Theory of Refraction

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Abstract

In this paper a theory of refraction has been presented.
Keyword : Theory of refraction.

1 LAW OF REVERSIBILITY OF MOTION

By impulse-momentum theorem, we can obtain that a body will follow exactly the same path if its direction of travel is reversed at any point of its path provided that force acting on the body remains unchanged at each point of the path.

2 SNELL'S LAW

When a light ray passes from medium 1 to medium 2,

$$\frac{\sin\theta_2}{\sin\theta_1} = k = \mu_{12} = \frac{\mu_1}{\mu_2}$$

3 SPEED RATIO

For the given pair of media, let the speed ratio of a photon in refraction

$$\frac{c_2}{c_1} = \beta_{21} \qquad (\beta_{21} > 0)$$

$$\Rightarrow c_2^2 = \beta_{21}^2 c_1^2$$

$$\Rightarrow c_{2x}^2 + c_{2y}^2 = \beta_{21}^2\left(c_{1x}^2 + c_{1y}^2\right) \qquad (i)$$

Now

$$\left(\frac{c_{2x}}{c_{1x}}\right) = \frac{c_2\sin\theta_2}{c_1\sin\theta_1} = \beta_{21}\frac{\sin\theta_2}{\sin\theta_1} = \beta_{21}\mu_{12}$$

$$\left(\frac{c_{2y}}{c_{1y}}\right) = \frac{c_2\cos\theta_2}{c_1\cos\theta_1} = \beta_{21}\frac{\cos\theta_2}{\cos\theta_1}$$

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Substituting in (i)

$$\Rightarrow \beta_{21}^2 \mu_{12}^2 c_{1x}^2 + c_{2y}^2 = \beta_{21}^2 \left(c_{1x}^2 + c_{1y}^2\right)$$

$$\Rightarrow \beta_{21}^2 (\mu_{12}^2 - 1) c_{1x}^2 = \beta_{21}^2 c_{1y}^2 - c_{2y}^2$$

$$\Rightarrow \beta_{21}^2 (\mu_{12}^2 - 1) c_{1x}^2 = c_{1y}^2 \left[\beta_{21}^2 - \left(\frac{c_{2y}}{c_{1y}} \right)^2 \right]$$

$$\Rightarrow \beta_{21}^2 (\mu_{12}^2 - 1) \times \left(\frac{c_{1x}}{c_{1y}} \right)^2 = \beta_{21}^2 \left[1 - \left(\frac{\cos\theta_2}{\cos\theta_1} \right)^2 \right]$$

$$\Rightarrow (\mu_{12}^2 - 1) \times \tan^2\theta_1 = \left[1 - \left(\frac{\cos\theta_2}{\cos\theta_1} \right)^2 \right] \qquad (ii)$$

Since equation (ii) is independent of the speed ratio, β_{21} need not be equal to μ_{12} .

4 CRITICAL ANGLE

At critical angle of incidence, angle of refraction is 90°, substituting in (ii)

$$\Rightarrow (\mu_{12}^2 - 1) \times \tan^2\theta_1 = \left[1 - \left(\frac{\cos 90^o}{\cos\theta_1} \right)^2 \right] = 1$$

$$\Rightarrow \tan^2\theta_1 = \frac{1}{(\mu_{12}^2 - 1)}$$

$$\Rightarrow \tan^2\theta_c = \frac{1}{(\mu_{12}^2 - 1)}$$

5 BREWSTER'S ANGLE

At Brewster's angle of incidence, angle between reflected ray and refracted ray is 90°

$$\Rightarrow \frac{\cos\theta_2}{\cos\theta_1} = \frac{\cos(90^o - \theta_1)}{\cos\theta_1} = \frac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 \qquad \left[\because \theta_1 + 90^o + \theta_2 = 180^o \right]$$

substituting in (ii)

$$\Rightarrow \mu_{12}^2 \tan^2\theta_1 - \tan^2\theta_1 = 1 - \tan^2\theta_1$$

$$\Rightarrow \tan^2\theta_1 = \frac{1}{\mu_{12}^2} = \mu_{21}^2$$

$$\Rightarrow \tan^2\theta_B = \mu_{21}^2$$

References

1. Hugh D. Young, Roger A. Freedman, Albert Lewis Ford, *“Sears’ and Zemansky’s University Physics with Modern Physics 13th edition.”*