a Collatz tree covers all positive integers. In this paper, we construct a Collatz tree with 1 as its root node by rearranging the perfect binary tree. In order to prove the Collatz conjecture, one must demonstrate that this tree covers all positive integers.[1].

2. Collatz tree with node 1 as its root node

A Collatz tree shown in Figure 1 is presented in [2]. This tree is arranged in levels 0 to ∞. There is only node 1 in level 0. For i $\geq$3, number of nodes in a level i is less than number of nodes in level i+1. There is no nontrivial cycle or divergence sequence in this tree. But in order to prove the Collatz conjecture, this tree must cover all positive integers

Figure 1. A Collatz tree

3. Rearranging the perfect binary tree

A perfect binary tree which covers all positive integers is shown in Fig. 2.

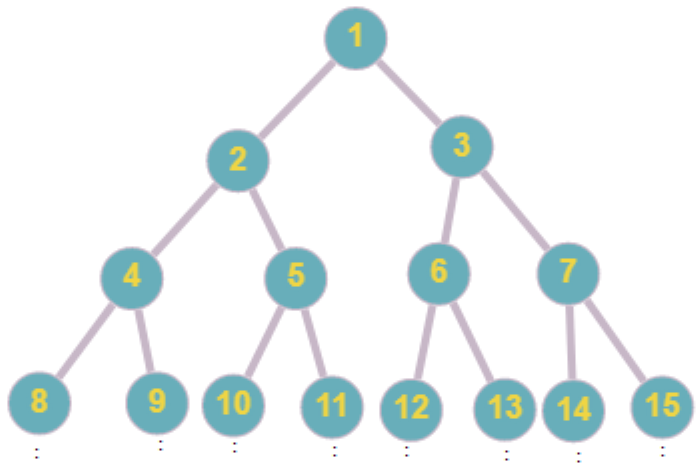

Figure 2. A perfect binary tree

A perfect binary tree can be rearranged to a Collatz tree according to the inverted Collatz sequences rules. Stages 1 - 5 of rearrangment are shown in Fig. 3 - 7.

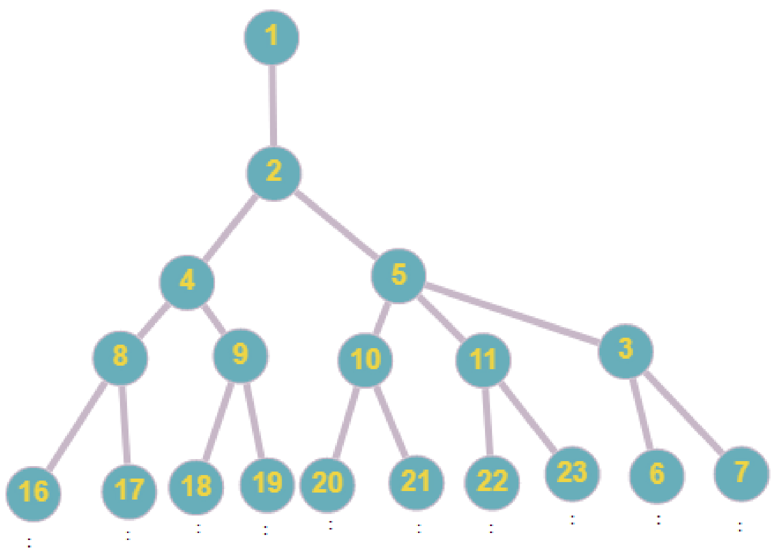

Figure 3. Stage 1 of rearrangement

Figure 4. Stage 2 of rearrangement

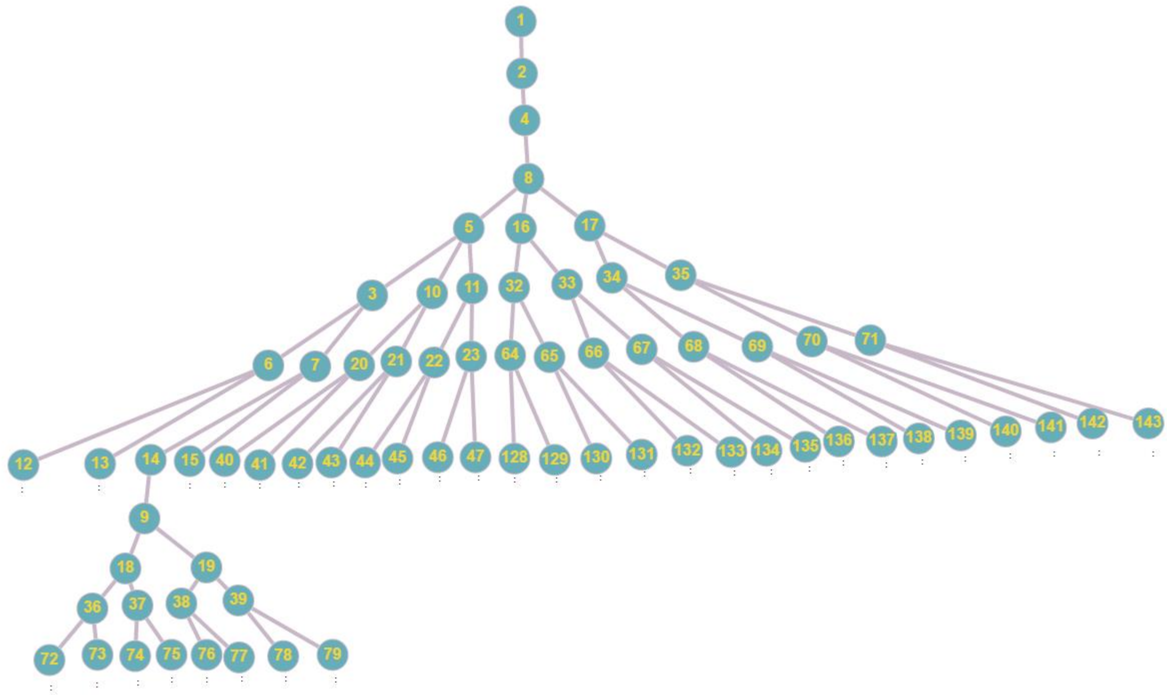

Figure 5. Stage 3 of rearrangement

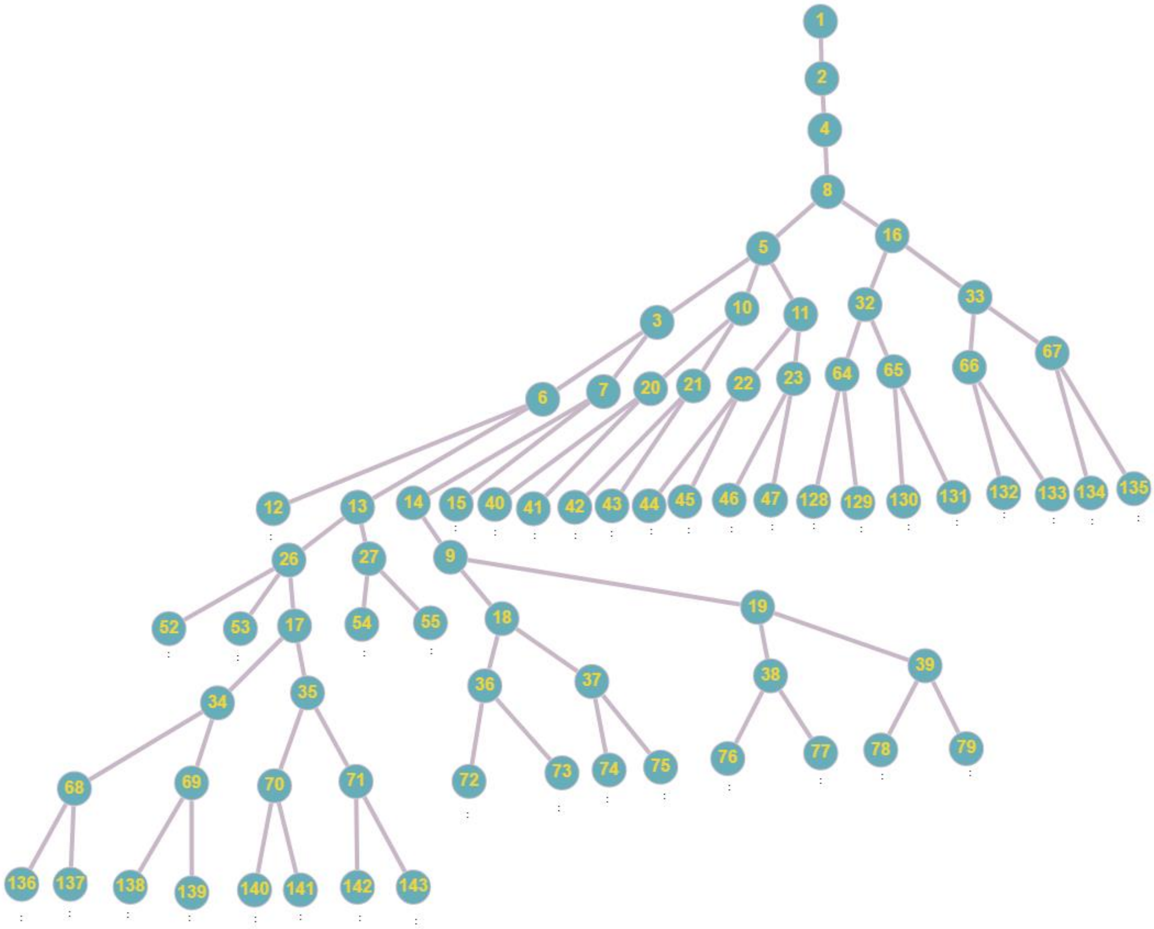

Figure 6. Stage 4 of rearrangement

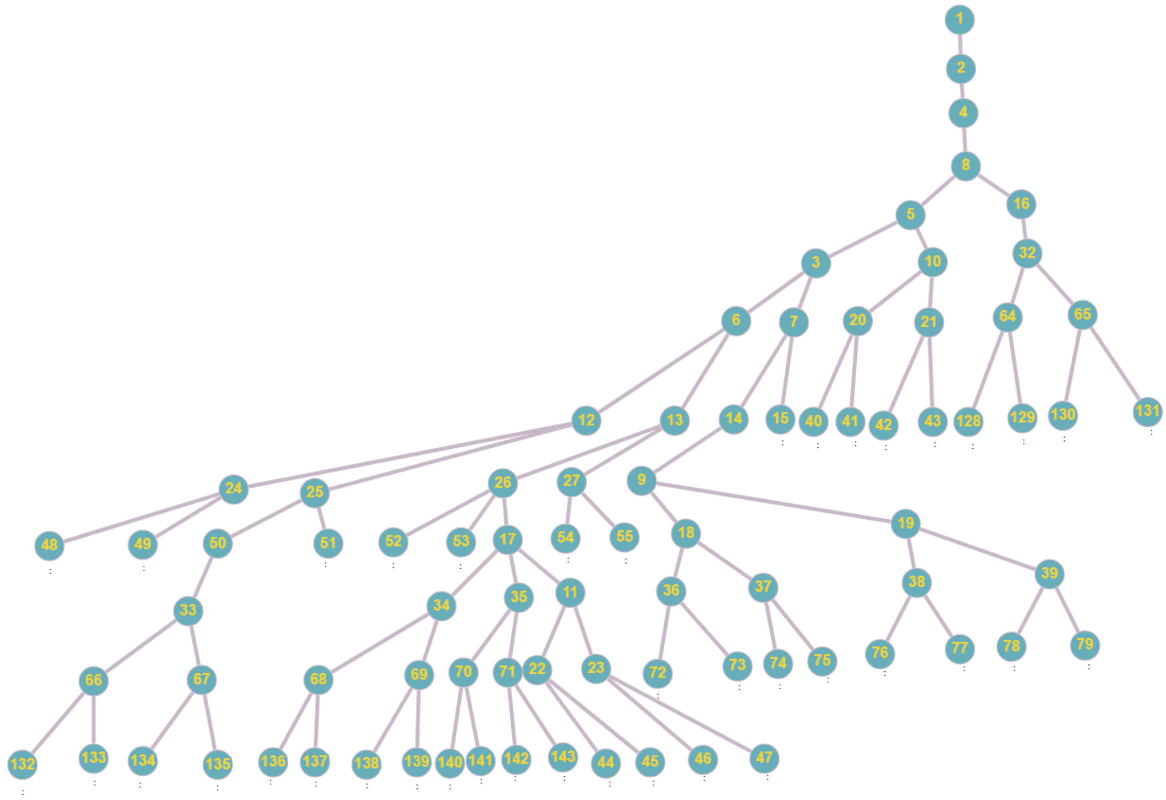

Figure 7. Stage 5 of rearrangement

By rearrangement of a perfect binary tree by following stage 1 through ∞ a perfect binary tree transformed to a Collatz tree.

4. Conclusion

Since nodes in a Collatz tree and a perfect binary tree are the same A connected Collatz tree shown in Figure 1 covers all positive integers. By starting at any node in a tree, there is a unique path from that node to a node 1.

References

[1] R . Terras, (1976). " A stopping time problem on the positive integers". Acta Arithmetica, 30(3), 241-252.

[2] W. Homsup and N. Homsup. " Proof of the Collatz conjecture". vixra: 2311.0105v2.