Elektrogravity Structure of a Particle

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Part: 1 Particle Evolution

Abstract: *Among all unanswered questions in Physics, like quantization of the gravity, fields unification, and so on, the question of the particle mass creation and its mass quantization deserve special attention. This paper is about the evolution of the physical particle-supported by the electro and gravity forces. Unexpectedly, electric charge and gravity unify into electro-gravity interaction in the presence of the Planck masses on a very fundamental level, making the electric charge, gravity, and mass inseparable entities. Moreover, electro-gravity necessarily polarises the Planck masses.*

Particles are created from the vacuum. The principle of the minimum of the static action creation introduces the evolution time derivative, which further defines the particle tangent space and its evolution kinetic energy. The particle self-interaction energy is its electro and gravity interaction energy. All of this is necessary and sufficient to write the first-order, nonlinear particle evolution equations for its electro and gravity interactions and the mass state variables.

The article contributes the particle polarization classification, two-dimensional particle internal geometry and its internal space and time. In addition, it contributes the implicit mass solutions for some particular interaction cases, and describes the character of the particle development in the general case. And finally, the article predicts the universal mass unit $1_{\mathrm{M}_o} = \mathbf{0.565}\left[meV\mathrm{C}^{-2}\right]$, *consistent with mass* $m_{\nu_e} = 1.17\,meV$, *of the lightest, the electron neutrino.*

Keywords: Electro-gravity, Planck masses, interaction time derivative, interaction variables, Planck particles, universal mass unit.

Sophisticated physical theories are developing to understand and explain many of the secrets of the resourceful nature of the physical particles. Special relativity, the theory of the space and time and the matter motion teaches that there is a particle energy minimum identical to the particle rest energy in the set $\mathbb{R}^+$. The experiments show that the particles are distinct objects distinguished by the discrete and fixed spectrum of the particle rest energies, and the set of all particles is countable.

All of this indicates that there is a natural law which systematically builds and distributes particles by their masses. However, the simplest and the most important questions: "What physical law builds and redistributes particles according to masses, and why the particles of the same kind have the same, strictly prescribed, masses" are without answer.

In Special Relativity, the theory of the motion of matter in the space and time, the particles are embedded objects of already a prescribed rest mass, and the Special relativity theory cannot tell anything about the particle structure and evolution. General relativity, the theory about the dynamics of the matter-energy and space-time does not tell anything, or we do not know it yet, about the structure of the particle and its mass formation.

Further, electrodynamics and quantum field theories acknowledge the discreetness of the particles and all their physical attributes in the already existing world, and do not have an inherited ability to say anything about the formation of the particles. All these theories are theories of motion of already formed particles and inherently not designed to reveal the reality of particles in the realm of the matter rest state.

All this suggests that there is a theory that describes the physical world of the particle rest state and that the world does not need to be at rest the way we expect. Thus, there may be a nontrivial theory of the particle evolution. This article is a naive attempt to construct such a theory. We are constructing physical particles supported only by the electric and gravity forces in the physical world spanned by the Planck universal constants: $\hbar, \mathrm{C}, \mathrm{G}$, and derived physical constants, the Planck charge, Planck mass, Planck length, and time

$$\mathrm{E}_p^2 = \hbar\mathrm{C}, \;\; \mathrm{M}_p^2 = \frac{\hbar\mathrm{C}}{\mathrm{G}}, \;\; \mathrm{L}_p^2 = \frac{\hbar\mathrm{G}}{\mathrm{C}^3}, \;\; \mathrm{T}_p^2 = \frac{\hbar\mathrm{G}}{\mathrm{C}^5}.$$

Observable electric charge $\mathrm{E}^2 = \alpha\hbar\mathrm{C} = \alpha\mathrm{E}_p^2$, where $\alpha = 1/137$ is the constant of the fine structure, its inverse $\overline{\alpha} = 1/\alpha = 137$, is smaller than the Planck charge. Since $\mathrm{E}_p^2 = \mathrm{M}_p^2\mathrm{G}$ it follows that $\mathrm{E}^2 = \alpha\hbar\mathrm{C} = \alpha\mathrm{M}_p^2\mathrm{G} = \mathrm{M}_{\mathrm{M}}^2\mathrm{G}$ and $\mathtt{M}_{\mathrm{M}} = \sqrt{\alpha}\mathrm{M}_p$ is the Maxwell mass of the electron. Numerical value of the Planck mass is $\mathtt{M} = 2.18 \cdot 10^{-5}$ gr so that the Maxwell mass of the electron is $\mathtt{M}_{\mathtt{M}} = 1.86 \cdot 10^{-6}$ gr.

The gravity radius of a massive particle m is $\mathrm{R}_g = 2m\mathrm{G}\mathrm{C}^{-2}$, and every particle collapses under its gravity if its mass is squeezed in a sphere of a radius $r < \mathrm{R}_g$. Thus, the condition of the collapse of the particle (m, r) is

$$\frac{\mathrm{R}_g}{r} = \frac{2m\mathrm{G}\mathrm{C}^{-2}}{r} = \frac{2m\mathrm{G}}{\mathrm{C}^2 r} = \frac{\mathrm{G}}{\mathrm{C}^2}\frac{2m}{r} < 1 \;\; \Rightarrow \;\; \frac{m}{r} < \frac{\mathrm{C}^2}{2\mathrm{G}}.$$

The gravity radius of the light is zero, and the massive bodies have huge gravity radiuses. Notice that the gravity radius of the Planck mass is double Planck length, and the pair $(\mathtt{M}, \mathrm{L}_p)$ cannot be a particle. The infimum of the particle pairs (m, L_p), of masses $m < \mathtt{M}$ that satisfies the non-collapse condition is the zero mass particle. Thus, we do not know if there is a nontrivial mass minimum of such particles. However, we may answer an essential question: "How much of the mass may be stored in a sphere of radius equal to the Planck length?" We will answer the question in the physical world supported by the fundamental Planck constants.

Corollary 0.1. *There is no particle of the Planck mass and radius of the Planck length. The largest particle of the radius equal to the Planck length may have at most mass* $\mathtt{M}_{:2} = \mathtt{M}/2$

□ The particle, pair $(\mathtt{M}, r)$ $r = \mathrm{L}_p$ has gravitation radius

$$\mathrm{R}_g = 2\mathrm{M}\mathrm{G}\mathrm{C}^{-2} = 2\mathrm{L}_p > r = \mathrm{L}_p.$$

Thus, the non-collapsing particle of the Planck mass must have a radius greater than the Planck length, and

$$\sup_m\{(m, \mathrm{L}_p\} = \mathtt{M}_{:2} = \mathtt{M}/2.$$

Consequently, the only particles of the Planck length radius are of the masses $m \leq \mathtt{M}_{:2}$. ■

Definition 0.2. *The mass* $\mathtt{M}_{:2} = \mathtt{M}/2$ *is the reduced Planck mass. The particle* $\Pi_p \sim (\mathtt{M}_{:2}, \mathrm{E}_p)$, $\mathrm{E}_p = \sqrt{\hbar\mathrm{C}}$ *is the Planck electron and* $\Pi_{\mathtt{M}} \sim (\sqrt{\alpha}\mathtt{M}_{:2}, \mathrm{E}))$, $\mathrm{E} = \sqrt{\alpha\hbar\mathrm{C}}$ *the Maxwell electron.*

The following sets the base of the Physical world based on the Planck constants.

> **A1:** *The smallest natural length unit is the Planck length* $\mathrm{L}_p = 1.612 \cdot 10^{-33}\,cm$*, and the fundamental base of the Physical world is the reduced Planck triplet* $\{\mathrm{L}_p, \mathrm{T}_p, \mathtt{M}_{:2}\}$.

Comments: *The task of the article is to find a possible description of the physical particle evolution. The change of the particle mass per unit of the electric charge may be estimated on the experimental masses of the particles appearing as the charged and neutral, if* N_a *is the Avogadro number,*

$$\triangle m \approx 7.72 \cdot 10^{-27} \approx \frac{3}{2\alpha}\frac{\mathrm{M}_p}{\mathrm{N}_a} \text{ gr.}$$

We assume that the physical particles form by the extraction from the vacuum. The vacuum is void space equipped with electro and gravity interactions, energy/masses, and other physical properties. Regardless of whether we think the particle forms over or in the vacuum, the particle distance from the vacuum is measured by the vector $\vec{r}$ *characterized by the size* $|\vec{r}| = r$. *The level of the particle extraction is measured by the instant statical action* $\vec{S} = \int_0^t m\vec{r}\,d\tau$ *in the presence of the electro-gravitational interactions or the general charge* $\mathbf{q} \sim (q, g)$. *We remark that the role of gravity in the formulation of the the existing quantum theory of the fields and particles plays a negligible role, and that it may play a significant role in the particle formation and evolution.*

0.1 Physical Particle

Physical object characterized by a mass m and a space configuration bounded by a sphere of a radius R in the presence of the electro and gravity interaction (q, g) is the physical particle. The particle is at rest, such is its center of the masses, and its duration is measured by its internal time τ, related to the observable time by a function $\tau = \tau(t)$.

> **A2:** *Electro-gravitational interaction is the entity* (q, g) *defined by the elementary charge* E *and the Newton gravitational constant* G *with the relations* $\mathrm{q} = \chi\mathrm{E}$ *and* $\mathrm{g} = \gamma^2\mathrm{G}$. *The functions* γ *and* χ *are intensities of the electro and the gravity interactions, and the coefficient* $\mathrm{A}_{\chi\gamma} = \chi^2\gamma^{-2}$ *is the relative strength of the electro-gravitational interactions.*
> *The Planck mass function* μ_p *is the representative of the equivalence class* $[\mathrm{g}, \mathrm{q}]$ *of the interaction set* $\mathrm{g} \otimes \mathrm{q}$ *such that*
>
> $$d_{\mathrm{g}}\mathrm{q}^2 = \pm\mu_p^2, \quad \forall \mu_p. \tag{A1}$$
>
> *The differential equation (A1) and its solutions*
>
> $$\mathrm{q}^2 \mp \mu_p^2\mathrm{g} = \mathrm{q}_{\mathrm{o}}^2 \in \mathbb{R}, \quad \forall \mu_p \tag{A1*}$$
>
> *are the canonical constitutional equations of the electro-gravity.*

Interactions and charges are related. For, $q_g^2 = \mu_p^2 g$, is the gravity in the charge representation so that $\mathbf{q} = (q, q_g) \sim (\mathrm{q}^2, \mu_p^2 g)$ is the general charge in the electric charge representation, and further, we may say either electro-gravitational interaction or general charge. The q_{o} is the proto charge, and $g_o = \mu_p^{-2}\mathrm{q}_{\mathrm{o}}^2$ is the proto charge gravitational measure.

While the electro and gravitational interactions have physical dimensions of the electric charge and gravitational constant, the constitution equations have the dimension of the electric charge square. The gravity electric charge equivalence $\mathrm{q}_g^2 = \mu_p^2 g$ identifies the gravity interaction and the gravity charge q_g^2, and the electro and gravity interactions (q, g) are the interaction properties of a particle, and may be joined to the particle's internal variables. Thus, the particle is $(m, R; q, g)$ at the rest at the coordinate origin. Further on, the electro-gravitational interaction and Planck masses are inseparable, and the triplet (q, μ_p, g) is the fundamental entity of the particle interaction.

At this place, we introduce the interaction operators, projectors $(\hat{q}, \hat{g}, \hat{\mu_p})$ of the interaction to the particle electro, gravitational, and mass sectors, and

$$\hat{q}^2 \hat{g}^{-1} = \hat{\mu_p}^2, \quad \hat{g} = \hat{\mu_p}^2 \hat{q}^{-2}, \quad \hat{q}^2 = \hat{\mu_p}^2 \hat{g}^{-1}. \tag{A}$$

Comments: *Two signs of the charge gravitation gradient in the electro-gravity constitution relations will be understood as the electro-gravity polarization of the Planck masses. We assume that* $\mathrm{q}_\mathrm{o}^2 \in \mathbb{R}$*, and require* $\mathrm{q}^2 \geqslant 0$. *In the case of the zero proto-electro charge, there are exactly two opposite solutions for the electric charge square. Conditionally, the solution with the negative sign we will call the antigravity solution.*

Planck Mass Polarization

Planck masses are polarized, and the gravity and antigravity solutions of the constitutional equation are just contributions of the polarization of the Planck mass square μ_p^2 or the rest energy square $\mu_p^2 \mathrm{c}^4$ by the electro-gravity. We introduce the gravity polarization $\sigma_g = (1, -1)$, the negative sign is for the antigravity, and write the final constitution equation

$$\mathrm{q}^2 - \sigma_g \mu_p^2 \mathrm{g} = \mathrm{q}_\mathrm{o}^2 \in \mathbb{R}. \tag{A0*}$$

The following corollary introduces/constructs the polarized masses.

Corollary 0.3. *The gravity* m_σ*, proto* $m_{\sigma p}$*, and totally polarized masses* $\overline{\mathsf{m}}_\sigma$ *are*

$$\mathsf{m}_\sigma \ : \ \mathsf{m}_\sigma^2 = \sigma_g \mu_p^2, \tag{A1*}$$

$$m_{\sigma p} : \ m_{\sigma p}^2 = \sigma_g \beta - 2\mu_p^2, \tag{A2*}$$

$$\overline{\mathsf{m}}_\sigma \ : \ \overline{\mathsf{m}}_\sigma^2 = \sigma_g \Phi \mu_p^2, \tag{A3*}$$

$$\Phi \ = (1 - \beta^2)^{-1}, \tag{A4*}$$

where $\beta^2 = \mathrm{q}_\mathrm{o}^2 \mathrm{q}^{-2} \in \mathbb{R}$ *is the coefficient of the proto charge, and* Φ *the polarization function. The polarizations add according to the polarization mass reciprocal rule*

$$\frac{1}{\mathsf{m}_\sigma^2} - \frac{1}{m_{\sigma p}^2} = \frac{1}{\overline{\mathsf{m}}_\sigma^2} \quad \Leftrightarrow \quad \overline{\mathsf{m}}_\sigma^2 = \frac{\mathsf{m}_\sigma^2 m_{\sigma p}^2}{m_{\sigma p}^2 - \mathsf{m}_\sigma^2}.$$

A mass is principally polarized if it is in the particle positive, $\sigma_g = +1$ *sector. The same mass is fundamental if its polarization is reduced to gravity polarization only.*

□ We introduce the polarized mass by the projector $\hat{g}$ of the equation (A0*) into the mass sector. Thus

$$\begin{aligned}
& \mathrm{q}^2 g^{-1} = \sigma_g \mu_p^2 + \hat{g}^{-1} \mathrm{q}_\mathrm{o}^2 = \ \sigma_g \mu_p^2 + \mathrm{q}_\mathrm{o}^2 \mathrm{q}^{-2} \cdot \mathrm{q}^2 \hat{g}^{-1} \\
\Rightarrow \quad & \overline{\mathsf{m}}_\sigma^2 = \mathsf{m}_\sigma^2 + \beta^2 \overline{\mathsf{m}}_\sigma^2 \ \Rightarrow \ \overline{\mathsf{m}}_\sigma^2 (1 - \beta^2) = \mathsf{m}_\sigma^2 \\
\Leftrightarrow \quad & \frac{1}{\mathsf{m}_\sigma^2} - \frac{\beta^2}{\mathsf{m}_\sigma^2} = \frac{1}{\overline{\mathsf{m}}_\sigma^2} \ \Leftrightarrow \ \frac{1}{\mathsf{m}_\sigma^2} - \frac{1}{m_{\sigma p}^2} = \frac{1}{\overline{\mathsf{m}}_\sigma^2} \\
\Rightarrow \quad & \overline{\mathsf{m}}_\sigma^2 = \frac{\mathsf{m}_\sigma^2 m_{\sigma p}^2}{m_{\sigma p}^2 - \mathsf{m}_\sigma^2} = \frac{\mathsf{m}_\sigma^2}{1 - \mathsf{m}_\sigma^2 m_{\sigma p}^{-2}} = \frac{\mathsf{m}_\sigma^2}{1 - \beta^2} = \mathsf{m}_\sigma^2 \Phi = \ \sigma_g \Phi \mu_p^2.
\end{aligned}$$

■

Remark: Electro-gravity interaction in the creation of the physical particles necessarily polarizes masses, and classifies all particles in the class of the *totally polarized particles* $\overline{\mathcal{P}} = [\mathbf{m}, \beta, \pm\mathbf{1}]$, the class of only the *gravity polarized particles* $\mathcal{P} = [\mathbf{m}, \mathbf{0}, \pm\mathbf{1}]$. The particles polarized in the positive gravity sector make the *principal class* $\overline{\mathcal{P}}_{+1} = [\mathbf{m}, \beta, +\mathbf{1}]$ class, and those there which are only the gravity polarized do make the *fundamental particle class* $\mathcal{P}_{+1}[\mathbf{m}, \mathbf{0}, +\mathbf{1}]$.

Remark: The electro-gravity constitutional equations in the polarized masses representation are

$$\mathrm{q}^2 = \overline{\mathsf{m}}_\sigma^2 g = \sigma_g(1-\beta^2)^{-1}\mu_p^2\, g = \mu_p^2 g\Phi. \tag{A5*}$$

We notice also that the prime charge operator $\hat\beta$ transforms the gravity-polarized mass into total polarized mass and

$$\hat\beta\colon\ \mathsf{m}_\sigma^2 \to \mathsf{m}_\sigma^2\,(1-\beta^2)^{-1} = \overline{\mathsf{m}}_\sigma^2.$$

Parametrization of the Constitutional Equations

Further, we use elementary charge E, gravity constant G and dimensionless interaction entities χ and γ and introduce gravitational charge $\mathrm{q_g}$ to unify into electro-gravity interaction in the representation of the electric charge.

Definition 0.4. *Gravitational charge is the electric charge equivalent of the gravity interaction, and*

$$\mathrm{q_g}:\quad \mathrm{q_g^2} = \mathrm{G}\mu_p^2\gamma^2.$$

Further on, the general or the electro-gravity charge is the pair $\tilde q = (\mathrm{q}, q_{\mathrm{g}})$. After this the canonical interaction equations in the electro-charge representation is

$$\mathrm{q}^2 \mp \mathrm{q_g^2} = \mathrm{q_o^2} \in \mathbb{R}.$$

We are showing that the electro-gravity equation has only three acceptable realizations. Depending on the sign of the $\mathrm{q_o^2}$ there are following four cases:

$$\begin{array}{ll} & \mathrm{q}^2 \mp \mathrm{q_g^2} = \mathrm{q_o^2} \in \mathbb{R} \\ \Leftrightarrow & \mathrm{q}^2 \mp \mathrm{q_g^2} = +\mathrm{q_o^2} \geq 0 \quad \Rightarrow \quad \mathrm{q}^2 - \mathrm{q_g^2} = +\mathrm{q_o^2} \ \cdots (\mathbf{a}); \quad \mathrm{q}^2 + \mathrm{q_g^2} = +\mathrm{q_o^2} \ \cdots (\mathbf{b}) \\ & \mathrm{q}^2 \mp \mathrm{q_g^2} = -\mathrm{q_o^2} \leq 0 \quad \Rightarrow \quad \mathrm{q}^2 - \mathrm{q_g^2} = -\mathrm{q_o^2} \ \cdots (\mathbf{c}); \quad \mathrm{q}^2 + \mathrm{q_g^2} = -\mathrm{q_o^2} \ \cdots (d) \end{array}$$

All cases except the (d) case are real. Further, the cases (a) and (c) with interchanged roles of the electro and gravity interactions are hyperbolic, and only the case (b) is elliptic. All solutions are

$$\begin{array}{ll} \mathrm{q} \vee \mathrm{q_g} = \mathrm{q_o}\cosh\theta,\ \ \mathrm{q_g} \vee \mathrm{q} = \mathrm{q_o}\sinh\theta, & -\infty < \theta < \infty, \\ \mathrm{q} = \mathrm{q_o}\cos\theta, \quad \mathrm{q_g} = \mathrm{q_o}\sin\theta, & 0 \leqslant \theta < 2\pi. \end{array}$$

We introduce $\alpha_p = \mathtt{M}^{-2}\mu_p^2$, $\chi_o^2 = \mathrm{q_o^2}\mathrm{E}^{-2}$, and present step-by-step transformation of the canonical electro-gravity constitution equation to the representation of the interaction intensities

$$\begin{array}{lll} \mathrm{q}^2 \mp \mathrm{q_g^2} = \mathrm{q_o^2} & \xrightarrow{\times \mathrm{E}^{-2}} & \chi^2 \mp \mathrm{GE}^{-2}\gamma^2 = \mathrm{E}^{-2}\mathrm{q_o^2} \quad \cdots \\ \cdots & \xrightarrow{\mathrm{E}^{-2} = (\alpha\hbar\mathrm{C})^{-1}} & \chi^2 \mp \mathrm{G}(\alpha\hbar\mathrm{C})^{-1}\mu_p^2\,\gamma^2 = \mathrm{E}^{-2}\mathrm{q_o^2} \quad \cdots \\ \cdots & \xrightarrow{\mathrm{G}(\alpha\hbar\mathrm{C})^{-1} = \mathtt{M}^{-2}} & \chi^2 \mp \bar\alpha\alpha_p\,\gamma^2 = \chi_o^2. \end{array}$$

Particle Interaction Energy

The particle interaction energy is the sum of the electro and gravity self-interaction energies. Exactly, for each $a \in \{\mathrm{q}, g\}$, the particle self-interaction energy is the sum of the potential energies between all of its parts a' and a'' separated for an r. We use the theorem of the summation average to find

$$E_a = \sum_{a',a''}^{a,a} \frac{a'a''}{r} = a \cdot \sum_{a'}^{a} \frac{a'}{\bar{\mathrm{R}}} = \frac{a^2}{R}. \tag{1}$$

A3: *At each moment the physical particle is an object of the high symmetry geometry on the $\mathbb{R}^4_{1,}$ space characterized by its mass m or the rest energy $m\mathrm{c}^2$, gravity* g *and electro* q *interactions and single space size variable R. The self-interaction energies of the particle are its electro and gravity potential energies*

$$E_{\mathrm{q}} = \frac{q^2}{R}, \quad E_{\mathrm{g}} = -\frac{m^2 g}{R}. \tag{B1}$$

and their sum

$$E_{\mathrm{gq}} = E_{\mathrm{g}} + E_{\mathrm{q}} = -\frac{m^2 g}{R} + \frac{q^2}{R}. \tag{B2}$$

is the particle interaction energy.

Shortly, the particle definition is global in the sense that all fine details of the natural laws reduce to an object of a mass m and a charge q, call it the particle of a radius R at a single point of the $\mathbb{R}^4_{1,}$ space. The particle mass, rest energy, radius electro, and gravitational energies are the particle's internal properties. Physical particles are all above. Finally, all particle physical variables are the interaction (q, g, μ_p) and the particle internal (m, $\mathbf{q}$, R) variables.

0.2 Particle Evolution Equations

Physical particle is the global object of prescribed mass/energy, electric charge, size, etc, placed at a single point in the space-time $\mathbb{R}^3 \times \mathbb{R}^1$. In this part of the article, we will formulate the particle evolution equations. First, we will give an intuitive motivation to define the essential evolution time derivative, which is not to be understood as the evolution derivative derivation.

We understand that the particle is extracted from the vacuum in or over the vacuum.

We assume that : ***1****. The mass μ_o of the vacuum directly involved in the particle creation is much larger than the particle mass, and* ***2****. That the extraction is negligibly slow.*

Further on, the particle of a mass $m(\tau)$ and a radius $R(\tau)$ creates in a time interval $[0, t]$. According to our original setting the particle at each moment of its creation is an object of high symmetry, and the measure of its creation is a function of its mass and radius only. We choose the creation function to be the particle static state formation action

$$\hat{\mathbf{m}}(\text{vacum}) = \quad S(t) \quad = \int_0^t (\mu_o - m) R \, d\tau.$$

The particle evolution is defined by the following variational principle and above vacuum conditions:

A4: *Evolution trajectories of the physical particle are the extremals of static action functional $S(t)$ with fixed boundaries, and the zero variable variations at the boundaries.*

Under all enumerated conditions with the $\delta\tau = 0$, the extremals must satisfy the variational equation:

$$\delta S(t) = \int_0^t [(\dot{\mu}_o - \dot{m})R + (\mu_o - m)\dot{R}]\delta\tau\, dt = 0$$

With the vacuum imposed conditions $\dot{\mu}_o \approx 0,\ \ \mu_o \gg m$ the under-integral function must be zero

$$\therefore \quad (\dot{\mu}_o - \dot{m})R + (\mu_o - m)\dot{R} = 0 \quad \approx \quad \dot{m}R - \mu_o\dot{R} = 0 \quad \Rightarrow \quad d_\tau R = \mu_o^{-1} d_\tau m\, R = \quad \dot{m}\mu_o^{-1}\, R.$$

Definition 0.5. *The evolution time derivative is the multiplication operator $\hat{\mathrm{D}}_\tau = \mu_{\mathrm{BC}}^{-1}\dot{m}\, \times$ defined on the particle mass and the mass μ_{BC} of its background, with the properties that for all functions A, B and $f(A)$ defined on the particle state:*

$$\begin{aligned}
\hat{\mathrm{D}}_\tau A \quad &\equiv \hat{\mathrm{D}}_\tau(m, A) = \mu_{\mathrm{BC}}^{-1} A\; d_\tau m = \mu_{\mathrm{BC}}^{-1}\dot{m}\, A, \\
\hat{\mathrm{D}}_\tau AB \quad &= \hat{\mathrm{D}}_\tau A\, B + A\, \hat{\mathrm{D}}_\tau B, \\
\hat{\mathrm{D}}_\tau f(A) \ &= \ f'_A \hat{\mathrm{D}}_\tau A, \\
\hat{\mathrm{D}}_\tau^{\,2} A \quad &= \hat{\mathrm{D}}_\tau(\hat{\mathrm{D}}_\tau A).
\end{aligned} \tag{ED}$$

The evolution time derivative is sufficient to construct the particle tangent space, its kinetic energy, and the particle evolution equations.

Definition 0.6. *The tangent space of the particle is two-dimensional space defined by the particle linear v and angular ω velocities*

$$\begin{aligned}
v &\equiv \hat{\mathrm{D}}_\tau R = \mu_{\mathrm{BC}}^{-1}\dot{m}\, R, \\
\omega &\equiv \hat{\mathrm{D}}_\tau \varphi = \mu_{\mathrm{BC}}^{-1}\dot{m}\, \varphi,
\end{aligned}$$

and the particle kinetic energy is

$$T \ \ \equiv \ \frac{k}{2}\, m(\hat{\mathrm{D}}_\tau R)^2 + \frac{k}{2}\, mR^2(\hat{\mathrm{D}}_\tau \varphi)^2 = \quad \frac{k}{2\mu_{\mathrm{BC}}^2}\, \dot{m}^2 mR^2(1 + \varphi^2).$$

The particle evolution proceeds in the presence of electro-gravitational force only, and the particle energy is the sum of its kinetic and interaction energy and

$$E \equiv T + E_{\mathrm{gq}} \ = \ \frac{k}{2\mu_{\mathrm{BC}}^2}\, \dot{m}^2 mR^2(1 + \varphi^2) + \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R}. \tag{2}$$

The evolution forces and torques consistent with the particle evolution energy are the generalized forces M, F :

$$\begin{aligned}
\mathsf{M} &\equiv \partial_\varphi T = \frac{k}{\mu_{\mathrm{BC}}^2}\, \dot{m}^2 mR^2 \varphi, \\
\mathsf{F} &\equiv \partial_{R} T = \frac{k}{\mu_{\mathrm{BC}}^2}\, \dot{m}^2 mR(1 + \varphi^2) - \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R^2}.
\end{aligned}$$

Corollary 0.7. *Non-rotating evolving particles achieve the energy minimum at the static equilibrium, and its size and energy are the following mass functions*

$$R \quad = \frac{3}{2}\, \frac{\mathrm{q}^2 - \mathrm{g}m^2}{E} \tag{3}$$

$$\mu_{B}^2 E^3 \quad = \dot{m}^2 m(\mathrm{q}^2 - \mathrm{g}m^2)^2. \tag{4}$$

$\square$ Conditions for the particle evolution energy extremum on the evolution space $\mathrm{R} \otimes \varphi$ are exactly the generalized forces equilibrium conditions $M = 0,\ F = 0$. For, the energy extremum necessary conditions are

$$\begin{aligned} \partial_\varphi T &\equiv \mathsf{M} = \frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R^2 \varphi = 0, \\ \partial_{R} T &\equiv \mathsf{F} = \frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R (1+\varphi^2) - \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R^2} = 0. \end{aligned}$$

Since there are no external torques, the rotation equilibrium condition $\mathsf{M} = 0$ is satisfied at $\varphi = 0$. The particle creates a non-rotational state. Further we use the equilibrium condition $\mathsf{F} = 0$ at $\varphi = 0$

$$\therefore\quad \frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R - \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R^2} = 0 \quad\Rightarrow\quad 2T \equiv \frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R^2 = \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R}. \tag{5}$$

We are confirming the minimum of the particle evolution energy on its evolution space. Exactly,

$$E = \frac{k}{2\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R^2 + \frac{\mathrm{q}^2 - \mathrm{g}m^2}{R} = 3T, \tag{6}$$

$$\partial_{R} E = \partial_{R} 3T = 3 \cdot 2T\, R^{-1} \sim R \;\Rightarrow\; \partial_{RR} E > 0. \tag{7}$$

Thus, the particle evolution energy achieves minimum $E = 3T$ on its evolution space.
Now, we are showing (3) and (4) statements. We use $T = E/3$ in the equation (5) to find the particle radius

$$R = \frac{\mathrm{q}^2 - \mathrm{g}m^2}{2T} = \frac{3}{2}\,\frac{\mathrm{q}^2 - \mathrm{g}m^2}{E},$$

which confirms the equation (3). Further, we substitute the explicit form of the kinetic energy of non-revolving particles in the $E = 3T$ to find

$$\begin{aligned} E &= 3T = 3 \cdot \frac{k}{2\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m R^2 = \frac{3}{2}\,\frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m \cdot \Big(\frac{3}{2}\,\frac{\mathrm{q}^2 - \mathrm{g}m^2}{E}\Big)^2 \\ \Rightarrow\qquad & E^3 = \frac{9}{8}\,\frac{k}{\mu^2_{\mathrm{BC}}}\,\dot{m}^2 m \cdot \big(\mathrm{q}^2 - \mathrm{g}m^2\big)^2. \end{aligned}$$

We introduce $\mu^2_{B} = \frac{3}{2}\frac{k}{\mu^2_{\mathrm{BC}}}$ and the last equation is equivalent to the equation (4). Undefined factor k is absorbed in the free mass factor μ_{B}. $\blacksquare$

Comments: *Altogether, the particle evolution description involves six variables, the interaction variables* (q, g, μ_p), *and three particles internal variable* (m, E, R). *We have at our disposal only three independent equations, constitutional equation (A1*), and the particle size and energy equations (3) and (4), and the particle description system of the equations is not complete. In the next section, we will give the explicit formulation of the evolution equations and finally make their rest energy completion.*

0.3 The Rest Energy Completion

The particle description system of the equations is not complete, and at least one additional equation without the introduction of the new new variables are necessary. At this moment, the particle interaction variables (m, E, R). are independent, and the only natural particle evolution equations completion is the following:

A6: *The evolution of non-revolving physical particles proceeds at its evolution rest state, and, at each moment of its evolution all the particle energy is its rest energy .*

The **A6** completes the system of the particle evolution equations (3) and (4), and energy substitution $E = m\mathrm{C}^2$ reduces the set of all independent variables to the interaction variables $\{m, R; \mathrm{q}, g\}$, and finally

$$
\begin{aligned}
(3) \implies \quad & R && = 3 \cdot 2^{-1}\mathrm{C}^{-2}\, m^{-1}(\mathrm{q}^2 - \mathrm{g}m^2). && (8)\\
(4) \implies \quad & \mu_{\scriptscriptstyle B}^2 m^3\mathrm{C}^6 && = \dot{m}^2 m(\mathrm{q}^2 - \mathrm{g}m^2)^2 &&\\
& \Leftrightarrow && \pm\mu_{\scriptscriptstyle B}\mathrm{C}^3 = \dot{m}\, m^{-1}(\mathrm{q}^2 - \mathrm{g}m^2). &&\\
& \Rightarrow && m - \textbf{evolucion particle equations:} &&\\
& m && = 0, && (9)\\
& \pm\mu_{\scriptscriptstyle B} m^2\mathrm{C}^3 && = \dot{m}\, m^{-1}(\mathrm{q}^2 - \mathrm{g}m^2) \qquad - \textbf{dual solutions}. && (10)
\end{aligned}
$$

The particle radius is the algebraic function of the particle mass, R-evolution equation, and the particle mass m the first-order differential, - m-evaluation equations. The m-evaluation equations have the $m = 0$ massless solution and the dual solutions of the first-order differential equations.

<u>**Remark**</u>: *While two mass polarizations are associated with gravity-antigravity, the duality is associated with the particle-anti particle properties of the physical particles.*

It is clear at the first look that the function $X = \mathrm{q}^2 - \mathrm{g}m^2)m^{-1}$, or its associated differential form Xdm, is central to the particle evolution equations, and that it deserves particular attention. Since $X\,dm = \mathrm{q}^2 - \mathrm{g}m^2)m^{-1}dm \sim \dot{m}\, m^{-1}(\mathrm{q}^2 - \mathrm{g}m^2)$, the complete system of the particle evolution equations are

$$
\begin{aligned}
0 &= m, && \text{(E0)}\\
0 &= dm\; X(m, \mathrm{q}, g) \mp \mu_{\scriptscriptstyle B}\mathrm{C}^3\, d\tau. && \text{(E1)}\\
R &= 3 \cdot 2^{-1}\, \mathrm{C}^{-2}\, X(m, \mathrm{q}, g), && \text{(E2)}
\end{aligned}
$$

equations, completed by the electro-gravity constitutional equations

$$
\begin{aligned}
&\mathrm{q}^2 - \sigma_g\mu_p^2\mathrm{g} = \mathrm{q}_{\mathrm{o}}^2, \quad \forall\mu_p, && \text{(C1)}\\
&d_g\mathrm{q}^2 = \sigma\mu_p^2. && \text{(C2)}
\end{aligned}
$$

Now, we look at the differential form $X\,dm = \mathrm{q}^2 - \mathrm{g}m^2)m^{-1}dm \sim \dot{m}\, m^{-1}(\mathrm{q}^2 - \mathrm{g}m^2)$.

Corollary 0.8. *The electric charge square differential form Xdm is the product of the electro interaction square q^2 and the differential $d\xi$ of the dimensionless particle mass function ξ. Exactly*

$$
\begin{aligned}
\xi = \ln m^2 - \frac{m^2}{\overline{m}_\sigma^2} &= \; \ln m^2 e^{-m^2\overline{m}_\sigma^{-2}} && \text{(X1)}\\
2\mathrm{X}\, dm \;\; &= \;\; q^2\, d\xi,. && \text{(E2)}
\end{aligned}
$$

☐ The physical dimension of the differential form Xdm is the dimension of the square of *the electric charge interaction function* $\overline{\mathcal{Q}}^2$. We confirm the Corollary statement by the following calculation

$$
\begin{aligned}
\mathrm{X}\, dm &= \frac{(\mathrm{q}^2 - \mathrm{g}m^2)}{m}\, dm \quad \rightarrow \quad \mathrm{q}^2\Big(\frac{1}{m} - m\frac{g}{\mathrm{q}^2}\Big)dm \\
&= \mathrm{q}^2\Big(\frac{1}{m} - \frac{m}{\mathsf{m}_\sigma^2}\Big)dm = \frac{\mathrm{q}^2}{2}\, d\Big(\ln m^2 - \frac{m^2}{\mathsf{m}_\sigma^2}\Big) = \frac{1}{2}\,\mathrm{q}^2 d\xi \qquad \text{(X0)}
\end{aligned}
$$

$$
\textbf{def}: \quad \xi = \ln m^2 - \frac{m^2}{\mathsf{m}_\sigma^2} = \ln m^2 e^{-m^2 \mathsf{m}_\sigma^{-2}} \qquad \text{(X1)}
$$

$$
\Rightarrow \quad 2\mathrm{X}\, dm = q^2\, d\xi. \qquad \text{(X2)}
$$

■

In the end, we underline that $\ln m^2 \equiv \ln\left(m^2 1_{\mathrm{M}_o}^{-2}\right)$, *the* 1_{M_o} is the universal mass unit, and that the particle evolution equations are (E0), (E1) and (E2), and finally, all the particle variables and parameters are $\{m, R; q, g; \mu_p; \mu_B, \mathrm{q}_o\}$.

Comments: *Together with the particle creation forms its geometry and the space and time. While the particle is a global object its geometry and the space-time are local objects placed at a particular point of the space-time* $\mathbb{R}^3 \times \mathbb{R}^1$. *The particle size is the generator of the particle geometry and its space-time.*

0.4 The Particle Geometry

The particle in the presence of electro-gravity is an object of high symmetry in the physical space and time. The geometry of such an object is its own geometry determined by the content of its mass and electro-gravitational interaction, characterized by a single-length variable radius.

$$
R = \frac{3}{2}\,\frac{\mathrm{q}^2 - \mathrm{g}m^2}{m\mathrm{C}^2} = \frac{3}{2}\,\frac{\mathrm{q}^2}{\mu_p^2 \mathrm{C}^2}\,\frac{\mu_p^2 - m^2}{m} \qquad \text{(E2)}
$$

The particle radius is zero at the Planck mass $\mu_p^2 = \mathrm{q}^2 g^{-1}$, changes the sign there, and approaches the infinity at the zero particle mass, the photon-like particles, and approaches minus infinity at the infinity particle masses, "mass-like particles".
While the negative radius does not have an understandable geometric meaning the curvature of the two-dimensional surface acquires both signees and it is a naturally suited variable to characterize the particle geometry.

We introduce the particle electro $R_{\mathrm{q}} = \mathrm{q}^2(m\mathrm{C}^2)^{-1}$ and gravitational $R_{\mathrm{g}} = \mathrm{g}m(\mathrm{C}^2)^{-1}$ radiuses. Notice that the gravity radius used here is the half of the Schwarzschild gravity radius and that the electro and gravity radiuses are identical at the Planck mass.

The particle geometry is naturally classified according to the interaction dominance into: electro `E` sector when $m < \mu_p$ or $R_{\mathrm{q}} > R_{\mathrm{g}}$ and the gravity sector `G` when $m > \mu_p$, or $R_{\mathrm{g}} > R_{\mathrm{q}}$. The Planck mass is the dividing point.

Further we introduce the electro κ_{q} and the gravity κ_{g} curvatures

$$
\begin{aligned}
&\kappa_{\mathrm{q}} \equiv \frac{1}{\mathrm{R}_{\mathrm{q}}} = \frac{m\mathrm{C}^2}{\mathrm{q}^2} \in \mathbb{R}^+, \quad \kappa_{\mathrm{g}} \equiv \frac{1}{\mathrm{R}_{\mathrm{g}}} = -\frac{\mathrm{C}^2}{m\mathrm{g}} \in \mathbb{R}^-, \\
\therefore \quad & R = \frac{3}{2}\left(\frac{1}{\kappa_{\mathrm{q}}} + \frac{1}{\kappa_g}\right) \quad \Rightarrow \quad \frac{1}{R} = \frac{2}{3}\,\frac{\kappa_g \kappa_{\mathrm{q}}}{\kappa_{\mathrm{q}} + \kappa_g}.
\end{aligned}
$$

Definition 0.9. *The internal geometry of the evolution particle is the geometry of the two-dimensional surface Σ. Each particle state corresponds to a point on the surface Σ of the local curvature identical to the particle electro-gravity curvature*

$$\kappa = \frac{1}{R} = \frac{2}{3}\frac{\kappa_{\mathrm{g}}\kappa_{\mathrm{q}}}{\kappa_{\mathrm{q}}+\kappa_{\mathrm{g}}} \in \mathbb{R}.$$

We recognize that $\mathrm{K} = \kappa_{\mathrm{g}}\kappa_{\mathrm{q}}$ is the local Gauss curvature, and the $\mathrm{H} = \kappa_{\mathrm{q}} + \kappa_{\mathrm{g}}$ the particle local mean curvature. An explicit calculation shows that

$$\begin{aligned} \kappa_{\mathrm{g}}\kappa_{\mathrm{q}} &= -\frac{\mu_p^2 \mathrm{c}^4}{\mathrm{q}^4} = -\mathrm{K}_{gp}^2 < 0, \quad \mathrm{H} = \kappa_q + \kappa_{\mathrm{g}} = \frac{\mathrm{c}^2}{\mathrm{q}^2}\frac{m^2-\mu_p^2}{m} \\ \Rightarrow \qquad \kappa &= -\frac{2}{3}\frac{\mathrm{K}_{gp}^2}{\mathrm{H}} = -\frac{\mu_p^2\mathrm{c}^2}{\mathrm{q}^2}\frac{m}{m^2-\mu_p^2} \in \mathbb{R}. \end{aligned}$$

The particle radius and curvature are shown in the Figure 1.

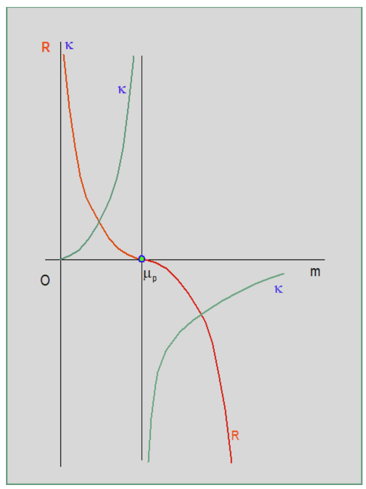

Figure 1: Particle Radius and Curvature

Remark: **a**) The particle curvature is zero when the particle mass is zero, the light-like particle, or infinitely large, the mass-like particle. However, only singular particles, the particles of the mass equal to the Planck mass, $m = \mu_p$, have $\kappa = \pm\infty$ curvatures.

b) The particles of zero mass have zero curvature and infinite radius, such particles are photons. The photon-like particles, the particles of the small masses, $m < \mu_p$, have small curvatures and huge radiuses and are completed in the electro sector E. On the other hand the mass-like particles, the particles of huge masses, $m > \mu_p$, have huge curvatures and small radiuses and complete in the gravity G sector.

c) The singular particles, $m = \mu_p$, have zero radiuses and singular curvatures. Electro-gravity interacting object of the Planck mass has the singular geometry.

Remark: The sign of the interaction curvature is opposite to the sign of the local mean curvature and the sign of the local mean curvature depends on the sign of the difference between the particle and the Planck masses. The following shows the dependence of the curvature sign on the sign of the mean curvature

$$\begin{aligned}
\mathrm{H} > 0 \quad &\Leftrightarrow |\kappa_{\mathrm{g}}| < \kappa_{\mathrm{q}} \Leftrightarrow m > \mu_p \quad \Rightarrow \quad \kappa < 0 \\
\mathrm{H} = 0 \quad &\Leftrightarrow |\kappa_{\mathrm{g}}| = \kappa_{\mathrm{q}} \Leftrightarrow m = \mu_p \quad \Rightarrow \quad \kappa = \infty \\
\mathrm{H} < 0 \quad &\Leftrightarrow |\kappa_{\mathrm{g}}| < \kappa_{\mathrm{q}} \Leftrightarrow m < \mu_p \quad \Rightarrow \quad \kappa > 0.
\end{aligned}$$

Remark: **1.** When $|\kappa_{\mathrm{g}}| < \kappa_{\mathrm{q}}$ the particle curvature is negative and the particle is stored in the gravity sector. An example would be the particles inside of the massive bodies.
2. When $|\kappa_{\mathrm{g}}| > \kappa_{\mathrm{q}}$ the particle curvature is positive, and the particle is stored in the electro sector Such particles are electro-dominated. An example would be the particles in our observable world.
3. When $|\kappa_{\mathrm{g}}| = \kappa_{\mathrm{q}}$ the particle curvature is infinite, the particle is stored in the vacuum.

Remark: The particle geometry is the union of the electro and gravity sectors However, our observable world shows that they are rather intertwined.

Comments: *At this point, we imagine the particle as a global object of a mass m and a radius R, bounded in a sphere, $S^3(R)$, placed as a local point in the $\mathbb{R}^4_1$ space,so that the particle space is $\mathbb{R}^4_1 \times S^3(R)$. The particle's internal physical geometry is described above, and its internal space-time, defined by its physical properties, is the subject of the next subsection.*

0.5 The Particle Space-Time

The differential form $X\,dm$, equation (E1), relates the particle's internal time to its physical properties. Explicitly

$$\begin{aligned}
&\dot{m}\,X = \pm\mu_B \mathrm{C}^3 \quad \Rightarrow \quad \mathrm{X}\,dm = \pm\mu_B \mathrm{C}^3\,d\tau \\
&\Rightarrow \quad \mathrm{C}\,d\tau = \pm\frac{1}{\mu_B \mathrm{C}^2}\,X dm = \ \pm\frac{1}{2\mu_B \mathrm{C}^2}\,\mathrm{q}^2\,d\xi && \text{(T1)} \\
&\Rightarrow \quad \mathrm{C}\,\tau = \pm\frac{1}{2\mu_B \mathrm{C}^2}\int \mathrm{q}^2\,d\xi + const. && \text{(T2)}
\end{aligned}$$

Observe that the differential measure $(2\mu_B \mathrm{C}^2)^{-1}\,d\xi$, equation (T1), folds the square of the electro-interaction in the distance element over the dimensionless variable ξ. In other words, the $(2\mu_B \mathrm{C}^2)^{-1}\mathrm{q}^3$ is the time density over the dimension one dimensionless space. Further, we construct the time-space linear differential form

$$\mathrm{C}d\tau \mp d\mathrm{Z} = d\mathrm{Z}_o + d\tilde{\mathrm{Z}} \Leftrightarrow \mathrm{C}d\tau \mp \frac{\mathrm{q}^2}{2\mu_B \mathrm{C}^2}\,d\xi = 0$$

Dimensionless variable $\xi = \xi(m)$ implies $d\xi = \xi'_m\,dm$, so that $(2\mu_B \mathrm{C}^2)^{-1}\,d\xi = (2\mu_B \mathrm{C}^2)^{-1}\xi'_m\,dm$, and

$$\mathrm{C}d\tau \mp \frac{\mathrm{q}^2}{2\mathrm{C}^2\mu_B}\,\xi'_m\,dm = 0, \tag{G}$$

is exactly the particle time differential equation in the mass presentation.

In the accord with the relativistic field theory, we understand that each particle is created as the particle Π and its antiparticle Π^* pair pair $\tilde{\Pi} = (\Pi, \Pi^*)$. Further on, the negative sign in the equation (G) is for the particle.

Definition 0.10. *The particle internal space-time* $\mathrm{z}_o \otimes \mathrm{z}$ *is spanned by the time-space differential elements* $dZ = (d\mathrm{z}_o, d\tilde{\mathrm{z}})$, $d\,\tilde{\mathrm{z}} = (d\mathrm{z}, d\mathrm{z}^*)$

$$d\mathrm{z}_o = \mathrm{c}d\tau, \quad d\mathrm{z} = +\mathrm{q}^2(2\mu_{\scriptscriptstyle B}\mathrm{c}^2)^{-1}\xi'_m\, dm, \quad d\mathrm{z}^* = -\mathrm{q}^2(2\mu_{\scriptscriptstyle B}\mathrm{c}^2)^{-1}\xi'_m\, dm.$$

Let $\mathrm{g}_{oo} = c^2\mathrm{q}^2$ and $\mathrm{g}_{mm} = \mathrm{q}^2(2\mathrm{c}^2\mu_{\scriptscriptstyle B})^{-1}$ are the metric tensors. Then

$$\begin{array}{llll} d\mathrm{z}_o & = \sqrt{\mathrm{g}_{oo}}d\tau & \Rightarrow & \mathrm{z}_o = \sqrt{\mathrm{g}_{oo}}\,\tau \quad + \mathrm{C}_o, \\ d\mathrm{z} & = -\sqrt{\mathrm{g}_{mm}}\,dm & \Rightarrow & \tilde{\mathrm{z}} \ = -\sqrt{\mathrm{g}_{mm}}\,dm + \mathrm{C}. \\ d\mathrm{z}^* & = +\sqrt{\mathrm{g}_{mm}}\,dm & \Rightarrow & \tilde{\mathrm{z}} \ = +\sqrt{\mathrm{g}_{mm}}\,dm + \mathrm{C}. \\ \therefore \ dZ : & \sqrt{\mathrm{g}_{oo}}\,d\tau - \sqrt{\mathrm{g}_{mm}}\,d\,\mathrm{m} = 0, & & \text{for } \textbf{particle} \\ d\tilde{Z} : & \sqrt{\mathrm{g}_{oo}}\,d\tau + \sqrt{\mathrm{g}_{mm}}\,d\,\mathrm{m} = 0, & & \text{for } \textbf{antiparticle}. \end{array}$$

Corollary 0.11. *The (particle, antiparticle) pairing introduces the differential metric distance form* $ds^2 = g_{ij}d\mathrm{z}^i d\mathrm{z}^j$ *consistent with the norm* $|\mathrm{Z}|^2$*, and*

$$|\mathrm{Z}|^2 \quad = Z\tilde{Z} \quad = g_{oo}\,\tau^2 \pm g_{mm}\,m^2 = \mathrm{c}\mathrm{c}^*.$$

□ We integrate the dual time equations to find

$$\begin{array}{ll} Z & = \sqrt{\mathrm{g}_{oo}}\,\tau - \sqrt{\mathrm{g}_{mm}}\,m - \mathrm{c}, \\ \tilde{Z} & = \sqrt{\mathrm{g}_{oo}}\,\tau + \sqrt{\mathrm{g}_{mm}}\,m - \mathrm{c}^* \\ \Rightarrow \quad Z\cdot\tilde{Z} & = \mathrm{g}_{oo}\tau^2 \ - \ \mathrm{g}_{mm}\,m^2 = \mathrm{c}\mathrm{c}^*. \end{array}$$

The integration constant $\mathrm{c}\mathrm{c}^*$ may be defined by specialization of the function $\xi = \ln m^2 e^{-m^2/\mu_{\scriptscriptstyle P}^2}$ to the Planck mass at the initial moment $\tau = 0$, so that

$$\mathrm{g}_{oo}\tau^2 \ \pm \ \mathrm{g}_{mm}\,m^2 = \mathrm{c}\mathrm{c}^* \quad \xrightarrow{\tau=0} \quad \mp\mathrm{q}^{2}(2\mu_{\scriptscriptstyle B}\mathrm{c}^2)^{-1}\xi'_m = \mathrm{cc}^*.$$

■

<u>Comments</u>: *We should recall that together with the creation of the physical particle creates the particle state* $\mathcal{S}$*, which by itself is the collection of all physical variables associated with the particle at each level of its development. Such variables are the mass and the charge and gravity interaction functions. The interaction state variables are mutually related by the operations among the projectors* $\{\hat{q}, \hat{g}, \hat{\nu}\}$*. In the next section, we will construct the particle state or, call it, the particle interaction variables.*

0.6 Particle State Variables

Differential element $X\,dm \in \mathbb{R}$, the real function of the dimensionless variable ξ in the equation (E1), is a physical variable of the electric charge square dimension. It is associated with the particle in the evolution and, therefore, must be the differential element of the particle state or the particle interaction variable. The next is the definition of the particle electro-stare variable.

Definition 0.12. *The function*

$$\overline{\mathcal{Q}}^2 \in \mathbb{R} \therefore\ \hat{\mathrm{q}} : \mathcal{S} \to d\overline{\mathcal{Q}}^2 = 2X dm = \mathrm{q}^2 d\xi \tag{E3}$$

$$\xi = \ln m^2 e^{-m^2 \overline{m}_\sigma^{-2}}, \tag{X1}$$

is the electric charge state variable, further on the electro-state variable, or the interaction charge of the evolving particle.

Remark: In conclusion, the particle state $\mathcal{S}$, is the collection of all physical properties of the evolving particle such that its projection to the electro-interaction sector E is exactly the charge interaction function $\overline{\mathcal{Q}}^2$. This is an implicit definition of the particle state $\mathcal{S}$. Since the mass gravity and charge projectors are mutually related by the equation (A), the interaction mass $\overline{\mathfrak{M}}^2$ and the interaction gravity $\overline{\mathcal{G}}$ variables are projections to the mass M and the gravity G sectors, and may be constructed from the state variable $\overline{\mathcal{Q}}$. For:

$$\overline{\mathcal{Q}}^2 \in \mathbb{R}: \; d\overline{\mathcal{Q}}^2 = \mathrm{q}^2 d\xi, \tag{E3}$$

$$\overline{\mathfrak{M}}^2 \in \mathbb{R}: \; d\overline{\mathfrak{M}}^2 = \hat{\mu}_p{}^2 \mathcal{S} = \hat{g}^{-1}\hat{q}^2\mathcal{S} = \hat{g}^{-1}d\overline{\mathcal{Q}}^2 = \hat{g}^{-1}\mathrm{q}^2 d\xi = \mu_p^2 d\xi, \tag{E4}$$

$$\overline{\mathcal{G}} \in \mathbb{R} \;: \; d\overline{\mathcal{Q}} = \hat{g}\,\mathcal{S} = \hat{\mu}_p{}^{-2}\hat{g}^2\mathcal{S} = \hat{\mu}_p{}^{-2}d\overline{\mathcal{Q}}^2 = \hat{\mu}_p{}^{-2}\mathrm{q}^2 d\xi = g d\xi. \tag{E5}$$

Dimensionless mass function ξ, induces the structure functions:

$$(\xi, \overline{\mathrm{F}}): \quad \xi \equiv \overline{\nu} = \ln \overline{\mathrm{F}} \tag{11}$$

$$\overline{\nu} = \ln m^2 - m^2 \overline{\mathsf{m}}_\sigma^{-2} = \ln m^2 - m^2 \mu_p^{-2}\Phi^{-1}, \tag{12}$$

$$\overline{\mathrm{F}} = m^2 e^{-m^2 \overline{\mathsf{m}}_\sigma^{-2}} = m^2 e^{-m^2 \mu_p^{-2}\Phi^{-1}} \tag{13}$$

$$\Phi = \sigma_g (1-\beta)^2)^{-1}. \tag{14}$$

Further on, we are constructing the primitive functions of the particle state variables:

$$\overline{\mathcal{Q}}^2 = \mathrm{q}^2\overline{\nu} = \mathrm{q}^2 \ln \overline{\mathrm{F}}, \tag{D1*}$$

$$\overline{\mathfrak{M}}^2 = \overline{\mathsf{m}}_\sigma^2 \overline{\nu} = \mu_p^2 \ln \overline{\mathrm{F}}, \tag{D2*}$$

$$\overline{\mathcal{G}} = g\overline{\nu} = g \ln \overline{\mathrm{F}}. \tag{D3*}$$

We notice that the particle state functions are mutually related at the elementary level, and

$$d_m \overline{\mathcal{Q}}^2 = g\, d_m \overline{\mathfrak{M}}^2, \tag{E6}$$

$$d_m \overline{\mathcal{Q}}^2 = \mu_p^2\, d_m \overline{\mathcal{G}}, \tag{E7}$$

$$g\, d_m \overline{\mathfrak{M}}^2 = \mu_p^2\, d_m \overline{\mathcal{G}}, \tag{E8}$$

The state variable $\{\overline{\mathcal{Q}}^2, \overline{\mathfrak{M}}^2, \overline{\mathcal{G}}\}$ are all real functions, the squares in the notation are not the algebraic operations but to signify the variable physical dimension. Notice that the common factor to all state variables is the dimensionless structure function $\ln \mathsf{F}$.

Corollary 0.13. *The evolutions of the particle state variables are governed by the following differential equations:*

$$\overline{\mathcal{Q}}: \quad d_m \overline{\mathcal{Q}}^2 = q^2\, d\overline{\nu} = q^2 d \ln \overline{F} = \pm 2\mu_{\scriptscriptstyle B} \mathrm{C}^3\, d\tau, \tag{D1}$$

$$\overline{\mathfrak{M}}: \quad d_m \overline{\mathfrak{M}}^2 = \overline{m}_\sigma^2\, d\overline{\nu} = \overline{m}_\sigma^2 \ln\, d\overline{F} = \pm 2\mu_{\scriptscriptstyle B}\, g^{-1}\mathrm{C}^3\, d\tau, \tag{D2}$$

$$\overline{\mathcal{G}}\;: \quad d_m \overline{\mathcal{G}} = g\, d\overline{\nu} = g\, d \ln \overline{F} = \pm 2\mu_{\scriptscriptstyle B}\, \overline{m}_\sigma^{-2}\mathrm{C}^3\, d\tau, \tag{D3}$$

$$d_m \overline{\mathcal{Q}}^2 - g\, d_m \overline{\mathfrak{M}}^2 = 0 \tag{D4}$$

□ We use the equations (X1) and (X2) together with the m-equation (E1) to produce the equation (D1).

$$\mathrm{X}\, dm - \mu_{\scriptscriptstyle B} \mathrm{C}^3\, d\tau \xrightarrow{\times 2 \text{ and use eqn. (E2)}} q^2\, d\xi \pm 2\mu_{\scriptscriptstyle B} \mathrm{C}^3\, d\tau.$$

The other state variable equations are the projections of the equation (D1). Explicitly:

$$\hat{\mu_p}^2\mathcal{S} \to \quad d\overline{\mathfrak{M}}^2 = \hat{g}^{-1}\overline{\mathcal{Q}}^2 \equiv \pm g^{-1}\cdot 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\,d\tau = \pm 2\mu_{\scriptscriptstyle B}\,g^{-1}\mathrm{C}^3\,d\tau$$
$$\mu_p{}^{-2}\mathcal{S} \to \quad d\overline{\mathcal{G}} = \mu_p{}^{-2}\overline{\mathcal{Q}}^2 \equiv \pm\mu_p{}^{-2}\cdot 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\,d\tau = \pm 2\mu_{\scriptscriptstyle B}\,\mu_p{}^{-2}\mathrm{C}^3\,d\tau$$

The differential equation (D4) is just one of the identities (E6),(E7) i (E8). ■

Now, it is straightforward to write the solutions for the particle state variable evolution equations:

$$\overline{\mathcal{Q}}^2 \equiv \mathrm{q}^2\overline{\nu} = \pm 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\tau + C^q_{\pm} \equiv \mathrm{q}^2\ln\overline{\mathrm{F}} \in \mathbb{R} \tag{D1.1}$$
$$\overline{\mathfrak{M}}^2 \equiv \overline{\mathsf{m}}^2_\sigma\overline{\nu} = \pm 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\tau + C^m_{\pm} \equiv \overline{\mathsf{m}}^2_\sigma\ln\overline{\mathrm{F}} \in \mathbb{R} \tag{D2.1}$$
$$\overline{\mathcal{G}} \equiv g\overline{\nu} = \pm 2\mu_{\scriptscriptstyle B}\overline{\mathsf{m}}_\sigma^{-2}\mathrm{C}^3\tau + C^{\mathrm{g}}_{\pm} \equiv g\ \ln\overline{\mathrm{F}} \in \mathbb{R} \tag{D3.1}$$
$$\overline{\mathcal{Q}}^2 - g\,\overline{\mathfrak{M}}^2 + const. \tag{D4.1}$$

Remark: We understand that $\pm\,\mu_{\scriptscriptstyle B}\mathrm{C}^3 d\tau = d\mathcal{Q}^2_{\mathrm{B}} = d(A_b)$ is the contribution of the particle background to the interaction charge $\overline{\mathcal{Q}}^2$. We should understand that the particle state variables are not the particle contents of charge, mass, and gravity, but like the particle energy, the functions of the particle evolution,

Remark: The elementary relations (E6), (E7) and (E8), are not only the state variable identities, but the differential conservation laws. Hence $d\overline{\mathcal{Q}}^2 = g\,d\overline{\mathfrak{M}}^2$ is the conversation law of the electro-gravity charge state variable/interaction in the differential form.

Corollary 0.14. *The electro-gravity charge state variable $\overline{\mathcal{Q}}^2\wedge g\overline{\mathfrak{M}}^2$ is preserved on the particle evolution, the $\mathbf{Q}^2_{\mathrm{O}}$ is an initial electro gravity charge state variable, and*

$$\overline{\mathcal{Q}}^2 - g\overline{\mathfrak{M}}^2 = C_{\mathrm{O}} \in \mathbf{Q}^2_{\mathrm{O}}. \tag{D4.1'}$$

□ The differential identity (D4.1) implies

$$d_m\overline{\mathcal{Q}}^2 = \mp\,2\mu_{\scriptscriptstyle B}\mathrm{C}^3 d\tau = g\,d_m\overline{\mathfrak{M}}^2 \ \Rightarrow\ \overline{\mathcal{Q}}^2 - g\overline{\mathfrak{M}}^2 = const.$$

■

Remark: The evolution equations (D1.1, D2.1, D3.1 i D4.1) of the state variables will be completed after the initial conditions to determine the integration constants are specified. We chose that at the moment $\tau = 0$

$$\overline{\mathcal{Q}}^2(0) \equiv \mathrm{q}^2\ln\overline{\mathrm{F}}(0) = C^q_{\pm} \tag{D1.1'}$$
$$\overline{\mathfrak{M}}^2(0) \equiv g\overline{\mathsf{m}}^2_\sigma\ln\overline{\mathrm{F}} = C^m_{\pm} \tag{D2.1'}$$
$$\overline{\mathcal{Q}}^2(0) - g\overline{\mathfrak{M}}^2(0) \equiv C^q_{\pm} - C^m_{\pm} = \mathrm{C}_{\mathrm{O}}. \tag{D4.1'}$$

Extremum of the Mass State Variable

The properties of the particle state variables are predetermined by the properties of the structure functions $\overline{\nu}$ and $\overline{\mathrm{F}}$. We are showing that the mass state variable achieves maximum at the polarized mass square function $\overline{\mathsf{m}}^2_\sigma$, and that

$$\mathfrak{M}_\star^2 = \overline{\mathsf{m}}_\sigma^2 \ln\left(\overline{\mathsf{m}}_\sigma^2 e^{-1}\right) \quad \text{at} \quad m\colon\, m^2 = \overline{\mathsf{m}}_\sigma^2 = \sigma_g \Phi \mu_p^2, \tag{M1}$$

□

$$\begin{aligned}
\mathfrak{M}^2 \quad & = \overline{\mathsf{m}}_\sigma^2 \ln \overline{\mathrm{F}} = \overline{\mathsf{m}}_\sigma^2 \ln\, m^2 e^{-m^2 \overline{\mathsf{m}}_\sigma^{-2}} = \overline{\mathsf{m}}_\sigma^2 (\ln\, m^2 - m^2 \overline{\mathsf{m}}_\sigma^{-2}) \\
\partial_{m^2}: \quad & \mathfrak{M}^2 \to \mathrm{m}_\sigma^2 (m^{-2} - \overline{\mathsf{m}}_\sigma^{-2}).\;\; \partial_{m^2}\mathfrak{M}^2 = 0 \Rightarrow m^2 = \overline{\mathsf{m}}_\sigma^2 \;\;\Rightarrow\;\; \mathfrak{M}^2(\overline{\mathsf{m}}_\sigma^2) = \overline{\mathsf{m}}_\sigma^2 \ln\left(\overline{\mathsf{m}}_\sigma^2 e^{-1}\right) \\
\partial_{m^2}^2: \quad & \mathfrak{M}^2 = -\overline{\mathsf{m}}_\sigma^{-4} < 0 \;\;\Rightarrow\;\; \text{maksimum.}
\end{aligned}$$

At the end the function $\mathfrak{M}_\star^2$, call it the extremal mass state variable is

$$\mathfrak{M}_\star^2 \quad = \mu_p^2 \Phi \ln\left(\mu_p^2 \Phi e^{-1}\right), \quad \Phi = \sigma_g (1 - \beta^2))^{-1}$$

The extremal mass state is the function of the Planck mass and as the evaluation of the state variable must be the mass state variable $\mathfrak{M}_\star^2(\mu_p)$ by itself .

Remark: The extremal mass states variable is associated with the class of the Planck mass, call it "particles". The function $\mathfrak{M}_\star^2(\mu_p)$ achieves minimum $\mathfrak{M}_\star^2 = 0$ at the Planck mass $\mu_p^2 = \Phi^{-1} = \sigma_g(1-\beta^2)$.

□

$$\begin{aligned}
\mathfrak{M}_\star^2 \quad & = \mu_p^2 \Phi \ln\left(\mu_p^2 \Phi e^{-1}\right) = \mu_p^2 \Phi (\mu_p^2 \Phi + *) \\
\partial_{\mu_p^2}: \quad & \mathfrak{M}_\star^2 \to \Phi \ln \mu_p^2 \Phi.\;\; \partial_{\mu_p^2} \mathfrak{M}_\star^2 = 0 \;\Rightarrow\; \mu_p^2 \Phi = 0 \Leftrightarrow \mu_p^2 = \Phi^{-1} \;\;\Rightarrow\;\; \mathfrak{M}_\star^2 = 0 \\
\partial_{Mp^2}^2: \quad & \mathfrak{M}_\star^2 = 1 > 0 \;\;\Rightarrow\;\; \text{minimum.}
\end{aligned}$$

■

0.7 Planck Particles

Starting from the Planck mass constant, over the electro-gravity constitutional equations Planck mass has been silently present in our consideration, and, finally, It enters the particle evolution equations through the polarization mass $\overline{\mathsf{m}}_\sigma^2 = \sigma_g \mu_p^2 \Phi$, $\Phi = (1-\beta^2)^{-1}$. However, the informal concept of the Planck particle enters in as a massive object of singular geometry and, finally, trough the evaluation of the extremum of the mass state variable. Namely, the universality of the state variables implies that the particle mass state variable $\overline{\mathfrak{M}}^2 = \overline{\mathsf{m}}_\sigma^2 \ln \overline{\mathrm{F}}$, $\overline{\mathrm{F}} = m^2\, e^{-m^2 m_\sigma^{-2}}$ prescribes to any particle, and hence, to an object of the mass equal to the Planck mass. Thus

$$\begin{aligned}
m^2 \overline{\mathsf{m}}_\sigma^{-2} \quad & \xrightarrow{m \to \mu_p} \quad \mu_p^2\, \sigma_g \mu_p^{-2} \Phi^{-1} = \sigma_g \Phi^{-1} \\
\mathfrak{M}^2 \quad & = \overline{\mathsf{m}}_\sigma^2 \ln\left(m^2 e^{-m^2 \overline{\mathsf{m}}_\sigma^{-2}}\right) \xrightarrow{m \to \mu_p} \quad \mathfrak{M}_{\mu_p}^2 && \text{(P1)} \\
\mathfrak{M}_{\mu_p}^2 \quad & = \;\; \mu_p^2\, \Phi \ln \mu_p^2 1_{\mathrm{M}_o}^{-2} e^{-\sigma_g \Phi^{-1}}. && \text{(P2)}
\end{aligned}$$

Definition 0.15. *An abstract object of the mass identical to the Planck mass, realized in any polarization in the equation (P2) is the total polarized Planck particle. The Planck particle polarized in the positive gravity sector is the Principal Planck particle, and the Principal Planck particle polarized only in the gravity sector is the fundamental Planck particle.*

Remark: At this place, we will classify the particles by their mass polarization.

$$
\begin{tikzcd}
(\overline{\mathcal{P}} : m, \mu, \sigma_g, \overline{\mathrm{F}}) \arrow[r, "\beta\to\ 0"] \arrow[d, "\hat{\mu}"'] & (\mathcal{P} : m, \mu, \sigma_g, \mathsf{F}) \arrow[d, "\hat{\mu}"] \\
(\overline{\mathcal{P}} : \mu, \mu, \sigma_g, \overline{\mathrm{F}}^{\mu}) \arrow[r, "\beta\to\ 0"] \arrow[d, "+\hat{1}"'] & (\mathcal{P}^{\mu} : \mu, \mu, \sigma_g, \mathsf{F}^{\mu}) \arrow[d, "+\hat{1}"] \\
(\overline{\mathcal{P}}^{\mu}_{\mathtt{princ}} : \mu, \mu, +1, \overline{\mathrm{F}}^{\mu}_{+}) \arrow[r, "\beta\to 0"'] & (\mathcal{P}^{\mu}_{\mathtt{princ}} : \mu, \mu, +1, \mathsf{F}^{\mu}_{+}).
\end{tikzcd}
$$

The set of all totally polarized particles is $\overline{\mathcal{P}}$, and its subset of only gravity polarized particles is $\overline{\mathcal{P}}^{\mu}$. The operation $\beta \to 0$ excludes the proto-charge polarization. Totaly polarized Planck particles $\overline{\mathcal{P}}^{\mu}$ are subset of the $\overline{\mathcal{P}}$, while only gravity polarized particles $\mathcal{P}^{\mu}$ are in the $\mathcal{P}$. The operator $\hat{\mu}$ reduces to the Planck particles and the operator $+\hat{1}$ reduces to the positive gravity sector.
We notice the extremal interaction mass $\mathfrak{M}^2_{\star} = \overline{\mathsf{m}}^2_{\sigma} \ln \mathsf{F}^*$ and the interaction mass of the Planck particle $\mathfrak{M}^2_{\mu_p} = \overline{\mathsf{m}}^2_{\sigma} \ln\ln \mathsf{F}^{\mu_p}$ differ in the structure functions $\mathsf{F}^* = \mu_p^2\, \Phi e^{-1}$ and $\mathsf{F}^{\mu_p} = \mu_p^2 1^{-2}_{\mathrm{M_o}} e^{-\Phi^{-1}}$. The extremal and the Planck particle have the same mass state variables when $\Phi = 1 \Leftrightarrow \beta = 0$ and $\sigma_g = 1$ Such particle is in the positive gravity sector, and $\mathsf{F}^* \equiv \mathsf{F}^{\mu_p} = \mu_p^2 1^{-2}_{\mathrm{M_o}}\, e^{-1} = \mathsf{F}_{\mathtt{fun}}$. Its structure-function is the structure-function of the Fundamental particle. Both the interaction charge and the interaction masses are proportional to the structure function $\mathsf{F}_{\mathtt{fun}}$ and

$$\overline{\mathcal{Q}}^2_{\mathtt{fun}} = \mathrm{q}^2 \ln \mu_p^2\, e^{-1}, \quad \mathfrak{M}^2_{\mathtt{fun}} = \mu_p^2 \ln \mu_p^2\, e^{-1} \tag{P3}$$

0.8 Some Predictions

The mass solutions of the particle evolution equations are hidden in the structural functions so that the mass implicit solution is in one of the structure functions, so that it is necessary to solve one of the evolution equations (D1.1, D2.1, D3.1) for one of the $\overline{\nu}$ or $\overline{\mathrm{F}}$ functions. We are solving the interaction charge equation (D1) for the structure-function $\overline{\mathrm{F}}$. Follows:

$$d_m \overline{\mathcal{Q}}^2 = q^2 d(\ln \overline{\mathrm{F}} = q^2 d\big(\ln m^2 1^{-2}_{\mathrm{M_o}} e^{-m^2 \overline{\mathsf{m}}_{\sigma}^{-2}}\big) = \pm 2\mu_{B}\, \mathrm{C}^3\, d\tau, \tag{D1}$$

$$\mathrm{q}^2 \ln \overline{\mathrm{F}} \equiv \mathrm{q}^2 \ln m^2 1^{-2}_{\mathrm{M_o}} e^{-m^2 \overline{\mathsf{m}}_{\sigma}^{-2}} = \pm 2\mu_{B}\, \mathrm{C}^3 \tau + C^{q}_{\pm} \tag{S1}$$

$$\overline{\mathrm{F}} \equiv m^2 1^{-2}_{\mathrm{M_o}} e^{-m^2 \overline{\mathsf{m}}_{\sigma}^{-2}} = e^{\pm 2\mu_{B} \mathrm{C}^3 \mathrm{q}^{-2} \tau + C^{q}_{\pm} \mathrm{q}^{-2}}. \tag{S2}$$

The following table presents the structure functions F of the polarized particles.

Table 2. Structure Function F of the Polarized Particles

$\overline{\mathrm{F}}$	$\overline{\mathrm{F}}^{+1}$	$\overline{\mathrm{F}}^{\mu_p}$	$\overline{\mathrm{F}}^{\mu_p}_{+1}$	
$\overline{\mathcal{P}}$	$m^2 e^{-m^2 \mu_p^{-2} \sigma_g (1-\beta^2)}$	$m^2 e^{-m^2 \mu_p^{-2} (1-\beta^2)}$	$\mu_p^2 e^{-\sigma_g (1-\beta^2)}$	$\mu_p^2 e^{-(1-\beta^2)}$

	F	F^{+1}	F^{μ_p}	$\mathsf{F}_{\mathsf{fun}}$
$\mathcal{P}$	$m^2 e^{-m^2 \mu_p^{-2} \sigma_g}$	$m^2 e^{-m^2 \mu_p^{-2}}$	$\mu_p^2 e^{-\sigma_g}$	$\mu_p^2 e^{-1}$

Solutions of the Mass Evolution Equations

To get some insight into the mass solutions of the particle evolution equations, we will look for the solutions of the equations (D1) and S1 specialized to the $(\mathcal{P}: m, \mu, \sigma_g = +1, \mathsf{F}^{+1})$ polarization class. Explicitly:

$$d_m Q^2 = d(\ln m^2 - m^2\mu_p^{-2}) = q^2 d\big(\ln m^2 1_{\mathrm{M_o}}^{-2} e^{-m^2\mu_p^{-2}}\big) = \quad \pm 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\, d\tau, \tag{D1'}$$

$$\mathsf{F} \quad \equiv m^2 1_{\mathrm{M_o}}^{-2} e^{-m^2\mu_p^{-2}} = \ln m^2 - m^2\mu_p^2 = \quad f(\tau, \mathrm{q}; \mu_{\scriptscriptstyle B}). \tag{S2'}$$

Two signs refer to the particle and its dual pair (Π, Π^*).

1. <u>Zero Proto Charge</u>

The interchange of the charge between a particle and its neighborhood is zero, the mass factor $\mu_{\scriptscriptstyle B} = 0$, and the particle holds only by the electro-gravity interaction. The particle mass is the solution of the equation

$$\mathrm{q}^2 d\ln\mathsf{F} = 0 \quad\Rightarrow\quad \ln\mathsf{F} = const \quad\Rightarrow\quad \mathsf{F} = m^2 e^{-m^2/\mu_p^2} = \mathrm{C}$$

Integration constant C is bounded above by the maximum $e^{-1}\mu_p^2$ of the structure-function F at a mass $m^2 = \mu_p^2$, see Figure 2. For $\mathrm{C}: 0 \le \mathrm{C} \le e^{-1}\mu_p^2$ there is at least one mass solution. If we exclude the infinite mass solution and accept the Planck mass as a double solution, the particle must have two (m_1, m_2), $0 < m_1^2 \le \mu_p^2 \le m_2^2 < \infty$ mass solutions.
The rest mass function F is universal in the sense that any particle mass is realized just on this function, and there are no other particle masses except those who obey the function F.

2. <u>Electro Neutral Particle</u>

Electro neutral particle, $\mathrm{q} = 0$, is held only by the interaction of the gravity, and its evolution is described by the following simple differential equation

$$-\,\mathrm{g}\, m\, dm \pm \mu_{\scriptscriptstyle B}\mathrm{C}^3\, d\tau = 0 \quad\Rightarrow\quad m^2 = m_o^2 \mp \frac{2\mu_{\scriptscriptstyle B}\mathrm{C}^3}{\mathrm{g}}\,\tau.$$

The particle either accumulates or loses mass/rest energy in the constant portions $\pm 2\mu_{\scriptscriptstyle \mathrm{BC}}\mathrm{C}^5\mathrm{g}^{-1}$ per unit of the time. The particle of a mass m_o decays in the $m_o^2\mathrm{g}/2\mu_{\scriptscriptstyle B}\mathrm{C}^3$ seconds. The same time is needed to create a particle of the same mass,

3. <u>Purely Charged Particle</u>

Well-defined particle excludes gravity only when the particle is massless. Thus, the purely charged particle is one of a mass very small compared to the Planck mass. For, such particles

$$\frac{\mathrm{q}^2\,\dot{m}}{m} = \mp\,\mu_{\scriptscriptstyle B}\mathrm{C}^3 \quad\Rightarrow\quad m = m_o e^{\mp\mu_{\scriptscriptstyle B}\mathrm{C}^3\mathrm{q}^{-2}\,\tau}, \quad m_o \ll \mu_p.$$

Purely charged particles are either the mass-increasing or the mass-decreasing.

4. <u>The Character of the Complete Mass Solution</u>

An implicit solution of the evolution equation is the structure function by itself. Thus

$$\mathsf{F} \quad = m^2 1_{\mathrm{M_o}}^{-2}\, e^{-m^2/\mu_p^2} = e^{\,C_\pm/\mathrm{q}^2}\, e^{\mp 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\,\tau/\mathrm{q}^2} = \quad \mathsf{A}^\pm\, e^{\mp 2\mu_{\scriptscriptstyle B}\mathrm{C}^3\,\tau/\mathrm{q}^2} \equiv \quad f(\tau). \tag{S2.1}$$

We are showing that the structure-function achieves maximum $e^{-1}\mu_p^2$ at the mass $m = \overline{\mathsf{m}}_\sigma \sim \mu_p$. Else the function is zero at the zero mass and approaches zero at the infinite mass.

$$\begin{aligned}
\mathsf{F}' \quad &= e^{-m^2/\mu_p^2}\big(1 - m^2\mu_p^{-2}\big), \qquad & \mathsf{F}' = 0 \;\;\Rightarrow\;\; m^2 = \mu_p^2\\
\mathsf{F}'' \quad &= -e^{-m^2/\mu_p^2}\big(2 - m^2\mu_p^{-2}\big)\mu_p^2, \quad & \mathsf{F}''(\mu_p^2) < 0 \Rightarrow F_{\max} = \mu_p^2 e^{-1}.
\end{aligned}$$

Thus, the structure-function F achieves maximum at the Planck masses and

$$\mathsf{F}_\star = \mathrm{f}^2(\mu_p) = e^{-1}\mu_p^2 \quad \Rightarrow \quad e^{-1}\mu_p^2 \;\geq\; 1^2_{\mathrm{M}_o}\, e^{C_\pm/\mathrm{q}^2}\, e^{\mp 2\mu_B \mathrm{c}^3\,\tau/\mathrm{q}^2}.$$

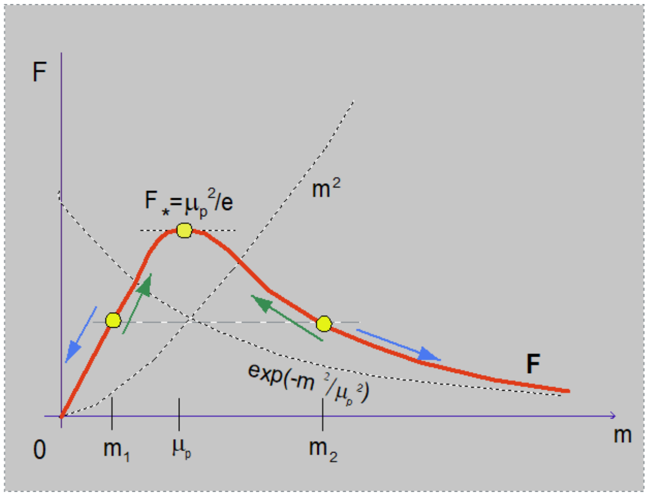

Figure 2: Universal Rest Mass/Energy Function

Corollary 0.16. *A particle of a finite mass either evolves into a Planck particle or a particle of the arbitrary large mass or decays into a particle of the* $\inf m$. *mass.*

□ At each moment $\tau \geq 0$ the equation $\mathsf{F} = f(\tau) < \mathsf{F}_\star = f(\mathtt{T}^*)$ has the dual mass pair solutions (m_1, m_2) : $\inf m \leq m_1 \leq \mu_p \leq m_2 \leq \sup m$, Figure 1. We follow the particle on the graphic of the structure-function F.

Each particle is the particle or dual particle of a mass before or after the Planck mass on the mass axes of the function F. If the particle is before the Planck particle and, its time flows forward the particle and its dual particle approach the Plank "particle". Else the particle decays to a particle of a mass $\inf m$, and its dual particle in a huge mass particle. That implies that the increase of the particle time is bounded by the time $\mathtt{T}_*$ the particle needs to reach the top of the structure-function F.

Reversed conclusions hold for the particles of a mass greater than Planck's mass on the mass axes. ■

Universal Unit of the Mass

The universality of the Planck mass is that it participates in the formation of all particles, and the universality of the particle state variable functions is that they correspond to the particles of all masses. Therefore, both statements hold on the class of the Planck particles in all mass polarisations. In particular they hold on the restriction to the class $(\mathcal{P}_{\mathsf{fun}} : \mu_{\mathrm{O}}, \mu_{,\mathrm{O}}\,, +1, 1)$ of the Fundamental Planck particles. The last restriction reduces to the structure-function $\mathsf{F} \to \mathsf{F}_{fun} = \mu_p^2 e^{-1} = \mathcal{Q}^2_{\mathtt{fun}}\mathrm{q}^{-2}$, and the interaction charge equation (S1) reduces to the

$$\mathcal{Q}^2_{\mathtt{fun}} \;= \mathrm{q}^2 \ln(\mu_{\mathrm{O}}^2 1^{-2}_{\mathrm{M}_o} e^{-1}) = e^{\pm 2\mu_B \mathrm{c}^3 \tau + C^q_\pm} \;\xrightarrow{\tau \to 0}\; e^{C^q_\pm} = \;\mathcal{Q}^2_{\mathrm{O}} \in \mathbb{R} \tag{S1.2}$$

$$\Rightarrow\; 1^2_{\mathrm{M}_o} = \frac{\mu_{\mathrm{O}}^2}{e}\, e^{-\mathrm{q}^{-2}\mathsf{Q}^2_{\mathrm{O}}} \tag{S1.3}$$

Consequently, the equation (S1.3) relates the Fundamental particle mass μ_{O}, the universal mass unit 1_{M_o} and an initial interaction charge $\mathcal{Q}_{\mathrm{O}}$. Further specialization is given in the next definition.

Definition 0.17. *The Fundamental particle is the Maxwell electron* $\Pi_{M} \sim (\sqrt{\alpha}\mathtt{M}_{:2},\, \mathrm{E})$ *under electro interaction* $\mathrm{q} : \mathrm{q}^2 = \alpha\hbar\mathrm{C}$.
The initial charge $\mathcal{Q}_{\mathrm{O}}$ *is the charge* $\hbar\mathrm{C}$ *of the Planck electron* $\Pi_p \sim (\mathtt{M}_{:2}, \mathrm{E}_p)$ *distributed by the symmetry group* Z_2*. Explicitly*

$$\mathcal{Q}_{\mathrm{O}}^2 = \mathbb{Z}_2\,\hbar\mathrm{C} \rightsquigarrow |-\hbar\mathrm{C},\, 0,\, +\hbar\mathrm{C}\rangle.$$

Corollary 0.18. *The universal mass unit in the physical word supported by the fundamental Planck constants is* $1_{\mathrm{M}_o} = \mathbf{0.565}\ \left[meV\mathrm{C}^{-2}\right]$.

$\square$ By definition $\mu_{\mathrm{O}} = \sqrt{\alpha}\mathtt{M}_{:2}$, the reduced Planck mass constant $\mathtt{M}_{:2} = \mathtt{M}/2$, electric charge $\mathrm{q} = \mathrm{E}^2 = \alpha\hbar\mathrm{C}, \alpha = 1/137$ and the initial charge $\mathrm{Q}_{\mathrm{O}}^2 \sim \mathbb{Z}_2\,\hbar\mathrm{C}$. Substitution to the equation (S1.3) leads to

$$1_{\mathrm{M}_o} = \frac{\mu_{\mathrm{O}}}{\sqrt{e}}\, e^{-\mathrm{q}^{-1}\mathrm{Q}_{\mathrm{o}}} = \frac{\sqrt{\alpha}\mathtt{M}_{:2}}{\sqrt{e}}\, e^{-\mathrm{q}^{-2}\mathbb{Z}_2\,\hbar\mathrm{C}} = \sqrt{\frac{\alpha}{e}}\,\mathtt{M}_{:2}\, e^{-((\alpha\hbar\mathrm{C})^{-1}\mathbb{Z}_2\,\hbar\mathrm{C}} = \sqrt{\frac{1}{137\,e}}\,\frac{2.18\cdot 10^{-5}}{2}\, e^{-\alpha^{-1}\mathbb{Z}_2}$$

$$\Rightarrow \qquad 1_{\mathrm{M}_o} = 5.65\cdot 10^{-7}\, e^{-\frac{137}{2}\cdot\left|1,0,-1\right\rangle} = \left|1.201\cdot 10^{-36},\, 5.65\cdot 10^{-7},\, 3.17\cdot 10^{23}\right\rangle\ [\mathrm{gr}]$$

$$\Leftrightarrow \qquad 1_{\mathrm{M}_o} = \left|\mathbf{0.565},\ \mathbf{3.17}\cdot 10^{29},\ \mathbf{1.78}\cdot 10^{59}\right\rangle\ \left[\mathrm{meVC}^{-2}\right].$$

The smallest mass $1_{\mathrm{M}_o} = \mathbf{0.565}\left[\mathrm{meVC}^{-2}\right]$, about half of the predicted mass $m_e = \mathbf{1.17}\left[\mathrm{meVC}^{-2}\right]$ of the lightest among the neutrinos, the electron neutrino [1], Is the universal unit of the mass in the physical world supported by the universal Planck constants? $\blacksquare$

Conclusion

Physics is on an endless road to completion through the unification of concepts and theories while checking and testing its roots. This article looks at the core of Physics to describe particle formation, which gives glimpses into understanding and explaining possible origins of some crucial physical concepts, like particle-antiparticle and gravity-antigravity concepts, and phenomena of the universe's expansion and possibly the origin of invisible matter, etc.

Further work would be done to verify and develop the imposed ideas. Quantized particle evolution equations should reveal the laws of the particle masses discretization.

Thanks for reading my paper.
Radomir M.

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[1] R. Majkic, *Neutrino Mass Prediction, JHEPGC, Vol.10 No.4, August 29, 2024*