Derivation of the fine structure constant and the Rydberg constant from Coulomb's law

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1. Abstract

The fine-structure constant is generally considered to be a coupling constant that represents the strength of the electromagnetic interaction of elementary particles, but its origin is unknown. Using the electron and proton model of the Energy Body Theory, I clarified the origin of the fine-structure constant from the relationship between Coulomb's law and Planck's constant. This equation expressing the origin of the fine structure constant further led to the Rydberg constant.

2. Overview

The electron and proton model of the Energy Body Theory has revealed that the fine structure constant is a different expression that applies to the same phenomenon, in which Coulomb's law and the Planck constant apply.

In other words, the energy applied by Coulomb's law and the Planck constant are equal. The energy of these two different expressions was combined in an equation and transformed to derive the fine structure constant.

The derived formula also showed that the fine structure constant is the ratio of 4π times the distance r between the bound electron and proton to the wavelength λ of light. The wavelength of light is 4π times the distance r between the bound electron and proton, and then multiplied by 137. The reason for this is that the wavelength of light is expressed by the following formula:

Wavelength of light = electron transition distance + photon speed (= electron transition speed approx. 2,000 km/sec) v × time t required for excitation

Note: For the difference between the speed of the photon and the speed of light, see 4.

Furthermore, I was able to derive the Rydberg constant using the formula for the fine structure constant this time. This proves the correctness of the fine structure constant derived this time and is one of the events that demonstrates the correctness of the elementary particle model of the Energy Body Theory.

3. Derivation

3.1. Derived constants

The constants derived from Coulomb's law are the fine structure constant and the Rydberg constant.

3.1.1 Fine structure constant

The fine structure constant is:

$$\alpha = \frac{\mu_0 c e^2}{2h} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \qquad \alpha = \frac{1}{137} \qquad (1)$$

The fine structure constant was introduced by Sommerfeld in 1916 to explain the fine structure that appears in the spectral lines of hydrogen-like atoms. It is now considered to be a coupling constant that describes the strength of electromagnetic interactions between elementary particles in a more general sense, independent of atomic structure. However, it has not been known why this value appears.

3.1.2. Rydberg constant

R is a constant called the Rydberg constant, and is expressed by the following formula:

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c} \qquad (2)$$

The Rydberg constant appears in the following formula, which describes the spectrum of the hydrogen atom:

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad (3)$$

3.1.3. Coulomb's law

Coulomb's law describes the mutual attraction and repulsion between two charges, and is expressed by the following formula:

$$f = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \qquad (4)$$

To calculate the energy of one electron, $q_1 q_2$ in (4) are set to e^2.

$$f = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \qquad (5)$$

3.2. Derivation

Multiply both sides of Coulomb's law (5) by r/2 to find the potential energy of one electron. Multiplying by r/2 is done to get the energy of one electron.

$$f \cdot \frac{r}{2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \cdot \frac{r}{2} \qquad (6)$$

The potential energy of the electron in the proton's orbit was calculated in (6). In an electronic transition, this potential energy is converted into kinetic energy and consumed.
Kinetic energy is a wave motion generated in the space in front of the electron. The electron

transits by being pulled by the kinetic energy, which is the wave motion of the electron. While the electron stops in the ground state orbit, kinetic energy leaves from the electron and becomes a photon.

This kinetic energy is equal to electronic transition energy.

The electronic transition energy is expressed by the following equation (7).

$$E = h\nu = \frac{hc}{\lambda} \qquad (7)$$

Therefore, since (6) and (7) are equal, we obtain the following equation (8).

$$f \cdot \frac{r}{2} == \frac{hc}{\lambda}$$

$$\frac{hc}{\lambda} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \cdot \frac{r}{2} \qquad (8)$$

Multiplying both sides of equation (8) by 4 π r and rearranging, we obtain equation (9). This is the fine structure constant. The fine structure constant is the ratio of 4 π r times the distance between an electron and a proton to the wavelength of a photon.

$$\frac{4\pi r}{\lambda} = \frac{e^2}{2\varepsilon_0 hc} \qquad (9)$$

Dividing both sides of the fine structure constant equation (9) by 4 π r gives us equation (10), which is the same as going back to equation (8) and rearranging it.

$$\frac{1}{\lambda} = \frac{e^2}{8\pi\varepsilon_0 hc} \cdot \frac{1}{r} \qquad (10)$$

Here, we need to find 1/r in equation (10). However, this method has been known for a long time and is as follows.

If we consider that the electron revolves around the proton's orbit, the Coulomb force and the centrifugal force are balanced, and we obtain the following equation (11).

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = \frac{mv^2}{r} \qquad (11)$$

Moreover, the condition for an electron to be stable in a proton orbit is given by the following (12), based on Bohr’s quantum condition and de Broglie’s relation.

$$2\pi r = n\lambda \qquad (12)$$

Moreover, de Broglie's relations are the following equation (13).

$$\lambda = \frac{h}{p} \quad \rightarrow \quad \lambda = \frac{h}{mv} \qquad (13)$$

Substituting (13) into (12), we obtain the following equation (14).

$$2\pi r = \frac{nh}{mv} \qquad (14)$$

Transforming (13) to find v gives us equation (15).

$$v = \frac{nh}{2\pi mr} \qquad (15)$$

Substituting (15) into (11), we obtain the following equation (16).

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} = \frac{m}{r}\left(\frac{nh}{2\pi mr}\right)^2 \qquad (16)$$

By calculating 1/r from equation (16), we obtain equation (17).

$$\frac{1}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \cdot \frac{1}{m}\left(\frac{2\pi mr}{nh}\right)^2$$

$$= \frac{\pi m e^2}{\varepsilon_0 h^2} \cdot \frac{1}{n^2} \qquad (17)$$

Substituting equation (17) into equation (10), we obtain the following equation (18).

$$\frac{1}{\lambda} = \frac{e^2}{8\pi\varepsilon_0 hc} \cdot \frac{\pi m e^2}{\varepsilon_0 h^2} \cdot \frac{1}{n^2}$$

$$= \frac{m e^4}{8\varepsilon_0^2 h^3 c} \cdot \frac{1}{n^2} \qquad (18)$$

Comparing equation (18) with Balmer's equation (3), the Rydberg constant R is found to be equation (19).

$$R = \frac{m e^4}{8\varepsilon_0^2 h^3 c} \qquad (19).$$

3.3. Concept

3.3.1. Coulomb's law and Planck's constant

Figure 1 explains that the energy to which Coulomb's law applies is equal to the energy to which the formula for the Planck's constant applies.

To understand this figure, you need to know the elementary particle model of the Energy Body Theory, so a simple explanation has been added to 4.

An electron and a proton have disklike shapes rotating as wave and widely spreading. When the foot of each waves comes into contact, both feet are distorted and restoring forces are generated. Depending on the direction of each waves at the contact point, if they are the same, it is an attractive force, and if they are different, it is a repulsive force.

Coulomb's law expresses the restoring force from the distortion of the electron's foot and the distortion of the proton's foot by multiplying both electric charges.

The equation for Planck's constant expresses the restoring force from the distortion of the electron's wave foot and the distortion of the proton's wave foot by adding both functions like angular momentum.

However, although the energy to which Coulomb's law applies is equal to the energy to which the formula for the Planck constant applies, there are some points to note. Planck's constant is the raw value of one electron and one proton, whereas Coulomb's law is a coarse-grained approximation of many, many electrons and protons, but when viewed as one electron and one proton, quantum mechanics and electromagnetism have the same value.

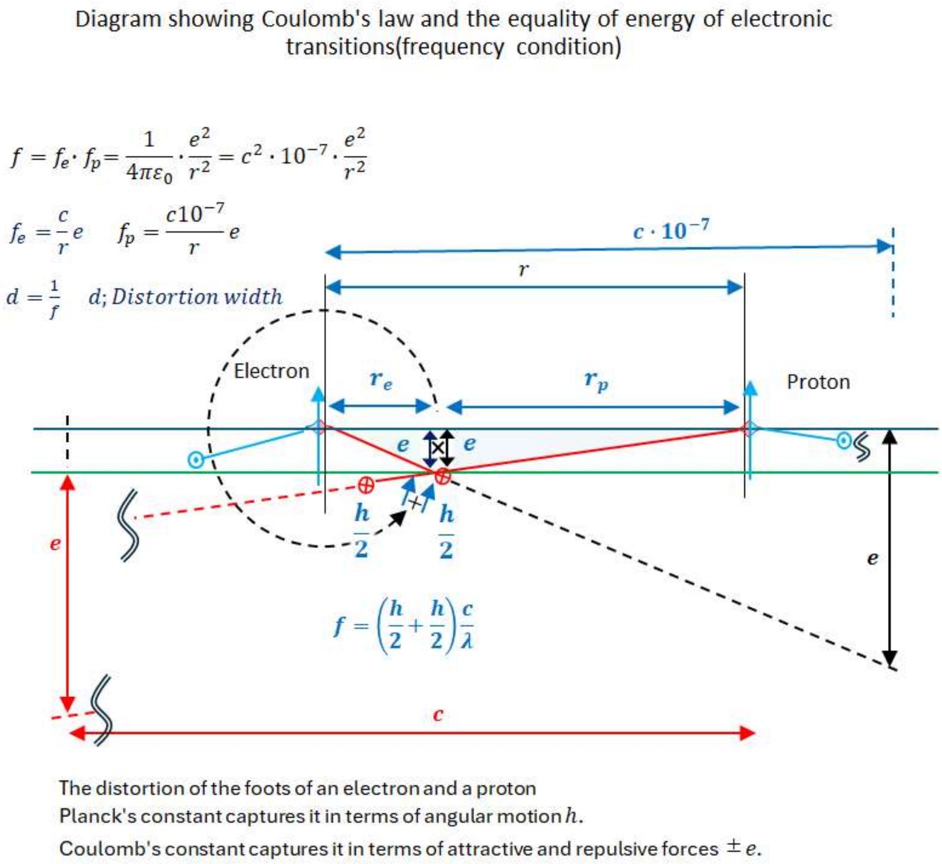


Fig1.

2.3.2. Electron-proton bonding and electronic transitions

A. Movement from ground state to excited state

When an electron is on the orbit of a proton in the ground state, the electron and proton are in a bonded state through electromagnetic interaction. Therefore, Coulomb's law applies to the bonding force (energy) between the electron and the proton. The reason that the bonding force works is that the rotating wave of the electron and the rotating wave of the proton move in the same direction, the energy decreases and an attractive force works. At this time, the foot of both are distorted as shown in Fig2, producing a restoring force. If the restoring energy of the distortion of the electron foot is E_e and the restoring energy of the distortion of the proton foot is E_p the two are equal. And if the total restoring energy is E, E is the sum of the restoring energy of the distortion of the electron foot E_e and the restoring energy of the distortion of the proton foot E_e.

$$E_e = E_p \qquad E = E_e + E_p$$

The restoring force during an electron transition is Coulomb's law multiplied by r/2. This value is equal to the energy of the electron transition, $h\nu$. The reason for multiplying by r/2 is to make it the amount of energy of the electron that transition. A detailed explanation is given in Fig2.

$$E = h\nu = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \cdot \frac{r}{2}$$

When an electron moves to an excited orbit, the potential energy increases by $h\nu$. This is the sum of the restoring energy of the electron foot distortion and the proton foot distortion. This is because the foot distortion of the proton and electron is linear.

Diagram of the relationship between Coulomb's law and energy quanta, which derives the "fine structure constant" and the "Rydberg constant"

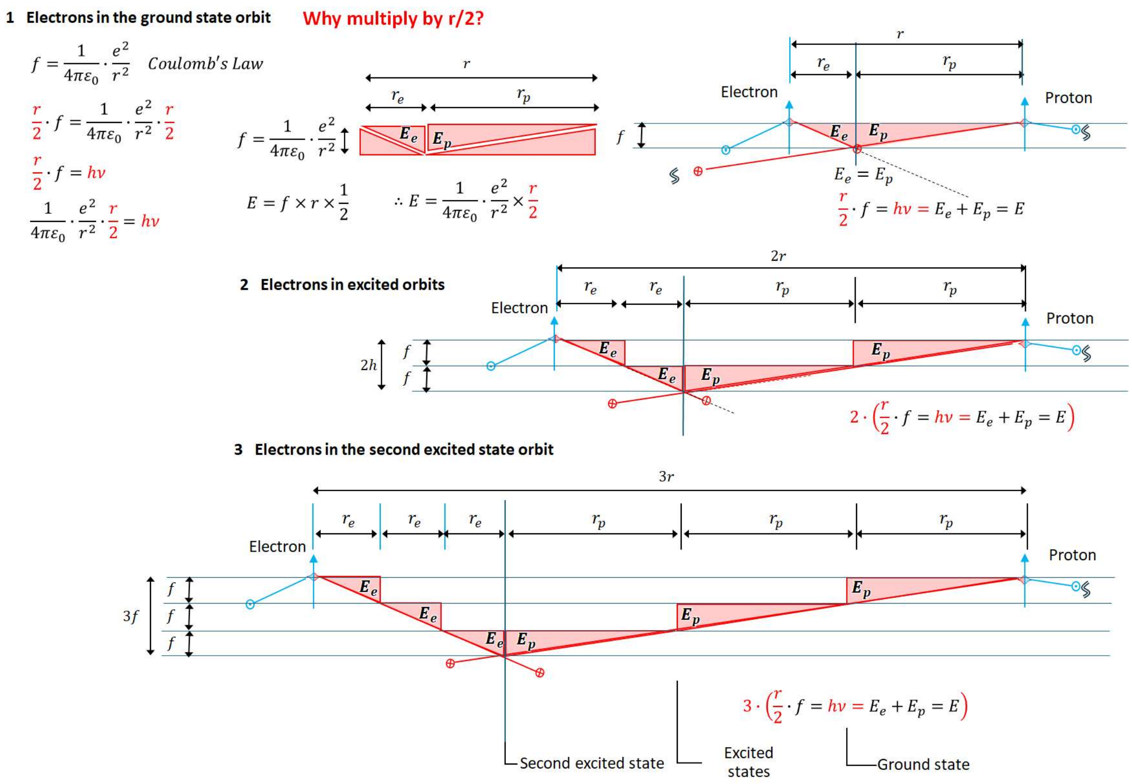


Fig2.

B. Transition from an excited state

Electron transition is the reverse process of the movement to an excited state. The transition occurs while emitting the restoring energy of the distortion of the electron's foot, which increased through the movement to the excited state, and the restoring energy of the distortion of the proton's foot hv. At this time, it is important to note that the electron does not move by its own energy. The electron generates a wave in the space in front of it in the direction of its axis of motion, and the electron is dragged by that wave. Therefore, the wave energy and kinetic energy are the same.

$$hv = \frac{1}{2}mv^2$$

When the electronic transition is complete and the electron stops moving, the kinetic energy continues to travel and becomes a photon.

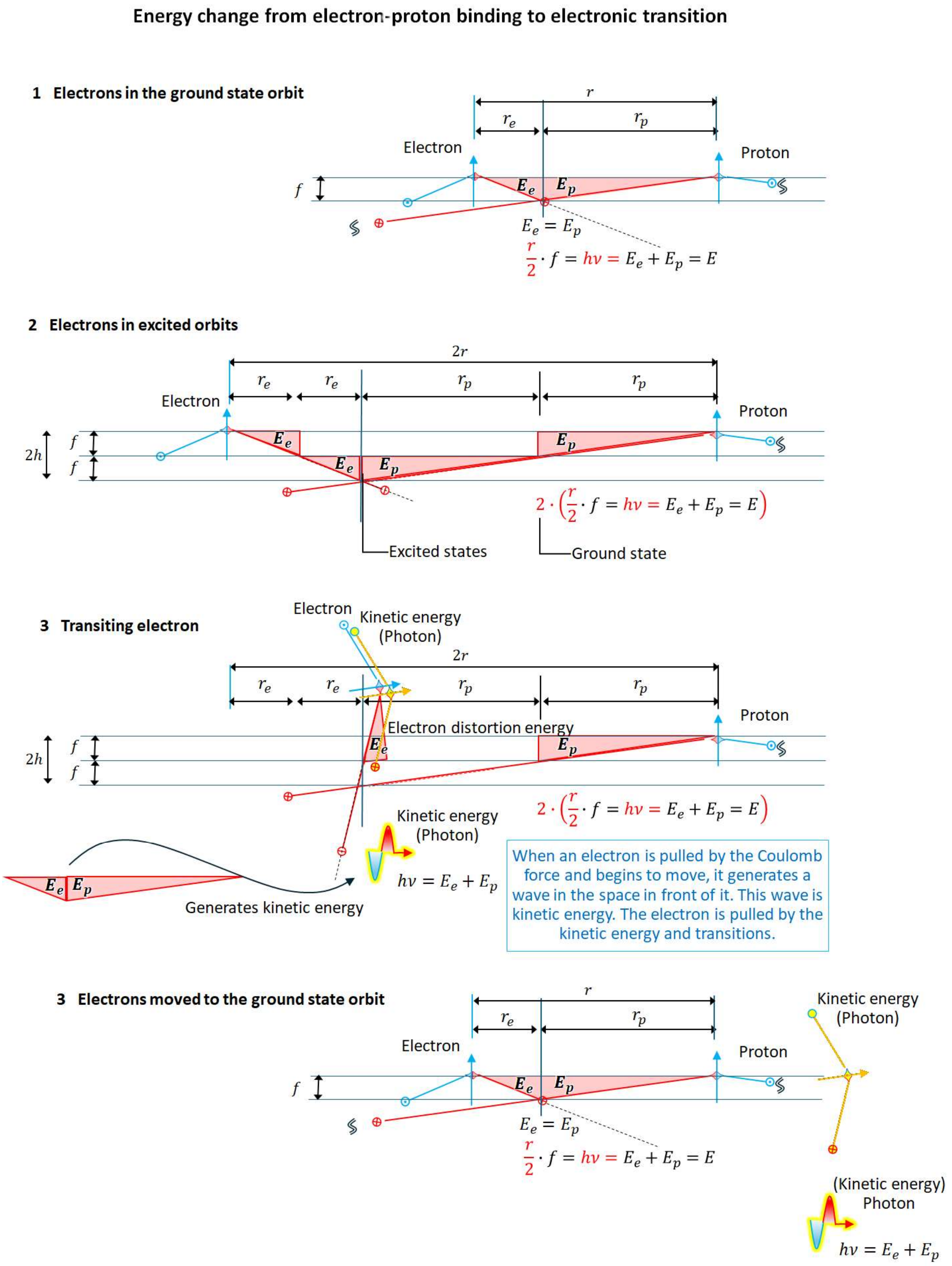


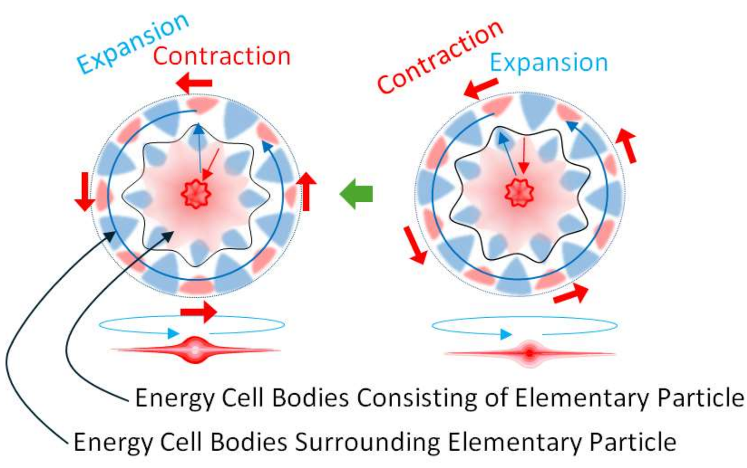
Fig3.

4. Particle model of Energy Body Theory

4.1. Overview

First, let me briefly explain what the elementary particle model based on the Energy Body Theory is. The elementary particle model of the energy body theory is a locally excited state of space. In the Energy Body Theory, it is considered that space is made up of energy cell bodies of the Planck length. The restoring force when energy cell bodies expand, or contract, is the source of energy. In other words, if one energy cell body contracts, the adjacent energy cell body will expand, and overall equilibrium is maintained. When the energy cell bodies receive pressure from all directions of the celestial sphere, they contract to the limit. However, if the pressure is greater than this, the energy has nowhere to go and starts to rotate. On the other hand, the energy cell bodies that have contracted to the limit expand because the pressure is deflected. In this way, the pressure that was moving toward the center is instantly converted into rotational energy, so the expansion and contraction rotate as vibrations. It is important to note here that the entire elementary particle that is formed does not rotate around like a top. This rotation of energy appears as the crests and troughs of a wave like wrinkles extending radially from the center, and the phase shifts in the direction of rotation. This rotation of the wave is a de Broglie wave. This is the cause of the spin of elementary particles. Due to the balance with this gravitational field and their own spin, elementary particles such as electrons and protons can continue to exist.

Elementary particle model of energy body theory



The energy cell bodies in space, which are much smaller than elementary particles, contract and expand radially. The expansion and contraction spin out of phase radially.

Fig4.

Elementary particles have a disk-like shape with the foot that spreads out into space. The particle in the center represents a particle, and the foot represents a field.

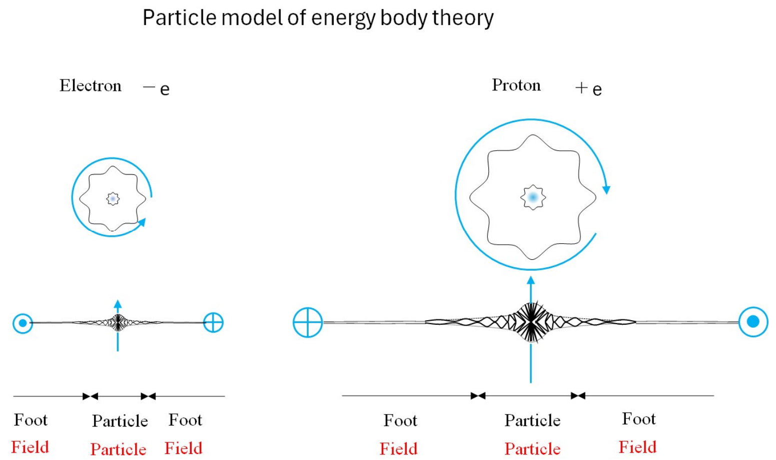


Fig5.

4.2. Electromagnetic Interaction

Electromagnetic interaction depends on the direction of the waves at the interaction; when the waves are in the same direction, an attractive force occurs, and when the waves are in opposite directions, a repulsive force occurs.

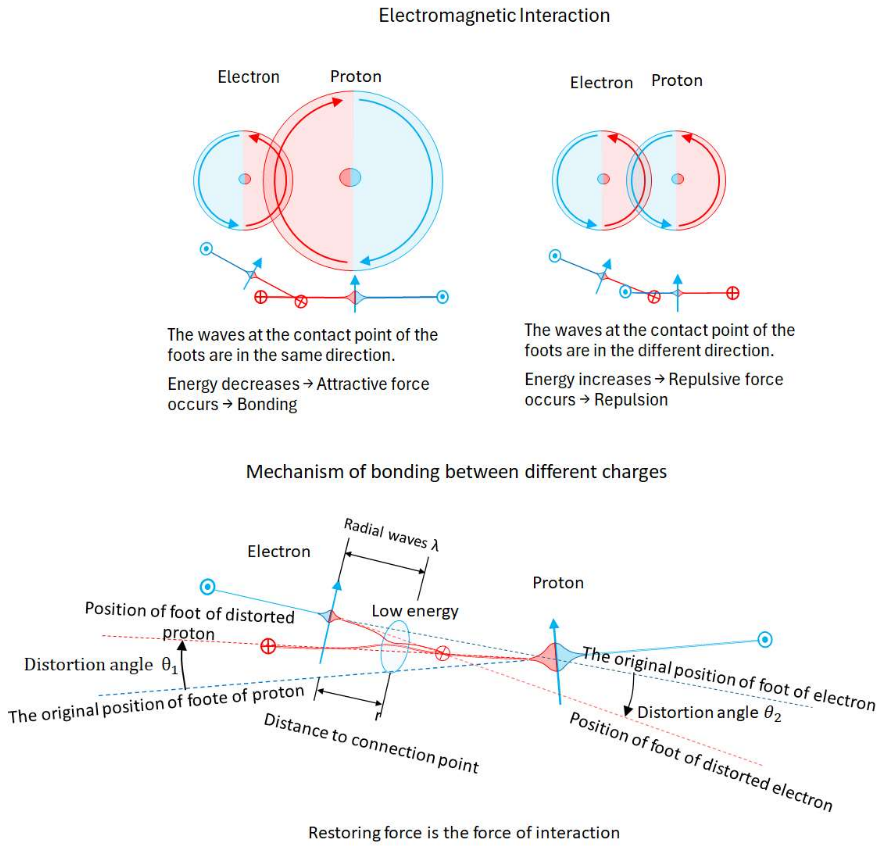


Fig.6

4.3. Derivation

Coulomb's law and Planck's constant are the same thing as the restoring energy of the distortion of the foot of the rotating waves of electrons and protons, expressed from different perspectives.

Coulomb's law regards the energy decrease (or increase) caused by the wave directions of electrons or protons as an attractive or repulsive force ±e.

In the space where an electron and a proton come into contact, the directions of the rotating waves of the electron and proton are the same. This increases the rotation speed and decreases the energy of the space at the contact point. As a result, restoring energy generates, and they attract each other. When the waves are opposite to each other, energy increases, and they repulsive each other.

Planck's constant regards the restoring direction of the distortion of the rotating waves caused by the wave directions of electrons or protons as angular motion h.

In the space where an electron and a proton come into contact, the directions of the rotating waves of the electron and proton are the same.

As a result, the rotation speed increases, and the wavelength of the rotating wave (de Broglie wave) extends.

Then, the foot of the rotating wave is distorted at an angle θ, and the energy of the space at the contact point decreases.

As a result, restoring energy generates and angular momentum 1/2 h comes into effect.

In other words, the restoring energy of Coulomb's law and the restoring energy of Planck's law are the same. By connecting these two laws with an equation, we can derive the fine structure constant α, which connects Coulomb's law of electromagnetism and Planck's law of quantum mechanics.

4.4. Photon generation

The electron on the proton orbit is bound to the proton in the position as shown in Fig7. At this time, the foot of the rotating wave of the electron and proton is distorted. Coulomb's law applies. When an electron in a ground state orbit moves to an excited state orbit and exceeds the excitation limit, kinetic energy is generated in front of the electron. At the same time, the excited state of the proton is released, and the electron transitions to the ground state orbit. When the electron stops in the ground state orbit, the kinetic energy is emitted as a photon. The release of the distortion (excitation) in the foot of the electron and proton and the generation of kinetic energy is the electron transition energy, and the Plank's constant applies.

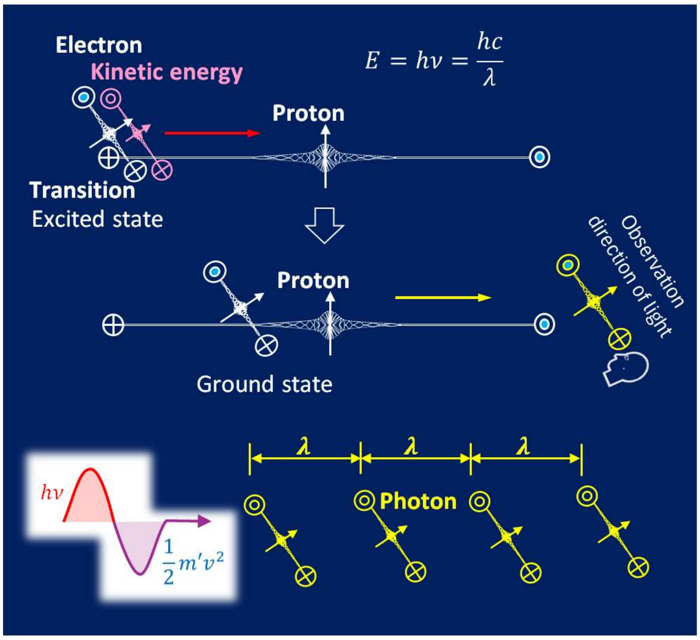


Fig.7.

Planck's constant h, Coulomb's constant, and electromagnetic interaction are the restoration from the distortion generated in the electron's foot. Also, the speed of light is the ratio of the delay in time it takes for a photon's foot to reach an observer to the distance to the photon. This is because the arrival of the foot of light is delayed due to the photon's foot distortion observing the side perpendicular to the direction of travel of the photon. It is shown in Fig8. The speed of the photon inherits the speed of the electron just before it is separated.

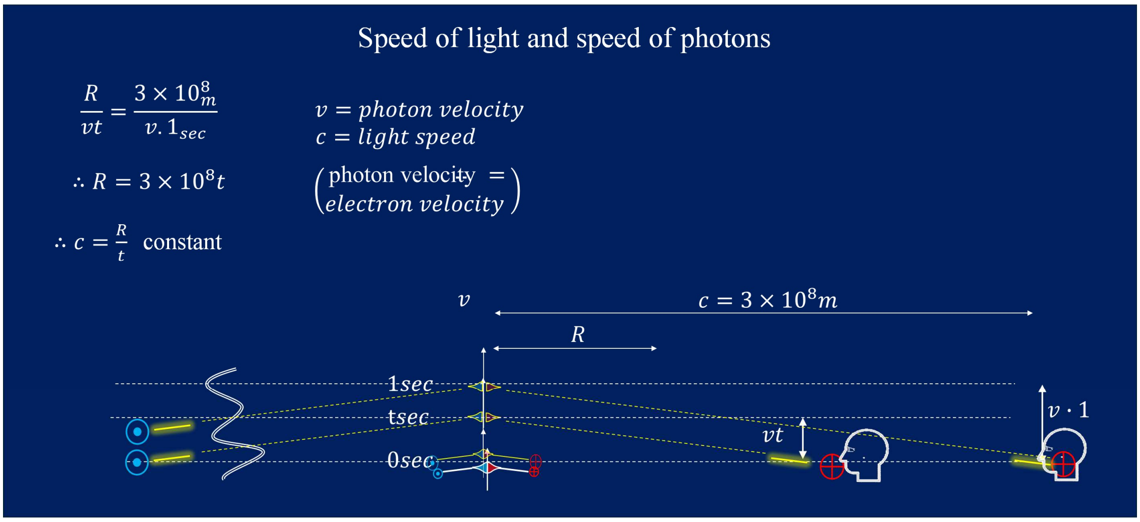


Fig.8

Also, as shown in Fig9., the direction of travel of the photon and the distortion of the photon's foot have the same relationship as an optical clock observation in an inertial system explained by the special theory of relativity.

Comparison of the distance traveled by the light in the optical clock with the distance of the distortion of the photon foot

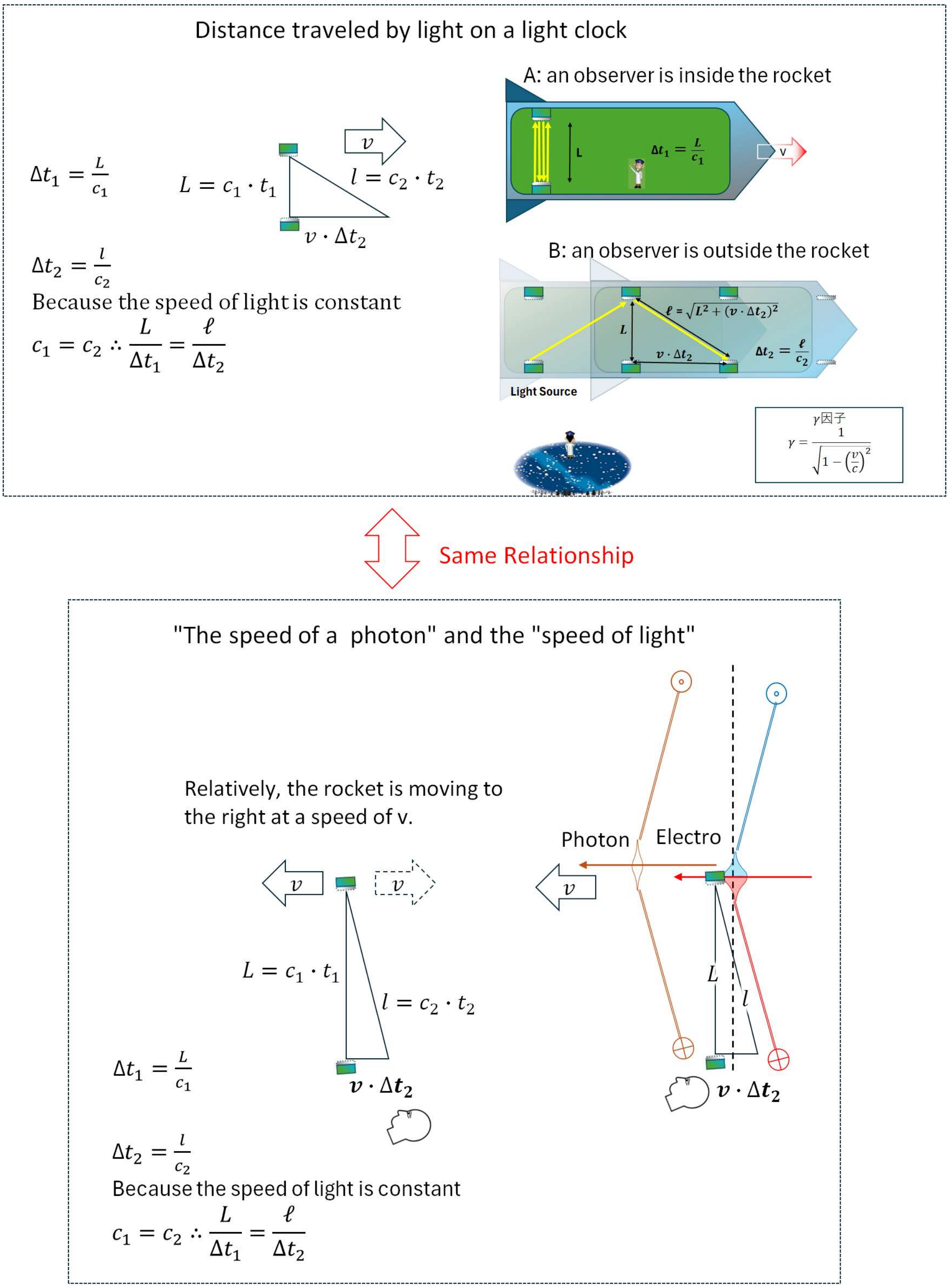


Fig9.

5. Conclusion

It has become clear that "Planck's law," a basic principle of quantum mechanics, the "Rydberg constant," a physical constant that is a dimensionless number expressing the strength of electromagnetic interactions and is also known as the "fine structure constant," and "Coulomb's law," a basic law of electromagnetism, are the same thing, originating from the restoring force of the distortion of the foot of the rotating wave of the electron and proton model of the Energy Body Theory. This proves that the elementary particle model of the Energy Body Theory is correct and can be said to be a result that calls for a fundamental review of current elementary particle theory, quantum theory, and electromagnetism.

6. Reference

・Wikipedia; fine structure constant

・viXra:2407.0099 ; This paper is a revised version of viXra:2407.0099 “Clarifying the Origin of the Fine-Structure Constant”

7. Acknowledgements

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